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COLLUSION IN SPATIALLY SEPARATED MARKETS
WITH QUANTITY COMPETITION

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Collusion in spatially separated markets with quantity competition

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This paper develops the incentives to collude in a model with spatially separated markets and quantity setting firms. We find that increases in transportation costs stabilize the collusive agreement. We also show that, the higher the demand in both markets the less likely will collusion be sustained. Gross and Holahan (2003) use a similar model with price setting firms, we compare their results with ours to analyze the impact of the mode of competition on sustainability of collusion. Further we analyze the impact of collusion on social welfare and find that collusion may be welfare enhancing.

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1 Introduction

In spatial markets the collusive behaviour among firms acting has been studied in a large number of papers. The first approach in this area use a linear city model with mill pricing firms and analyzes the relationship between firm locations and sustainability of collusion. For example Chang (1991) and Hackner (1995) use this kind of model with symmetrically located firms and find that collusion is easier to sustain if firms are located further apart.\(^1\) However, Gupta and Venkatu argue that

“Typically when transport cost is large relative to total production cost, for example as in the ready-mix cement industry, spatial discriminatory (or delivered) pricing is more appropriate than mill pricing” (Gupta and Venkatu (2001), p. 52).

Therefore, these authors study collusive behaviour in a linear city model where firms are able to adopt spatial price discrimination. Gupta and Venkatu (2001) show that in their model a smaller firm dispersion is more likely to sustain the collusive agreement. The assumption of price competition is not appropriate for all industries. If quantity is less flexible than the price of the good, then using competition in quantities is more adequate. Colombo (2010) analyzes collusion with quantity setting firms that spatially discriminate in two different models: in a linear and in a circular city model. He shows that higher transportation costs increase collusion sustainability. On top of that he investigates the welfare implications and finds that collusion leads to a lower social welfare in a linear city model. In contrast to this in a circular city model a higher welfare might occur.

All these approaches make use of the standard assumption of uniformly distributed consumers. However, this assumption does not capture all relevant geographic distributions of population, e.g. consider two large cities that are connected with a highway. In that case, the assumption of uniformly distributed consumers does not reflect reality. Because of that Hwang and Mai (1990) introduce a spatial model with concentrated demand at the two endpoints of a line.\(^2\)

Using the model setting of Hwang and Mai (1990), Gross and Holahan (2003) study collusion in a model with spatially separated markets, assuming that firms compete in prices in both markets. They show that increasing transportation costs tend to destabilize collusion.

In this article we study sustainability of collusion with concentrated demand and quantity setting firms. We use the model setting of Hwang and Mai (1990) and introduce two firms that are able to spatially discriminate, i.e. they are able to sell different quantities at each location. Therefore our paper is closely related to Gross and Holahan (2003).

\(^{1}\)Also see Chang (1992) and Jehiel (1992) among others.

\(^{2}\)Their model has been developed further by Gross and Holahan (2003) and Liang et al. (2006), among others.
Comparing the results of price vs. quantity competition yields some fruitful insight into the role of the mode of competition on stability of the collusive agreement in a spatial world. Our results indicate that with quantity setting firms an increase in transportation costs leads to a more stable collusion. Furthermore we get the result that rising demand destabilizes collusion with quantity but not with price competition. On top of that we compare social welfare in the case of collusion with the case of competition and show that collusion may lead to higher welfare if transportation costs are relatively high.

The paper is organized as follows. Section 2 describes the model and the main results. Section 3 presents the result of the welfare comparison and Section 4 concludes the paper.

2 The model

According to Hwang and Mai (1990) we use a linear spatial model with concentrated demand at the corners of the line. This concentrated demand can be interpreted as a city, so we are able to represent a geographical situation corresponding to two major cities that are connected by a transportation route, e.g. a highway. The population density between these spatially separated markets is zero. The two cities located at the left and the right endpoint will be labeled as market 1 and market 2. For simplicity, the distance between the cities is normalized to 1. An example of such a geographical situation are the city pairs Hamburg/Berlin in Germany or Dallas/Houston in the US.

We assume that a single firm is located in each city, firm A in market 1 and firm B in market 2. These firms produce a homogeneous good and are faced with identical and constant marginal costs which are normalized to zero, without loss of generality. In addition, we make the assumption that firms set quantities à la Cournot. Moreover, firms are able to conduct spatial discrimination, which means that they are able to deliver different quantities at different locations in space.

Goods are delivered to the consumers by the firms that have to pay transportation costs of \( t \) per unit of output and unit of distance. To guarantee that both markets can be served by both firms, we assume \( 0 < t \leq \frac{a}{2} \). Further, we consider transportation costs within one city to be negligible.

Consumers in both cities act rationally and therefore pursue the goal of utility maximization. Further, we assume that consumers can purchase the good only at their respective locations. There is no opportunity for consumers to exploit spatial price differences through direct shipments.

The inverse demand function in market 1 is given by \( p_1 = a - bQ_1 \) and in market 2 \( p_2 = a - bQ_2 \), where \( p_i \) indicates the price and \( Q_i \) the total quantity in market \( i \) \( (i=1,2) \); \( a \) and \( b \) are positive constants.

To analyze stability of collusion we suppose that the firms interact repeatedly in a game

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3This assumption has been used by Hamilton et al. (1989) among others. For a discussion of the spatial Cournot model with higher transportation costs, see Chamorro-Rivas (2000), Benassi et al. (2007) and Scrimitore (2010).
that takes place over an infinite horizon, where firms maximize the present discounted value of their payoffs. We assume that firms follow a grim trigger strategy as in Friedman (1971). Under this strategy firms start by adhering to the collusive agreement. The firms adhere to this agreement until one firm has deviated in the previous period. If one firm deviates at time \( \tau - 1 \), from \( \tau \) onward, both firms play the non-cooperative game.

### 2.1 Competition

We begin our investigation with the outcome of a standard one-shot non-cooperative game if firms set quantities simultaneously. In the standard Cournot case both firms sell their good in both markets. Single period competitive profits\(^4\) can be written as

\[
\begin{align*}
\pi_A^C &= (a - b(q_{A1}^C + q_{B1}^C))q_{A1}^C + (a - b(q_{A2}^C + q_{B2}^C))q_{A2}^C - tq_{A2}^C, \\
\pi_B^C &= (a - b(q_{A1}^C + q_{B1}^C))q_{B1}^C - tq_{B1}^C + (a - b(q_{A2}^C + q_{B2}^C))q_{B2}^C,
\end{align*}
\]

where \( q_{ji}^C \) indicates quantity of firm \( j \) (j=A,B) in market \( i \) (i=1,2).

Standard calculation of the Cournot-Equilibrium yields

\[
q_{A1}^C = q_{B2}^C = \frac{a + t}{3b}, \quad (3)
\]

\[
q_{B1}^C = q_{A2}^C = \frac{a - 2t}{3b}. \quad (4)
\]

Substitution of (3) and (4) into (1) and (2) gives the one period competitive profits

\[
\pi_A^{C*} = \pi_B^{C*} = \frac{2a^2 - 2at + 5t^2}{9b}. \quad (5)
\]

### 2.2 Collusion

In this subsection we analyze the situation in which firms engage in collusion. If they collude, each firm decides to sell only in their respective market and neither firm exports (“invades”) the other market. Therefore, each firm has a monopoly position in the home city. The one-shot profits\(^5\) are given by

\[
\begin{align*}
\pi_A^M &= (a - bQ_{1}^M)Q_{1}^M, \\
\pi_B^M &= (a - bQ_{2}^M)Q_{2}^M.
\end{align*}
\]

Since firm A and B behave as monopolists, they choose quantities

\[
Q_{1}^M = Q_{2}^M = \frac{a}{2b}. \quad (8)
\]

\(^4\)The superscript “C” denotes the competitive case.

\(^5\)Superscript “M” is used to refer to the monopoly case.
Substituting (8) into (6) and (7) yields profits in the case of collusion. These are

\[ \pi_M^{A*} = \pi_M^{B*} = \frac{a^2}{4b}. \]  

(9)

If firms can establish a long run collusive agreement the resulting profits are

\[ \Pi_M^{A*} = \frac{1}{1-\delta} \pi_M^{A*} = \frac{a^2}{4b(1-\delta)}. \]  

(10)

where \( \delta \) denotes the discount factor. We define \( 0 < \delta < 1 \) to express each firm’s weight placed on future profits.\(^6\) Because of symmetry, (10) holds for firm B as well.

Suppose now that firm A breaks the collusive agreement. This means, that firm A continues to sell the monopolistic output at market 1, but invades the other market and sells positive quantities in market 2 as well. Given that firm B still produces the collusive outcome \( Q_M^2 \) the deviation profit\(^7\) is given by

\[ \pi^D_A = (a - bQ_M^1)Q_M^1 + (a - b(q_A^D Q_M^2))q_A^D - tq_A^D. \]  

(11)

Standard calculation yields the optimal quantity in market 2, we get

\[ q_A^D = \frac{a - 2t}{4b}. \]  

(12)

Plugging (8) and (12) into (11) yields the following deviation profit

\[ \pi^D_A = \frac{5a^2 - 4at + 4t^2}{16b}. \]  

(13)

If one firm\(^8\) decides to break the collusive agreement, it will enjoy one period of high profits due to invasion, but then will earn competitive profits for all future periods.

Therefore the present value\(^9\) of deviation is

\[ \Pi_A^D = \pi_A^D + \frac{\delta}{1-\delta} \pi_A^C \]

\[ = \frac{a^2(45 - 13\delta) + 4at(\delta - 9) + 4t^2(9 + 11\delta)}{144b(1-\delta)}. \]  

(14)

---

\(^6\)Note that the present value of a monetary unit received every period in the future beginning with the first period equals \( \frac{1}{1-\delta} \).

\(^7\)Superscript “D” is used to indicate the deviation profit.

\(^8\)Due to symmetry it does not matter which firm deviates, therefore we use firm A.

\(^9\)The present value of a monetary unit received every period in the future beginning with the second period equals \( \frac{1}{1-\delta} \).
2.3 Stability of collusion

Collusion is sustained as an equilibrium if

\[ \Pi^M_A \geq \Pi^D_A. \]  

(15)

This is equal to \( \frac{1}{1-\delta} \pi^M_A \geq \pi^D_A + \frac{\delta}{1-\delta} \pi^C_A \). Solving this expression for \( \delta \) yields the critical discount factor

\[ \delta^* = \frac{\pi^D_A - \pi^M_A}{\pi^D_A - \pi^C_A}. \]  

(16)

If the discount factor is larger than this critical discount factor collusion is a stable equilibrium, otherwise it is not sustainable. Therefore, the critical discount factor measures the sustainability of collusion. The larger \( \delta^* \), the smaller the set of discount factors which support the collusive agreement and vice versa.

Substitution of (5), (9) and (13) into (16) yields the critical discount rate

\[ \delta^* = \frac{9(a - 2t)}{13a + 22t}. \]  

(17)

Collusion is sustained if \( \delta \geq \delta^* \). The critical discount rate is determined by the size of demand and transportation costs.

First we analyze the effect of transportation costs on \( \delta^* \). The result is summarized in proposition 1.

**Proposition 1:** An increase in transportation costs leads to a larger set of discount factors which support collusion.

**Proof:** Taking the derivative of \( \delta^* \) with respect to transportation costs, we get

\[ \frac{\partial \delta^*}{\partial t} = - \frac{432a}{(13a + 22t)^2} < 0. \Box \]  

(18)

This result indicates that collusion is more likely to be sustained for higher transportation costs. The reason for this effect is that \( \pi^M_A \) is independent of \( t \) and therefore remains unaffected by an increase in \( t \), while \( \pi^D_A \) and \( \pi^C_A \) decrease in \( t \). Therefore the difference \( (\pi^D_A - \pi^M_A) \) decreases faster with a rising \( t \) than the difference \( (\pi^D_A - \pi^C_A) \). This leads to a lower critical discount rate. Deviating from collusion is getting less attractive with high transportation costs compared to a situation with a lower \( t \), since the additional sold quantity in market 2 is getting lower with a higher \( t \). In this model of quantity competition transportation costs enhance the stability of collusion, because the local market is getting more protected. This result is in sharp contrast to Gross and Holahan (2003), where an increase in transportation costs leads to a lower stability of collusion.

In figure 1 we show the critical discount factors with price and quantity competition as a function of \( t \). Note that the critical discount factor with price competition is taken from Gross and Holahan (2003).\(^\text{10}\) For simplicity we use \( a = b = 1. \)

\(^\text{10}\)See Gross and Holahan (2003), eq. (20), p. 308.
The dotted line represents the critical discount factor with price competition and the black line the quantity competition case. Figure 1 makes clear that the black line is falling with rising \( t \), while the dotted line rises with rising \( t \). Therefore, with high transportation costs collusion is stable in the case of quantity competition and unlikely with competition in prices. With low transportation costs, \( t \in \left[ 0, \frac{10}{19} - \frac{1}{9}\sqrt{79} \right] \), the picture changes and collusion is more likely with price competition. However, figure 1 makes clear that over the whole range of \( t \) collusion is much more stable with quantity competition.

Next, we consider the effect of demand on the sustainability of collusion. The following proposition summarizes the result.

**Proposition 2:** The higher the demand in both markets, the less likely collusion will be sustained.

**Proof:** Standard calculation yields the derivative

\[
\frac{\partial \delta^*}{\partial a} = \frac{432t}{(13a + 22t)^2} > 0. \square
\]

According to equation (19) the critical discount factor is increasing in the size of demand \( a \). However, this result contradicts with Gross and Holahan (2003).\(^\text{11}\)

\[^{11}\text{Gross and Holahan do not analyze the impact of demand on stability in their paper. By using their critical discount factor } \delta_{GH}^* = \frac{a}{2(a-bt^2)} \text{ (Formula (20), p. 308) and differentiating it with respect to } a, \text{ we get } \frac{\partial \delta_{GH}^*}{\partial a} = -\frac{bt}{2(a-bt^2)} < 0. \text{ Therefore, rising demand increases stability in this setup.}\]
3 Welfare

In this section we analyze the effect of collusion on social welfare. We are interested in the comparison between the collusive and the competitive outcome, since in equilibrium both firms either decide to collude or engage in competition. To simplify the analysis, we only look at one period results. Social welfare is defined as the sum of producer and consumer surplus. We denote consumer surplus in market $i$ with $CS_i$.

In the case of competition social welfare is given by

$$W^C = \pi^C_A + \pi^C_B + CS^C_1 + CS^C_2 = \frac{8a^2 - 8at + 11t^2}{9b}. \quad (20)$$

If firms engage in collusion, we get

$$W^M = \pi^M_A + \pi^M_B + CS^M_1 + CS^M_2 = \frac{3a^2}{4b}. \quad (21)$$

The difference in social welfare is calculated as

$$\Delta W = W^M - W^C = -\frac{5a^2 - 32at + 44t^2}{36b}. \quad (22)$$

If $\Delta W > 0$ then welfare is larger in the case of collusion and vice versa.

The result of our analysis is summarized in proposition 4.

Proposition 4: Social welfare is larger under collusion if transportation costs are sufficiently large. For low values of $t$ competition leads to a higher social welfare.

Proof: Solving $\Delta W = 0$ with respect to $t^{12}$, we get the following critical transportation cost rate

$$t^W = \frac{5}{22} a. \square \quad (23)$$

For all $t < t^W$, welfare is larger in the case of competition, while for all $t > t^W$ welfare in the collusive case is higher than with competition. This means that with low transportation costs the competitive case yields a higher welfare. If transportation costs are sufficiently large, the collusive outcome results in higher social welfare. This result seems surprisingly first. The reason for this result is that with collusion each firm serves only the consumers in the home market and therefore transportation costs are eliminated. With high transportation costs this effect is large enough to overcome the loss in competition.

However, Colombo (2010) shows that in a linear city model with quantity setting firms social welfare is always lower with collusion. This is an interesting result, since we as-

\[12\] However, the second solution equals $t = \frac{a}{2}$, the upper bound of transportation costs.
sume symmetric, but concentrated, demand and find a possibly higher social welfare. In a circular city Colombo (2010) is able to show higher welfare when transportation costs are high and firms are distant enough.

4 Conclusion and policy implication

This article extends the existing literature on collusion sustainability in a spatial setting by introducing firms that are able to set location specific quantities in spatially separated cities using the model setup developed by Hwang and Mai (1990). The natural benchmark of our model is the model of Gross and Holahan (2003), who analyze collusion in spatially separated markets with price setting duopoly. We find that collusion as a combination of monopoly pricing in the respective home city combined with non-invasion of the foreign city is a subgame perfect Nash equilibrium for a determinable set of discount factors. We show that an increase in transportation costs leads to an increase in sustainability of collusion. This result is in contrast to the result derived by Gross and Holahan (2003) with price setting firms, where an increase in transportation costs makes collusion less stable.

How can these results be used in politics? Suppose a government decides on an infrastructure project which leads to falling transport costs. With quantity competition the realization of the project has two positive aspects, on the one hand it reduces transport costs and on the other hand the probability of collusion is reduced. In the case of price competition the last effect is reversed so that the realization of the infrastructure project bears the risk of a higher probability of collusion.

Further we show that increasing demand leads to a destabilization of the collusive agreement, another result that contradicts with Gross and Holahan (2003). A higher demand can be interpreted as relatively richer cities in contrast to a lower level of \( a \) that corresponds to the case of relatively poorer cities. Due to equation (19) collusion is more likely in the case of poorer cities, because deviating from collusion is getting more attractive with richer markets, since the additional profit is absolutely larger. In the case of price competition the result is reversed and collusion in richer cities is more stable than in poorer cities. This result shows that in a developing country (characterized by a relatively low level of \( a \)) collusion is more likely with quantity competition and less likely with price competition.

Therefore our results indicate that the mode of competition (price or quantities) has an important influence on sustainability of collusion in a model with spatially separated cities, because the effects of increasing transportation costs and demand is reverted in our setup compared to Gross and Holahan (2003).

Further we derive the impact of collusion on social welfare and find that collusion may be welfare improving. If transportation costs are relatively large collusion leads to a higher welfare than competition, while for low values of \( t \) we get a lower social welfare under collusion.
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