Dynamical simulation of the “velocity-porosity” reduction in observed strength of stellar wind lines

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I use dynamical simulations of the line-driven instability to examine the potential role of the resulting flow structure in reducing the observed strength of wind absorption lines. Instead of the porosity length formalism used to model effects on continuum absorption, I suggest reductions in line strength can be better characterized in terms of a velocity clumping factor that is insensitive to spatial scales. Examples of dynamic spectra computed directly from instability simulations do exhibit a net reduction in absorption, but only at a modest 10-20% level that is well short of the ca. factor 10 required by recent analyses of PV lines.

1 Spatial clumping and porosity

Historically the wind clumping that is the focus of this workshop has been primarily considered for its effect on diagnostics that scale with the square of the density, e.g. free-free emission, and emission or absorption from atomic states that are populated by collisional excitation or recombination. The strength of such diagnostics is enhanced in a clumped medium, leading to an overestimate of the wind mass loss rate that scales with \( \sqrt{\ell/f} \), the inverse square-root of the clump volume filling factor \( f \).

Over the past couple of years, there also has been considerable attention to an additional “porosity” effect that can reduce the strength of single-density diagnostics. In particular, Oskinova et al. (2006) have claimed a porous reduction in bound-free continuum absorption of X-rays emitted from wind shocks can help explain the unexpected relative symmetry of X-ray emission lines observed by Chandra and XMM. However, Owocki & Cohen (2006) have argued this requires an unrealistically large value for the wind porosity length \( h \equiv \ell/f \), defined by the ratio the clump scale \( \ell \) to their volume filling factor \( f \).

A recent follow-on preprint by Oskinova et al. (2007) now applied an analysis based on this porosity length to argue for a reduction in the strength of line absorption in structured stellar winds. If substantiated, such a reduction might help explain the unexpected weakness of PV lines observed by FUSE (Fullerton et al. 2004), which otherwise might require a substantial, factor-ten or more reduction in wind mass loss rate.

A key point of the present paper is to argue that a description based on the spatial porosity that is central to reducing continuum absorption is not well suited to characterizing the effects on line absorption, which instead depend on a kind of “velocity porosity” (or “vorosity”?) that is relatively insensitive to the spatial scale of wind structure. The next section suggests a simple analytic scaling based on a simplified description of the velocity structure arising in numerical hydrodynamics simulations of the line-driven instability. The follow-on section uses line-absorption profiles formed in actual dynamical instability models to show that the reduction in overall line absorption is typically quite modest, only about 10-20%.

2 The ‘velocity clumping factor’

The left panel of figure 1 illustrates the typical result of 1D dynamical simulation of the wind instability, plotted here as a time-snapshot of velocity vs. a mass coordinate \( M(r) \) instead of radius \( r \). The intrinsic instability of line-driving leads to a substantial velocity structure, with narrow peaks corresponding to spatially extended, but tenuous regions of high-speed flow, which bracket dense, spatially narrow clumps/shells that appear here as nearly flat, extended velocity plateaus in mass.

The right panel of figure 1 illustrates a simplified, heuristic model of such wind structure for a representative wind section, with the velocity clumping now represented by a simple “staircase” structure, compressing the wind mass into discrete sections of the wind velocity law, while evacuating the regions in between. The structure is characterized by a “velocity clumping factor” \( f_c \), set by the ratio between the internal velocity width \( \delta v \) to the velocity separation \( \Delta v \) of the clumps. The straight line through the steps represents the corresponding smooth wind flow.

The effect of the velocity structure on the line-absorption profile depends on the local Sobolev optical depth, which scales with the inverse of the mass derivative of velocity, \( \tau_s \sim 1/(dv/dm) \), evaluated at
a resonance location \( r_s \) where the velocity-scaled, observer-frame wavelength \( x = -v(r_s)/v_\infty \). In a smooth wind with Sobolev optical depth \( \tau_x \), the absorption profile is given simply by

\[
A_x = 1 - e^{-\tau_x} .
\]  

(1)

In the structured model, the optical thickness of individual clumps is increased by the inverse of the clumping factor \( f_v \), but they now only cover a fraction \( f_v \) of the velocity/wavelength interval. The net effect on the averaged line profile is to reduce the net absorption by a factor

\[
R_A(\tau_x, f_v) = f_v \frac{1 - e^{-\tau_x/f_v}}{1 - e^{-\tau_x}} .
\]  

(2)

Note that for optically thick lines, \( \tau_x \gg 1 \), the reduction approaches a fixed value, given in fact by the clumping factor, \( R_A \approx f_v \). If the smooth-wind line is optically thin, \( \tau_x \ll 1 \), then \( R_A(\tau_x, f_v) \approx (1 - e^{-\tau_x/f_v})/(\tau_x/f_v) \), which is quite analogous to the opacity reduction for continuum porosity (cf. Owocki & Cohen 2006, eqn. 4), if we just substitute for the clump optical depth, \( \tau_x \rightarrow \tau_x/f_v \).

But a key point here is that, unlike for the continuum case, the net reduction in line absorption no longer depends on the spatial scale of the clumps. Instead one might think of this velocity clumping model as a kind of velocity form of the standard venetian blind, with \( f_v \) representing the fractional projected covering factor of the blinds relative to their separation. The \( f_v = 1 \) case represents closed blinds that effectively block the background light, while small \( f_v \) represent cases when the blinds are broadly open, letting through much more light.

3 Line-absorption profile from instability simulations

Because the line-driven instability occurs at scales near or below the Sobolev length, dynamical simulations of the resulting structure cannot rely on the standard CAK/Sobolev form for the line-force, but rather must use expressions that require a non-local spatial integration of the line-optical depth at a sample of observer-frame wavelengths \( x = -v/v_\infty \). The output of such simulations thus readily provide dynamical, non-Sobolev results for the total radial optical depth from the stellar surface to an observer at the outer boundary,

\[
\tau_x = \int_{R^*}^{R_{\text{max}}} dr' \kappa \rho(r') \phi(\tilde{x}(r')) .
\]  

(3)

Here \( \kappa \) is the mass-absorption coefficient of the line, \( \rho \) is the density, and \( \tilde{x}(r') = (x v_\infty - v(r'))/v_{th} \) is the local co-moving-frame wavelength at radius \( r' \), in units of the thermal velocity width \( v_{th} \) of the line profile function \( \phi \). Use of eqn. (3) in place of the Sobolev optical depth in eqn. (1) then provides a full non-Sobolev computation of the line-absorption trough in this dynamical model.

Figure 2 shows results for line-absorption spectra from a typical wind instability simulation, which starts with a smooth, CAK initial condition, and is then evolved forward using the standard “Smooth Source Function” (SSF) formulation for the line-force (Owocki & Puls 1996, 1999). Although there are no explicit perturbations, the intrinsic, self-excited nature of the instability leads to extensive wind structure above a radius of about \( r \approx 1.5 R_\star \). The upper panels of figure 2 show the corresponding effect on the dynamic spectra for a weak, medium,
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Figure 2: Lower panels: Absorption trough of time-averaged P-Cygni line profile plotted versus velocity-scaled observer wavelength \( x = -v/v_\infty \) from line-center, for a weak, medium, and strong line. The smooth curves correspond to the smooth, CAK initial condition, while the jagged curves represent results for 1D dynamical instability simulations using the Smooth Source Function (SFF) method. Upper panels: Color-scale plots of the associated dynamical spectra, with time increasing vertically from the CAK initial condition. Note that, although for simplicity the re-emission from scattering is ignored, the line absorption profiles here are not the result of a Sobolev model, but instead are computed from the full spatial integral of the line absorption from the stellar surface to the outer boundary (set here to \( R_{\text{max}} = 100R_\star \)).

and strong line. The lower panels compare the associated time-average profile with that of the smooth CAK initial condition.

Because of the restriction here to pure-absorption along a single radial ray in a 1D instability model, the synthetic profiles here have an artificially fine level of structure and variability, but they illustrate quite vividly the key effects that can alter line absorption. The high level of velocity clumping leads to many tracks of enhanced, even saturated absorption, while at the same time exposing channels between the clumps that allow for increased transmission of the stellar surface flux. The time-averaged profiles in the lower panels thus show a general reduction in the absorption compared to the smooth, CAK model, most notably at middle wavelengths \((-x = v/v_\text{th} \approx 0.3 - 0.8)\) relative to blue edge for the CAK terminal speed. On the other hand, the unstable flow faster than the CAK \( v_\infty \) extends the absorption beyond \( x = -1 \), leading to notable softening of the blue edge.

The net effect is still to reduce the overall absorption equivalent width of the medium and strong lines, though only by about 10-20%. Thus while the dynamical simulations of wind instabilities do confirm the basic velocity porosity effect, they indicate the net reduction in line absorption may be quite modest, and thus may only play a minor part in the factor 10 reduction thought to be necessary to explain the observed strength of PV lines.

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References

Townsend: Is it not likely that the velocity porosity you discuss will be augmented by spatial porosity?

Owocki: In principle it could be. But it would require relatively large spatial porosity lengths \((h > R)\), which might be unrealistic. A key result of this analysis is that “velocity porosity” can occur even when the spatial porosity for continuum absorption is entirely in the microclumping limit \((h \ll R)\), for which there is little or no reduction in the absorption. For example, Wolf-Rainer quoted in his talk that the Oskivova et al. analysis implies a notable reduction in line absorption even with a spatial porosity length \(h = 0.25 R^*\). Indeed, for a moderately strong line, i.e. with a smooth wind Sobolev depth \(\tau > 1\), velocity clumping on any scale should in principle lead to an absorption reduction.

Hamann: It is very interesting how you have modeled the porosity effect for line opacity. I want to emphasize that your “vorosity” is equivalent to the model of Oskinova et al. which I have presented in my talk this morning. Some details, however, are slightly different. You are using a Sobolev opacity over a clump which has a velocity gradient, and we assume a Gaussian distribution of velocities inside the clump for our cmf optical depth.

Owocki: I agree there are some similarities, but in my eyes the scaling picture I emphasize is conceptually distinct from the “porosity length” formalism followed in Oskinova et al. I do not assume, or propose, that the clump has a positive velocity gradient. Rather the key parameter in my picture is the “velocity clumping factor”, effectively the ratio of the clump velocity width (which might be turbulent, using your “Gaussian” picture) over the velocity separation between clumps. A key point is that this parameter is, unlike the porosity length, entirely independent of any assumed spatial length scale. Finally, I would emphasize that, however one chooses to describe the effect in terms of simple scaling parameters, the latter half of my talk uses radiation hydrodynamic models in which the optical depth is computed not from any Sobolev model, but from numerical integration of the profile-weighted line optical depth through the complex velocity and density structure. In this model we do indeed see clearly that moderately strong lines have an effective absorption reduction of a few tens of percent.

Puls: I think one can actually use this step-function (maybe with a single velocity “step” at different locations) to get a correct description of the radiation field, where the step function is constructed from averaging over the hydrodynamic simulations.

Owocki: That is an interesting idea. I am in the process of trying to develop a formalism to derive an effective “velocity porosity” for any given complex structure obtained from instability simulations.

Feldmeier: The “venetian blind” distance in your model is apparently a stochastic quantity. We discussed yesterday the statistical properties of cloud turbulence, i.e. its length distribution according to Kolmogoroff’s law. What is the statistical distribution function of your velocity turbulence?

Owocki: That is a good question. I think the key here is to focus not on the spatial distribution and whether it follows Kolmogoroff, but really the velocity distribution. That is what my “velocity clumping” parameterization attempts to capture for a simple two-component medium, but I agree that a more realistic model requires some statistical description of how much material is clumped into a distribution of velocity bins. Such a description is still to be developed.