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Price responses to market entry with and without endogenous product choice
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Abstract

Textbook wisdom says that competition yields lower prices and higher consumer surplus than monopoly. We show in two versions of a simple location-product differentiation model with and without endogenous choice of products that these two results have to be qualified. In both models, more than half of the reasonable parameter values lead to higher prices with duopoly than with monopoly. If the product characteristics are exogenous to the firms, consumers may even be better off with monopoly in average.

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JEL-codes: L12; L13; L41; D43

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1 Introduction

It is almost a common fact that market entry and competition lead to a decrease in prices. The emergence of a new competitor forces incumbent firms to fight for their market shares by decreasing prices, so the argument goes. In addition, firms will try to increase efficiency and trigger innovation, which enables them to lower prices even further. Except for the persuasiveness of these intuitive arguments, the result also shows up in some theoretical models, most importantly in the textbook model of Cournot oligopoly. Therefore, one might conclude that the issue is not worth being considered more closely. We decidedly think that this view is not correct in general, though.

To support our claim, we use a spatial-differentiation model in the spirit of Hotelling (1929) with elastic demand. In order to assess the effects of market entry in a stylized fashion, we compare the outcome with respect to prices and consumer welfare in two different settings, respectively for the cases of monopoly and duopoly. Although we assume specific demand and transport cost functions, our results hold under quite general conditions.

One striking result we obtain is that consumer prices may well be higher under competition than in the case of a monopoly. In our interpretation, this is due to the fact that firms specialize on consumers whose preference for the considered firm’s good is pronounced and whose price elasticity is thus relatively low in the case of a duopoly. This effect exhibits that tastes and goods are relatively strongly differentiated. On the other hand, the degree to which goods differ must not be too high, since otherwise the firms may not compete at all.

We consider two versions of the model and their implications on prices and consumer surplus. The first model assumes that product characteristics are being given to the firm(s), while the second allows for endogenous product choice. In the latter case, the firms take into account that a higher price renders it optimal to the competitor to produce a more similar good. Since this would harm profits, prices are lower in the case of endogenous product choice, but might still be higher than with monopoly. Consumer surplus may even be higher in the case of a monopoly, but only if firms cannot choose freely the product they supply.

The following section describes the basic model and derives the results for the reference case of a monopoly. Section 3 is devoted to an analysis of the effects of market entry if the product characteristics are exogenous to the firms. Section 4
allows for firms that choose prices and product characteristics so as to maximize profits, which changes the results to some extent, and section 5 concludes.

2 Basic model setup and related literature

Consider a linear market of length $Z$, bounded at both ends. Each point is characterized by its distance from the left market boundary (see fig. 1). Consumers are distributed evenly across the market with unit density and have the same linear demand function

$$q = \begin{cases} a - bp(r) & \text{if } p(r) < \frac{a}{b} \\ 0 & \text{if } p(r) \geq \frac{a}{b} \end{cases}$$

(1)

where $p(r)$ denotes the delivered price (mill or f.o.b. price $m$ plus transportation costs) at the consumer’s location $r$. Transportation costs per unit quantity of the good and per unit distance are constant and denoted $t$, i.e. $p(r) = m + t|r - r_i|$ if the firm’s location is at $r_i$. Transportation costs may be interpreted as a monetary measure for the disutility that a consumer experiences because the good is different from the ideal choice.

Constant marginal costs of production are denoted $c$, and there are no fixed costs. Firms choose the mill price and location that yield a profit maximum, considering the other firm’s location and mill price as being given, respectively. This constitutes what sometimes is referred to as 'Bertrand competition', although this attribution is admittedly somewhat dubious (see Puu, 2001, p. 1).
Already Hotelling has assumed transportation costs that are linear in distance. Together with inelastic demand this yields that firms may increase profits if they move closer to the competitor’s location, hence the ‘principle of minimum differentiation’. D’Aspremont, Gabszewicz and Thisse (1979) proved that eventually (in the symmetric case when firms locate at the quartiles of the market) a point is reached when it becomes profitable for both firms to undercut the rival and serve the entire market, which destroys the stability of the equilibrium. They then show that a stable equilibrium exists when transportation costs are quadratic in distance. Economides (1986) has generalized their results for ‘less-than-quadratic’ convex transportation costs. In this paper we refrain from a discussion of whether convex or concave transportation costs are more plausible (for a recent contribution to this issue see Hammoudi and Moral, 2005). Two arguments have forced us to use linear transportation costs. First, there is a long tradition of this assumption in industrial and spatial economics, which makes it a benchmark case (see for instance Hotelling (1929), Lerner and Singer (1937), Smithies (1941), and Salop (1979)). Our somewhat counter-intuitive results probably raise more interest if we derive them in a model framework that is standard in the literature. Second, convex transportation costs effectuate excessive product differentiation (‘principle of maximum differentiation’) in equilibrium (D’Aspremont et al., 1979). As we show later in this essay, this implies that it is even more likely that duopoly prices are higher than the monopoly price. Hence, by assuming linear transportation costs we avoid to predetermine what will only be one possible result of several opponent forces.

One major difference with respect to the bulk of the related literature is the assumption of an elastic individual demand function\(^1\). The reason why we use an elastic demand function is that most of our results are driven by changes of the price elasticity of aggregate demand. Although the latter may vary even if individual demand is isoelastic or inelastic, the results come up much more clearly if the individual demand function is elastic. The specific form of the assumed demand function – as in the case of the transport costs – has no effect on the qualitative results, however.

In order to meaningfully compare the outcome of monopoly and duopoly, the length of the market must constrain the duopolists, i.e. be smaller than twice the monopoly market. The size of the monopoly market can be calculated by using the

\(^1\)Although there are, over the years, quite a few examples, see e.g. Smithies (1941), Beckmann (1968), and Puu (2002). The assumption of elastic demand is also one major difference with respect to Perloff, Suslow and Seguin (1996) who focus on a similar issue.
condition that demand must drop to zero at the boundary in distance $R_M$ from the monopolist’s site:

$$a - bm_M - btR_M = 0$$

$$R_M = \frac{a - bm_M}{bt}$$  \hspace{1cm} (2)$$

The index $M$ stands for 'monopolist'. We also consider the case that the monopolist is constrained by an exogenous boundary in the distance $R_f \leq R_M$ at one side.

Under the assumptions made, profits $\pi_M$ are

$$\pi_M = (m_M - c) \cdot \left\{ \int_0^{R_f} [a - bm_M - bt(R_f - r)]dr + \int_{R_f}^{R_f + R_M} [a - bm_M - bt(r - R_f)]dr \right\}$$  \hspace{1cm} (3)$$

From the first-order condition the monopoly mill price obtains as

$$m_M = \frac{1}{3b} (2a + 2bR_f t + cb - \alpha)$$  \hspace{1cm} (4)$$

with \[\alpha = \sqrt{(a - bc - bR_f t) \cdot (3bR_f t + a - bc) + 10b^2R_f^2t^2} > 0\]

(4) gives the solution to the firm’s optimizing problem for any value of the boundary $R_f \leq R_M$. If the latter is non-binding, the profit maximizing price becomes

$$m_M = \frac{a + 2bc}{3b}$$  \hspace{1cm} (5)$$

which serves as a reference in the case of endogenous location choice. The extension of the market at both sides is then given by

$$R_M = \frac{2(a - bc)}{3bt} = \frac{2}{3} \kappa$$  \hspace{1cm} (6)$$

where the compound variable

$$\kappa \equiv \frac{1}{t} \left( \frac{a}{b} - c \right)$$

serves to express our results more concisely, see Puu (2002). Otherwise, inserting (4) in (2) yields the extension of the monopoly market

$$R_M = \frac{1}{3bt} (a - cb - 2bR_f t + \alpha)$$  \hspace{1cm} (7)$$
3 Exogenous products

In this section, we start by calculating the optimum prices of symmetric duopolists whose locations (products) are exogenously given and compare the outcome to the monopoly price (4) for different market extensions (measured by the distance $S$ between the competitors) and locations (measured by the firms’ distance to the market boundary $R_f$). In addition, we compare consumer surplus in both cases.

Effects on prices

If a competitor (firm 2) enters at distance $S < 2R_M$ from the incumbent firm 1, full prices at the boundary between their market shares are equal: $m_1 + (R_b - R_f)t = m_2 + (S + R_f - R_b)t$, where $R_b$ is the distance of the boundary from the incumbent’s site. Solving for $R_b$ yields

$$R_b = \frac{m_2 - m_1}{2t} + \frac{2R_f + S}{2}$$

(8)

In the symmetric case, when firms charge identical mill prices, $m_1 = m_2$, the boundary is at the midpoint between their sites.

Once the newcomer has entered the market, profits of the former monopolist become

$$\pi_1 = (m_1 - c)\left\{ \int_0^{R_f} [a - bm_1 - bt(R_f - r)]dr + \int_{R_f}^{R_b} [a - bm_1 - bt(r - R_f)]dr \right\}$$

(9)

Maximization of (9) gives

$$m_1 = \frac{1}{4b} \left( 2a + 2cb + 3btS + 8bR_ft - \beta \right)$$

(10)

with

$$\beta = \sqrt{4(a - bc) \cdot (a - btS - bc) + b^2t^2 \cdot (13S^2 + 48R_fs + 80R_f^2)} > 0$$

A second solution does not fulfill the second-order condition and is thus a profit minimum.²

Setting equal (4) and (10) and solving for $S$ yields two distances between the firms’ sites at which the incumbent firm fetches the same price with or without market entry:

$$S' = \frac{-2}{3bt} (a - cb + 4bR_ft - 2\alpha)$$

(11)

$$S'' = \frac{2}{3bt} (a - cb - 2bR_ft + \alpha) = 2R_M$$

(12)

²This implies that the corresponding corner solution $m_1 \to \infty$ possibly dominates the local maximum. Yet, the assumed composite demand function (1) secures that (10) is globally optimal.
Fig. 2: Price effects of market entry
(see (7)). The second solution is trivial, since at this distance the firms are actually two local monopolies. The following proposition states how entry impacts on the incumbent firm’s price, depending on the distance of the newcomer.

**Proposition 1**

At distances $S = S' \leq R_M$ and $S \geq S'' = 2R_M$ between the incumbent’s and the newcomers sit the incumbent’s mill price remains constant. At all distances $S \in [0, S')$ it decreases due to market entry (if we assume homogenous goods, $S = 0$, the usual result comes up), and at all distances $S \in (S', S'')$ it increases (Fig. 2 illustrates this for the case $a = b = t = 1, c = 0.1$).

**Proof:** It is trivial that firms have no impact on each others prices if the distance between them is greater than $S''$. Proposition 1 is correct if the second derivative of $m_1$ with respect to $S$ is negative, i.e. if the function is concave. Then, there is a single maximum between $S'$ and $S''$, which is thus above the monopoly price. The first two derivatives of $m_1$ with respect to $S$ are

$$\frac{dm_1}{dS} = \frac{t}{4\beta} (3\beta - 2cb - 13btS - 24bR_f t + 2a)$$

$$\frac{d^2 m_1}{dS^2} = -\frac{4t^2b}{\beta^3} \left[29b^2t^2R_f^2 + (3a - 3cb) (a - cb + 2bR_f t) \right] < 0 \quad (13)$$

Finally, we show that $S' \leq R_M$. It is straightforward to show that $S' = R_M$ if
\( R_f \in \{0, R_M\} \). Because the difference \( R_M - S' \) is concave in \( R_f \),
\[
\frac{d^2(R_M - S')}{dR_f^2} = \frac{-6bt(a - cb)^2}{\alpha^3} < 0
\]
it follows that \( S' < R_M \forall R_f \in (0, R_M) \). This signifies that the incumbent’s mill price rises due to entry of a competitor for more than half of the meaningful values \( S \in [0, 2R_M] \) if \( R_f \in (0, R) \) (see Fig. 2). Entry causes an increase of the incumbent’s price if the newcomer locates outside or close to the boundary of the former monopoly market.

With the assumed demand function the further away a consumer is from the firm, the more elastic her demand. Entry has two opponent effects on the price. On the one hand, the optimum price is lower because higher prices lead to a loss of market share. On the other hand, the remaining consumers’ demand is relatively inelastic in average, which works towards a higher price. The latter effect prevails if the entry point is beyond \( S' \) and vice versa. At point \( S' \) both effects are equally strong.

**Effects on consumer surplus**

Consumers fare better with monopoly if the lower mill price outweighs the disadvantage of higher average transport costs. If the newcomer locates in distance \( S' \), total consumer surplus increases due to entry because the mill price of the incumbent remains stable (see proposition 1), and all consumers that become customers of the newcomer, i.e. with preferences in the interval \((R_b, R_f + R_M]\) are better off because of lower transport costs. Therefore, only if entry takes place beyond the distance \( S' \), consumer surplus may be higher in monopoly.

To infer whether consumers that have been supplied by a monopolist fare better with competition or not, we must neglect the effect of a larger number of consumers. Therefore, we have to calculate aggregate consumer surplus (\( \Psi_C \)) within the formerly monopolistic area. Individual consumer surplus is \( \psi_j = 1/(2b)q_j^2 \). Aggregation over the relevant market yields:

\[
\Psi_C = \frac{1}{2b} \left\{ \int_0^{R_f} [a - bm_1 - bt(R_f - r)]^2 dr + \int_{R_f}^{R_b} [a - bm_2 - bt(R_f + S - r)]^2 dr \right\}
\]
The first two integrals collect consumer surplus within the market area that is still served by the incumbent firm. The third integral sums up surplus within the part of
the formerly monopolistic market that is now served by the newcomer. We obtain
the following proposition:

**Proposition 2**
There is a distance between the firms’ sites $S^*$, with $S' < S^* < S''$, above which total
consumer surplus decreases due to market entry.

$$\Psi_C < \Psi_M \forall S \in (S^*, 2R_M)$$  \hspace{1cm} (16)

**Proof:** If a newcomer enters at distance $2R_M$ from the incumbent’s site, consumers’
surplus within the incumbent firm’s market area is not affected, since the firm remains
a local monopoly. To check whether consumers may be better off with monopoly, it
suffices to calculate the derivative of (15) with respect to $S$ at distance $S'' = 2R_M$. If
the derivative is positive, consumer surplus must have been lower than with monopoly
for some $S < 2R_M$. The total derivative of (15) with respect to $S$ is composed of two
effects. One direct effect, which corresponds to the partial derivative with respect to $S$, and one indirect effect, which arises through the impact of a larger distance $S$ on the price $m$.

$$\frac{d\Psi_C}{dS} = \frac{\partial \Psi}{\partial S} + \frac{\partial \Psi}{\partial m_i} \cdot \frac{\partial m_i}{\partial S}$$  \hspace{1cm} (17)

Since $\partial \Psi/\partial m_i < 0$ and $\partial m_i/\partial S < 0$ in the surrounding of $S''$ (proposition 1), the second term is unambiguously positive at $S'' = 2R_M$. To prove that the total derivative is positive, it suffices to show that $\partial \Psi/\partial S$ is nonnegative. This derivative is

$$\frac{\partial \Psi}{\partial S} = \frac{1}{2} \left( a - bm - bt \frac{S'}{2} \right)^2 - \frac{7}{8} b^2t^2 (S - 2R_M)^2$$  \hspace{1cm} (18)

At $S = 2R_M$ the value is zero, thus the total derivative (17) is positive.  \hspace{1cm} \square

To summarize, market entry possibly has counter-intuitive effects. Not only mill
prices may be higher than the monopoly price, it may even occur that competition has
detrimental effects on consumer surplus. The former effect is due to a decrease of the
aggregate price elasticity of demand following the entry of a competitor, since both
firms are able to specialize on consumers with a high preference for their respective
good. The latter effect arises if the increase in prices due to market entry more than
offsets that the purchased goods are in average closer to the consumers ideal choice.

This is not to say that the result that more than half of the economically mean-
ingful values for $S$ and $R_f$ produce higher prices in the duopoly case means that this
result is more likely. In order to make such a judgement one would have to infer
the values of $S$ and $R_f$ that yield endogenously from the firms’ mutually interacting optimal behavior, which is at issue in the following section.

4 Endogenous product choice

A newcomer entering into the market could and would choose its location/product such that profits are maximized. Maintaining the assumption of Bertrand competition, i.e. that the competitors choose the profit-maximizing mill price for a given price of the other firm, it is natural to assume that the firms also take the location of the other firm as given. However, at least after some delay, the former monopolist can and will react to the entry of the newcomer. This reaction comprises price changes as well as possible relocation. If the firms would not be constrained by market borders, they would locate such that each firm possesses its own monopolistic market, i.e. entry would lead to two spatially separated monopolies. Therefore, it only makes sense to analyze market entry in a spatial market whose extension is less than twice the extension of a monopoly market: $Z < 4 R_M$ where $R_M$ stands for the radius of the monopoly market. Before entry of the competitor, the then monopoly locates at $r_M = Z/2$, i.e. at the midpoint of the total interval.

If we allow for relocations under the firms’ conjecture that the competitor’s location remains unchanged, a well-known problem arises. Namely, because firms neglect possible price reactions of the competitor, undercutting always seems to pay. That is, if one symmetric firm moves to the location of the rival and undercuts him by an arbitrarily small amount, profits within the former rival’s market are the same as the firm’s own former profits in the limit, and in addition it accrues some profits within its own former market (for the same intuition see e.g. Novshek (1980) and Anderson (1986)). If the firm chooses a different optimal undercutting location, as in Puu (2002), profits must be even higher. Therefore, if we want to maintain our relatively simple analytical framework (e.g. without relocation costs), undercutting strategies must be obviated via assumption. This assumption is based on the rationale that “a firm cannot rationally believe it is possible to eliminate a rival without retaliation” (Anderson, 1986, p. 24) and is known as the “no-mill-price-undercutting assumption” (Eaton and Lipsey, 1978), but even goes back to Lerner and Singer (1937).

Two cases have to be distinguished regarding the pre-entry situation, namely that the monopolist may or may not be constrained by market boundaries. If the total market area is small, the boundaries restrict the extension of the monopoly
(Z < 2R_M), so that the size of the total market equals the size of the monopoly market. If the total market area is large, the monopolist can choose the profit-maximizing extension (Z ≥ 2R_M). The total market area is then larger than the area that is served by the monopoly.

Effects on Prices

If the market area is sufficiently large that the monopolist is not constrained by the extension of the total market area, the mill price and extension of the monopoly are given by eq. (5) and (6).

If, however, the monopoly is constrained because the extension of the total market area is small, the result differs from the previous section because the monopoly is constrained at both ends. The profit function in this case (Z < 2R_M) becomes:

$$\pi_M = 2 (m_M - c) \int_0^{Z/2} (a - bm_M - brt) dr$$

The first-order condition for a profit maximum gives

$$m_M|_{\text{constr.}} = \frac{1}{2} \left( \frac{a}{b} + c \right) - \frac{tZ}{8}$$

The next question we address is when the monopoly is actually constrained. In the limiting case, the extension of the unconstrained monopoly market equals the size of the total market area:

$$Z' \equiv 2R_M|_{\text{unconstr.}} = \frac{4}{3t} \left( \frac{a}{b} - c \right) = \frac{4}{3} \kappa$$

Therefore, the monopoly price is

$$m_M = \begin{cases} \frac{1}{3} \cdot \left( \frac{a}{b} + 2c \right) & \forall Z \geq Z' \\ \frac{1}{2} \cdot \left( \frac{a}{b} + c \right) - \frac{tZ}{8} & \forall Z < Z' \end{cases}$$

Figure 3 shows the monopoly price as a function of the size of the market area Z.

The following values have been assumed: a = b = t = 1; c = 0.1. With these parameter values, the monopoly is constrained by the extension of the total market area if Z < Z' = 1.2. If Z > 1.2, the size of the market area that is actually served by the monopoly is constant, and so is the price. If, however, the total market size is smaller than Z’, the downward sloping price function becomes effective. A smaller market area leads to a higher mill price because the price elasticity of aggregate demand is lower the smaller the market is. The line segments that are respectively relevant are highlighted by diamonds.
Now consider the entry of one firm. Without loss of generality, we assume that the newcomer (firm 2) locates to the right of the incumbent’s location (firm 1), \( r_1 < r_2 \) (see fig. 1). Profits of the incumbent firm after the entry of one newcomer become

\[
\pi_1 = (m_1 - c) \left\{ \int_0^{r_1} [a - bm_1 - bt(r_1 - r)]dr + \int_{r_1}^{R_b} [a - bm_1 - bt(r - r_1)]dr \right\} \quad (23)
\]

Note that \( R_b \), the boundary between the incumbent’s and the newcomer’s market area depends on the choice variable product/location, \( r_1 \) and \( r_2 \). As before, it yields from the condition that at the market boundary consumer prices must be equal, i.e. \( m_1 + t \cdot (R_b - r_1) = m_2 + t \cdot (r_2 - R_b) \):

\[
R_b = \frac{m_2 - m_1}{2t} + \frac{r_1 + r_2}{2} \quad (24)
\]

Again, it can be verified easily, that the market boundary is at half distance between the competitors, if prices are equal. If the newcomer (firm 2) raises its mill price, the border is further to the right, and if the incumbent (firm 1) increases the price, the border is further to the left.

Solving the integrals in eq.\((23)\), using eq. \((24)\), and building the first-order condition for a profit maximum with respect to the location of the incumbent firm \( r_1 \) yields the following best response function:

\[
r_1 = \frac{1}{5t} \left( m_2 - 3m_1 + r_2t + 2 \frac{a}{b} \right) \quad (25)
\]
An analogous procedure yields

\[ r_2 = \frac{1}{5t} \left( 3m_2 - m_1 + 4tZ + r_1 t - 2 \frac{a}{b} \right) \tag{26} \]

Substituting eq. (24) in the profit functions, using the two best response functions eq. (25) and (26) to replace \( r_1 \) and \( r_2 \), and differentiating with respect to the mill prices yields respectively two solutions, of which the following fulfill the second-order condition for a profit maximum \(^3\):

\[ m_1 = \frac{1}{9} \left( 4 \frac{a}{b} + 3c + 2r_2 t + 2m_2 \right) - \frac{1}{18} \sqrt{\alpha_1 - \beta_1} \tag{27} \]

and

\[ m_2 = \frac{1}{9} \left( 4 \frac{a}{b} + 3c + 2tZ - 2r_1 t + 2m_1 \right) - \frac{1}{18} \sqrt{\alpha_1 - \beta_2} \tag{28} \]

with

\[ \alpha_1 = 46 \left( \frac{a}{b} \right)^2 - 48 \frac{a}{b} c + 36 c^2; \]

\[ \beta_1 = \left[ 24c + 44 \frac{a}{b} + 34 (m_2 + r_2 t) \right] \cdot (m_2 + r_2 t) \]

\[ \beta_2 = \left[ 24c - 44 \frac{a}{b} + 34 (m_1 - r_1 t + tZ) \right] \cdot (m_1 - r_1 t + tZ) \]

In a symmetric Nash-Bertrand equilibrium all four best response functions, eq. (25), (26), (27), and (28) must be fulfilled simultaneously. Substituting eq. (26) for \( r_2 \) in eq. (25) and vice versa yields:

\[ r_1 = \frac{1}{6t} \left[ 2m_2 - 4m_1 + tZ + 2 \frac{a}{b} \right] \tag{29} \]

and

\[ r_2 = \frac{1}{6t} \left[ 4m_2 - 2m_1 + 5tZ - 2 \frac{a}{b} \right] \tag{30} \]

The point halfway between the location of the incumbent, \( r_1 \), and the location of the newcomer, \( r_2 \), is

\[ \frac{r_1 + r_2}{2} = \frac{1}{2t} (tZ + m_2 - m_1) \tag{31} \]

If firms are symmetric, which we assume throughout the following analysis, \( m_1 = m_2 = m_0 \), eq. (31) simplifies to \( (r_1 + r_2)/2 = Z/2 \). This implies that firms are located symmetrically around the center of the market (see e.g. Puu, 2002). Then, the locations of the two firms are given by

\[ r_1 = \frac{1}{6t} \left[ -2m + tZ + 2 \frac{a}{b} \right] \tag{32} \]

\(^3\)It can be easily checked that corner solutions are not dominating.
and

\[ r_2 = \frac{1}{6t} \left[ 2m + 5tZ - \frac{2a}{b} \right] \] (33)

The results for \( r_1 \) and \( r_2 \) are only valid if \( r_1 < \frac{Z}{2} < r_2 \), which has been assumed for the profit function, see eq. (23). From

\[ \frac{1}{6t} \left[ -2m + tZ + 2 \frac{a}{b} \right] < \frac{Z}{2} \]

we get

\[ \frac{a}{b} < m + tZ \] (34)

The left-hand side of the inequality is the prohibitive price. The right-hand side is the mill price plus the transportation costs from one boundary of the total market area to the other. Hence, our results are only valid if one firm cannot serve the entire market area from one endpoint. How restrictive this condition is, will be at issue once we have derived the mill prices of the oligopolists.

The identical price charged by both symmetric firms can be calculated by setting \( m_1 = m_2 = m_O \) in eq. (27) or (28), plugging in eq. (29) and (30), and solving for \( m \). The result is

\[
m_O = \frac{1}{20} \left[ 8 \frac{a}{b} + 12c + 16tZ - 3 \sqrt{34t^2Z^2 + 16 \left( \frac{a}{b} - c \right)^2 - 16 \left( \frac{a}{b} - c \right) tZ} \right] \\
= c + \frac{4}{5}tZ + \frac{2}{5} \kappa t - \frac{3}{20} t \gamma; \quad \gamma \equiv \sqrt{34Z^2 - 16 \kappa Z + 16 \kappa^2} \] (35)

Inserting this price in condition (34), it may be rewritten as

\[ Z > \frac{8}{11t} \left( \frac{a}{b} - c \right) = \frac{8}{11} \kappa \] (36)

If this condition is fulfilled, the locations of both symmetric firms and their prices correspond with the results given in eq. (32), (33), and (35). If, however, the market extension is smaller, both firms crowd at the center of the market, as in the original Hotelling article. The upper limit to \( Z \) is given by the situation when the market area is large enough to accommodate two adjacent disjoint monopolies, i.e. \( Z \leq 4 \cdot R_M \big|_{unconstr.} = \frac{8}{3} \kappa \). Thus, of all parameter values that are consistent with oligopoly only a fraction of

\[ \frac{8}{3} \kappa = \frac{3}{11} = 0.27 \]

lead to Hotelling’s result of 'minimum differentiation' regarding the firms’ choice of location with the assumed demand function.

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Figure 4 compares the mill price two symmetric competitors charge, eq. (35), with the monopoly price eq. (22) for the standard parameter values \((a = b = t = 1, c = 0.1)\). The monopoly price is a composed function, taking account of whether the monopoly is constrained by the size of the entire market or not. The figure reveals that – for the employed parameter values – proposition 1 continues to hold: Oligopoly prices (curve labelled with circles) may be higher than monopoly prices (curve labelled with diamonds).

Fig. 4: Monopoly vs. duopoly price as a function of market extension

For which parameter values does the result that the oligopoly mill price is higher than the monopoly mill price hold? The following proposition states that this result is actually valid for more than half of the economically meaningful parameters.

**Proposition 3**
The monopoly mill price is lower than the duopoly mill price in the interval

\[
Z \in \left(1.22407 \kappa, 2.5 \kappa\right)
\]

**Proof:** Setting equal the mill price of the unconstrained monopoly, eq. (5) and the mill price of one symmetric oligopolist, eq. (35), and solving for the extension of the market yields two solutions:

\[
Z = \left[\frac{16}{15} \left(\frac{a}{b} - c\right)\right] = \left[\frac{16}{15} \kappa\right] = \left[\frac{8}{3} \kappa\right] \tag{37}
\]

The second solution is twice the extension \(Z'\) which means that the firms are adjacent
unconstrained monopolists rather than oligopolists. Of course, prices must be equal for even larger markets as well. The first solution is relevant if the monopoly is actually unconstrained at this market size.

Equating the oligopoly price with the price of the constrained monopoly, eq. (20), and solving for $Z$ gives

$$Z = \left[ \frac{4(6\sqrt{7}-7)}{29t} \left( \frac{a}{b} - c \right) \right] = \left[ \frac{4(6\sqrt{7}-7)}{29t} \kappa \right]$$

(38)
of which the second solution is negative and thus meaningless, whereas the first solution corresponds with the intersection of the downward sloped monopoly price and the oligopoly price in fig. 4. Since

$$\frac{16}{15} \kappa < \frac{4(6\sqrt{7}-7)}{29} \approx 1.22407 \kappa < \frac{4}{3} \kappa$$

the oligopoly is constrained at the intersection between oligopoly price and monopoly price, i.e. the intersection is always left from the kink of the monopoly price, as depicted in fig. 4. At this intersection, the price-increasing “specialization effect” (firms are closer to consumers in average, which decreases aggregate price elasticity) exactly equals the price-decreasing “contention effect” (firms that increase prices lose market share). Next, we show that the function of the oligopoly price, eq. (35), is concave:

$$\frac{d^2m_O}{dZ^2} = \frac{-72t^2 \left( \frac{a}{b} - c \right)^2}{\left[ 4 \left( 2 \frac{a}{b} - 2c - tZ \right)^2 + 30 t^2 Z^2 \right]^\frac{3}{2}} < 0$$

Concavity of the oligopoly price implies that the oligopoly price is higher than the monopoly price between the two relevant points of intersection.

Proposition 3 shows that, as with exogenous locations, more than half of the meaningful parameter values lead to higher, rather than lower, duopoly prices. This result may explain to some extent, why the evidence on the effects of market entry are mixed. For instance, Perloff et al. (1996) found that prices in the anti-ulcer drug market increased in the period 1977–1993, when brand-name entry occurred. In other markets, where consumers perceive the goods as more homogenous (low distance between firms), prices decreased, which is fully compatible with our analysis.

**Effects on consumer surplus**

We have shown that the first result from the model with exogenous products/locations, namely that oligopoly may lead to higher prices than monopoly, continues to hold if
the firms’ location is endogenous. As in the previous section, we restrict the analysis to those consumers whose preferences are in the range that the monopoly would serve.

Since the welfare comparison is related to the respective extension of the monopoly, we need to distinguish the cases when the monopoly respectively is or is not constrained by the extension of the total market area, i.e. \( Z < Z' \) or \( Z \geq Z' \). In the latter case, the extension of the monopoly market is \( 2R_M|_{\text{unconstr.}} \). First, we calculate the surplus in the cases of an unconstrained and a constrained monopoly. In the unconstrained case (\( Z \geq Z' \)), aggregate consumer surplus is:

\[
\Psi_M|_{\text{unconstr.}} = \frac{1}{b} \int_0^{R_M} (a - bm_M - br)^2 \, dr
\]

where \( m_M \) is given by eq. (5), and \( R_M \) by eq. (6). Solving the integral and substituting variables yields the following aggregate consumer surplus in the case of an unconstrained monopoly:

\[
\Psi_M|_{\text{unconstr.}} = \frac{8}{81} \left( \frac{a}{b} - c \right)^3 = \frac{8}{81} \kappa^3
\]

This surplus does not depend on the size of the market \( Z \) because the range of coverage is smaller than \( Z \).

If, however, the monopolist is constrained by the size of the given market area (\( Z < Z' \)), aggregate consumer surplus becomes:

\[
\Psi_M|_{\text{constr.}} = \frac{1}{b} \int_0^Z (a - bm_M - br)^2 \, dr
\]

where \( m_M \) is given by eq. (20). Solving the integral and substituting the mill price yields the consumer surplus in the case of a constrained monopoly:

\[
\Psi_M|_{\text{constr.}} = \frac{bZ}{384} \left[ 48 \left( \frac{a}{b} - c \right)^2 - 24 tZ \left( \frac{a}{b} - c \right) + 7 t^2 Z^2 \right]
= \frac{b t^2 Z}{384} \left( 48 \kappa^2 - 24 \kappa Z + 7 Z^2 \right)
\]

The oligopolists are always constrained by the exogenously given size of the market area. If the market were large enough to accommodate two unconstrained monopolists, there would be no reason for the firms to compete for market shares. But, since consumers’ surplus is calculated within the market area that a monopolist would serve, we have to take account of whether the monopolist would be constrained
for the comparison to be meaningful. Total surplus within the area that corresponds with an unconstrained monopolist reads

\[
\Psi_{O|\text{unconstr.}} = \frac{bt^2}{324000} \left( 428976 Z^3 - 18688 \kappa^3 - 243 \gamma^3 - 66096 \gamma Z^2 \\
-8784 \gamma \kappa^2 + 234432 \kappa^2 Z - 395496 \kappa Z^2 + 48384 \gamma \kappa Z \right)
\] (41)

Consumers’ surplus in oligopoly but within the market area of a monopolist that is constrained by the extension of the total market area reads

\[
\Psi_{O|\text{constr.}} = \frac{bt^2}{12000} \left( 19268 Z^3 - 2304 \kappa^3 - 9 \gamma^3 - 2988 \gamma Z^2 \\
-432 \gamma \kappa^2 + 12096 \kappa^2 Z - 20808 \kappa Z^2 + 2592 \gamma \kappa Z \right)
\] (42)

Eq. (41) gives aggregate consumer surplus under oligopoly in the cases \( Z \geq Z' \), and eq. (42) gives consumers’ surplus if \( Z < Z' \) (see eq. (21)). Depending on the size of the market area, respectively eq. (39) and eq. (41), or eq. (40) and eq. (42) have to be compared. Since the oligopoly price is lower than the monopoly price in small market areas (see fig. 4), and consumers have the additional advantage that transportation costs are lower in average, only in the case of large market areas consumers may fare better with monopoly. The economic reason is that aggregate demand is more elastic in large market areas, which works as a discipline on the pricing behavior of the monopoly. In this model, however, consumers always fare better with oligopoly in aggregate, as the following proposition states:

**Proposition 4**

Aggregate consumer surplus \( \Psi \) is higher in oligopoly for all relevant parameter values.

**Proof:** See the appendix

Figure 5 illustrates proposition 4 and its proof for the parameter values \( a = b = t = 1, c = 0.1 \). The solid lines depict aggregate consumer surplus in the case of a monopoly, and the dotted lines depict surplus in the case of duopoly. In both cases, the lines representing surplus in the case of a ’constrained monopoly’ are valid in the interval \( Z \in ((8/11)\kappa, (4/3)\kappa] \). For higher values of \( Z \), i.e. if \( Z \in [(4/3)\kappa, (8/3)\kappa] \), lines corresponding with ’unconstrained monopoly’ are relevant. The respectively valid lines are printed in bold. If the extension of the total market area is \( Z = (4/3)\kappa \) both pairs of lines intersect. The figure reveals that, if one compares the lines that are respectively relevant, consumers never fare better with monopoly. Only in the limiting case of \( Z = (8/3)\kappa \), consumers’ surplus is equal under monopoly and oligopoly, but at this market extension the duopolists indeed become adjacent monopolists.
What are the economic reasons for our result and for the difference with respect to our findings regarding the case of exogenous products? As proposition 3 states, the mill price is higher in large market areas under oligopoly than under monopoly. But this does not render monopoly beneficial to consumers because under oligopoly the average distance to the next firm is smaller, which economizes transportation costs, and because the differences in prices are smaller than in the model with exogenous locations. The latter follows from the best response functions eq. (29) and eq. (30). If one firm increases its mill price, the other firm’s optimum location is closer to the location of the first firm, which harms profits. Therefore, the firms charge lower prices than if they would consider the locations as given. This negative effect on the mill prices with endogenous location choice prevents that the difference to the monopoly’s mill price becomes large enough to overset the advantage in transport costs.

Of course, all results regarding aggregate consumers’ surplus have to be interpreted with some caution. Not only that the concept of consumer surplus is disputable⁴. The fact that consumers always fare better with oligopoly does not mean that all consumers are better off. Instead, consumers close to the boundaries profit more than proportionately, and consumers at the center of the total market area lose because they are faced with higher transportation costs in addition to the possibly

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⁴Chipman and Moore (1980) showed that restrictive and empirically untenable assumptions on preferences are necessary in order to make consumer’s surplus a valid measure (see also Armstrong (2004)).
higher mill prices under oligopoly. Because the respectively most distant consumers locate closer to the nearer oligopolist’s site than to the monopolist’s site, there are less differences between the consumer surplus of individuals in oligopoly. Hence, in addition to efficiency considerations, distributional effects may also favor oligopoly in this model.

5 Summary and discussion

We have shown that in both versions of the model, with and without endogenous choice of products, competition does not necessarily lead to lower prices. The mill prices of the oligopolists may be higher than the monopoly mill price if the total market area is large and the competitors’ sites far apart. In comparison, oligopoly prices are always lower in the case of small markets where the firms’ sites are at close quarters. Our results challenge an almost common knowledge, which can be derived e.g. in the standard Cournot oligopoly model. We claim that our model increases realism with respect to the latter because markets are actually differentiated. Taking this into account gives rise to a trade off between the “specialization effect” and the “contention effect”, which have opposite directions.

Even more remarkably, consumers may be better off with monopoly if the (characteristics) space is sufficiently large. Hence, the lower monopoly price possibly overcompensates the disadvantage of higher average transportation costs. The latter result only comes up if products/locations are exogenous, however. Endogenous locations effectuate that consumers fare better with oligopoly. The rationale for this difference between the two models is that with endogenous locations firms take into account that a higher price would render it advantageous for the competitor to locate closer. Since this would harm profits, the firm chooses a lower mill price than with exogenous locations. As a consequence, consumer surplus is higher and monopoly is never advantageous for consumers in aggregate. In spite of this result, some consumers in the center of the total market area may be better off in the case of a monopoly because of lower transport costs. In average transport costs are lower with oligopoly, however. Since this effect is the strongest in large market areas, where the oligopoly price eventually is higher than the monopoly price, it explains why consumers nevertheless fare better with oligopoly.

The reasoning used in the previous paragraph may also be depicted as a trade off between two types of inefficiencies in oligopoly. In small markets, products are
excessively similar and prices are low. This means, that competition is effective, which yields the efficient outcome with respect to prices. The chosen products are not efficient, however, because firms only insufficiently account for consumers’ preferences. In comparison, large market areas lead to almost-efficient locations/products. The monopoly power that firms attain through the distance from the competitor yet leads them to charge higher prices than in the former case.

References


Appendix

Proof of proposition 4

To prove the proposition, we proceed in 3 steps: First, we show that $\Psi$ is higher in the case of an oligopoly at the start point of the relevant interval, and that the difference is even increasing with the extension of the total market area. Then we show that $\Psi$ is equal at the end point of the relevant interval, but that consumers fare better with oligopoly if the extension of the total market area is marginally smaller. Finally we show that there is only one extremum of $\Psi_O$ within the relevant interval of market extensions, which is thus a maximum.

Step 1: If the mill price in oligopoly (35) is lower than the monopoly price, consumers are always better off with oligopoly. Also if the size of the total market area is such that the mill price of one oligopolist and of the monopolist coincide (see eq. (38)), consumers prefer oligopoly in aggregate because they have lower average transportation costs. The relevant interval is thus, when the monopoly is not constrained by the size of the total market area, i.e. $Z \in [Z', 2Z']$.

Subtracting eq. (39) from eq. (41), and substituting $Z = Z' = (4/3)\kappa$ yields:

$$\left( \frac{5996}{3375} - \frac{116}{375} \sqrt{31} \right) b\kappa^3 t^2 > 0$$
Substituting $\gamma$ in eq. (41), building the derivative with respect to market extension $Z$, and reusing the compound variable $\gamma$, we get:

$$
\frac{d \Psi_O}{dZ} \bigg|_{unconstr.} = \frac{bt^2}{9000 \gamma} \left( 35748 \gamma Z^2 + 26048 \kappa^3 - 112912 \kappa^2 Z + 181356 \kappa Z^2 \right. \\
\left. -210681 Z^3 + 6512 \gamma \kappa^2 - 21972 \gamma \kappa Z \right) 
$$

(43)

Substituting $Z = Z' = (4/3)\kappa$ in eq. (43) obtains

$$
\frac{d \Psi_O}{dZ} \bigg|_{unconstr.}(Z') = \frac{8}{34875} \left( 19747 - 3533 \sqrt{31} \right) \kappa^2 t^2 \approx 0.017454 \kappa^2 t^2 > 0
$$

Since consumer surplus in the case of an unconstrained monopoly does not depend on the extension of the total market area, this means that the difference $\Psi_O - \Psi_M$ increases in the neighborhood of $Z'$.

Step 2: In the limiting case $Z = 2 Z' = \frac{8}{5} \kappa$ consumers’ surplus in the cases of oligopoly and monopoly must coincide because the two oligopolists become disjoint monopolists. Next, we build the first derivative of $\Psi_O |_{unconstr.}$ at $(Z = 2 Z')$. If the derivative is positive, consumers must be worse off with oligopoly for some $Z < 2 Z'$ because consumer surplus in the case of an unconstrained monopoly does not depend on $Z$.

Substituting $\frac{8}{5} \kappa$ for $Z$, eq. (43) simplifies to

$$
\frac{d \Psi_O}{dZ} \bigg|_{unconstr.}(2 Z') = \frac{-4 \kappa^2 t^2}{33} < 0
$$

The negative sign of the derivative implies that if the market extension decreases, starting from the largest meaningful value $2 Z'$, oligopoly becomes increasingly advantageous for consumers in aggregate.

Step 3: To detect local extrema of $\Psi_O$, we set the first derivative, eq. (43), equal to zero. Rearranging and taking squares to get rid of the roots (see the definition of $\gamma$ in eq. (35)) yields the fifth-order polynomial equation

$$
-5001216 \kappa^5 + 8024064 \kappa^4 Z - 15139584 \kappa^3 Z^2 \\
+21133008 \kappa^2 Z^3 - 20470968 \kappa Z^4 + 7497765 Z^5 = 0
$$

This equation has two pairs of complex roots, and one real root in the relevant interval, at approximately $Z = 1.5968049 \kappa$. Since the derivative is positive at $Z = (4/3)\kappa$ and negative at $(8/3)\kappa$, we conclude that there is one maximum of the consumer surplus at $Z = 1.5968049 \kappa$. Because there are no other local extrema in the relevant interval, consumer surplus must always be higher under oligopoly than under monopoly. □
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