

The stellar Eddington limit

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It is often assumed that when stars reach their Eddington limit, strong outflows are initiated, and that this happens only for extreme stellar masses. We discuss here that in models of up to $500 M_{\odot}$, the Eddington limit is never reached at the stellar surface. Instead, we argue that the Eddington limit is reached inside the stellar envelope in hydrogen-rich stars above $\sim 30 M_{\odot}$ and in Wolf-Rayet stars above $\sim 7 M_{\odot}$, with drastic effects for their structure and stability.

1 Introduction

Massive stars are powerful engines and strongly affect the evolution of star forming galaxies throughout cosmic time (Langer 2012, Szécsi et al. 2015). In particular the most massive ones produce copious amounts of ionising photons (Doran et al., 2013), emit powerful stellar winds (Kudritzki & Puls, 2000, Smith 2014) and are thought to produce the most energetic and spectacular stellar explosions, as pair-instability supernovae (Kozyreva et al., 2014), superluminous supernovae (Gal-Yam et al. 2009), and long-duration gamma-ray bursts (Larsson et al., 2007, Raskin et al., 2008).

2 The Eddington limit in main sequence stars

The most massive stars are close to the so called Eddington limit. The Eddington luminosity is traditionally considered as the maximum luminosity which a star may have to avoid that the radiative acceleration at its surface exceeds gravity, when considering only electron scattering as opacity source (Eddington 1926):

$$L_e = 4\pi cGM/\kappa_e. \quad (1)$$

Massive main sequence stars, which we understand here as those which undergo core hydrogen burning, have a much higher luminosity than the Sun, as they are known to obey a simple mass-luminosity relation, $L \sim M^{\alpha}$, with $\alpha > 1$. However, whereas this relation is very steep for low mass stars ($\alpha \simeq 5$), it is shown in Kippenhahn & Weigert (1990) that $\alpha \rightarrow 1$ for $M \rightarrow \infty$, and Köhler et al. (2015) find $\alpha \simeq 1.1$ for $M = 500 M_{\odot}$. In fact, Kato (1986) showed that zero-age main sequence models computed only with the electron scattering opacity do reach the Eddington limit at a mass of about $\sim 10^5 M_{\odot}$. Whereas it is debated in the literature whether real main sequence stars do reach the Eddington-limit (Langer 1998, Crowther et al. 2010, Maeder et al. 2012, Langer & Kudritzki 2014, Bestenlehner et al., 2014), we argue below that this is indeed the case.

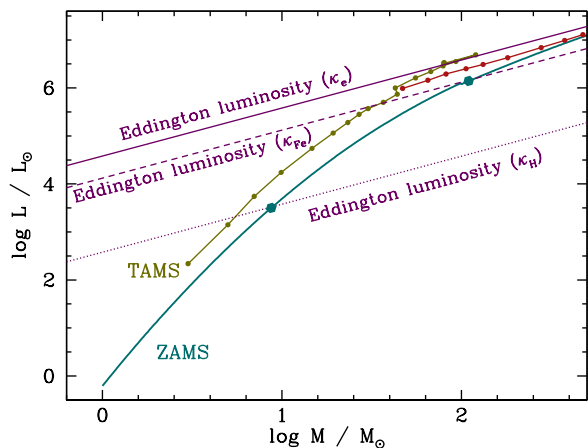


Fig. 1: The stellar Eddington limit (cf., Eqs. 1 and 2) in the mass-luminosity plane, assuming three different dominant opacity sources. The first one, referring to the electron scattering opacity κ_e (full-drawn straight line), is valid for a Solar hydrogen and helium mass fraction and assuming complete ionisation, such that $\kappa_e \simeq 0.34 \text{ cm}^2 \text{ g}^{-1}$. The second one (dashed straight line) refers to a Rosseland mean opacity dominated by iron, assumed here as $\kappa_{\text{Fe}} \simeq 1 \text{ cm}^2 \text{ g}^{-1}$. The third one (dotted straight line) refers to the Rosseland mean opacity in the hydrogen recombination zone and is assumed here as $\kappa_{\text{H}} \simeq 100 \text{ cm}^2 \text{ g}^{-1}$ (see text). The curved blue line gives the location of zero-age main sequence stellar models, with dots marking the crossing of the H- and Fe-Eddington limit. The brown line marks the terminal-age main sequence (TAMS) location of the models of Brott et al. (2011) and Köhler et al. (2015) for LMC composition rotating initially with $\sim 100 \text{ km/s}$, and the red line marks those models at which a surface helium enrichment is starting to occur.

Figure 1 illustrates the situation in the mass-luminosity plane. The zero age main sequence (ZAMS) bends such that the electron-scattering Eddington limit is only met far outside the figure. I.e., real massive stars never encounter this limit. However, the true opacity can be substantially larger than the electron scattering opacity. We thus follow Sanyal et al. (2015; SGLB15) and define a local

Eddington luminosity anywhere inside the star as

$$L_{\text{Edd}}(r) = 4\pi cGM(r)/\kappa(r), \quad (2)$$

where r is the radial coordinate and $\kappa(r)$ now represents the local Rosseland mean opacity. We consider L_{Edd} locally inside the star, since SGLB15 showed that, again, the condition $L = L_{\text{Edd}}$ according to Eq. (2) is not met by any stellar model of Köhler et al. (2015) at the stellar surface. This implies that, based on these models, we do not expect stars of up to $500 M_{\odot}$ to drive a super-Eddington wind.

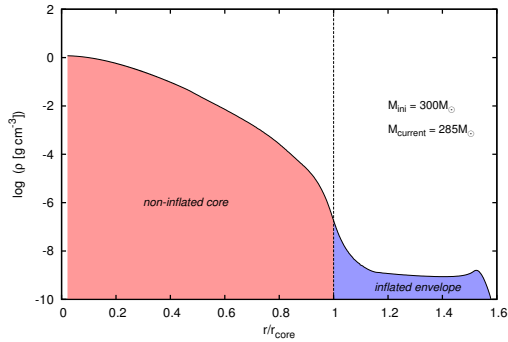


Fig. 2: Density profile of a non-rotating $285 M_{\odot}$ star with $T_{\text{eff}} = 46600 \text{ K}$ and $\log(L/L_{\odot}) = 6.8$, showing an inflated envelope and a density inversion. The X-axis has been normalised by the core radius r_{core} of $25.3 R_{\odot}$.

However, when assuming that even in hot stars, the Rosseland mean opacity may become as large as $1 \text{ cm}^2 \text{ g}^{-1}$ due to the iron opacity peak (cf., fig. 9 in SGLB15), the Eddington luminosity according to Eq. (2) is about a factor of three smaller than that according to Eq. (1). Correspondingly, the ZAMS line crosses it at about $M = 100 M_{\odot}$ (upper blue dot in Fig. 1). When looking at the luminosity increase during core hydrogen burning by including the terminal age main sequence (TAMS) in Fig. 1, we see that even core hydrogen burning stars with $M = 30 M_{\odot}$ may violate the local Eddington limit in their envelopes. The iron opacity peak is metallicity and density dependent, such that the corresponding Eddington limit is only approximate. However, we show below that the expectation derived from Fig. 1 is actually met by detailed stellar models.

The largest opacities are found in the hydrogen recombination zones of cool stars. Thus, for illustration, we plot a third Eddington limit in Fig. 1, which is again approximate, by assuming $\kappa_{\text{H}} = 100 \text{ cm}^2 \text{ g}^{-1}$, which then we expect to be violated by cool giants with $M \gtrsim 5 M_{\odot}$.

SGLB15 demonstrated that when the local radiative luminosity in the stellar envelope exceeds the

Eddington luminosity as defined by Eq. (2), the envelope inflates, as shown for an example in Fig. 2. As the iron opacity decreases for lower densities, inflation leads to an increase of the Eddington luminosity and generally stops for the hot models when the Eddington limit is not violated any more. Figure 3 shows that, for LMC metallicity, inflation occurs for $\log L/L_{\odot} \gtrsim 5.5$, and exceeds a factor of 2 for models which are located to the right of the hot edge of the S Doradus instability strip (see also Gräfener et al. 2012).

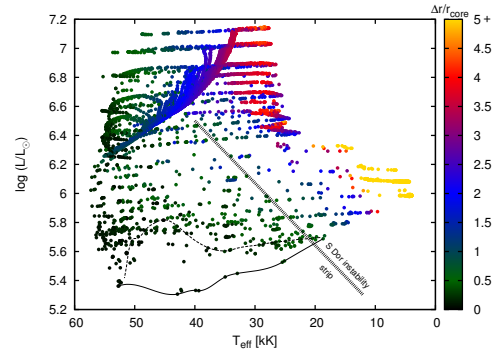


Fig. 3: Hertzsprung-Russell diagram showing core-hydrogen burning inflated models from the LMC grid of Köhler et al. (2015). The color indicates the degree of inflation as defined through $\Delta r/r_{\text{core}}$, where r is the stellar radius, r_{core} would be the stellar radius if inflation was absent, and $\Delta r = r - r_{\text{core}}$ (cf., Fig. 2). Dots corresponding to models with $\Delta r/r_{\text{core}} > 5$ are colored in yellow. Below the solid black line, no inflated models are found, and above the dotted black all models were found to be inflated. The hot part of the S Dor instability strip (Smith et al., 2004) is also marked. Cf., SGLB15.

Since the inflated envelopes are convectively unstable (Langer 1997), and the convectively transported fraction of the stellar luminosity does not contribute to the inflation, the extent of inflation depends on the assumed convection theory. The shown results are obtained with the standard Mixing Length Theory (cf. SGLB15).

Observational evidence for inflation in Galactic main sequence stars, as predicted by the extremely extended main sequence band for $5.5 \lesssim \log L/L_{\odot} \lesssim 6.5$ of the Köhler et al. (2015) models, is found in fig. 5 of Clark et al. (2014) and fig. 1 of Castro et al. (2015). These figures show an abundance of B supergiants which is unexpected if they would have to be explained by core helium burning stars. The inflated envelopes are also likely to be pulsationally unstable (cf. Grassitelli et al., this volume), which may lead to observable consequences.

Inflated stars may be found well below the quoted Eddington luminosities for various reasons. E.g., stars which lost a significant amount of their hydrogen-rich envelope to a wind or to a close binary companion will, to first order, keep their luminosity, but since their mass is reduced their Eddington luminosity will be reduced. Similarly, rotation may reduce the Eddington luminosity (Langer 1997, 1998), even though its effect is latitude dependent and vanishes at the poles. SGLB15 suggested that latitude dependent inflation may give rise to the B[e] phenomenon in supergiants (Zickgraf et al. 1985).

3 The Eddington limit in Wolf-Rayet stars

So far, the effect of envelope inflation has been discussed most extensively for Wolf-Rayet stars (Ishii et al. 1999, Petrovic et al. 2006, Gräfenner et al. 2012). Figure 4 demonstrates the situation in the HR diagram on the basis of zero-age helium star models. Using standard Rosseland mean opacities, we find inflation to occur for models above $\sim 7M_{\odot}$ at solar metallicity (Grassitelli et al., this volume), and above $\sim 12M_{\odot}$ for the SMC metallicity.

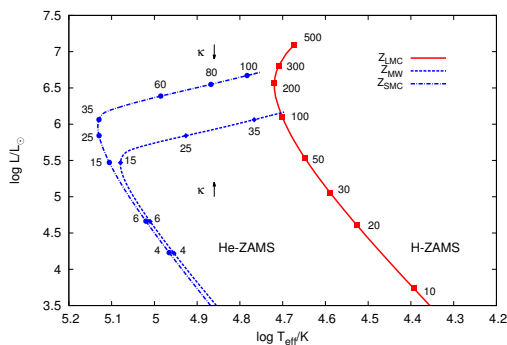


Fig. 4: Zero-age main sequence of the models of Köhler et al. (2015) for LMC metallicity (full drawn red line) together with lines for zero-age main sequence helium star models for Galactic (blue, dashed line) and SMC (blue, dash-dotted line) metallicity. Labelled dots represent masses in units of the Solar mass. The labels “ $\kappa \downarrow$ ” and “ $\kappa \uparrow$ ” indicate the vertical shift in the kink of the He-ZAMS lines if the opacity in the inflated stars would decrease (the Eddington luminosity would move up) or increase (the Eddington-limit would move down), respectively.

It has been suggested that the opacity in the inflated layer may come down because of porosity effects (Owocki et al. 2004), which would lead to an

increase of the Eddington-luminosity and to an occurrence of the kink in the He-ZAMS at higher luminosity. An increase of the opacity has been suggested, on the other hand, by Gräfenner et al. (2012), as this might lead to a better agreement of the surface temperatures of Wolf-Rayet stars with the observations. We conclude that the main parameters of the Wolf-Rayet stars are strongly affected by their proximity to the Eddington limit.

References

- Bestenlehner J M, Gräfenner G, Vink J S, et al., 2004, *A&A*, 570, A38
- Brott I, de Mink S E, Cantiello M, et al., 2011, *A&A*, 530, A115
- Crowther P A, Schnurr O., Hirschi R., et al. 2010, *MNRAS*, 408, 731
- Doran E.I., Crowther P.A., de Koter, A., et al., 2013, *A&A*, 558, A134
- Eddington A.S., 1926, *The Internal Constitution of Stars*
- Gal-Yam A, Mazzali P, Ofek E O, et al., 2009, *Nature*, 462, 624
- Gräfenner G, Owocki, S P, Vink, J S 2012, *A&A*, 538, A40
- Ishii M, Ueno M, Kato M, 1999, *PASJ*, 51, 417
- Kato M, 1986, *Ap&SS*, 119, 57
- Kippenhahn R., Weigert A., 1990, *Stellar Structure and Evolution* (Springer, Berlin)
- Köhler K., Langer N., de Koter A., et al., 2015, *A&A*,
- Kozyreva A, Blinnikov S, Langer N, Yoon S-C, 2014, *A&A*, 566, A146
- Kudritzki R.-P., Puls, J, 2000, *ARA&A*, 38, 613
- Langer N, 1997, in *ASP Conf. Ser. Vol. 120*, p. 83
- Langer N, 1998, *A&A*, 329, 551
- Langer N, 2012, *ARA&A*, 50, 107
- Langer N., Kudritzki R P, 2014, *A&A*, 564, A52
- Larsson J, Levan A J, Davies M B, Fruchter A S, 2007, *MNRAS*, 376, 1285
- Maeder A, Georgy C, Meynet G, Ekström, S. 2012, *A&A*, 539, A110
- Owocki S P, Gayley K G, Shaviv N J, 2004, *ApJ*, 616, 525
- Petrovic J, Pols O, Langer N. 2006, *A&A*, 450, 219
- Raskin C, Scannapieco E, Rhoads J, Della Valle M, 2008, *ApJ*, 689, 358
- Sanyal D., Grassitelli L., Langer N., Bestenlehner J.M., 2015, *A&A*, 580, A20
- Smith N, 2014, *ARA&A*, 52, 487
- Smith N, Vink J S, de Koter A. 2004, *ApJ*, 615, 475
- Szécsi D., Langer N., Yoon, S.-C., et al., 2015, *A&A*, 581, A15
- Zickgraf F-J, Wolf B., Stahl O., et al., 1985, *A&A*, 143, 421

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Dany Vanbeveren: What is the effect of a companion on the inflation?

Norbert Langer: The companion may suck off the inflated envelope (RLOF), but it will regrow, such that the mass transfer rate will be close to the critical mass-loss rate above which inflation is suppressed.

