Sustainable Urban Growth:

Technology Supply and Agglomeration Economies in the City

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This City is what it is because our citizens are what they are.

Platon, n.d.

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Executive Summary

This dissertation explores the determinants for sustainable and socially optimal growth in a city. Two general equilibrium models establish the base for this evaluation, each adding its puzzle piece to the urban sustainability discourse and examining the role of non-market-based and market-based policies for balanced growth and welfare improvements in different theory settings.

In a first step, a non-spatial model sheds light on the role of innovation supply decisions for growth and welfare. It discusses an environment where a research and development (R&D) sector can improve the trinity of standard measures for encountering environmental damages: (1.) technology to directly compensate for damages, (2.) adaptation to reduce the impact of environmental damages, and (3.) abatement efforts to reduce damaging pollution. While monopolistic price markups and a labor allocation based on the private value of innovation bias the innovation intensity, the effort allocation among alternative technologies remains sustainable and socially optimal if agents have full access to information and technology. If such access is restricted and the environmental damage elasticity exceeds unity, sustainable growth is impossible without corrective policies. For lower damage elasticities, growth is sustainable but not socially optima,l such that policies have a considerable welfare effect. Irrespective of the scenario, non-market-based policies such as investments in education and improvements in the design for innovation-friendly patent laws are similarly suitable to internalize the environmental externality as standard market-based policies such as an environmental tax.

In a second step, a spatial model evaluates regional challenges for sustainable growth. The attention is on two features: First, the role of population and construc-

tion density for urban growth. Second, the consequences of exogenous pollution-intensive energy, which is often a challenge for cities. This framework reveals that sustainable urban growth is subject to two markets: (1.) the energy market determining the energy source and hence the pollution intensity of production and commuting, (2.) the consumption market in which the demand for a numéraire good, apartments, and a hinterland good (e.g., nature) determines the relative price of morphological capital and land. This relative price affects the construction density and has repercussions on the pollution intensity. The combination of both effects determines the quality of agglomeration economies and accordingly growth and welfare.

If energy is ‘’brown” and its price remains constant, pollution grows to an unsustainable extent. An energy price rise can stop this development as it reduces energy demand. However, rising energy prices increase commuting costs. Citizens consequently locate closer to the center, what elevates central density. Countermeasures such as commuting subsidies or commuting energy-saving infrastructure are required to avoid a destabilizing development. Sustainable urban growth hence either calls for policy actions or a green energy transition. Thereby, non-market-based infrastructure to save energy in commuting and improve pollution abatement is less efficient than market-based price policies related to commuting subsidies and environmental taxes. Further, calibration reveals that a green energy transition is socially optimal. This transition is the more likely, the higher the brown energy price.

In all, the non-spatial and the spatial models both detect alternative R&D market failures that can pose severe challenges to the sustainability of urban growth and the social optimality of decentralized allocation decisions. Still, both frameworks demonstrate that a careful (holistic) combination of policy instruments can achieve sustainable growth and even be first best, providing hope for the sustainability of urban growth and the power of policies to improve urban welfare.

Zusammenfassung

Diese Dissertation untersucht die Determinanten für ein nachhaltiges und sozial optimales städtisches Wachstum. Zwei endogene Wachstumsmodelle untersuchen hierzu die Rolle von nichtmarktbasierten und marktbasierten Politikeingriffen. Jedes Modell fügt dabei dem städtischen Nachhaltigkeitsdiskurs sein eigenes Puzzleteil hinzu.

Ein nicht räumliches Modell konzentriert sich auf Innovationsentscheidungen, wenn ein Forschungs- und Entwicklungssektor ein Portfolio der üblichen Strategien zur Bekämpfung von Umweltschäden bereitstellt: (allgemeine) Technologien, Anpassungsmaßnahmen und Maßnahmen zur Minderung der Verschmutzung.

Das Modell stellt fest, dass eine intrinsische Motivation zu nachhaltigen und sozial optimalen Innovationsentscheidungen führt, unter der Bedingung, dass es vollen Zugang zu Informationen und Technologien gibt. Wenn ein solcher Zugriff eingeschränkt ist, hat die Bildungspolitik einen vergleichbaren Effekt wie Umweltsteuerprogramme.

Ein räumliches Modell fügt die Dimension der Dichte und einen exogenen, umweltfreundlichen braunen Energiesektor der nachhaltigen Wachstumsdiskussion hinzu. Es zeigt, dass Konsumentscheidungen die Baudichte beeinflussen, die dann sowohl Netzwerkeffekte als auch Umweltverschmutzung antreiben. Letztere bestimmen dann das Wirtschaftswachstum einer Stadt. Das Zusammenspiel von einem versperrten Zugang zum braunen Energiesektor, der Knappheit von Boden und einer unelastischen Energienachfrage macht urbanes nachhaltiges Wachstum auf Basis von brauner Energie ohne staatliche Maßnahmen unmöglich.

Bei brauner Energienachfrage und konstanten Energiepreisen steigt die Umweltverschmutzung. Bei Energiepreisanstiegen steigt die Baudichte (um Pendelkosten zu sparen), oder die Wirtschaft beginnt mit der Produktion von grüner Energie. Grüne

Energie ist nachhaltig, sie führt jedoch zu starken Verzerrungen im Innovationssektor, sodass politische Eingriffe die Wachstumsraten erhöhen. Eine numerische Anwendung zeigt, dass der unelastische Energiebedarf dann zu große Profite abwirft und daher den Arbeitsmarkt destabilisiert. Infrastruktur und Steuern können jedoch beide zu sozial effizienten Ergebnissen führen, wobei letztere effizienter sind. Eine Kalibrierung zeigt zudem, dass grüne Energien sozial bevorzugte Alternative sind.

Insgesamt weisen beide Modelle auf Verzerrungen im Forschungsmarkt hin, zeigen jedoch, dass eine (ganzheitliche) Kombination verschiedener politischer Instrumente zu einem sozial optimalen Resultat führen kann.

Abbreviations

BEA	Bureau of Economic Analysis
BGP	Balanced Growth Path
CBD	Central Business District
CES	Constant Elasticity of Substitution
DTC	Directed Technical Change
EPS	Energy and Pollution Saving (in reference to an EPS regime)
ES	Energy Saving (in reference to an ES regime)
F.O.C.	First Order Conditions
FUA	Functional Urban Area
GE	Green Energy (in reference to an GE regime)
GK	Generalized Knowledge (in reference to a GK regime)
MSA	Metropolitan Statistical Area
OMB	Office of Management
PI	Personal Income
PS	Pollution Saving (in reference to an PS regime)
R&D	Research and Development
SBGP	Spatial Balanced Growth Path
SDG	Sustainable Development Goal
SK	Specialized Knowledge (in reference to an SK regime)
UPC	Unified Patent Court
WTP	Willingness To Pay

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Chapter 1

Introduction

1.1 Impetus

We form cities in order to enhance interaction, to facilitate growth, wealth, creation, ideas, innovation, but in doing so, we create, from a physicists viewpoint, entropy.

Geoffrey West (2011)

The industrial revolution has not only raised the wealth of nations to unprecedented levels, but has also intensified urbanization to a degree never seen in the history of humankind. This trend is far from end. The current urbanization rate of about 50% is expected to increase to 75% by 2050 (Habitat, 2015). However, this path has been fueled by the exhaustion of natural resources. Consequently, economies are beginning to pay the price: The quality of the environment diminishes while land becomes increasingly scarce. These developments start threatening the laboriously developed economic prosperity. Corrective actions are required. The sooner, the better.

Urban agglomeration economies nourish innovations and increase the concentration of production factors in the cities. Yet, it is precisely the concentration of economic activity, which is particularly prone to ecological threats. Environmental pollution shows its toxic side via smog and heat island effects exacerbate the consequences of climate change. In cities, both affect a comparatively large number of people in a confined space. Hence, the core question is whether urban growth is sustainable. And if so, whether it is socially optimal. In this direction, little is known. Beyond, there is no proper theoretical foundation to study the concept of sustainable growth in an urban context. This dissertation intends to fill this gap and develop theories addressing these challenges of urban growth.

1.1.1 Civitas agglomerata oeconomica: The roots of the urban sustainability discussion

The history of cities and civilizations can hardly be separated. Having the same semantic root in the Latin word ‘civitas’, their symbiosis began with the Neolithic Revolution when our ancestors sacrificed their nomadic lifestyle as hunters and fishermen to become indigenous, build farms and raise cattle, see, e.g., Mumford (1961), Bairoch (1988) or LeGates and Stout (2015) for historical retrospectives.

In Latin the action of collecting in a mass is called ‘agglomeration’ and the mass of settlers in a local community disclosed the benefits of urbanization which are nowadays discussed as agglomeration economies[1]: local settlements made it possible to use production factors more efficiently, to offer a greater variety of goods and to spread new production techniques quickly in the communities. Further, the villages and cities provided a good protection against external threats (e.g., wild species or savage barbarians), significantly improved social life and had a great impact on the political culture[2].

However, the cost of agglomeration also revealed early; diseases were transmitted faster, it was no longer possible to reduce the effects of floods, droughts or temperature waves through relocation, and the concentration of activity had the conse-

[1] Actually, the concept describes proximity related benefits such as labor market pooling, input sharing, and knowledge spillovers, see, e.g., Rosenthal and Strange (2004) and the later discussion.

[2] The Greek word polis from which our understanding of politics emerges literally means ‘city’ and defined the administrative, cultural and religious center of the Greek, see, e.g., Hansen (1993).

quence that local environmental damage affected relatively large numbers of people and livestock, see, e.g., Rosen and Tarr (1994), Vlahov et al. (2004) or McDonnell and Niemelä (2011) (on ecology) for historical reviews. The decision to settle in cities has therefore been subject to cost-benefit assessments right from the start.

1.1.2 The challenge with urban sustainability

Thousands of years after the first settlements, industrialization initiated a new era of urbanization. The industrial revolution raised the rates of innovation, production and urbanization to unprecedented levels, urban health conditions improved (Galea and Vlahov, 2005) and the increasing diversification of the economy strengthened the resilience of production to any kind of shock be it economic, political or ecological (see Jones, 2000, for a profound historical retrospective on economic growth). While this made cities less prone to drought or temperature waves, new challenges arose. The increasing land scarcity elevated population density (Gerber, Hartmann, and Hengstermann, 2018), pollution started becoming a serious threat (Markham, 2019), and increasing greenhouse gas emissions paved the way for anthropogenic climate change (see Haumann, Knoll, and Mares, 2020, for a retrospective), whose temperature effect is especially severe in cities due to heat island effects .

The fundamental question of every city is how to organize a growing and environmentally vulnerable activity within a restricted space. Despite a strong political will to tackle these challenges[3], there is little theoretical knowledge. In particular, there is no plain theory reference for studying both, the sustainability and the social optimality of urban growth.

This lack of theory motivates this dissertation to delve deeper into these fascinating questions and develop models explaining how the interplay of density and the environment affects growth and welfare within cities.

[3]Prominently, the 11^{th} Sustainable Development Goal of the United Nations requires that cities are inclusive, safe, resilient and sustainable.

1.2 The subject areas of this dissertation

Sustainable urban growth is an extensive research area that deals with aspects of urban economics, environmental economics, public economics, and environmental growth theory. The lack of a simple framework for this discussion incites this dissertation to unite various fields of economic theory for an assessment of the conditions for sustainability. It is, hence, essential to clarify this thesis's interpretation of sustainability and explain the methodological context that evaluates it.

Sustainability refers to a far-reaching and differently approached concept. Stemming from the Latin word *sustinere*, meaning 'to hold', it originates in *tenere* referring to 'hold', 'keep', 'comprehend', 'represent', and 'support'. These are meanings that all relate to maintaining a status quo. Probably the most widely used definition of sustainability comes from the report of the World Commission on Environment and Development (1987), which famously describes it as:

'*development that meets the needs of the present without compromising the ability of future generations to meet their own needs*' (WCED, 1987).

While the scientific literature structures sustainability through the trinity of a social, ecological, and economic dimension (see e.g. Thiele, 2016), economists usually address all three areas with a single discussion whereby they are interested in distributional issues and questions about dynamic efficiency. The former are a subject of research in the direction of ''sustainable development''. Thereby, ''development'' is an overarching theme that incorporates dynamic efficiency problems. A sub-area of the debate is sustainable growth, which is the fundamental topic of this work.

Conceptually, this thesis follows an approach by Arrow et al. (2004), which understands by sustainability that the intertemporal social welfare must not decrease over time. Sustainability is hence a dynamic criterion and depends on the development of the 'productive base', which combines manufactured capital, human capital, natural capital, and technology. The *value* of investments and disinvestments in these assets (calculated via the shadow values of the respective assets) is interpreted as *genuine investment* (GI) whose aggregate provision has to remain nondecreasing to achieve non-decreasing genuine wealth, a condition which describes the sustainability criterion, see also Hanley, Dupuy, and McLaughlin (2015) and Ferreira and Vincent (2005). Since capital and technology can compensate for natural capital exhaustion, the concept may be interpreted as a weak sustainability criterion. Howbeit, as the value of genuine wealth must not decrease, a complete depletion of all natural capital causes such enormous damages that it needs to be avoided. A more restrictive condition is the demand for strong sustainability requiring that all individual assets at least remain constant, see, e.g., Dedeurwaerdere (2014).

Notably, the sustainability criterion is not an optimality criterion. Numerous alternative sustainable growth paths are conceivable. The exciting question is which of these paths is socially optimal, and what determines this social optimality.

This dissertation considers these questions by illuminating research incentives for alternative technologies. It discusses how environmental quality as a non-market good affects market goods due to damages on production. Against this background, three subject areas dominate the evaluation: technology, natural capital, and density. The role of these three features will be explained in more depth subsequently.

1.2.1 Sustainable growth and technology

There is a clear consensus among economists that long-run growth demands continuous innovation. Nonetheless, if the innovation-driven increase in production deteriorates the environmental quality, innovation per se is not sufficient for sustainability. Sustainable growth either calls for enough innovations to compensate for increasing environmental damages or avoid the continuous exhaustion of natural capital.

In this thesis, innovation can improve physical technologies or knowledge as they are treated interchangeably and represent many facets of the economy, including infrastructure. Actually, innovations can increase the efficiency of the deployment of resources, compensate for the exhaustion of resources, support the employment of new resources, or improve the capability to combine alternative resources in the production process. Thereby, improvements in technology and an increase in manufactured capital can compensate for a reduction in natural capital if the value of the productive base is non-decreasing. That begs two questions: Firstly, how to interpret natural capital? Secondly, how to interpret the damage caused by its depletion? These questions are addressed subsequently in more detail.

1.2.2 Sustainable growth and natural capital

Natural capital essentially contains all goods that do not have anthropogenic origins, see Barbier (2019) for an overview. It is thus a heterogeneous asset, the provision of which is subject to two types of challenges: scarcity and property rights.

Scarcity is a problem of non-renewable resources and is a serious concern to sustainable growth if the resource is an essential factor in production[4], see Dasgupta and Heal, 1974. Technology may be a key to overcoming such problems. The question is how to use scarce resources over time. In this regard, the famous Hotelling's rule states that efficient use of non-renewable resources requires the resource price to increase with the discount rate[5], leading to a continuously decreasing rate of resource extraction and hence socially efficient resource extraction, see, e.g., Gaudet (2007).

The second second type of challenge with natural capital are lacking property rights. Many natural capital goods such as the natural capacity to absorb pollution and emissions have the character of public goods; i.e. goods characterized by non-excludability and non-rivalrous consumption. They are sometimes discussed as common-pool resources (Gardner, Ostrom, and Walker, 1990) or environmental sinks (Andersen, 2007) which are limited and essential for the sustainability of economic growth. For this reason, their careful handling is crucial. According to the Samuelson condition (Samuelson, 1954), a socially optimal allocation of public goods requires that the marginal social benefit and the marginal costs of their provision (or maintenance) coincide. Since public goods are not in private possession, there are no clear property rights for their use which usually leads to the externality of overuse, see, e.g., Laffont (1989).

In this context, an interesting case are energy-related natural capital goods such

[4]Note that physical entropy laws limit the availability of all natural resources, see accordingly Smulders (1995).

[5]While there is empirical evidence for the actual observability of the rule (Templeton and Wood, 2017), there is also critical evidence that casts doubt on its practical validity (Livernois, 2009).

as oil, coal, or gas whose employment causes pollution. In principle, such non-renewable energies confront the economy with two types of scarcity. First, there is resource scarcity. Second, the use of these resources causes pollution and emissions that exhaust scarce environmental sinks. Any laissez-faire economy charging a scarcity rent based on Hotelling's rule prices the first scarcity efficiently. However, the second scarcity requires assessing the social costs of pollution. These costs hardly find their way into privately owned energy resources without any environmental policy in place. The necessary policies are detailed in more depth in the following.

Policies for managing natural capital

There is a rich set of policy instruments to address different environmental externalities and intense theoretical and practical knowledge about the costs and benefits of alternative policy measures. Anyway, environmental policy cannot solve all market distortions. For example, there are often practical policy constraints that limit the applicability of policy measures. The emitter of a pollutant may be in a different region or country so that local decision-makers are limited in their policy instruments to internalize environmental externalities. An illustration for such a scenario is found in Sussman (2004) who discusses the role of the USA to promote global environmental protection. Another example is stranded assets, which occur if, for instance, a coal-fired power plant gets shut down for reasons of climate policy before initially planned. As Kalkuhl, Steckel, and Edenhofer (2020) assess, such a scenario causes significant (social) costs.

In any case, environmental economic literature usually emphasizes the potential of market-based instruments such as taxes, charges, and subsidies, emissions trading, and other tradable permit systems to internalize environmental externalities (see, e.g., Edenhofer et al., 2010, on emissions). In addition, there are also non-market-based policies, usually distinguished among voluntary agreements and regulatory instruments ('command-and-control'). Decisively, the environment's ability to absorb local environmental pollution describes a local public good, which mainly raises questions about the local challenges for sustainability and intergenerational equity.

The ability to absorb greenhouse gases characterizes a global public good which requires to assess intergenerational and cross-country equity concerns on a global scale, see Loehman et al. (1979), Schnaiberg, Watts, and Zimmerman (1986), or Tol et al. (2004) for early discussions. For both challenges, a single theory cannot deal with all complexities of natural resource management. This dissertation concentrates on regional aspects, as detailed subsequently.

Sustainability interpretation

Urban growth is regional growth. For a differentiated perspective on the determinants for local sustainable growth, this dissertation evaluates two alternative models, each with its distinct focus. The first model takes an aggregate view and considers environmental damages caused by dirty numéraire production. The second model goes deeper into the sectoral structure of an urban economy and evaluates damages proportional to the brown energy employment in production and commuting. Both models describe closed economies. Hence, the agents responsible for environmental externalities get confronted with the related social costs directly. The specifications are thus more suitable for local common problems related to environmental pollution than global commons problems akin to climate change. Yet, they can technically assess both types of environmental challenges, enabling an evaluation beyond the one presented here, a feature detailed in the chapters.

A related question is whether to assess the damage of excessive use of natural capital via a stock or a flow variable. Although there is no clear demarcation line, literature often describes environmental pollution damages proportional to the flow of current emissions and climate change damages proportional to the stock of greenhouse gas emissions (see, e.g., Perman et al., 2003). In this dissertation, the first model presents a setting where the environmental damages are proportional to numéraire output, which is a flow variable. Though, numéraire production is proportional to the technology stock and increases with innovations. As is detailed in the chapter's discussion section, the very same model can be associated with a flow or a stock challenge, while the corresponding distinction is rather a question of how to interpret the respective model[6]. In light of this, it is next essential to explain this thesis' interest in land and density for sustainable growth.

1.2.3 Sustainable growth and density

Local land is not simply a factor of production, but rather describes an additional dimension of the analysis as it scales the efficiency of other factors and technologies involved in the production process. The actual metric for this dimension is density, be it building density or population density, a characteristic that has not gained much attention in the economic discourse so far.

In principle, density represents an environmental sink, namely the capability of a location to accommodate economic activity. Cities exist because the concentration of production offers advantages in terms of increasing economies of scale. Such benefits are called agglomeration economies and explained by network effects, labor matching and improvements in local value chains, see Ciccone and Hall (1996), Baptista (2003), De Groot, Poot, and Smit (2009). While denser cities encourage these effects, there is a critical point where congestion jeopardizes the benefits of local concentration. Examples are traffic jams or too little work and living space for citizens to be productive.

The fundamental question is whether urban growth is sustainable if the urban space is constrained. In this direction, relatively little is known. While there is literature on critical city sizes, see Henderson (1974), Fujita (1989) or Duranton and Puga (2004), the spatial dimension of this discussion is barely addressed, especially in the environmental growth literature. The lack of models motivates this thesis to develop a new theory that assesses the role of density in an environmental context. While the third chapter discusses this role in-depth, the second chapter ignores the spatial dimension to first explain the determinants for technology supply in general, a differentiation explained subsequently.

[6]In this context, there is early literature indicating that environmental growth effects are qualitatively hardly affected by whether the environmental effect describes a stock or a flow variable, see e.g. Keeler, Spence, and Zeckhauser (1971), Maler (1974), Brock (1977), Becker (1982) and Tahvonen and Kuuluvainen (1993).

1.3 Outline

There are two main questions this thesis reviews: (1.) how environmentally vulnerable economies choose their direction of technical change and (2.) how this decision is influenced by being taken in a city. Each question is approached in one chapter.

Chapter (2) initiates the sustainable urban growth discussion with an evaluation of endogenous research decisions. It assesses the innovation criteria when alternative directions of technical change are available. The research question is whether and under which conditions endogenous incentives are sufficient to achieve sustainable, balanced growth and what determines a socially optimal growth path. While environmental economic theory usually focuses on the demand for technology and evaluates a binary decision between brown and green innovations, the chapter investigates the supply of technology and the potential to combine multiple technologies. For simplicity, the chapter refers to a non-spatial vertical innovation model in which an R&D sector offers general technologies and adaptation and abatement knowledge to compensate for environmental damage. This setting reveals an intrinsic incentive for environmentally friendly production, which is hardly considered in the literature[7]. The model doubts the assumption that a lack of property rights to natural resources leads to environmental externalities, showing that these externalities result from restrictions in information or access to technology. Simplified access to technologies and knowledge can hence lead to sustainable and even socially optimal growth. For this purpose, non-market-based policies like education can be similarly effective as market-based environmental policies such as taxes and quotas.

Chapter (3) adds the spatial dimension to the debate and introduces an exogenous brown energy sector. The research question is how spatially organized growth is sustainable if it faces land scarcity and potential environmental damage due to an exogenous supply of dirty technologies. With the determinants of innovations addressed in Chapter (2), Chapter (3) shifts the focus to the challenges of sustainable growth under restricted space with exogenous brown energy supply. The allocation

[7]Yet, endogenous incentives for environmental friendliness are investigated in other strands of the literature, see, e.g., Fransen (2015), Fransen (2018), and Lambin and Thorlakson (2018).

of labor, manufactured capital, and land determines the density profile of an economy, which in combination with the decision on the energy source affects the quality of agglomeration economies and thus the pace of innovation.

The chapter thereby discloses that environmental externalities occur if agents lack access to either technology, innovation, or sectoral production and are the consequence of a moral hazard problem since the exogenous brown energy sector (e.g., a foreign or neighboring city) ignores the environmental consequences of brown energy use. While resource scarcity can lead to sufficient price increases to initiate an endogenous, sustainable, green energy transition, abundance in the brown energy source leads to a growth-threatening environmental externality. Without a green energy transition, either pollution or central densities increase excessively, making growth unsustainable without policy action. However, a holistic policy strategy combining environmental, urban planning, and the research market policies can be first-best.

To synthesize results, Chapter (4) critically compares the findings of Chapter (2) and Chapter (3) and concludes with a combined urban policy proposal that summarizes the core results of both evaluations.

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Chapter 2

The Power of Spillover Effects

Abstract Economists are worried that the lack of property rights to natural capital goods jeopardizes the sustainability of economic growth. This article questions this position. A vertical innovation model with a portfolio of technologies for abatement, adaptation, and general (Harrod-neutral) productivity improvements reveals that environmental damage spillovers have a comparable effect on research profits as technology spillovers so that the social costs of using natural capital are internalized. As long as there is free access to information and technology, growth is sustainable and the allocation of research efforts among alternative technologies is socially optimal. While there still is a need to internalize externalities from monopolistic research markets, no environmental policy is necessary. These results suggest that environmental externalities may originate in restricted access to information and technology, demonstrating that education has a similar effect as an environmental tax and knowledge transfers have an impact comparable to that of subsidies for research in green technology.

2.1 Introduction

Whenever a theory appears to you as the only possible one, take this as a sign that you have neither understood the theory nor the problem which it was intended to solve.

Karl Popper (1972)

It was the power of innovations like the steam engine and electricity that brought our generation to a standard of living that previous generations had only dreamed of. The price of this progression is a burden on the environment. Vivid examples are the pollution that marks urban and rural landscapes, the smog that obscures the view in the global metropolises, and the rise in sea levels around the proverbial pristine island due to climate change.

Economists have not been tired raising concerns about the sustainability of this development for decades, see e.g., Boulding and Jarrett (1966), Meadows et al. (1972), Stern, Common, and Barbier (1996), Arrow et al. (2004), or a recent overview in Drupp et al. (2020). The quality of the environment is a public good, a lack of property rights can lead to overuse and thus to an environmental externality that must be internalized, especially with a view to sustainable development. While economists typically call for a rethinking of production processes, some have hope in the potential of technology to compensate for the use of natural capital. Technologies can reduce both actual pollution, and the use of natural resources, or at least make the economy more resilient to environmental damage. To achieve such a (green) technology transition, the literature emphasizes the importance of environmental policy, see Kemp and Never (2017).

Yet, this view does not pay attention to decentralized incentives for green production and innovation. In recent decades, consumers have become much more sensitive to their individual ecological footprints, and the producers are aware of their environmental responsibility, see, e.g., Fransen (2015), Fransen (2018) and Lambin and

Thorlakson (2018). Surprisingly, environmental growth literature hardly considers that environmental friendliness is an essential quality feature of products today. The standard literature focus is on how new technologies can be employed to internalize predetermined environmental externalities. This chapter follows a different approach and develops a stylized model, indicating that a change in perspective from the demand side to the supply side of technologies can have a deep impact on this discussion. It is shown that agents who have complete access to information and can combine alternative technologies choose a socially optimal environmental research strategy. Environmental externalities only occur if either a limited access to technology leads to lock-in effects or if the information about the environmental impact of individual actions is incomplete. In the former case, one-time research subsidies can shift innovation in the preferred direction, given they do not have to augment different technologies. In the latter case, information policies (respectively education) and Pigouvian environmental taxation are qualitatively identically effective.

These strong results go hand in hand with a highly stylized theory and are hence not intended to provide unequivocal evidence. Rather, they point out that the usual focus of the literature on the demand for technology can underestimate the potential of an educated society to solve environmental challenges. Non-market-based policies such as improving access to information and technologies is similarly effective as market-based policy such as environmental taxes.

This evaluation is organized as follows: Section (2.2) gives an overview of how sustainable growth and depletion of natural capital are assessed in the literature. Section (2.3) presents the model, Section (2.4) evaluates possible equilibria and balanced growth. Section (2.5) discusses the sustainability and social optimality of balanced growth, whereas Section (2.6) assesses decentralized economy results and evaluates measures to improve welfare. Finally, Section (2.7) critically discusses the model and its implications, while Section (2.8) concludes.

2.2 Sustainable growth and natural capital

The assessment of the sustainability of growth has a long tradition in economic theory and combines, among other things, debates about resource scarcity, public goods, and technical change. While the sustainability concept has been introduced in Chapter (1), this chapter presents a first application of it. Here, a model is developed which illuminates the role of natural capital and technologies for sustainable growth on a rather abstract level. The intention is to introduce the practical challenges of sustainability, with a special focus on the importance of information and access to technology for sustainable growth. To understand the role of this analysis in the economic discourse, it is worthwhile to begin with a short historical literature review. While a detailed retrospective is found in Pearce (2002), the focus is here on some key articles about the evaluation of technology in this context.

2.2.1 Sustainability and the history of economic thought

From an intellectual point of view, Aristotelian ethics laid the foundation stone for the sustainability discussion. They regard a good life as the greatest good for human beings. Henceforth, practicing virtues such as justice is essential (Crisp, 2014). The sustainability debate is all about justice as it examines the fair allocation of limited resources over time.

Economists gained increasing interest in these considerations with the development of environmental economics in the 1950s (see, e.g., Sandmo, 2011 for an overview). The famous fishery model by Gordon (1954) was an early spark for the debate, as it was among the first to discuss renewable resource depletion. A little later, Barnett and Morse (1963) evaluated the use of non-renewable resources. The authors provided an optimistic assessment of the potential of technological progress to compensate for the exhaustion of non-renewable resources. Soon, an article by Boulding and Jarrett (1966) and the famous 'Report of the Club of Rome on the State of Humanity: The Limits to Growth' by Meadows et al. (1972) questioned this perspective. These voices initiated a debate on how resource scarcity threatens growth, see Daly and Daly (1973), Georgescu-Roegen (1975), Georgescu-Roegen

(1977), Holdren John et al. (1971), Novak (1973). About a decade later, the famous Brundtland Report (WCED, 1987) provided the widely accepted definition of sustainability which was cited in Chapter (1). The report raised considerable public attention and shed light on the responsibility of politics in natural resource conservation, see G. M. Heal (1998) for a review. As well-reviewed in Drupp et al. (2020), the discussion on sustainability has not come to an end to this day. One essential component of this discourse is technology, a core subject of this dissertation. The following subsection sheds light on the history of theory on this topic.

2.2.2 Sustainability and technology

At its core, sustainability is about intertemporal consumption options, for which technology plays a fundamental role. Technologies can compensate for natural capital exhaustion when considering weak sustainability and support the conservation of each asset in a strong sustainability scenario. The academic assessment of technology in this context has evolved over recent decades, as detailed in the following.

Theory on technology and sustainability Economists began investigating the role of technology to encounter challenges related to the exhaustion of natural capital in the 1970s on a broader scale. Intrigued by the debate on possible limits to growth mentioned above, they were initially interested in the potential of (exogenous) technological progress to eliminate such limits (see, e.g., Stiglitz, 1974, Dasgupta and G. Heal, 1974, or Solow, 1974). A considerable strain of the literature assessed the critical relation between technical growth and environmental pollution[1] (see, e.g., Keeler, Spence, and Zeckhauser, 1971, Maler, 1974, or Brock, 1977).

These assessments became more nuanced with the introduction of endogenous growth theory[2], which was especially helpful to elucidate alternative roles of environmental policy in a dynamic context, see Jaffe, Newell, and Stavins (2000) for a retrospective. For example, Bovenberg and S. Smulders (1995) and Bovenberg

[1]Usually based on Ramsey-Cass-Koopmans (and also Solow) models.

[2]With Romer (1986), Robert (1988) and Rebelo (1991) (human capital), Romer (1987) (horizontal differentiation), and Philippe Aghion and Howitt (1990) (vertical differentiation).

and S. A. Smulders (1996) demonstrated that ambitious environmental policy can promote long-run growth if technological change[3] enables less pollution-intensive production. Tahvonen (1997) investigated that an optimal energy consumption strategy involves a fossil fuel reduction and an increase in the use of a backstop technology (green energy) whereby a decrease in the backstop price reduces the initial level of the optimal emission tax. Beyond, L. H. Goulder and S. H. Schneider (1999) demonstrated that policy measures to reduce carbon dioxide have ambiguous effects on industry-specific R&D, so that the social benefits of environmental policy are complex to assess.

Only a little later, the newly emerging theory on directed technical change (DTC, see Acemoglu, 1998 and Acemoglu, 2002) enabled more differentiated assessments of the substitution possibilities between technologies with alternative environmental effects. The basis of this literature are constant elasticity of substitution (CES) production functions, described with $P = [ax^p + by^p + cz^p]^{\frac{1}{p}}$ whereby x, y, z denote inputs and a, b, c input weights or technologies. Inputs are often related to further production functions, combining additional production factors. No arbitrage conditions and decreasing returns in final production factor use bring about that the elasticity of substitution, $\sigma = 1/(1-p)$, determines the direction of technical change. If $0 \leq \sigma < 1$, factors are complements in final output generation, so the sector with the lower technology growth attracts all factors. If $1 < \sigma$, the inputs are substitutes, so the branch with the higher technology growth rates attracts all factors.

This type of modeling was used by S. Smulders and De Nooij (2003) to show that policies that affect the level of energy demand have no growth effects, while policies that affect the rate of growth in energy demand have growth effects. Hart (2004) used a DTC framework to demonstrate that, given sufficient substitutability in input technologies, an environmental tax (sales tax) has the potential to improve environmental research, can shift vintage technology in a newer (cleaner) direction, and can increase the pace of innovations. Beyond, Grimaud and Rouge (2008) used

[3]This is labeled pollution-augmenting technological change. While *technological* change describes the entire process of invention, innovation and technology diffusion, see Jaffe, Newell, and Stavins (2002), *technical* change describes the shift from one technological focus to another.

DTC theory to elaborate that, given sufficient substitutability in input technologies, an optimal environmental policy delays the extraction of non-renewable resources and promotes environmentally-friendly research. Further, Acemoglu, Philippe Aghion, et al. (2012) and Acemoglu, Akcigit, et al. (2016) elaborated that environmental taxes and research subsidies can achieve (weakly) sustainable growth if dirty and clean input technologies are sufficiently substitutable.

A fundamental similarity from environmental growth literature in general, and the directed technical change literature in particular, is the view that sustainable or socially optimal growth requires policy action. Rare literature indicating that environmental policy[4] is not needed for socially optimal environmentally-friendly research is Schou (2000) and Schou (2002). A reason for this result is that, similar to this theory, the two papers focus on market goods and accordingly ignore the direct effects of the environment on utility. As will be considered in Section (2.7), there are good arguments for this ignorance. What distinguishes this chapter from their work is that this theory evaluates the use of a technology portfolio rather than just one technology and, for this purpose, refers to two special cases discussed in the DTC theory. First, a Cobb-Douglas-case which offers broad comparability with findings of not DTC related literature. This case is achieved with $p \to 0$ so $\sigma \to 1$ which leads to $I = x^a y^b z^c$ and describes an economy with free access to technology. An elasticity of substitution below unity is not further assessed for simplicity because its consequences are comparable to the specialization scenario. The only difference is that such a scenario leads to a concentration of production factors and research efforts in the lower growing direction, while specialization attracts these efforts in the faster growing direction. Beyond, there are no qualitative differences to assess, so the discussion is kept parsimonious. With this knowledge, the model can be introduced.

[4]Acemoglu, Philippe Aghion, et al. (2012) and Acemoglu, Akcigit, et al. (2016) also indicate that such a path is possible but only if non-renewable resources are scarce.

2.3 The theory

This chapter's theory refers to the vertical innovation literature introduced by Grossman and Helpman (1991) and Philippe Aghion and Howitt (1990). It thereby follows a methodology by Grimaud and Ricci (1999), which offers a simple possibility to aggregate between disaggregated innovation sectors. As a result, along a balanced growth path, innovations are viewed as a continuous process where the relationship between leading technologies and average technologies remains proportional.

Before going into the details, it is worth starting with an overview of the model. There is a perfectly competitive final production sector that produces a numéraire good, a monopolistic competitive intermediate sector that produces alternative intermediate goods, and a representative household. The numéraire can either be consumed or saved, the latter through conversion into manufactured net capital[5], which implicitly adjusts for depreciation. Production requires natural capital that lacks property rights. Natural capital exhausts in proportion to the total production intensity. This causes damage, which decreases with abatement efforts.

A household can either work in production or do research and development (R&D). Any successfully innovator in the latter branch becomes an intermediate good provider in a particular industry until new innovators take over. Innovations are drastic and can improve technologies in three dimensions: (1.) the general productivity, (2.) the capability to adapt to environmental damages, in the following referred to as *adaptation*, and (3.) the potential to abate the environmental effect, in the following referred to as *abatement*. Knowledge and technology describe the same. Time indexes are ignored if possible and the focus is on a per capita representation.

2.3.1 Final production

There is a perfectly competitive final production sector that produces a numéraire , y, that can be consumed or saved. This production process requires a labor share, $1-n \in (0,1)$, and a continuum of intermediate goods, x_i, with a production technology

[5] This is in line with the Kaldor facts discussing income net of depreciation in the national account identity, see Rognlie (2016) who deliberately suggest the reference to net capital.

following:

$$y = (1-n)^{1-\alpha} N \int_0^1 \mathcal{T}_j x_j^{\alpha} dj \tag{1}$$

where $\alpha \in (0,1)$ denotes an elasticity and $\mathcal{T}_j$ refers to an intermediate gross production technology with

$$\mathcal{T}_j = A_j R_j, \tag{2}$$

where a general technology, A_j, scales an environmental robustness technology, R_j, both detailed later. Further,

$$N = \frac{1}{\bar{E}^{\phi}} \leq 1, \quad \textit{with} \;\; \phi = \iota\omega \geq 0$$

represents a natural capital stock which reduces if an environmental effect, $\bar{E}$, increases. As a standardization, $\bar{E} \geq 1$, so that if there is no environmental effect, $N = 1$. From a technical point of view, $\bar{E}$ can describe any deterioration in natural capital. The damage sensitivity is then subject to the environmental damage elasticity, ϕ, combining the net degeneration elasticity, ω, with the natural capital impact channel, ι. Therefore, neutral environmental damage occurs with $\iota = 1$, labor-biased damage with $\iota = 1-\alpha$ (e.g., health effects) and capital-biased damage with $\iota = \alpha$ (e.g., physical damage to the capital stock). The environmental effect is proportional to the production activity, while an abatement technology, G_j, can reduce the environmental footprint of intermediate products. Therefore,

$$\bar{E} = (1-n)^{1-\alpha} \Big(\int_0^1 \frac{\mathcal{T}_j}{G_j} x_j^{\alpha} dj \Big) \geq 1. \tag{3}$$

Note that even without abatement, it is required that $\left(\frac{R_j}{\bar{E}^{\phi}}\right) \leq 1$ because adaptation efforts can eliminate at most all damage caused by natural capital degeneration, but productivity will not be further increased. In view of this, final producers solve:

$$\max_{n, x_j} (1-n)^{1-\alpha} \int_0^1 \Big(\frac{\mathcal{T}_j}{\bar{E}^{\phi}}\Big) x_j^{\alpha} dj - \int_0^1 (1-T_p) p_j x_j dj - w\big(1-n\big)$$

with p_j as the intermediate price, T_p as a price subsidy to address markup pricing, and w as a wage. This gives the two factor demand equations:

$$w = (1-\alpha)(1-n)^{-\alpha} \int_0^1 \left(\frac{\mathcal{T}_j}{\bar{E}^{\phi}}\right) x_j^{\alpha} dj \tag{4}$$

$$(1-T_p)p_j = \alpha(1-n)^{1-\alpha} \left(\frac{\mathcal{T}_j}{\bar{E}^{\phi}}\right) x_j^{\alpha-1} dj. \tag{5}$$

2.3.2 Intermediate production

Each intermediate producer provides an intermediate that is equipped with up to three types of technology: First, a general technology, A_j, summarizing all technology not used to directly reduce environmental damage. In addition, there are two technologies explicitly addressing environmental damage. On the one hand, there is an environmental robustness technology, R_j, including any knowledge that protects against damage without reducing pollution, for instance indoor filters, medicine, different materials, etc., which is called adaptation. On the other hand, there is knowledge about how to reduce the pollution, which is called abatement and is denoted by G_j. Examples include emission filters or measures to improve energy efficiency. It is assumed that the respective knowledge scales the *technology intensity* of an intermediate according to $\mathcal{I}_j = A_j R_j F_j$ so that F_j measures the proportional abatement efforts. The greater the effort to reduce the environmental footprint of the production process, the higher the technological intensity of production, $\mathcal{I}_j$. Similarly, the higher the gross productivity, $\mathcal{T}_j = A_j R_j$, the higher the technological intensity of production for a certain intensity of the abatement effort, F_j. For the sake of simplicity, there is a directly proportional relationship between the abatement efforts and the actual abatement intensity, described by $F_j = G_j$. Therefore,

$$\mathcal{I}_j = A_j R_j G_j.$$

Following literature standards, intermediate production is capital-intensive and proportional to the technology intensity, here described with $\mathcal{I}_i$. Thus,

$$x_j = \frac{k_j}{\mathcal{I}_i} \tag{6}$$

so $K(t) = \int_0^1 \mathcal{I}_i x_j dj$. An intermediate producer who has a patent for the portfolio of technologies included in $\mathcal{I}_j$ is faced with a profit function that follows

$$\pi_{j,i} = \big(p_j - r(1+T_{E_j})\mathcal{I}_j\big)\, x_j$$

with $T_{E,j}$ as an environmental tax. The tax follows Grimaud and Ricci (1999) and affects the provision costs which are related to r. The intention to maximize monopoly rents results in[6]

$$p_j = \frac{r(1+T_{E_j})\mathcal{I}_j}{\alpha} \tag{7}$$

$$x_j = (1-n)\Big(\frac{\alpha^2}{r\bar{E}^{\iota\omega}}\Big)^{\frac{1}{1-\alpha}}\Big(\frac{\mathcal{T}_j}{\varrho_j\mathcal{I}_j}\Big)^{\frac{1}{1-\alpha}}, \quad \textit{with} \quad \varrho_j := (1-T_p)(1+T_{E_j}) \tag{8}$$

so that

$$\pi_j = \frac{\Lambda(1-n)}{r^{\frac{\alpha}{1-\alpha}}\bar{E}^{\frac{\iota\omega}{1-\alpha}}}\Big(\frac{\mathcal{T}_j}{(\varrho_j\mathcal{I}_j)^{\alpha}}\Big)^{\frac{1}{1-\alpha}}, \quad \textit{with} \quad \Lambda := (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}. \tag{9}$$

2.3.3 Research and development

This theory examines how two characteristics affect the innovation decisions: (a) access to information on the environmental impacts of innovations, (b) access to technologies. Both are described in detail subsequently.

(a) Access to Information

Stiglitz (1985) emphasizes that imperfect information creates a moral hazard problem, as people increase their exposure to risks if they do not anticipate that they bear the full costs of that risk. Such scenarios are often observed in the context of climate change and environmental pollution. One related example are citizens

[6] With (5), this is based on $\max_{p_j}\big(p_j - r(1+T_{E_j})\mathcal{I}_j\big)\frac{(1-n)\alpha^{\frac{1}{1-\alpha}}(A_jR_j^{\iota})^{\frac{1}{1-\alpha}}}{((1+T_p)p_j)^{\frac{1}{1-\alpha}}\bar{E}^{\frac{\iota\omega}{1-\alpha}}}$.

doubting the existence of anthropogenic climate change for different, mostly unscientific reasons. In the literature they are called *climate deniers*, see, e.g., Corry and Jørgensen (2015), Ploeg and Rezai (2019) or Krishna (2021). The interesting question is consequently how to address such individuals with neoclassical theory.

One possibility is to suggest that their information set is restricted, so they do not have the information necessary to anticipate the complete environmental consequences of their actions. Therefore, this theory distinguishes between agents that know their innovation will exhaust natural capital, indicated with $\mathcal{I} = 1$, and agents that lack information about their environmental footprint, indicated with $\mathcal{I} = 0$. While both types are exposed to a certain current environmental damage when determining their research strategy, only the former group envisages that their innovation will contribute to the depletion of natural capital if it is not accompanied by abatement. The reason for a lack of information is not explained further. It is possible that it is simply too costly to take into account the environmental impact of innovations, or that socio-cultural and political forces hinder access to the relevant information. In any case, the benchmark scenario relates to $\mathcal{I} = 1$.

(b) Access to technologies

Another important distinction is whether and how technologies can be combined and how the research sector can provide these technologies. For example, a car manufacturer has not the skills to produce solar panels and a civil engineer cannot produce lithium batteries. However, a car manufacturer may be able to switch from using gasoline to using batteries. Likewise, a civil engineer can possibly move from using concrete (which is relatively robust to environmental damage but emission-intensive) to using wood or other environmentally-friendly materials (which are less robust to environmental damage and less emission-intensive).

The question is thus whether innovators use all technologies and benefit from combined spillover effects, or whether they have to specialize so that the spillover effects become path-specific. While specialization is traditionally discussed with regard to comparative advantages in trade (see Laursen, 2015), this theory investigates its role on innovation in a closed economy. Thereby, specialization is not only of a technical nature, but can also be caused by very restrictive patent laws (so it can be used to assess various innovation constraints). To investigate this feature in more depth, this theory distinguishes two regimes:

(i) A **Generalized Knowledge (GK) regime** describes the benchmark scenario where the $R\&D$ sector has access to a shared pool of knowledge to improve alternative technologies. This, e.g., represents a civil engineer who can switch from using concrete to using wood, or combine both as a hybrid.

(ii) A **Specialized Knowledge (SK) regime** refers to an alternative setting where the $R\&D$ sector can improve the general productivity based on shared knowledge, but further needs to specialize for either improving adaptation or abatement. A combination of both technologies is thus not possible. This, e.g., represents a civil engineer who can either use concrete or wood, but no hybrid.

Research labor

A fraction $n \in (0,1)$ of the available labor allocates to research based on a standard no-arbitrage condition

$$w = \lambda(1 + T_V)V \tag{10}$$

where $\lambda > 0$ presents a Poisson parameter for the likelihood to innovate, T_V denotes a price instrument (tax or subsidy) to influence the profitability of R&D, and

$$V(s) = \int_0^\infty e^{-\int_0^t r(s,t)+\lambda n(s,t)dt}\pi(s,t)dt \tag{11}$$

represents the value of all innovations in the market, where r represents the interest rate and $\pi(s,t)$ denotes the profits of period s innovations in $t \geq s$. These profits are capitalized until a new drastic innovation takes over, for what λn measures how many vintage technologies are replaced by new innovations.

Research efforts

Each researcher considers how to improve the available technology package, $\mathcal{I}_i$. It is thus necessary to assess how to use a fraction of the research time, $\eta_i \in (0,1)$, to improve general productivity, A_j, $\kappa_i \in (0,1)$ of the time to improve abatement technologies R_j, while the remaining $1-\kappa_i-\eta_i$ is used for abatement activities G_j. Since it was simplified that $G_j = F_j$, the innovation function is directly related to G_j. In total, this leads to the following three path specific innovation difference equations

$$\dot{A}_i = \lambda n_i \varsigma_A \eta_i^{\theta} A_i \tag{12}$$

$$\dot{R}_i = \lambda n_i \varsigma_R \kappa_i^{\theta} R_i \tag{13}$$

$$\dot{G}_i = \lambda n_i \varsigma_G \big(1-\kappa_i-\eta_i\big)^{\theta} G_i \tag{14}$$

where $\theta \in (0,1)$ is responsible for decreasing returns in efforts and $\varsigma_j > 0$, $j = A, R, G$ denotes a path-specific research efficiency. The number of innovations (and thus the increase in the respective technology inventory) is linear in the existing technology and scaled by the respective research efforts κ and η, the probability of innovations λ and the share of researchers n. The allocation principles for research efforts and research labor, however, are regime-specific and discussed next.

Research in a GK regime

Every researcher knows that if an innovation is successful, its research efforts will be immediately visible in the next period. After an innovation, a researcher becomes an entrepreneur, until a new drastic innovation replaces the invention. Researchers, therefore, allocate efforts to maximize the potential next period profit increase, $\dot{\pi}$, so that if there is access to all technologies, the research effort allocation principles are described with

$$\kappa_i = \arg\max_{\kappa_i \in [0;1]} \dot{\pi}(t)(\kappa_i, \eta_i, n) \tag{15}$$

$$\eta_i = \arg\max_{\eta \in [0;1]} \dot{\pi}(t)(\kappa_i, \eta_i, n_i). \tag{16}$$

In aggregate $\int_0^1 \kappa_i di = \kappa$, $\int_0^1 \eta_i di = \eta$ and $\int_0^1 n_i di = n$, so (10) actually follows

$$w = \lambda n(1 + T_V)V(\kappa, \eta, n). \tag{17}$$

Research in an SK regime

With specialization, researchers must determine the direction of research at an early stage of their career. As a result, a fraction $\gamma \in (0, 1)$ of n choose *adaptation*. They face $\eta+\kappa = 1$ and decide upon η and κ. The remaining $1-\gamma$ researchers then choose *abatement* so that they face $\kappa = 0$ and select the profit maximizing η. The research effort allocation principles thus follow

$$\kappa_i = \begin{cases} \arg\max_{\kappa_i \in [0;1]} \dot{\pi}_{i,R}(t) & \textit{if } \; i = R \;\; (\textit{adaptation}), \textit{relevant for } \; \gamma \\ 0 & \textit{if } \; i = G \;\; (\textit{abatement}), \textit{relevant for } \; 1-\gamma, \end{cases} \tag{18}$$

$$\eta_i = \arg\max_{\eta_i \in (0,1)} \dot{\pi}_i(t). \tag{19}$$

The allocation of γ follows with the nested no-arbitrage condition

$$(1 + T_V)V_R\Big(\kappa_R, \eta_R, (1-\gamma)n\Big) = \lambda(1 + T_V)\Big(1 - T_{V,G})V_G(\eta_G, \gamma n\Big) \tag{20}$$

with $T_{V,G}$ as an abatement-specific tax or subsidy (described in detail later). The allocation of the research fraction n follows with the no-arbitrage condition

$$w = \lambda n \max\Big\{(1+T_V)V_R\Big(\kappa_R, \eta_R, (1-\gamma)n\Big), \lambda(1+T_V)(1-T_{V,G})V_G\Big(\eta_G, \gamma n\Big)\Big\}. \tag{21}$$

2.3.4 Households

There is an infinitely living representative household that offers its labor inelastically, owns all capital, and gains utility by using the unsaved output for consumption. Its intertemporal consumption preferences are described with a standard CRRA function. Capital is the only source for savings[7] (denoted by $i = \dot{K}$). With (10), the

[7]So the interest on consumption corresponds to the return on capital (if the savings market clears).

per capita consumption saving decision follows with the solution to

$$\max_{\{c(t)\}_{t=0}^{\infty}} \int_{t=0}^{\infty} e^{-\rho t} \frac{c(t)^{1-\epsilon}}{1-\epsilon} dt \quad s.t. \quad \dot{K}_t = w(t) + r(t)K(t) - c(t) - P(t) \tag{22}$$

where $\epsilon > 1$ scales the degree of risk aversion[8], $\rho > 0$ denotes time preferences, and $P(t)$ is a tax expenditure or income channel which finances the governmental budget

$$P(t) = (1 + T_V)V + T_p \int_0^1 x_j dj - r \int_0^1 T_{E_j} \mathcal{I}_j x_j dj.$$

For simplicity, population growth is ignored. Assuming a no Ponzi scheme condition $lim_{s\to\infty} e^{(-\int_0^t r(t)dt)} K(t) \geq 0$ and the initial asset equipment $K(0) = K_0 > 0$, this leads to the Euler equation

$$g_c(t) = \frac{r(t) - \rho}{\epsilon}. \tag{23}$$

In view of this, the next section assesses the model's balanced growth properties.

2.4 Balanced growth and sustainability

The definition of an equilibrium follows literature standards:

Definition 2.1. *A competitive equilibrium is a sequence of labor and effort allocations* $\{n(t), \eta(t), \kappa(t)\}_0^{\infty}$*, endowments* $\{x(t), K(t), c(t), P(t), A(t), D(t), G(t)\}_0^{\infty}$*, policy instruments* $\{T_p, T_E, T_V, T_{V,R}\}_0^{\infty}$ *and prices* $\{r(t), w(t), p(t)\}_0^{\infty}$ *that arise with the clearing of the markets for production factors, labor, and goods and services. Thereby final producers hire labor and capital to maximize profits, taking* $w(t)$ *and* $p(t)$ *as given, researchers choose their efforts and labor* $\eta(t)$*,* $\kappa(t)$*,* $\gamma(t)$*,* $n(t)$ *to maximize expected innovation values taking* T_p*,* T_E*,* T_V *and* $T_{V,R}$ *as given. If researchers become entrepreneurs, they rent capital* $K(t)$ *to maximize profits, taking* $r(t)$ *and* T_p *and* T_E *as given. Household optimize their utility subject to their budgets, whereas saving market clearing equates the interest on consumption with the investment rate.*

[8]Thereby, $\epsilon > 1$ satisfies the necessary conditions for balanced growth in the baseline Hewitt and Aghion (1998) specification this theory relates to and is set as a precondition for simplicity.

In line with the reference theory by Grimaud and Ricci (1999), the focus is then on the existence and characteristics of a balanced growth path (BGP). Transition paths are not addressed, as this would overload the discussion without making a significant theoretical contribution[9]. The definition of a BGP is standard:

Definition 2.2. *A balanced growth path (BGP) is a trajectory along which g_y, g_k, g_c and technologies grow at a constant positive (not necessarily the same) rate.*

Before examining whether a balanced growth path exists, some of its properties must be elaborated first. This is approached with Lemma (2.1).

Lemma 2.1. *If $1 > \phi$, a BGP can exist in two alternative scenarios of resource use*

(a) $g_E > 0$ (exhaustion scenario), characterized by

$$y(t) = (1-n)^{(1-\alpha)(1-\phi)} K(t)^{\alpha(1-\phi)} \bar{b}(t) \quad \textit{with} \quad \bar{b}(t) := \frac{B(t)}{1+\mathcal{B}}, \tag{24}$$

$$\mathcal{B} := (1-\phi)(1-\alpha)(\varsigma_A(\kappa\eta)^\theta + \varsigma_R((1-\kappa)\eta)^\theta) + (\phi - \alpha(1-\phi))\varsigma_G(1-\eta)^\theta, \tag{25}$$

$$B(t) := R(t)^{(1-\alpha)(1-\phi)} A(t)^{(1-\alpha)(1-\phi)} G(t)^{\phi-\alpha(1-\phi)}, \tag{26}$$

$$g_y = \lambda n \frac{\mathcal{B}}{1-\alpha(1-\phi)}, \tag{27}$$

with $\eta \in (0,1]$, $\kappa \in (0,1]$, and $n \in (0,1)$ all constant.

(b) $g_E = 0$ (non-exhaustion scenario), characterized by $g_G \geq g_A$ and

$$y(t) = (1-n)^{(1-\alpha)} K(t)^{\alpha} \hat{\bar{b}}(t) \quad \textit{with} \quad \hat{\bar{b}}(t) = \frac{\hat{B(t)}}{1+\hat{\mathcal{B}}}, \tag{28}$$

[9] The theory is based on a Ramsey-Cass-Koopmans model, so that if a BGP exists, there is also a stable trajectory leading to that path. Whether an economy is on such a trajectory depends on the initial supply of production factors. In this theory, this relates to manufactured capital. As elaborated in the appendix, it is generally possible to find a capital stock associated with a stable trajectory. Discussing this is important for a detailed, data-based cost-benefit analysis of alternative policy pathways to long-term sustainability, but plays a subordinate role for the theoretical analysis of this work.

$$\hat{\mathcal{B}} := \frac{(1-\alpha)\varsigma_A\varsigma_G}{\left(((1-\alpha)\varsigma_A)^{\frac{1}{\theta}} + \varsigma_G^{\frac{1}{\theta}}\right)^{\theta}}, \tag{29}$$

$$\hat{B}(t) := \frac{A(t)^{(1-\alpha)}}{G(t)^{\alpha}}, \tag{30}$$

$$g_y = \lambda n \hat{\mathcal{B}}, \tag{31}$$

with $n \in (0,1)$ *constant.*

If $\phi > 1$ *only* $g_E = 0$ *(non-exhaustion scenario) as described with (b) is possible.*

Proof. See Appendix (A).

The lemma differentiates two alternative balanced growth scenarios: either the environmental effect increases, $g_{\bar{E}} > 0$, or remains constant, $g_{\bar{E}} = 0$. An increasing effect states that innovations exhaust the natural capital stock. The intensity of this exhaustion is scaled with the environmental damage elasticity, ϕ.

Within both scenarios, we learn that disaggregated innovation activity along a balanced growth path can be interpreted as a sequential aggregated innovation process. Hence, technologies can be described via the relation among leading technologies, bundled with $B(t)$, respectively $\hat{B}(t)$, and average technologies, bundled with $\bar{b}(t)$, respectively $\hat{\bar{b}}(t)$. The proportionality among both bundles is sensitive to the net research efforts, $\mathcal{B}$, respectively $\hat{\mathcal{B}}$, which describe how the combination of research efforts along alternative innovation pathways affects net productivity growth. Higher net efforts have two effects: on the one hand, they widen the gap between average and leading technology stocks[10], on the other hand, they increase the production growth rate, g_y. Note here that the net research efforts are the higher, the higher the technology weight in production, $1-\alpha$, and the higher the research efficiencies ς_A, ς_G, and ς_R if $g_{\bar{E}} > 0$. If $g_{\bar{E}} > 0$, the elasticity of the environmental damage relativizes the efficiency of the research effort combination in $\mathcal{B}$. So, the higher ϕ, the smaller the net effects of research, but the more effective the abatement efforts.

[10] An increase in $\mathcal{B}$, respectively $\hat{\mathcal{B}}$, has a positive effect on $\bar{b}(t)$, respectively $\hat{\bar{b}}(t)$, since its effect on the net efforts dominates the scaling effect on the average bundle.

The output growth rate, g_y, is proportional to the number of new inventions which are measured through the product of the probability of an innovation, λ, and the fraction of researchers, n. The higher they are, the more innovations occur. Thereby, $\frac{1}{1-\alpha(1-\phi)}$ adjusts for the amount of production that is reinvested in manufactured capital and technology to compensate for the exhaustion of natural capital. The higher α, the less this adjustment, while the lower ϕ, the higher the output growth rate. The latter holds since a higher ϕ leads to greater environmental damage and reduces the elasticity of the manufactured capital used in production so that a larger part of the numéraire output must be used for compensation.

As soon as $g_E = 0$, the production elasticities are no longer weighed with ϕ as there is no increasing environmental effect. Consequently, there is no need for adaptation so that $\kappa = 1$. Yet, $0 < \eta < 1$ is required to achieve $g_E = 0$, which will be explained in more detail after having discussed the determinants of a BGP next.

Proposition 2.1. *If a BGP exists, it is unique and characterized by*

$$\hat{g} = g_y = g_\pi = g_V = g_w = g_{\bar{\iota}} + g_x = g_k = \begin{cases} \lambda n \frac{\mathcal{B}}{1-\alpha(1-\phi)} & if \;\; 1 > \phi \\ \lambda n \frac{\hat{\mathcal{B}}}{1-\alpha} & if \;\; \phi \geq 1. \end{cases}$$

If $1 > \phi$, *a BGP exists if* $n \in (0, 1)$. *If* $\phi \geq 1$, *a BGP exists if* $\in (0, 1)$ *and* $g_G \geq g_A$.

Proof. See Appendix (A).

The proposition summarizes the necessary and sufficient conditions for a balanced growth path, highlights the corresponding growth characteristics, and emphasizes the uniqueness of the path. Specifically, balanced growth requires that $0 < n < 1$ and is sensitive to whether $\phi \geq 1$ or $1 > \phi$.

If $\phi \geq 1$, an increase of the environmental effect cannot be sustained, so positive balanced growth is only feasible if the natural capital stock remains constant. This requires that $g_G \geq g_A$. On this occasion, note that it would be possible to further differentiate a scenario with $\phi = 1$ and evaluate a non-growth path. Although such a case is technically sustainable, it is not separately assessed for the sake of simplicity. The entire sustainable growth discussion sets its focus on positive growth.

It combines the assessments for $\phi = 1$ and $\phi > 1$ by demanding that a growth path is strongly sustainable, a condition discussed in detail in the proceeding subsection.

Finally, the proposition reveals that on a BGP, the Kaldor facts hold, stating that the expenditure shares among capital and labor, the growth rate for capital and output per worker, the capital-output ratio, and the interest rate are all constant. Standard literature usually expects theory to satisfy the Kaldor facts to be empirically relevant[11].

In view of the above results, the fundamental aspect to understand is the sustainability of balanced growth, which depends on the elasticity of the environmental damage and the technical potential to compensate for the exhaustion of natural capital. These determinants are considered in depth subsequently.

Sustainable growth path

According to the genuine investment concept, weak sustainability requires that the *value* of the productive base is non-decreasing. Distinctively, strong sustainability requires to keep *each asset* of the productive base non-decreasing. The theory of this chapter can assess both sustainability criteria based on the elasticity of environmental damage, ϕ, which is detailed in the following.

Weak Sustainability ($1 > \phi$): Although new innovations increase production and exhaust the environment, this development is weakly sustainable if the damage rate is below the innovation rate. Henceforth, as long as $1 > \phi$, both technology and manufactured capital can compensate for the depletion of natural resources[12]. Accordingly, it is possible to decouple the environmental impacts from the production process in the sense that the exhaustion of natural capital does not endanger[13] a BGP. As discussed later in more detail, such a decoupling can be achieved through

[11] Appendix (B) relating to Chapter (3) discusses this aspect in depth.

[12] This perspective focuses on the sustainability of the economic value generation. Actually, physical entropy laws restrict the unbounded use of nature, see therefore S. Smulders (1995).

[13] One indicator to see this is the return on investment given by the marginal product of capital in production, $r = \alpha(1-\phi)\frac{y}{K}$. A positive r requires that $1 > \phi$. Hence, the environmental damage caused by an additional unit of capital is less than the positive effects on production.

alternative research strategies which describe different directions of technical change.

Strong Sustainability ($\phi \geq 1$): A strong sustainability criterion refers to a case where the environmental damage of production is so severe that balanced growth requires to avoid any depletion of the natural capital stock. This is the case if $\phi \geq 1$ and calls for $g_{\bar{E}} = 0$.

Which of the two sustainability criteria to use is a question of the application of the model. For example, when being interested in health effects of (air) pollution, a conservative interpretation of the theory would suggest that a strong sustainability criterion must be met, since agents can hardly survive an ever-increasing pollution without damage. A way to assess this scenario is to focus on environmental degradation in labor productivity with[14] $\iota = 1 - \alpha$ and then look at $\omega > \frac{1}{1-\alpha}$. Alternatively, it is also possible to assume that technologies enable life with low pollution growth. This is the case if $\omega < \frac{1}{1-\alpha}$, which satisfies the weak sustainability criterion.

Against this background, two questions arise: First, whether a decentralized economy achieves sustainable growth. Second, whether this growth path is socially optimal. These questions require knowledge of the actual parameter values of the model and the socially optimal reference scenario. The subsequent section will, therefore, first derive the social planner solution. This solution is then the basis to assess the sustainability and social optimality of a decentralized growth path based on a calibrated version of the theory.

2.5 Social planner results

This theory understands a social planner as a benevolent force that maximizes the infinite utility of a representative household given the technology available in the decentralized economy. The intriguing question is how to organize research.

Proposition 2.2. *Suppose a social planner can select κ, η, and n to maximize the utility of a representative household.*

[14]Remember here that $\phi = \iota\omega$.

(i) If $1 > \phi$*, then* $j^{**} = j^* \in \{brown, gray, green\} \mid g_{j^{**}} \geq g_{j^*}$*, with*

(a) $j^* = green$*, characterized by* $\kappa^* = 0$*,* $\eta^* = \frac{1}{1+\left(\frac{\varsigma_A}{\varsigma_G}\right)^{\frac{1}{\theta}}}$*, and* $g^*_{\bar{E}} = 0$*.*

(b) $j^* = gray$*, characterized by* $\kappa^* = \frac{\varsigma_R^{\frac{1}{1-\theta}}}{\varsigma_A^{\frac{1}{1-\theta}} + \varsigma_R^{\frac{1}{1-\theta}} + (\Gamma\varsigma_G)^{\frac{1}{1-\theta}}}$*,* $\eta^* = \frac{\varsigma_A^{\frac{1}{1-\theta}}}{\varsigma_A^{\frac{1}{1-\theta}} + \varsigma_R^{\frac{1}{1-\theta}} + (\Gamma\varsigma_G)^{\frac{1}{1-\theta}}}$*, and* $g^*_{\bar{E}} > 0$*, only an alternative if* $\Gamma := \frac{\phi}{(1-\phi)(1-\alpha)} - \frac{\alpha}{1-\alpha} > 0$ *and* $\frac{\varsigma_A^{\frac{1}{1-\theta}} + \varsigma_R^{\frac{1}{1-\theta}}}{\varsigma_G^{\frac{1}{1-\theta}}} > \left(\frac{\phi}{(1-\phi)(1-\alpha)} - \frac{\alpha}{1-\alpha}\right)^{\frac{\theta}{1-\theta}} \left(\frac{\alpha}{(1-\alpha)} + (1-\phi)(1-\alpha)\right)$*.*

(c) $j^* = brown$*, characterized by* $\kappa^* = \frac{1}{1+\left(\frac{\varsigma_R}{\varsigma_A}\right)^{\frac{1}{1-\theta}}}$*,* $\eta^* = 1 - \kappa^*$*, and* $g^*_{\bar{E}} > 0$*.*

Thereby, $g^*_{y,j} = \lambda n^*_j \mathcal{B}^*_j$ *with*

$$\mathcal{B}^*_j := \begin{cases} \frac{(1-\phi)(1-\alpha)}{1-\alpha(1-\phi)} (\varsigma_A^{\frac{1}{1-\theta}} + \varsigma_R^{\frac{1}{1-\theta}})^{1-\theta} & \text{if } j^* = brown \\ \frac{(1-\phi)(1-\alpha)}{1-\alpha(1-\phi)} \left(\varsigma_A^{\frac{1}{1-\theta}} + \varsigma_R^{\frac{1}{1-\theta}} + \Gamma\varsigma_G^{\frac{1}{1-\theta}}\right)^{1-\theta} & \text{if } j^* = gray \\ \frac{(1-\alpha)\varsigma_A\varsigma_G}{\left(((1-\alpha)\varsigma_A)^{\frac{1}{\theta}} + \varsigma_G^{\frac{1}{\theta}}\right)^{\theta}} & \text{if } j^* = green, \end{cases}$$

$$n^*_j = \begin{cases} \left(\frac{1+\mathcal{B}^*_j - \frac{\rho}{\lambda\mathcal{B}^*_j}(1-\alpha)(1-\phi)}{\frac{\phi+\epsilon(1-\alpha)(1-\phi)}{1-\alpha(1-\phi)} + \mathcal{B}^*_j(\phi+\alpha(1-\phi))}\right) & \text{if } j^* \in \{brown, gray\} \\ \left(\frac{(1-\alpha)}{\epsilon(1-\alpha)+\alpha}\right)\left(\frac{(1+\mathcal{B}^*(1-\alpha))}{(1-\alpha)} - \frac{\rho}{\lambda\mathcal{B}^*(1-\alpha)}\right) & \text{if } j^* = green. \end{cases}$$

(ii) If $\phi \geq 1$ *,* $j^{**} = green$ *with* κ^* *and* η^* *following (i)(a).*

In $j^* \in \{brown, gray\}$*,* $n^* \in (0,1)$ *if* $\frac{\left(\frac{(1-\epsilon)^2}{1-\alpha(1-\phi)^2} + 4\frac{\rho}{\lambda}\right)^{\frac{1}{2}} - \frac{(1-\epsilon)}{1-\alpha(1-\phi)}}{2(1-\alpha(1-\phi))} > \mathcal{B}^*_j > \frac{(1+\frac{4\rho(1-\alpha)(1-\phi)}{\lambda})^{\frac{1}{2}} - 1}{2(1-\alpha(1-\phi))}$.
In $j^* = green$*,* $n^* \in (0,1)$ *if* $\frac{\left(\frac{(1-\epsilon)^2}{(1-\alpha)^2} + 4\frac{\rho}{\lambda}\right)^{\frac{1}{2}} - \frac{(1-\epsilon)}{(1-\alpha)}}{2} > \mathcal{B}^*_j > \frac{(1+\frac{4\rho(1-\alpha)(1-\phi)}{\lambda})^{\frac{1}{2}} - 1}{2}$.

Proof: See Appendix (A).

The proposition presents the socially optimal research strategy for satisfying a weak and a strong sustainability criterion. The respective strategy combines research efforts with research labor and needs to abate any increase in the environmental effect whenever strong sustainability is required. For a weak sustainability criterion, alternative innovation strategies are available as long as $\eta^* > 0$. The proposition identifies three characteristic directions of technical change for this latter scenario, depending on whether the planner combines general technology with abatement, with adaptation, or with both. Referring to such stylized scenarios is necessary since cases with no adaptation or abatement require differentiating the solution. Note thereby that a scenario that only addresses general innovations related to $\kappa^* = 0$ and $\eta^* = 1$ is never socially optimal and therefore ignored.

A *brown* direction of technical change is free of any abatement ($\eta^* + \kappa^* = 1$) so the environmental effect increases steadily. The planner concentrates on general innovations and adaptation and selects κ^* and η^* independently of ϕ because improvements in both technologies compensate for the damage caused by pollution. Hence, the only parameters to consider when allocating efforts are the research efficiencies ς_A and ς_R.

A *gray* direction of technical change describes a scenario where research combines improvements in all technologies without stopping pollution growth so that an increasing environmental effect remains. The path is only a social planner option if $\Gamma := \frac{\phi}{(1-\phi)(1-\alpha)} - \frac{\alpha}{1-\alpha}$ is positive. This factor describes the efficiency of abatement efforts and is worth being discussed in more depth. First, $\frac{\phi}{(1-\phi)(1-\alpha)}$ measures the net benefits of abatement. Hereby, ϕ measures the direct benefits of abatement as it reduces damages. This effect is rescaled by $1-\phi$ and $1-\alpha$. The former accounts for that environmental damage reduce production, the latter that abatement-focused research reduces production. A reduction in production lowers the depletion of the natural capital stock and thus the benefit of abatement. In parallel, the direct costs of abatement are measured with $\frac{\alpha}{1-\alpha}$. Thereby, α describes that abatement-focused research efforts cannot be used for general productivity improvements while $1-\alpha$ rescales this effect since the improvements are the lower, the lower production. Crucially, abatement efforts face an upper limit. When $\frac{\varsigma_A^{\frac{1}{1-\theta}} + \varsigma_R^{\frac{1}{1-\theta}}}{\varsigma_G^{\frac{1}{1-\theta}}} = \left(\frac{\phi}{(1-\phi)(1-\alpha)} - \frac{\alpha}{1-\alpha}\right)^{\frac{\theta}{1-\theta}} \left(\frac{\alpha}{(1-\alpha)} + (1-\phi)(1-\alpha)\right)$, abatement is so effective that $g_{\bar{E}} = 0$. In this case, the direction of technical change is not longer gray but green, a path detailed subsequently.

A *green* direction of technical change relates to a scenario where the planner abates any increase in the environmental effect, so $g_E = 0$. There is thus no need for adaptation, what leads to $\kappa^* = 1$. For all dynamic equations, this has the consequence that $\phi = 0$, so the assessment of the abatement efficiency variable, Γ, is no longer relevant for η^*.

It was discussed that if weak sustainability is required, all three directions of technical change are feasible. However, there is only one socially optimal direction of technical change. This direction is signaled with '**' and determined by the planner research path that leads to the highest productivity growth rate. This rate is the relevant criteria since it leads to the highest consumption growth rate and thus the highest rate of utility growth. The utility growth rate dominates the representative household utility profile and is consequently the appropriate determinant for the socially optimal direction of technical change. Its selection is subject to various parameter values and therefore assessed with a calibration in the next subsection.

For now, note that whenever the gray direction of technical change is feasible, it dominates the brown direction, as the combination of general innovations, adaptation, and abatement is more efficient due to decreasing returns in research efforts. These benefits of combining technologies is also why other strategies such as only using general innovations or focusing on general innovations and abatement without eliminating an increase in the environmental effect are never an alternative for the planner (see appendix for details).

Finally, for the research labor allocation, the planner weighs the advantages of using labor to either generate direct consumption by employing labor in production or to improve consumption growth rates by employing labor in the research sector. Thereby, $\frac{\partial n_j^*}{\partial \mathcal{B}_j^*} > 0 \forall j$ since the larger the (marginal) productivity of research, the stronger the social value of research. Further, $\frac{\partial n_j^*}{\partial \rho} < 0$ since greater impatience increases the benefits of immediate consumption and thus the value of labor in production. In addition, $\frac{\partial n_j^*}{\partial \lambda} > 0$ since a higher likelihood to innovate increases the consumption growth rate and thus the social value of research labor.

In view of this, the subsequent section calibrates the theory and calculates the welfare maximizing direction of technical change for increasing damage elasticities in a weak sustainability scenario.

Calibration of the social planner results

Figure (2.1) reveals the socially optimal net research efforts, $\mathcal{B}^*$ (top left), the socially optimal research labor, n^* (top right), the socially optimal depletion rate of natural capital, $g^*_{\bar{E}.}$ (bottom left), and the corresponding socially optimal production growth rate, g^*_y (bottom right), for all three directions of technical change at different intensities of the environmental damage elasticity, $\phi \in (0, 1)$, in a calibrated version of the model. The appendix explains that in this calibration, $\alpha = \frac{1}{3}$, $\rho = 0.015$, $\lambda = 0.5$, and $\varsigma_A = \varsigma_R = \varsigma_G = 0.14$. So the focus is set on a benchmark scenario where research is equally efficient among alternative research directions.

As a general observation, note that since the green direction of technical change eliminates environmental damage, the marginal product of labor in production is relatively high along this path. The social planner consequently allocates more labor to the production sector than in a brown or gray direction of technical change. As a result, the latter two directions have a higher proportion of research labor and thus a higher growth rate until the environmental damage (accounted for via the environmental damage elasticity) becomes so severe that the production growth rate of the green direction starts dominating the evaluation. Any damage elasticities above roughly 55 % lead to such green dominance.

To be more specific about the damage elasticities, note that for relatively low values up to 25 %, the brown direction of technical change is socially optimal. At these damage elasticity levels, the gray direction of technical change is not efficient ($\Gamma < 0$) while the green direction is characterized by a lower research labor fraction and, accordingly, a lower innovation rate. At $\phi \approx 0.25$, a gray direction of technical change becomes efficient ($\Gamma > 0$) and uses the available research efforts more effectively than the brown direction, since profiting by three sources of research returns. The gray direction of technical change also employs the most research labor and accordingly generates the highest productivity growth rates. This observation holds until the damage elasticity reaches $\phi \approx 0.55$. At this level, the marginal effect of

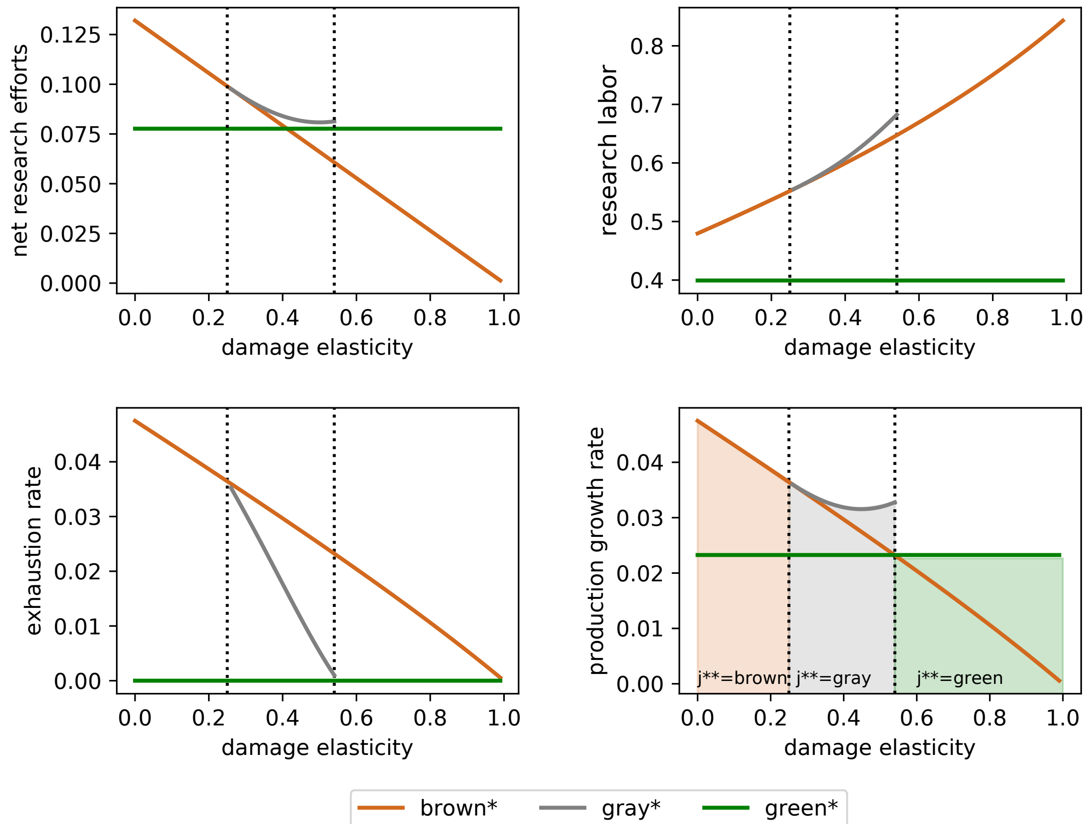

Figure 2.1: Social planner's net efforts, $\mathcal{B}^*$ (top left), research labor, n^* (topright), natural capital depletion rate, $g^*_{\bar{E}}$ (bottom left), and production growth rates, g^*_y (bottom right), at different damage elasticities, ϕ, reaching from 0 to 1.

using research for abatement is high enough to eliminate all pollution, so the gray direction turns into a green direction of technical change. Finally, the green direction of technical change remains socially optimal for all $\phi > 0.55$ since the environmental damages start having a relatively strong impact on the brown innovation rates[15].

[15]Note some details: (1.) If $j^* = gray$, $\mathcal{B}^*$, initially decreases with ϕ, but then increases because marginal abatement effects increase with damages, making this research direction more effective, while damage reduces it. The latter effect dominates for relatively low ϕ, the former dominates for relatively high ϕ. (2.) In a brown direction, $\mathcal{B}^*$ decreases monotonically in ϕ because A and R compensate for damage so that they are less efficient in increasing net productivity. (3.) A green direction is not affected by ϕ. (4.) There is a sharp decrease in the growth rates when the gray strategy turns into a green direction around $\phi \approx 0.55$ since the green direction eliminates all

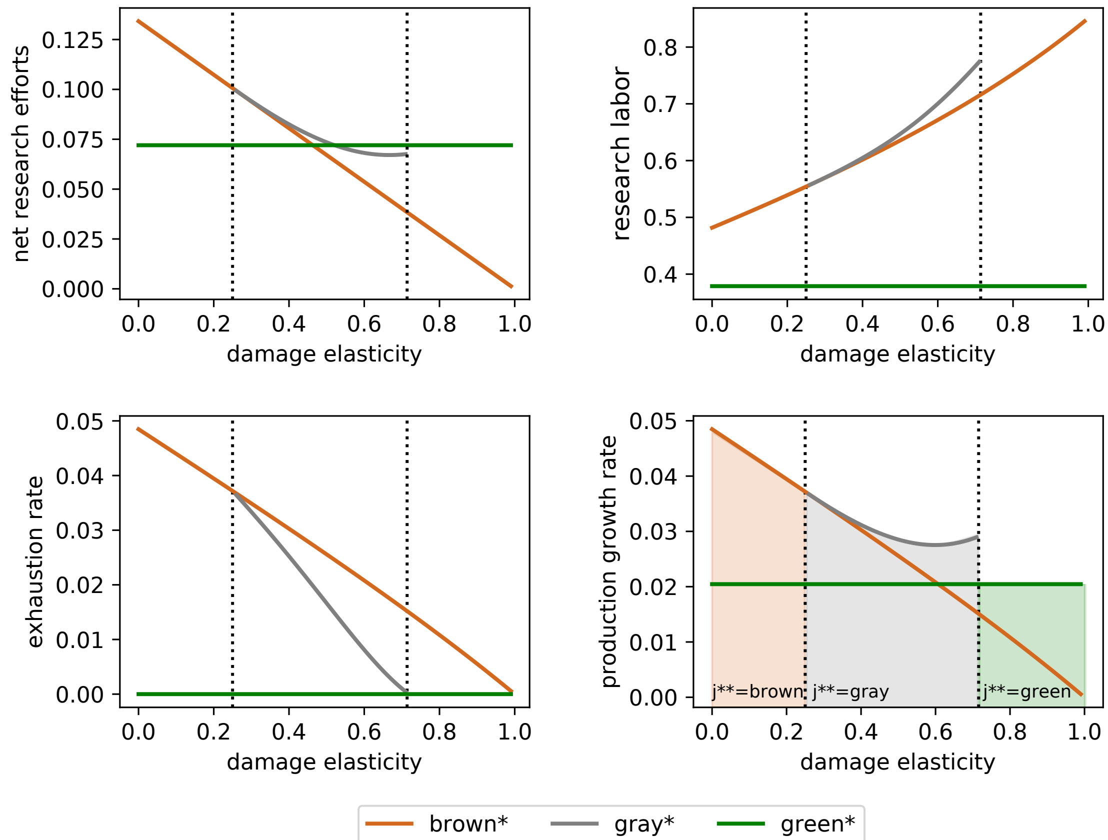

Figure 2.2: Social planner's net efforts, $\mathcal{B}^*$ (upper left), research labor, n^* (upper right), natural capital depletion rate, $g^*_{\bar{E}}$ (lower left), and production growth rate, g^*_y (lower right), when adaptation and abatement focused research is only 50% as efficient as general research.

As a robustness test, Figure (2.2) presents the results of a scenario where adaptation and abatement focused research efforts are only 50% as efficient as research in general technologies ($\varsigma_A = 0.18$, $\varsigma_R = \varsigma_G = 0.09$, while the remaining parameters are identical to the benchmark specification, see appendix for details). The adjustment does not affect the qualitative results, only the critical level at which the gray direction turns green, which is now associated with a damage elasticity close to 70%. This critical level is higher than in the benchmark, since the adjusted parameter weights

environmental damages, what increases the marginal product of labor in numéraire production and accordingly shifts labor from research to production. Therefore, this is simply a tipping point.

increase the absolute and relative efficiency of research in general innovations. Since all other results are qualitatively identical to the benchmark calibration, this chapter will directly proceed with an assessment of both the sustainability and social optimality of a decentralized growth path.

2.6 Decentralized economy results

This chapter's theory extends a Ramsey-Cass-Koopmans model. In the original model, innovations are exogenous, markets are perfectly competitive, and decentralized economies achieve a welfare-maximizing growth path when interpreting the representative household utility as an indicator of social welfare. This theory augments this setting with natural capital and a monopolistic intermediate sector. These extensions can cause environmental and innovation externalities. Before assessing whether these externalities exist and if they exist, in which intensity, this section will detail their qualitative origins.

Environmental Externalities: In this theory, numéraire production creates an environmental effect that depletes the natural capital stock. Whenever improvements in gross productivity, $\mathcal{T}$, are not neutralized with sufficient abatement efforts, G, the environmental effect, $\bar{E}$, increases and the natural capital stock, N, exhausts. Crucially, the exhaustion of natural capital is only an externality if its depletion rate is not socially optimal. Whether they only reduce welfare or jeopardize balanced growth is subject to the sustainability criterion and several structural conditions. In both scenarios, they cause a static inefficiency as the technology inventory is not socially optimal (respectively, the environmental damages are too high). In addition, they cause a dynamic inefficiency, as the innovation rate is not socially optimal.

Research Externalities The research sector is the potential source of several externalities. Innovations create positive externalities, as each new invention expands the stock of publicly accessible knowledge used for subsequent innovations. However, there are only incentives for innovation if there is sufficient compensation for the opportunity costs of research. Creating such incentives is sometimes called the 'appropriability effect' while the compensation follows via patent protection[16]. The challenge with patents is that they privatize the returns to public knowledge and create a monopolistic competitive research sector. Thence, patents lead to two types of distortion: First, successful researchers who become intermediate good providers take advantage of patent protection by charging a price mark-up on their goods. The high prices cause a static inefficiency since intermediates become too expensive so that too little are purchased. Further, there is a dynamic inefficiency since mark-up prices inflate the value of innovations, what incentivizes research.

A second cause of bias is that the R&D sector bases its innovation decisions on private rather than social research returns. The sector ignores the benefits of technology spillovers for future production and discounts innovation values above the social discount rate, resulting in insufficient research[17]. Even so, researchers ignore the business stealing effect that occurs when their innovation replaces a vintage technology, what leads to too excessive research. Which of both effects dominates is widely debated in the literature and estimated with a calibration a little later.

For evaluating the existence and intensity of both types of externalities, this section will first assess the allocation of research effort and labor in a benchmark specification characterized by full access to information and technology. Afterward, the section investigates how restrictions in access to innovation and technology affect the results.

2.6.1 Growth in a GK regime

The benchmark specification of this chapter refers to a general knowledge regime (GK regime). Agents have open access to information ($\mathcal{I} = 1$) and technology, and can combine general technologies with adaptation and abatement. A decentralized R&D sector evaluates how to allocate research efforts and labor to maximize indi-

[16] See Philippe Aghion, Akcigit, and Howitt (2015) for a short overview and Decker (2014) and Akcigit and Kerr (2010) for a discussion.

[17] The focus on individual profits can affect the evaluation of labor productivity effects which is profound if labor works with distinct technologies or distinct skill intensities, see Aghion and Hewitt (1998) for details.

vidual innovation profits. The corresponding factor allocations are summarized in Proposition (2.3).

Proposition 2.3. *Suppose an economy is characterized by a GK regime and $\mathcal{I} = 1$, then along a BGP $\mathcal{B} = \mathcal{B}^{**}$. Further, n is only socially optimal in an arbitrary case and given with*

$$n = \frac{\lambda(1+T_V)(1-\alpha)-\rho}{(\lambda\mathcal{W}_j + (1+T_V)(1-\alpha))},$$

$$\mathcal{W}_j := \begin{cases} \frac{\mathcal{B}_j^{**}\left(\epsilon + \frac{\phi}{(1-\phi)(1-\alpha)}\right)}{1-\alpha(1-\phi)} + 1 - \frac{\phi(\phi-\alpha(1-\phi)\varsigma_G(1-\eta_j-\kappa_j)^{\theta}))}{(1-\phi)(1-\alpha)} & \textit{if } j^{**} \in \{brown, gray\} \\ \epsilon\frac{\mathcal{B}_j^{**}}{1-\alpha} + 1 & \textit{if } j^{**} = green. \end{cases}$$

Proof: See Appendix (A).

The proposition reveals that complete information about the environmental impact of innovation and the capability to access and combine alternative technology creates an environment that internalizes environmental externalities. Decentralized economies not only find sustainable growth, but a socially optimal direction of technical change. Accordingly, $\mathcal{B} = \mathcal{B}^{**}$. Hence, access to information and technology is crucial for the social optimality of decentralized research.

For this result, note that the damage to natural capital is proportional to the available technology stock. An agent not only anticipates the research spillovers, but also how innovation influences the depletion of natural capital. Patent protection hence privatizes both the technology spillovers and the costs of natural capital exhaustion. Both affect the sales of intermediates and the value of innovation. This quality leads to socially optimal research efforts and internalizes potential environmental externalities.

In any case, the research labor allocation is biased in all but an arbitrary scenario since decentralized economies do not weigh the benefits of knowledge spillover effects against the business stealing effect. To give some details, similarly to the social planner, $\frac{\partial n}{\partial \rho} < 0$ since a larger impatience reduces the value of an innovation. Distinct to the planner, $\frac{\partial n}{\partial \lambda} < 0$ because a stronger replacement rate no longer stimulates research and $\frac{\partial n}{\partial \mathcal{B}} < 0$ because the no-arbitrage condition in the labor market draws labor from innovation sector to production[18]. So while the planner and the decentralized economy both shift the more labor to research the higher the discount rate ρ, decentralized n decreases with λ and $\mathcal{B}$, while the socially optimal n^* increases with both. For λ, these different reactions occur since a high replacement rate reduces the private value of innovations but increases their social value. For $\mathcal{B}$ the responses follow since the social planner compares instant consumption effects and consumption growth rate effects, leading to a distinct assessment of the value of labor[19].

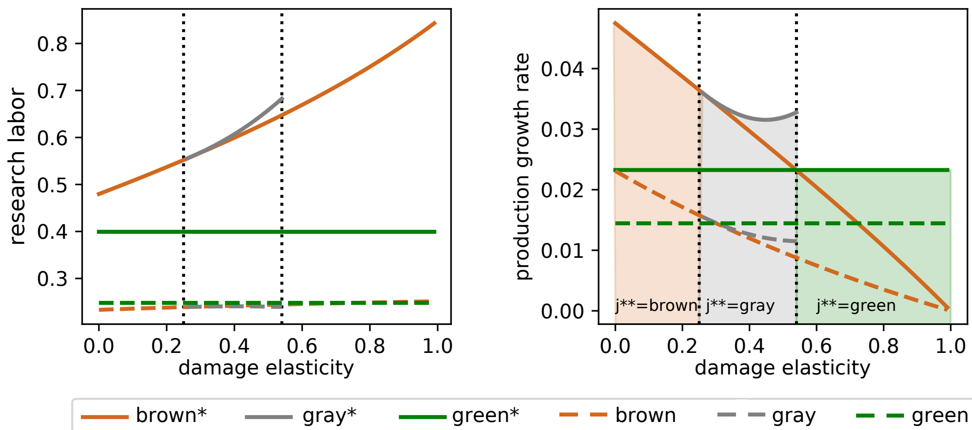

Figure 2.3: Comparison of the decentralized and social planner growth rates and research labor allocation under the three directions of technical change within the benchmark specification.

Figure (2.3) illustrates these differences by sketching the calibrated decentralized and social planner research labor allocation (right figure) and the corresponding production growth rates (left figure) for the three alternative directions of technical change. The figure presents the corresponding results for different environmental damage elasticities between 0 and 1, referring to a weakly sustainable growth scenario. Note that there is no discussion about the research effort allocation, since the

[18]Note that the marginal product of production labor increases with $(1-\alpha)$, whereas a lower α reduces the net effect of research labor, so the aggregate effects of α are ambiguous.

[19]Whenever $1 > \eta > 0$, $\mathcal{W}$ is scaled by $\varsigma_G(1-\eta)^\theta$ which accounts for that an increase in future intermediate demand elevates the environmental effect. Abatement focused research therefore also reduces the environmental footprint of a vintage intermediate good as long as it is provided. This reduces $\mathcal{W}$ and thus partially offsets the positive impact of the net efforts $\mathcal{B}$.

decentralized net research efforts of the benchmark economy, $\mathcal{B}$, are socially optimal (see above).

The calibrated parameter values reveal that the decentralized economy consistently provides too little labor for research. So compared to the planner, it experiences lower production growth rates regardless of the direction of technical change[20]. These results are qualitatively identical when adaptation and abatement are only 50% as effective as general research (as was considered in the robustness test previously), so it is next essential to assess the potential of policy for welfare improvements.

Corollary 2.1. *Suppose an economy is characterized by a GK regime and $\mathcal{I} = 1$, then $n = n^{**}$ is achieved with*

$$T_V = \begin{cases} \left(\frac{(1+\mathcal{B}_j)\lambda\mathcal{B}_j - \rho(1-\alpha)}{\rho(1-\alpha)-(1-\epsilon+(1-\alpha)\mathcal{B}_j)\lambda\mathcal{B}_j}\right)\frac{\lambda\mathcal{W}+\rho}{(1-\alpha)} - 1 & \text{if } j^{**} \in \{brown, gray\} \\ \frac{\left(1+\frac{\mathcal{B}_j}{1-\alpha}-\frac{\rho(1-\alpha)^2}{\lambda\mathcal{B}_{green}}\right)\left(\lambda\mathcal{W}+\rho\right)}{(1-\alpha)\left(\frac{\rho(1-\alpha)^2}{\lambda\mathcal{B}_{green}}+\epsilon-1-\mathcal{B}_{green}\right)} - 1 & \text{if } j^{**} = green, \end{cases}$$

while $T_p = \frac{1-\alpha}{\alpha}$ eliminates intermediate demand distortions due to markup pricing. In combination, these policies are first best.

Proof See Appendix (A).

The corollary details the design of the price subsidy, T_p, and the research subsidy, T_V, required to achieve static and dynamic efficiency. The latter increases with $\mathcal{B}$ and λ because of the labor market distortions discussed above. Since there are no environmental externalities while the cited policies eliminate all monopolistic market externalities, they are first best.

[20]Note that when a gray direction turns green at $\phi \gtrapprox 0.55$, the proportion of research labor increases as a greater abatement decreases the efficiency of research efforts. This is since the no-arbitrage condition in the labor market compensates for this development by shifting labor from production to research.

Environmental externalities in a GK regime

This chapter's observation that there are no environmental externalities in the benchmark setup is contrary to standard literature findings. While there are different explanations, there is little doubt in the literature that environmental externalities are the reason for socially harmful environmental pollution and climate change. Proposition (2.4) shows that a straightforward argument of this model for environmental externalities is a lack of information.

Proposition 2.4. *Suppose an economy is characterized by a GK regime and $\mathcal{I} = 0$, then along a BGP $j = brown$, which is*

(i) not sustainable if $\phi \geq 1$.

(ii) socially not optimal if $1 > \phi$ and $j^{**} \in \{green, gray\}$.

(iii) socially optimal if $1 > \phi$ and $j^{**} = brown$.

Further, n is only socially optimal in an arbitrary case as

$$n = \frac{(1 + T_V)(1 - \alpha) - \rho}{(\lambda \mathcal{W}_{anticipated} + (1 + T_V)(1 - \alpha))},$$

$\mathcal{W}_{anticipated} := \epsilon^{\frac{\mathcal{B}_{anticipated}}{(1-\alpha)}} + 1$, $\mathcal{B}_{anticipated} := (1 - \alpha)(1 - \alpha)(\varsigma_A^{\frac{1}{1-\theta}} + \varsigma_R^{\frac{1}{1-\theta}})^{1-\theta}$.

Proof See Appendix (A).

The proposition demonstrates that agents lacking information on the environmental impact of innovations ($\mathcal{I} = 0$) cause an environmental externality whenever the socially optimal direction of technical change is not brown. Thereby, the agents ignore the environmental damage on the innovation rate, $1 - \phi$, so they base their labor allocation decision on an overestimated research efficiency. As a consequence, $\mathcal{B}_{anticipated}$ is biased, what augments the general bias in the decentralized labor allocation discussed above. Yet, market-based policies can help. These policies are introduced with Corollary (2.2).

Corollary 2.2. *Suppose an economy is characterized by a GK regime and* $\mathcal{I} = 0$*, then*

(i) $\mathcal{B} = \mathcal{B}^{**}$ *is achieved with*

$$T_E(t) := \alpha(1 + T_p)\bar{E}_0 e^{(g_{\Gamma_E} + \Omega g_E)t} - 1,$$

whereby

(a) $T_E(t) = 0$ *if* $j^{**} = brown$*.*

(b) $\Omega := \frac{1}{1+\alpha} + \alpha$ *and* $g_{\Gamma_E} := \frac{\Omega\phi}{(1-\alpha)}\big(\frac{1}{1-\alpha(1-\phi)} g_G - (g_A + g_R)\big)$ *if* $j^{**} = gray$*.*

(c) $\Omega := \frac{1}{\alpha + \left(\frac{1-\phi}{\phi - \alpha(1-\phi)}\right)}$ *and* $g_{\Gamma_E} := 0$ *if* $j^{**} = green$*.*

(ii) $n^{**} = n$ *is achieved with*

$$T_V = \begin{cases} \left(\frac{(1+\mathcal{B}_j)\lambda\mathcal{B}_j - \rho(1-\alpha)}{\rho(1-\alpha) - (1-\epsilon+(1-\alpha)\mathcal{B}_j)\lambda\mathcal{B}_j}\right)\frac{\lambda\mathcal{W}_{anticipated}+\rho}{(1-\alpha)} - 1 & if\ j^{**} \in \{brown, gray\} \\ \frac{\left(1+\frac{\mathcal{B}_{green}}{1-\alpha} - \frac{\rho(1-\alpha)^2}{\lambda\mathcal{B}_{green}}\right)\left(\lambda\mathcal{W}_{anticipated}+\rho\right)}{(1-\alpha)\left(\frac{\rho(1-\alpha)^2}{\lambda\mathcal{B}_{green}} + \epsilon - 1 - \mathcal{B}_{green}\right)} - 1 & if\ j^{**} = green. \end{cases}$$

(iii) markup pricing is eliminated with

(a) $T_p = \frac{\Gamma_0 \bar{E}_0^{\Omega}}{\alpha} - 1$ *if* $j^{**} \in \{gray, green\}$*.*

(b) $T_p = \frac{1-\alpha}{\alpha}$ *if* $j^{**} = brown$*.*

Proof See Appendix (A).

The corollary presents policies to correct the allocation biases that occur when agents lack information about the environmental impact of their innovations. Thereby, monopolistic market externalities are addressed with research subsidies, T_v, and intermediate price subsidies, T_p, while if $j^{**} \in \{gray, green\}$, an environmental tax eliminates the environmental externality.

The environmental tax addresses the social costs of innovations and can thus be understood as a dynamic Piguvian tax. Its design is subject to the socially optimal direction of technical change. If $j^{**} = gray$, the environmental tax must increase proportionally to $\bar{E}(t)$ but must be accompanied by cost adjustments for vintage providers. These adjustments are denoted by g_{Γ_E} and necessary to reduce the operating costs of older providers. Such providers would otherwise be crowded out, as they would be disproportionately affected by a constantly increasing tax burden[21]. If $j^{**} = green$, the natural capital stock remains constant. In this case, the environmental tax does not need dynamic adjustments[22], so $g_{\Gamma_E} = 0$.

Distinct to the effort allocation, the research labor allocation principles are the same as those for agents who have complete information on the environmental impact of innovations. As a result, the research subsidy (or tax) T_V used to achieve $n = n^{**}$ follows the same structure as the one outlined in Corollary (2.1). However, its value is larger if $j^{**} = brown$, since $\mathcal{B}_{anticipated}$ is above $\mathcal{B}_{brown}$ (see above). In addition, as in the benchmark with access to information, a price subsidy, T_p, is required to eliminate price markups.

With this knowledge at hand, we can subsequently consider the role of specialization for decentralized research decisions.

2.6.2 Research with specialization (SK regime)

In the discussion so far, the research sector has been free to combine technologies. This is no longer the case when innovation requires specialization. In such a scenario, researchers need to decide on whether to focus on adaptation or abatement, so research spillover effects become path-specific. The consequences of these restrictions are illustrated with Proposition (2.5).

Proposition 2.5. *Suppose an economy is characterized by an SK regime and* $\mathcal{I} = 1$*,*

[21] If $j^{**} = gray$, it is socially optimal to improve abatement, but not at an intensity that stops the depletion of natural capital. With new innovations, each unit of emissions becomes more expensive, although older providers have not added any further emissions what increases their operating costs, so they are crowded out. Therefore, the environmental tax rate of vintage suppliers needs to be adjusted to stabilize the technology distribution.

[22] Note that if $j^{**}\{gray, green\}$, then $\Gamma_{E,0} = \alpha \frac{1+T_p}{\bar{E}_0^{\Omega}}$, so $\Gamma_{E,0}$ and T_p need to be set in a fixed proportion but are not further identified.

then

$$\gamma = \begin{cases} 1 & \textit{if } R(t)^{(1-\alpha)(1-\phi)} > (1+T_{V,R})G(t)^{\phi-\alpha(1-\phi)} \\ 0 & \textit{if } R(t)^{(1-\alpha)(1-\phi)} < (1+T_{V,R})G(t)^{\phi-\alpha(1-\phi)} \\ \gamma \in [0,1] & \textit{if } R(t)^{(1-\alpha)(1-\phi)} = (1+T_{V,R})G(t)^{\phi-\alpha(1-\phi)} \;\; \textit{and } \varsigma_R = \varsigma_G = \phi. \end{cases}$$

Along a BGP $\mathcal{B} = \mathcal{B}^{**}$ *if* $\phi \geq 1$*, while if* $1 > \phi$*,* $\mathcal{B} = \mathcal{B}^{**}$ *only if:*

(i) $\varsigma_R = \varsigma_G = \phi$*,*

(ii) $j^* = brown$ *and* $R(t)^{(1-\alpha)(1-\phi)} > (1+T_{V,R})G(t)^{\phi-\alpha(1-\phi)}$*,*

(iii) $j^* = green$ *and* $R(t)^{(1-\alpha)(1-\phi)} < (1+T_{V,R})G(t)^{\phi-\alpha(1-\phi)}$*.*

Further, the social optimality of n *is ambiguous and its allocation follows the principle established in Proposition (3).*

Proof: See Appendix (A).

The proposition sheds light on the fundamental role of access to technology for the sustainability and the social optimality of decentralized research. Before detailing the results, note that since there is no gray direction of technical change, it is critical to refer to this direction as a reference for socially optimal research. For the analysis, however, it is appealing to clarify the costs of restricted access to technologies. Thereupon, the gray direction of technical change will remain a reference scenario, although specialization cannot lead to a gray direction of technical change.

In light of this, the proposition reveals that if a strong sustainability criterion is relevant, agents specialize in abating all pollution growth. Such a strategy is sustainable and socially optimal. If a weak sustainability criterion is relevant, two scenarios lead to suboptimal research: (1.) if a gray direction of technical change is socially optimal, (2.) if technology-related lock-in effects keep the economy from shifting to a welfare superior growth path. While the former scenario straightforwardly follows with restricted access to technology, the latter occurs since spillover effects shift research in the direction that has access to a higher technology stock.

To detail the latter, note that an innovator compares the innovation values of adaptation-oriented research with abatement-oriented research. Thereby, researchers consider how path-specific spillover effects impact the value of an innovation in the next period. As several determinants of the value of an innovation are identical, this assessment results in a comparison of the net technology effects of adaptation, $R(t)^{(1-\alpha)(1-\phi)}$, with the tax-weighted net technology effects of abatement, $(1 + T_{V,R})G(t)^{\phi-\alpha(1-\phi)}$. The higher the weighted technology inventory, the higher the returns on research in a specific direction. In the arbitrary case where these two components are equivalent, research typically chooses the direction that is more efficient. Thereupon, γ is indefinite only in the stylized case where $\varsigma_R = \varsigma_G = \phi$. In any other scenario, the economy is tied to the direction in which the weighted technology stock dominates.

The allocation of research labor, n, follows the same principles as the ones observed in a GK regime, discussed with Proposition (2.3). This similarity occurs because research efforts allocate independently of labor so that the labor allocation decisions are qualitatively not affected by research effort decisions. However, quantitative differences are possible since $\mathcal{B}$ determines n so that the fraction of research labor may differ in both regimes.

To proceed, it is not reasonable to discuss specialization in more detail with a calibration because its main peculiarity is the decentralized decision on the innovation direction. This decision is subject to the technology stocks which were standardized previously. Although it is possible to expand the evaluation and assess data for reasonable initial technology stocks, such an evaluation is highly hypothetical and goes beyond the scope of this discussion. In view of this, it is more accurate to proceed with an analytical focus on the potential of policies to make decentralized research strategies socially optimal. Corollary (2.3) presents such policy.

Corollary 2.3. *Suppose an economy is characterized by an SK regime with $\mathcal{I} = 1$ while lock-in effects prevent a socially optimal direction of technical change, then*

(i) $j^{**} = gray$ cannot be achieved but $T_{V,R} = \frac{R(t)^{(1-\alpha)(1-\phi)}}{G(t)^{\phi-\alpha(1-\phi)}} - 1 < (>) 0$ improves welfare if $R(t)^{1-\alpha} > (<) G(t)^{\frac{1}{1-\alpha}}$ and $g_{green} > (<)\ g_{brown}$.

(ii) if $j^{**} = \{brown, green\}$, $\mathcal{B} = \mathcal{B}^{**}$ can be achieved with

$$T_{V,R} = \begin{cases} \frac{R(t)^{(1-\alpha)(1-\phi)}}{G(t)^{\phi-\alpha(1-\phi)}} - 1 < 0 & \text{if } j^{**} = green \\ \frac{R(t)^{(1-\alpha)(1-\phi)}}{G(t)^{\phi-\alpha(1-\phi)}} - 1 < 0 & \text{if } j^{**} = brown. \end{cases}$$

In addition, T_v and T_p that follow the principles discussed in Corollary (2.1)

(i) are first best, leading to $n = n^{**}$ if $j^{**} \in \{brown, green\}$.

(ii) can improve welfare if $j^{**} = gray$.

Proof See Appendix (A).

The corollary demonstrates that whenever technology lock-in effects hold a decentralized specialized economy on a socially inefficient path, a path-specific research subsidy, $T_{v,R}$, can shift research in the preferred direction. Evidently, if $j^{**} = gray$, such a shift is not possible. In this case, policies can still improve welfare if lock-in effects keep the economy from the faster-growing research direction. In addition, the corollary shows that the policy recommendations for research labor, T_V, and markup pricing, T_p, follow the principles outlined in Corollary (2.1). As a consequence, when $j^{**} \in \{brown, green\}$, policies can be first best. In light of this, it is essential to understand how specialization leads to environmental externalities.

Environmental externalities in an SK regime

Proposition (2.5) has revealed that if in an SK regime, lock-in effects prevent an economy from achieving a socially optimal direction of technical change, the depletion of the natural capital stock is socially suboptimal. The consequences are then environmental externalities. In addition, similar to a GK regime, environmental externalities can occur due to restricted access to information on the environmental impacts of innovations. This second cause for externalities is addressed subsequently.

Proposition 2.6. *If $\mathcal{I} = 0$, research efforts and labor allocation principles are not affected by whether the economy is characterized by an SK regime or a GK regime.*

Proof See Appendix (A).

The proposition emphasizes that agents that lack information about the environmental impact of their innovation choose a brown innovation strategy ‘by default’. As discussed in-depth for a general knowledge regime, they specialize in abatement irrespective of whether they can combine technologies what leads to environmental externalities whenever the socially optimal direction of technical change is not brown. Accordingly, any policy measure intending to improve the efficiency of research needs to account for both a lack of information and specialization, as explained next.

Corollary 2.4. *Suppose an economy is characterized by an SK regime and* $\mathcal{I} = 0$. *If lock-in effects prevent a BGP from being socially optimal, then*

(i) combining T_E *introduced in Corollary (2.2) with* $T_{V,R} = \max\{0, \frac{R(t)^{1-\alpha\Omega}}{G(t)^{\frac{\Omega}{1-\alpha}}} - 1\}$

  (a) achieves $\mathcal{B} = \mathcal{B}^{**}$ *if* $j^{**} = green$.

  (b) can improve welfare if $j^{**} = gray$.

(ii) T_v *and* T_p *following the principles outlined in Corollary (2.1)*

  (a) lead to $n = n^{**}$ *if* $j^{**} \in (brown, green)$.

  (b) can improve welfare if $j^{**} = gray$.

Proof See Appendix (A).

The corollary underlines that the knowledge regime can affect the design of policy aimed at improving decentralized research decisions. While with specialization, there is no policy to reach $j^{**} = gray$, intentions to reach $j^{**} = green$ require the use of an environmental tax that follows the principles outlined in Corollary (2.2). Further, additional subsidy for green innovation, $T_{v,R}$, is necessary in case that the preexisting stock of adaptation knowledge is so large that the economy is locked-in. Beyond, all scenarios require the R&D policy introduced in Corollary (2.1), i.e. T_v for socially optimal research labor and T_p to eliminate price markups, both according to the patterns discussed.

In summary, if $j^{**} = gray$, policies can only lead to a second best scenario, while if $j^{**} \in \{brown, green\}$ policies are first best. With this knowledge at hand, it is promising to conclude the section with a summary of the above policy results and, on this occasion, consider an alternative policy recommendation that expands the scope of the policy discussion.

2.6.3 The role of education

Table (2.1) presents the policy-related core results of this theory, summarizing on whether there are externalities requiring policy action and whether policies can be first best.

	GK Regime		**SK Regime**	
	$\mathcal{I}=1$	$\mathcal{I}=0$	$\mathcal{I}=1$	$\mathcal{I}=0$
no env. externality	$\surd$	$\surd$ if j^{**}=brown $\times j^{**}$=gray, green	$\surd$ if j^{**}=brown, green & if no lock-in $\times$ if j^{**}=gray	$\surd$ if j^{**}=brown $\times$ if j^{**}=gray, green
no R&D externalities	$\times^{i}$ i=except in an arbitrary case	$\times^{i}$	$\times^{i}$	$\times^{i}$
policies first best	$\surd$	$\surd$	$\surd$ if j^{**}=brown, green $\times$ if j^{**}=gray	$\surd$ if j^{**}=brown, green $\times$ if j^{**}=gray

Table 2.1: Decentralized market characteristics and the potential of policy.

In this table, two features stand out: First, environmental policy is unnecessary as long as access to information and technology is not restricted. Second, research sector policies are necessary to internalize externalities of the monopolistic competitive R&D sector. While the second finding is consistent with standard literature, the first is not. Its fascinating characteristic is that access to information and technology internalizes environmental externalities. Henceforth, any measures that support such access have a comparable effect as tax-based environmental policy. This result puts non-market-based policies into the spotlight. A good example is investments in education infrastructure (schools, universities, and public research laboratories, etc.). However, there are other alternatives to improve knowledge exchange, such as simplified patent and cooperation agreements.

The promising feature with educational policies is that although they have considerable implementation costs, they have great potential to reduce income inequality, see, e.g., Biggs and Dutta (1999), Sylwester (2002), or Abdullah, Doucouliagos, and Manning (2015). In parallel, environmental taxes reduce household income and are well-known for their disproportional effect on lower-income groups, see, e.g., Büchs, Bardsley, and Duwe (2011), Fullerton (2017), or Edenhofer, Franks, and Kalkuhl (2021), what raises the question of a fair tax burden on higher-income groups.

In light of this, there are convincing arguments to assess the potential of education-based measures to improve the access to information and knowledge as alternatives to market-based environmental policy. A promising next step in this direction is a detailed cost-benefit analysis of both policy types, with a strong focus on distributional aspects. However, especially the latter assessment goes well beyond the economic growth discussion of this chapter and is likely better served with a distinct theory framework distinguishing income groups and alternative policy schemes. Such an analysis is, therefore, left to future research, so this chapter proceeds with a critical discussion of the theory.

2.7 Chapter discussion

This chapter demonstrates that economies that combine alternative technologies and access all available information find socially optimal environmental research, although production exhausts natural capital. Since these results differ from other literature findings, discussing their basis is crucial and the subject of this section.

An essential feature of this theory is that it assesses how the depletion of natural capital impacts production. Its specification is silent on how the environmental quality affects utility. Such effects are usually called 'amenity effects' (see, e.g., Shechter, 1991) and are commonly included in environmental growth models. This theory is not interested in consumer demand, but in technology supply. Henceforth, it assesses features which other theories would explain via amenity effects via production effects. This quality needs a more in-depth clarification.

To begin with, plenty of economic models consider amenity effects without giving households a choice between products with different ecological footprints. Such modeling typically leads to environmental externalities by default, as agents suffer from environmental damage but have no possibility to change the situation. A nuanced consumption-focused framework should give households a choice among a portfolio of goods with different environmental impacts.

This chapter considers such a choice, but shifts its focus from the demand side for consumer goods to the supply side of technology. Thereby, the production damages represent the environmental amenity effect[23], and the alternative technologies symbolize the consumption decisions. On this basis, the theory reveals that there are endogenous incentives for socially optimal environmental research. A framework with environmental amenity effects and households that choose between products with different ecological footprints could have led to similar insights.

Against this background, an important attribute of this theory are the explanations for environmental externalities. One is that agents do not have full access to information. As referenced in the above assessment, this rationale can be related to Stiglitz, 1985, who points at the moral hazard problem that the decisions of agents who lack information are usually too risky. In this chapter, a lack of information about the environmental impact of innovations leads to a risk of excessive pollution, which follows the same line of reasoning.

This chapter's second explanation for environmental externalities is technology-related lock-in effects. As well discussed in the literature, such lock-in effects create path-dependencies with considerable welfare effects, see, e.g., Fouquet (2016). This theory discusses such path-dependencies in the context of specialization and shows that they can lead to environmental externalities if a research direction is not socially optimal. This reference is helpful as it underscores the importance of being able to combine technologies when growth is prone to depletion of natural capital. Usually, the literature assesses specialization in the context of comparative trade advantages among economies (see, e.g., Laursen, 2015) or cities (see, e.g., Becker and Henderson,

[23]Note that many topics that are commonly associated with utility effects can also be analyzed via production effects. Examples are health effects or indirect consumption impacts (the environmental effect 'rescales' consumption-based utility).

2000). Hence, embedding this theory's specialization discourse into established trade theory provides a promising area for future research. Yet, such plans require an adaptation of this theory, since it describes a closed economy.

This chapter's closed economy perspective is helpful to shed light on endogenous incentives for socially optimal environmental research. However, it is critical whether a closed economy helps to understand all challenges with urban growth. Cities exchange goods and services with other regions. An important example is the employment of energy. While urban regions can supply several renewable energies, such as wind or solar power, they also import non-renewable energies, such as oil, coal, and gas (at least nowadays). The utilization of the latter energy source is pollution-intensive, whereby its suppliers do not experience all the environmental damage associated with their use. The subsequent chapter addresses exactly this moral hazard problem in depth. Although it presents a scenario where exogenous brown energy providers are the source for environmental externalities, this does not state that the findings of this chapter are not valid when applied to open economies. If an R&D sector in one region anticipates that environmental damages in another region affect intermediate sales, there is no qualitative distinction between a closed and an open economy. As will clarify with the ongoing discourse, the two chapters of this thesis simply present two alternative perspectives on this subject.

Another feature of this chapter is that the damages of natural capital depletion are proportional to production and thus represented via a flow variable. Literature often models climate change damages proportional to the stock of greenhouse gases, see therefore the discussion in Chapter (1). Still, this chapter's distinction between flow and stock representations is of subordinate importance, as it is straightforward to reinterpret this theory to discuss environmental degradation as a stock challenge instead of a flow challenge without changing the model. To be precise, this chapter assumes that pollution and related damages are proportional to production. Although production is proportional to technology stocks, such a specification describes a flow representation. A stock representation would require a dynamic pollution channel that aggregates to a pollution stock, which then causes damage. If the chapter had alternatively stated that pollution increases with the formation of manufactured capital so that the pollution stock is proportional to the capital stock, pollution damages were similarly proportional to the technology stock, but would then describe a stock representation. This example illustrates that it is mainly a question of the interpretation of the model and not the actual theory specification that distinguishes the stock and flow representation of environmental damage.

With this knowledge, it is exciting to consider which aspect has not yet been included in the assessment but would improve this chapter's evaluation. One is digitalization, which has become increasingly relevant, especially since the pandemic health crisis. Digitalization facilitates access to information and the application of new technologies. According to this theory, both are essential for internalizing environmental externalities. In this regard, digitalization may present an alternative to environmental policy or educational policy. Likewise, it is possible to interpret digitalization as an additional technology that rescales the efficiency of the technologies discussed, so it defines a promising avenue for future research.

To conclude this section, it is intriguing to consider alternative uses for this chapter's theory. One promising domain is patent protection. The above evaluation has emphasized that too strict patent protection can hinder access to innovation, what creates environmental externalities. In parallel, the chapter has also discussed that some patent protection is required to initiate research. It is possible to assess this tradeoff with a reinterpretation of the presented theory. Thereby, R could relate to improvements in the patent protection of individual products. Though, the higher R, the more difficult it is to combine products, reducing the ability of the economy to innovate, resulting in a damage denoted by $\bar{E}$. To ease such damages, G could represent efforts to circumvent the strictness of patent protection through clever technological adaptations not hindered by patent law. The greater G, the lower the damage caused by patent protection. This evaluation could be related to Schovsbo, Riis, and Petersen (2015) who assess the design of the Unified Patent Court (UPC) as the new patent judiciary for enforcement of European patents. For a more general discussion see further Lanjouw and Mody (1996) analyzing the role of patent protection for environmentally friendly technology and Haber (2015) for a general assessment on how patents affect innovation. Leaving this promising debate to future research, it remains to conclude the chapter.

2.8 Chapter conclusion

There were various fundamental discoveries in environmental economic growth theory that have greatly improved our understanding of the determinants of sustainable and socially optimal growth. However, the main focus of the debate seems on how policies should incentivize innovations to internalize environmental externalities. This chapter reveals that such a view can be misleading, as it disregards decentralized R&D incentives for initiating green research.

With a theory shift on the supply-side of innovations, this chapter shows that a decentralized economy finds a sustainable growth path with socially optimal environmental research efforts if two conditions are satisfied: (1.) there is extensive information on the environmental impact of economic actions, (2.) there is access to a technology portfolio that can combine general technologies with adaptation and abatement knowledge. Environmental externalities can emerge if one of both conditions is not satisfied, depending on the socially optimal research strategy. This strategy is subject to the elasticity of environmental damage, which describes the sensitivity of production to a production-related depletion of natural capital. As long as this elasticity is below unity, a weak sustainability criterion is relevant. In this case, the economy can compensate for the exhaustion of natural capital through general innovations, adaptation, and abatement. If the elasticity is above unity, strong sustainability is required. In that case, abatement efforts must increase at the same rate as general technologies to keep the representative natural capital stock constant. A Pigouvian-type environmental tax can internalize externalities related to a lack of information. Research subsidies help to address environmental externalities that come with restricted access to technology. In some scenarios, the subsidies internalize these externalities. In others, they reduce the related welfare costs.

Other externalities occur due to a monopolistic R&D sector. They affect both the fraction of researchers and the provision of intermediates. A calibration of the model demonstrates that these externalities lead to too little research and too little innovation. Yet, they neither influence the sustainability of decentralized growth nor the social optimality of the environmental research efforts. Further, price subsidies that reduce monopolistic intermediate price markups and research subsidies that attract sufficient research labor internalize these externalities. As a consequence, a combination of research market policies with environmental policies (if needed) is first best in most scenarios.

The chapter's assessment of the sustainability of decentralized growth and the social optimality of decentralized research is unique and in contrast to many environmental economic literature discussions. This theory's findings thereby shift the attention from policies dealing with a lack of property rights on natural capital goods to policies aiming to make information and technology widely available. Consequently, the chapter concludes by putting educational policies into the spotlight and calls for a more detailed analysis of the corresponding policy impacts. Practical policy recommendations in this direction include educational programs, less restrictive patent laws, better information networks, and open-access platforms for technology, to name a few. These policy proposals refine the rich discussion on the role of knowledge for economic growth and development (see, e.g., Van den Berg, 2016 for an overview) with an environmental perspective. As this debate is mainly a discussion of economic development and not a concern of the determinants for economic growth, it is not further addressed in this dissertation and left to future research.

In line with this, there are several areas where this chapter's theory can be improved or augmented, be it addressing open economies, detailing the role of patent protection for innovation, or including digitalization in its assessment. Hopefully, this chapter serves as the first step for further research in these promising directions.

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Chapter 3

The Power of Density

Abstract While urbanization has led to astonishing prosperity, there is little evidence about its sustainability. The positive effects of agglomeration economies involve the risk of overpopulation, congestion, and pollution. Therefore, it is necessary to understand the interplay between residence density and environmental quality, both public goods. So far, there has been no theory discussing their relation, so this chapter develops new theory. Its model demonstrates that without a green energy transition, either a pollution or a density externality jeopardize sustainable growth. If energy is brown and not scarce, its use triggers a growth-threatening pollution externality. If it is scarce, a scarcity rent raises energy-sensitive commuting costs, creating a growth-threatening density externality as people allocate to the center. The benefit of the latter scenario is that it will initiate a green energy transition. Still, if green energy includes nuclear energy, there may be arguments to remain in a brown energy scenario. Irrespective of the scenario considered, if needed, market-based policy is welfare superior to non-market-based policy while a holistic combination of policy instruments can be first best.

3.1 Introduction

The axis of the earth sticks out visibly through the center of each and every town or city.

Oliver Wendell Holmes (1895)

Urbanization challenges economies worldwide to organize a growing spatial concentration of activities. This venture is accompanied by an intensifying cluster of environmental damage and a rising scarcity of land. The benefits of urbanization are increasing productivity and income levels, but its consequences are grave, diverse, and show up in the rise of numerous indicators, be it rents, smog, or density and heat stress, to name just a few.

Intrigued by the potential of urbanization and alerted by the related environmental threats, institutions and governments have placed urban policy as a top priority on their agendas[1]. Accordingly, the economic literature puts many efforts into understanding the potential of urban agglomeration economies and goes to great lengths to comprehend transition paths towards environmentally friendly urban growth. Though, research usually separates the discussion on both subject areas. In addition, it has a tendency to associate agglomeration economies to urban size and not urban density. As a consequence, the economic discourse remains remarkably silent about the interplay of population density and the environment that this dissertation considers as a core determinant for whether urban growth is both sustainable and socially optimal.

This chapter, therefore, presents a new theory that combines endogenous environmental growth literature with urban economics in a simplified model in which the use of brown energy in production and commuting causes pollution. Meanwhile, the

[1]There are several prominent examples like the sustainable city objectives by the UNDP, the Sustainable Development Goal 11 about "sustainable cities and communities" by the United Nations General Assembly, the 'Cities & Climate Change' science conference of the IPCC in 2018, or regular events as the City Futures conference organized by the European Urban Research Association (EURA) and the Urban Affairs Association (UAA).

interaction between population density-related network effects and pollution determines the quality of agglomeration economies and thus the sustainability and social optimality of urban growth. This type of modeling is new and reveals that without a green energy transition, there is either a dynamic environmental externality or a dynamic density externality that jeopardizes sustainable urban growth. Beyond, any growth path suffers from biased land, labor, and research effort allocations, causing a considerable welfare effect. In this direction, the theory elaborates that several market-based policies can achieve a growth scenario that is first best.

These findings are derived with the following structure: Section (3.2) browses through the literature to better understand the interaction of density with the environment and their role for sustainable urban growth. The theory is then discussed in Section (3.3). Section (3.4) presents the model, Section (3.5) general equilibrium conditions, Section (3.6) decentralized equilibrium properties and growth. Section (3.7) elaborates the social planner result, Section (3.8) presents numerical solutions, while Section (3.9) details the design of the holistic policy approach for socially optimal growth. Section (3.10) discusses these findings, while Section (3.11) concludes.

3.2 Literature

Although sustainable urban development has made its way into the United Nations General Assembly' 11^{th} Sustainable Development Goal (SGD), there is surprisingly little economic research that deals with sustainable urban growth specifically. The environmental economic literature has an interest in urban growth, but hardly discusses the related problems, with a particular focus on the concept of sustainability. Regardless, discussions about 'urban sustainability' are popular in the non-economic literature, especially in urban planning and development and regional studies, see Kooshki, Shokoohi, Bazvand, et al. (2015), Kee et al. (2019), MacLennan and Stacey (2016), for an overview see further Bibri and Krogstie (2017).

An early part of the urban planning literature began to discuss sustainability in connection with *Compact Cities* by emphasizing the advantages of compactness, arguing that denser cities have lower energy consumption per capita and less pollution from commuting, see Breheny (1992), Allmendinger, Prior, and Raemaekers (2000), or Haughton and Hunter (2004) for details.

A little later, sustainability started being taken into consideration within a broader discussion about *Smart Cities*. This buzzword was already referred to in Chapter (2) and describes the intelligent organization of urban activities to promote the development of the city. The literature on Smart Cities is interested in practical support for sustainability. Examples include the potential of green e-mobility or the efficient coordination of urban traffic flows, see e.g. Angelidou et al. (2018).

In principle, the Smart City concept combines aspects of technical improvements with agglomeration economies. These two features are the core determinants for urban development, and hence a good orientation on how the economic literature addresses sustainable urban growth subject areas. Since the role of technology in this discourse was highlighted in the last chapter of the dissertation, this chapter concentrates on literature on agglomeration economies, with a focus on density. Hereby, three areas are relevant: (1.) the role of density in economic theory, (2.) the interrelation of density and natural capital, and (3.) the role of density for sustainable urban growth. All three qualities are addressed in depth in the following.

3.2.1 The role of density in the economic literature

Economists assume the grounds of the attraction of cities are agglomeration economies, introduced by Marshall et al. (1920). They describe increasing returns achieved with a spatial concentration of economic activities and are commonly explained by the advantages of tight value chains, labor market matching, and network effects, see e.g. Ciccone and Hall (1996), Baptista (2003), De Groot, Poot, and Smit (2009) for an introduction. Agglomeration economies are often related to city size (e.g. population size) while density is given little attention.

The importance to consider density

The size of a city is undoubtedly the dominant force for industry-specific value chain effects on a macro, meso, and micro level as larger cities unite a larger number of different industries, specialized companies, and skilled employees. Regardless, population density is the driving force behind network-related agglomeration advantages, see e.g. Cheshire and Magrini (2009) and Andersson and Larsson (2016) for empirical evaluations, and Ahlfeldt and Pietrostefani (2019) for an overview on this discourse.

Notably, the literature has been discussing the importance of density for decades, but usually examined the matter indirectly and mainly in a static context. In the 70s, J. V. Henderson (1974) was among the first to emphasize that cities can be too big to be efficient so that the benefit curve of agglomeration concentration is hump-shaped. This finding is in line with Hagen (1975), who collected crude empirical evidence that income growth and density are positively correlated up to a critical level, while very high densities can lower income growth rates. Similarly, Fujita (1989) suggests that the average returns of a city concerning its (population) size are single-peaked, a characteristic a little later also suggested in Duranton and Puga (2004). The problem with this literature is that it does not take into account the actual physical dimension of urban population or construction density.

The physical dimension of density

Physical density measures the mass per volume unit, and population density relates to the number of persons in a particular area. While the economic theory is interested in the mass, it usually ignores the volume, respectively, the land where economic activity occurs. This ignorance is not only a concern for urban economic theory, but for growth theory in general, as Gaffney, 2008, and Homburg, 2014, critically point out. Individual industries, firms, and professionals need to be connected via space. The further away they are, the more inefficient this connection is due to opportunity costs. At the same time, concentration leads to congestion costs, see e.g. Pines and Sadka (1985), Brueckner (2000), Anas and Rhee (2006), or Swapan and Khan (2019) for an overview of the empirical literature. Therefore, a sensitive assessment of space is indispensable[2], see Wheaton (1998) for a discussion.

[2]Note that land is not only relevant with regard to the density dimension. The omission of land in economic theory distorts the production functions and, consequently, the marginal product of the production factors. This effect biases the market clearing, especially when production factors

Space is also crucial for research and development. For instance, Carlino and Saiz (2019) find that 2,200 jobs per square mile lead to the highest rates of innovation in the US, indicating literal stepping-on-toes effects. Standard endogenous growth theory interprets such effects metaphorically, implying that the more researchers, the more similar the ideas (see Jones and Williams, 2000). Yet, there is a critical point where the more researchers in a research lab, the less efficient their work. The labs can be too small to use all the necessary research tools, the researchers may simply be less productive under the exhausted air quality in their crowded offices, plenty of explanations are possible. In light of all this, Adam Smith's famous invisible hand (Smith, 1776) should possibly be complemented by invisible feet, since economic potential depends on the space on which it stands.

3.2.2 Density and natural capital

The trade-off between network and congestion effects motivated Chapter (1) to interpret density as an abstract form of an environmental sink, describing the capability of a place to accommodate economic activity. If cities become too dense, this ability is exhausted. Notably, the density sink interacts with other environmental sinks like environmental quality or nature's ability to accommodate greenhouse gases. Several complex interactions need to be considered. For instance, denser cities have shorter commuting distances, a quality that technically reduces pollution as long as there is no counterfactual congestion-related pollution (e.g., traffic jams). Furthermore, densely populated areas expose large numbers of citizens to a certain level of pollution, which Eriksson and Zehaie (2005) refer to as the 'reach of pollution'. That being the case, there is an interaction between density and natural capital which will shape the sustainability of urban growth. A theory specifically addressing this interlink is lacking, what motivates this dissertation to specify a new framework. Nonetheless, there are several promising articles investigating the interaction of urban economic activity with natural resources. Although this chapter will not follow their approach,

allocate over different sectors with different land-intensive production. Since a bias in the marginal product of capital affects the savings rate, ignoring land creates both static and dynamic biases.

some of those articles serve as a rough orientation and are introduced subsequently.

3.2.3 The city and the environment

A way literature has investigated the role of environmental quality on urban production and growth relates to endogenous city formation models. Thereby, agents move location or create new cities when the agglomeration benefits reach critical levels, see Black and V. Henderson (1999), Rossi-Hansberg and Wright (2007), or Rossi-Hansberg, Sarte, and Schwartzman (2019) for general applications of these models. Quaas and Smulders (2018) use such a model to demonstrate that along a brown growth path, characterized by pollution-intensive production, too few and too large cities arise, leading to increased pollution and socially suboptimal growth rates. By contrast, a green growth path can achieve socially optimal growth rates with a reduced number of cities and reduced pollution. Evidently, the relevant message of these models is their implicit discussion about the optimal city size. In reality, the abandonment of individual cities is associated with social barriers and high capital depreciation and is, thence, a rare event. Furthermore, the establishment of new cities can hardly be observed (especially in the West).

Another promising urban research field refers to the Fujita and Ogawa (1982) theory on the spatial location of businesses and households. This basis was used in Arnott, Hochman, and Rausser (2008) to evaluate the distribution of industrial and residential areas given the costs of commuting-related pollution. Kyriakopoulou and Xepapadeas (2017) expand this assessment to weight pollution and production externalities with commuting costs. Their focus is on the socially optimal allocation of scarce land, considering that land use and commuting affect environmental pollution.

One of the rare articles that directly examine the interaction of density and pollution with local growth rates is by Eriksson and Zehaie (2005). The authors use a model from Copeland and Taylor (1994) to discuss how perceived pollution affects the rate of growth, taking into account the effect of population density on pollution damage. While they even propose a one-sector semi-endogenous growth theory with a focus on a social planner economy, they do not discuss the direct impact of density on innovation, a subject this work is particularly interested in.

In all, assessing how the interaction of density with natural capital affects sustainable growth requires the development of a new model. The literature discussed offers a good orientation that helps specify new theory, as explained subsequently.

3.2.4 Synthesizing results

In a nutshell, the above literature identifies two characteristic qualities of urban growth. First, a decline in the quality of the environment reduces urban growth. Second, density has a positive effect on production and growth until it achieves a critical level. These findings can be summarized to two assumptions that will be fundamental for this chapter's model specification:

Assumption 1. *A reduction in the urban environmental quality reduces urban production growth.*

Assumption 2. *The impact of central urban population density on urban production growth is hump-shaped.*

The next section will discuss the difficulty of validating these assumptions (and urban economic growth theory in general) with accessible data. The section will also show that existing data reveal correlations that comply with the two.

3.3 Empirical basis

This chapter is interested in modeling the urban density profile within a monocentric city model. This model is detailed in the next section and assumes that the innovation potential of a city can be located in the central business district (CBD) while households settle around. The interest is then in the building density and environmental quality in this center. However, specific data on the CBD characteristics is not available. It is, therefore, neither possible to accurately calibrate the model nor to validate Assumptions (1) and (2). Yet, a later calibration will refer to the data available, so it is essential to discuss them and emphasize their restrictions. Such an evaluation will follow subsequently. Afterward, this section presents correlations among the characteristic indicators for Assumptions (1) and (2) and discusses how to interpret them in the context of this thesis.

3.3.1 Accessible data

Because of the country's size, the homogeneity of its cities in terms of socio-economic indicators, and the heterogeneity in terms of pollution, density, and productivity growth rates, United States data are most useful for validating urban theory. While they are relatively large, there is no systematically collected data on the city center, to begin with. The most detailed resolution comprises the entire city and is provided by the US Census Bureau. These data are not only difficult to classify as there is no clear definition of a city (see Batty and Ferguson, 2011), but they are also only available for population density, not for productivity and pollution. One option is to consider data on Metropolitan Statistical Areas (MSA) defined by the United States Office of Management and Budget (OMB), which encompass 392 regions and understand a metropolitan area as a region with at least one urban core of at least 50'000 residents. These areas are characterized by communities with a high degree of economic and social integration into this core. An alternative is data by the OECD on functional urban areas (FUAs) composed of a city and its commuting zone[3]. Among the 167 FUAs, 100 are indexed as monocentric. Nevertheless, both city references are too large to draw any conclusions about the CBD density[4].

Further, the available time series are too short and too volatile to make reasonable predictions about the long-term trends this theory is interested in. The collection of data started during a period of an economic boom in early 2000, followed by a deep recession in 2007 and a resulting strong recovery driven by central banks' low-

[3] The classification is based on census tracts, whereas the city boundaries and commuting zones adapt to county boundaries, see https://stats.oecd.org for details.

[4] An example is the Los Angeles (LA) area where the FUA states a population density of roughly 200 people per km^2, which is below the average density of all (available) FUAs of around 300 people per km^2, although around 9,000 people per km^2 live in central areas like Maywood. Hence, the LA center has one of the highest population densities in the entire US.

interest policies that have pushed market indexes to record highs. The variation in data patterns is hence rather shock-driven than trend-based. Figure (3.1) shows real GDP per capita growth rates (in USD, the base year 2015) from 2001 to 2017, of the five largest FUAs indicated as monocentric (Chicago, Dallas, Houston, Atlanta, and Phoenix). As can be seen, the yearly growth rates (connected by lines for a better visibility) are subject to considerable fluctuations in these years. Figure (3.2) exhibits

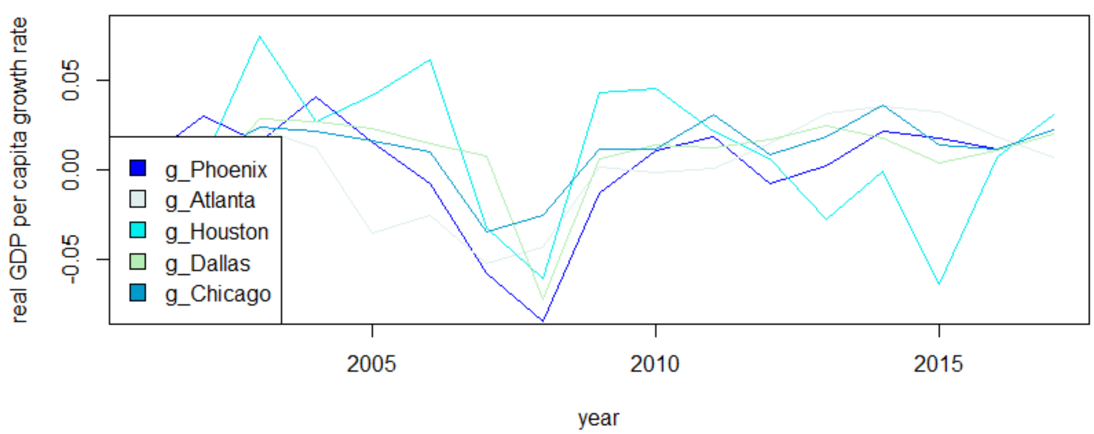

Figure 3.1: Yearly real FUA GDP growth rates from 2001 to 2017. Data: US FUA units defined by the OECD, accessible (as by April 25, 2021) via https://stats.oecd.org.

the corresponding population densities. Notably, these densities are all increasing (at different rates). This theory will infer that sustainable growth requires a stable urban density profile, so the current data basis would also be insufficient if more detailed data were available.

A further problem is that there is no sufficient data for robust estimates. On the one hand, the history of urban economic growth is determined by the development of specific industries, modes of transport (rail systems, ports, etc.), and various geographical components such as natural resources or the local climate, see Giersch (2012). Therefore, a distinction is made between agglomeration effects and regional effects, see Ahlfeldt and Pietrostefani (2019) for details. Nonetheless, such regional

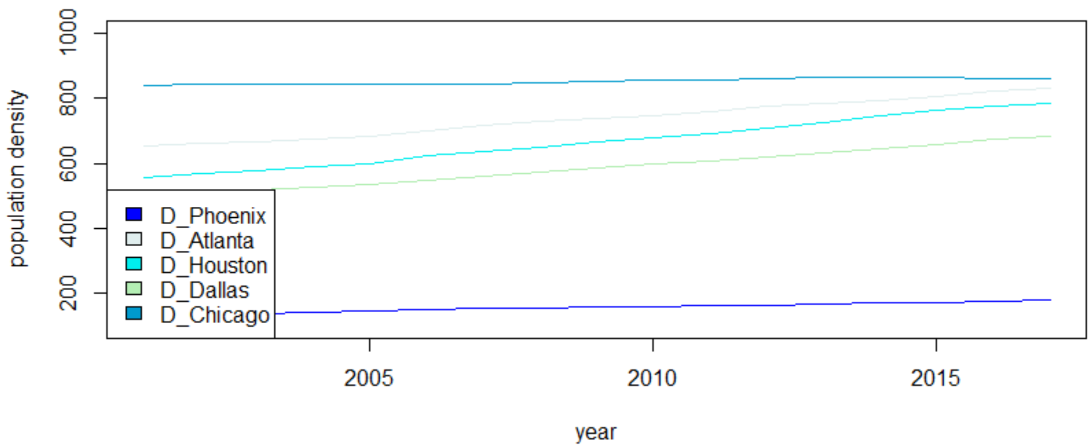

Figure 3.2: FUA population density (per km^2) from 2001 to 2017. US FUA units defined by the OECD, accessible (as by April 25, 2021) via https://stats.oecd.org.

data is not systematically collected, making it impossible to control for regional fundamentals. Similarly, when considering the impact of density and pollution on productivity growth rates, there is a severe endogeneity problem with regard to both variables, so it is necessary to find suitable instruments. While geographic factors or historical data could be captured with great effort (see, e.g., the work by Combes, Duranton, Gobillon, and Roux, 2010), the quality of such approaches remains questionable.

In any case, this dissertation is interested in a theoretical, not an empirical discussion. The section, therefore, concludes with a quick look at how production correlates with pollution and density, which is relevant for Assumptions (1) and (2).

3.3.2 Correlations in urban core indicators

To obtain a first impression of how the available data support Assumptions (1) and (2), this subsection plots productivity against density and pollution. In the spirit of *post hoc ergo propter hoc*, possible correlations do not state causations, so the plot only gives an ad hoc idea on potential relations among economic indicators.

Figure (3.3) plots OECD data on real GDP per capita (as a proxy for productivity, in USD, the base year 2015) against urban area (in m^2) per person (indicating density), with the GDP level on the left and the GDP growth rates on the right. The sample exhibits data for 2014 as they are the most recent for that this density information is available. The plot only includes the 100 FUAs that the data set indicates as monocentric.

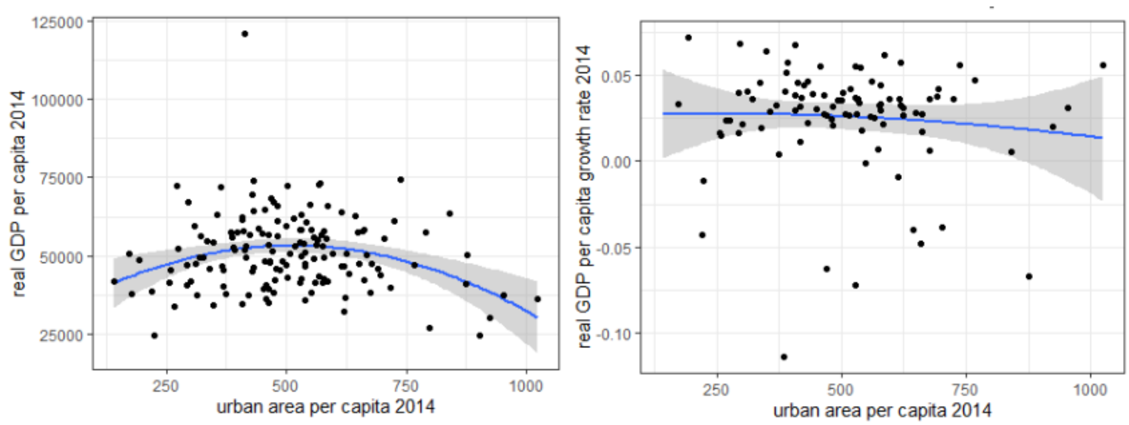

Figure 3.3: Local real GDP per capita (left) in USD in 2014 and real GDP growth rate from 2014 to 2015 (right) on the urban area (m^2) per capita in 2014. Data: US FUA units defined by the OECD (accessed on April 25, 2021, via https://stats.oecd.org.)

In Figure (3.4), the same GDP representatives are plotted against population density, measuring population per km^2 in the metropolitan region, again in the year 2014 for comparability. Both figures contain a 2^{nd} degree polynomial curve approximation. As can be shown, this curve describes the data better than a linear or logarithmic approximation. In both cases the curve is hump-shaped, which (ignoring serious endogeneity concerns) is in line with Assumption (2).

Figure (3.5) plots the average population exposure to fine particles (counted as PM_{25} in ug/m3) as a proxy for pollution against the GDP representatives. Again, based on the OECD FUA data from 2014 for reasons of comparability. While this data is rather noisy, it also indicates that a hump-shaped 2^{nd} order polynomial approximates the data best. At first sight, the correlation at low pollution levels con-

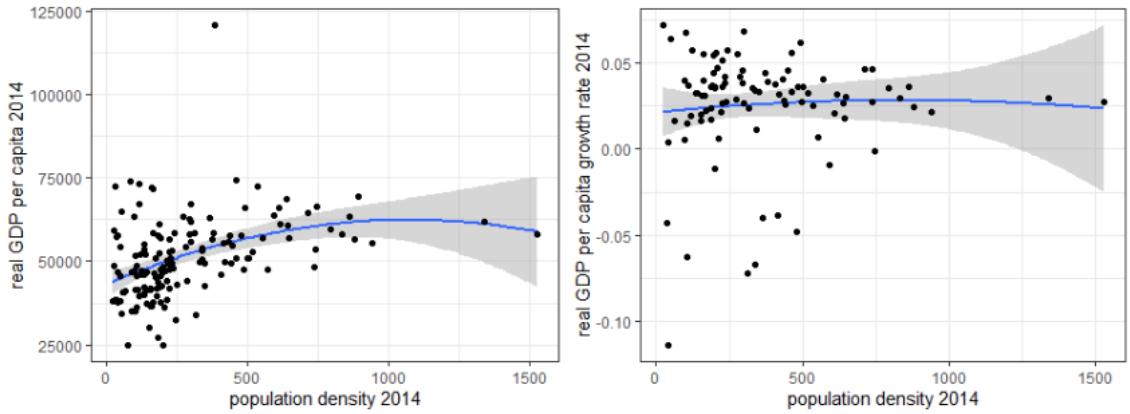

Figure 3.4: Real GDP per capita (in USD, base year 2015) on the left and the real GDP growth rate from 2014 to 2015 on the right, plotted on population density (citizen per km^2) in 2014, for US FUA units (accessed by April 25, 2021, via https://stats.oecd.org).

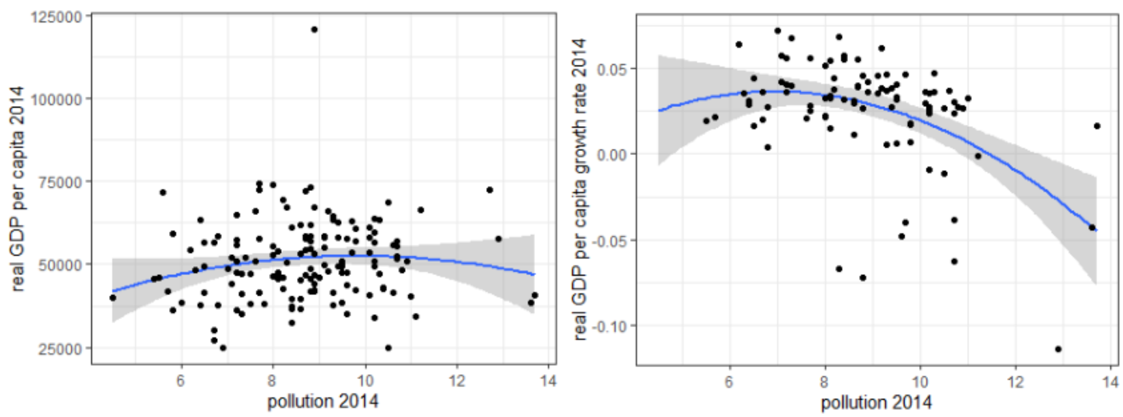

Figure 3.5: Real GDP per capita (in USD, the base year 2015) on the left and the real GDP growth rate from 2014 to 2015 on the right, plotted on average exposure of the population to fine particles (counted as PM_{25} in ug / m3) in 2014, for US FUA units (accessed by April 25, 2021, via https://stats.oecd.org).

trasts Assumption (1) and is hard to explain by intuition. A likely explanation for this observation is that it is related to the environmental Kuznets curve. This curve states that pollution-intensive activities initially have a positive effect on productivity (and production growth) until the pollution is so intense that damages predominate, see, i.e., Dinda (2004), Agras and Chapman (1999), or De Bruyn, Heintz, et al.

(1999). If this were the case, the data would not contradict Assumption (1). In any case, it is not of great value to speculate about these properties with the available data. Henceforth, the theory accepts that correlations do not state causation and accordingly follows the above-cited literature evidence. It, therefore, bases its specifications on Assumption (1) and (2), as described in depth in the following.

3.4 The theory

This theory combines a monocentric city model from Alonso et al. (1964), Mills (1967), and Muth (1969) with an endogenous version of a Ramsey-Cass-Koopmans economy in the tradition of a Romer (1990) horizontal product variety setting. For a well-structured discussion, growth is initially treated as exogenous but endogenized afterward. Following the corresponding literature, the economy is closed, symmetric, and spreads out from a central business district (CBD) over a width of $\hat{l}$, only illuminating one side of the spatial[5] land distribution.

There is a numéraire good that represents the entire urban production of goods and services, a real estate sector that provides apartments for citizens, and a hinterland which either represents nature or non-urban output such as manufacturing or agrarian production. The numéraire is produced in the CBD, while the citizens live around it up to the city fringe $\hat{l}$. In the tradition of the monocentric city model, the CBD area is dimensionless and coordinated at $l = 0$, the residents then occupy a width of $l \in (0, \bar{l}]$, after which the hinterland extends on the remaining area $l \in (\hat{l}, \bar{l}]$, as illustrated in Figure (3.6).

There is a brown energy sector and a green energy sector, both offer energy for production and commuting. Beyond, a local government can provide abatement and commuting infrastructure, levy taxes, or finance subsidies. All land belongs to the households that assign it through two types of auctions. First, a hinterland-to-city auction that determines the expansion of the city, $\bar{l}$. Second, a land auction within the city which follows Alonso et al. (1964) as urban settlers bid for land in sensitivity

[5]There are also circular specifications, e.g., in Arnott, Hochman, and Rausser (2008) and Lucas and Rossi–Hansberg (2002). Howbeit, they have no significant qualitative impacts.

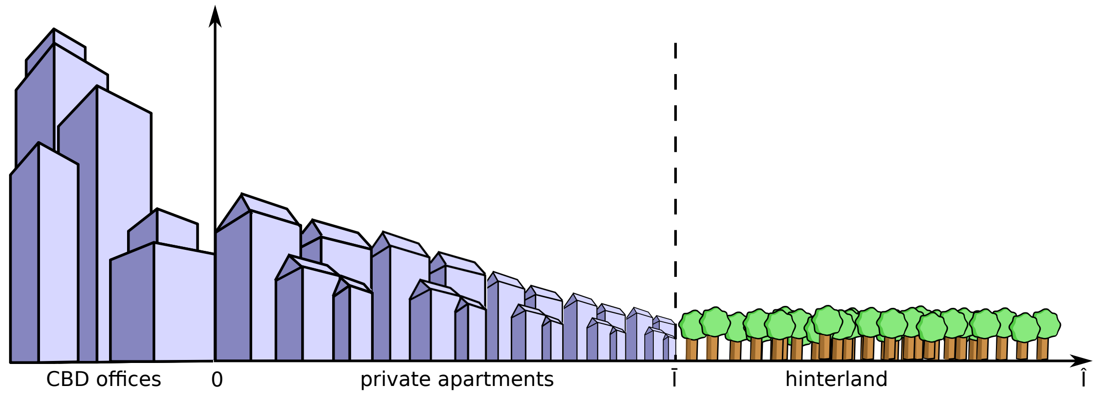

Figure 3.6: Illustration of the spatial organization of the economy.

to the local commuting routes. The model is solved on a per capita basis, specifies only key features, and omits time indexes whenever possible.

3.4.1 Households

There is a representative household that consumes a numéraire , c, apartments, a, and a hinterland good, x. The household's consumption-based utility is described by

$$U(t) = \int_{s=t}^{\infty} e^{-(\rho - n(s))s} ln\Big(\bar{c}(s)\Big) ds, \tag{1}$$

with

$$\bar{c}(s) = c(s)^{\epsilon} a(s)^{\iota} x(s)^{1-\iota-\epsilon} \tag{2}$$

as a consumption aggregate that weights preferences with $\epsilon > 0$ and $\iota > 0$, whereby $1 > \epsilon + \iota$. Further, $\rho - n > 0$ discounts future consumption by $\rho > 0$, representing standard time preferences, and $n \geq 0$, the population growth rate. Each household earns a wage, w, a land rent income, $L = \int_0^{\bar{l}} \beta(l)\bar{\omega} dl + \bar{\omega}(\hat{l} - \bar{l})$ (detailed later), and an interest r on assets S. The latter is associated with manufactured capital, k, that can be lent to the real estate sector, k_a, and numéraire sector, k_y, so that

$S = k = k_a + k_y$. Given this, the household income, $\hat{I}$, reads

$$\hat{I} = (r-n)S + (1-T_w)w + L + G \tag{3}$$

where $r-n$ adjusts for an effective return on savings, G represents the government budget discussed later and T_w represents a wage tax. Household income is either consumed or invested. Investments are denoted by $i = \dot{k} = \frac{\partial k}{\partial t}$. Hence,

$$\hat{I} = C + i, \tag{4}$$

$$C = (1+T_h)p_a a + (1+T_c)c + (1+T_x)p_x x \tag{5}$$

denotes aggregate consumption expenditures. Thereby, p_x prices the hinterland good, p_a apartments, while T_c, T_a and T_x refer to taxes on consumption, apartments, and the hinterland good, respectively. Household optimization leads to the inter and intra Euler equations[6]

$$\frac{\dot{\bar{c}}}{\bar{c}} = g_{\bar{c}} = r - \rho, \tag{6}$$

$$(1+T_c)\frac{c}{\epsilon} = (1+T_a)\frac{p_a a}{\iota} = (1+T_x)\frac{p_x x}{(1-\iota-\epsilon)}. \tag{7}$$

3.4.2 Production

Production is perfectly competitive. A labor share $1-\lambda \in (0,1)$ allocates to numéraire production, the remaining $\lambda \in (0,1)$ allocates to the real estate sector. In both sectors, capital is net capital so that depreciation is not further addressed.

Urban production

The numéraire good is produced in the city center with a Cobb-Douglas-technology using a labor fraction, $1-\lambda$, manufactured capital, k_y, and energy, e_y, according to

$$y = (1-\lambda)^{1-\varkappa-\varepsilon}\mathcal{A}^{1-\varkappa-\varepsilon}k_y^{\varkappa}e_y^{\varepsilon}. \tag{8}$$

[6]With a no Ponzi Game condition $\lim_{t^{‘}\to\infty} S(t^{‘})e^{\int_{s=t}^{t^{‘}}(r(s)-n)ds} \geq 0$ and initial assets $S_0 > 0$.

Thereby, $\mathcal{A}$ describes the general technology level and $\varkappa > 0$, $\varepsilon > 0$ elasticities, whereby $1 > \varkappa + \varepsilon$. Production can either use clean, *green* energy, $e_{y,g}$, or dirty, *brown* energy, $e_{y,b}$. Both are perfect substitutes. Hence, $e_y = e_{y,g} + e_{y,b}$. There is an environmental tax, T_e, on brown energy, so the production accounting identity reads

$$y = w(1-\lambda) + (1+T_r)rk_y + p_{e,g}e_{y,g} + (1+T_e)p_{e,b}e_{y,b} \tag{9}$$

with $p_{e,g}$ as the green energy price, $p_{e,b}$ as the brown energy price. The F.O.C. yield $k_y = \frac{\varkappa y}{(1+T_r)r}$, $w = \frac{(1-\varkappa-\varepsilon)y}{(1-\lambda)}$ and $e_y = max\{\frac{\varepsilon y}{(1+T_e)p_{e,b}}, \frac{\varepsilon y}{p_{e,g}}\}$ (where "max" indicates the cheaper energy being selected).

Real estate

A real estate sector produces apartments, a, for citizens. This provision depends on the spatial willingness to pay (WTP) and the available construction technology. The focus is on commute-related energy demand as a representative for commute-related opportunity costs. These expenditures then determine the WTP for a specific real estate unit at a specific spatial coordinate[7]. Urban theory often refers to iceberg-type commuting costs which reduce the WTP for real estate in proportion to commuting expenditures, see Samuelson (1954), Puga and A. J. Venables (1996), and Fujita, P. R. Krugman, and A. Venables (1999). Further, it has become popular to describe urban characteristics with power-law relations, see Bettencourt et al. (2007), Shalizi (2011) and Sarkar (2019). While there is some ambiguity as to whether this can be applied to urban energy use (see Bettignies et al., 2019), power-law relations considerably simplify the analytical tractability. Therefore, the commuting energy demand is described with $e_{\bar{c}\bar{c}}(l) = \frac{\varsigma_{\bar{c}\bar{c}} l^\theta}{\mathcal{I}}$, with $\theta > 0$, $\varsigma_{\bar{c}\bar{c}} > 0$, where $l \in (0, \bar{l}]$ denotes the distance to the CBD while $\mathcal{I}$ denotes a commuting technology. This leads to

$$p_a(l) = \frac{p_a \mathcal{I}}{(1-T_{\bar{c}\bar{c}})p_e \varsigma_{\bar{c}\bar{c}} l^\theta} \quad where \;\; p_e = \in \{(1+T_e)p_{e,b}, p_{e,g}\} \tag{10}$$

[7]This does not give a full picture of commuting expenditures, but helps to raise awareness regarding the role of energy for spatial density profiles. For a broader view on commuting costs see, e.g., Combes, Duranton, and Gobillon (2017) and Delloye, Lemoy, and Caruso (2020).

with p_a as the gross housing price. Commuting costs rise with the energy price and reduce with the commuting technology, $\mathcal{I}$, and commuting subsidies, $T_{\bar{cc}}$.

Since the value of real estate (e.g., quality, style, maintenance costs, etc.) is difficult to assess, the urban literature usually interprets prices and quantities of apartments as latent variables and focuses on the production function of housing services, see Epple, Gordon, and Sieg (2010), Duranton and Puga (2015), or Combes and Gobillon (2015) for details. As a result, there is relatively little agreement on how a morphological real estate production function should be modeled. This theory refers to urban planning articles by Ball et al. (2018), El-Haram and Horner (2002), and Bromilow and Pawsey (1987), who show that a broad variety of determinants affect housing supply decisions. To simplify the discussion, it is then suggested that the provision of real estate is similarly capital-intensive as labor-intensive (examples are tasks related to finance, physical construction, maintenance, administration, and legal issues). As a consequence, this theory assumes that capital and labor are strong complements in real estate provision while there is managed capital called *morphological capital* described by

$$b(l) := \min\{\varsigma_a \lambda(l), k_a(l)\} \tag{11}$$

costing $p_b = w + \frac{(1+T_r)r}{\varsigma_a}$ per unit with $\varsigma_a > 0$ as a technology parameter. In aggregate, the morphological capital intensity reads $b = \int_0^{\bar{l}} b(l)dl = \int_0^{\bar{l}} \min\{\varsigma_a \lambda(l), k_a(l)\}dl$, whereby $\lambda = \int_0^{\bar{l}} \lambda(l)dl$ and $k_a = \int_0^{\bar{l}} k(l)dl$. The theory then follows Epple, Gordon, and Sieg (2010), Duranton and Puga (2015), and Combes and Gobillon (2015) to describe the local apartment production technology with[8]

$$a(l) = \mathcal{B}b(l)^{\gamma} \;\; \textit{with} \;\; \gamma \in (0,1), \;\; \mathcal{B} > 0, \tag{12}$$

so that the real estate break-even condition reads

$$p_a(l)a(l) = (1+T_b)p_b b(l) + (1+T_l)\omega(l) \qquad \forall \;\; l \in (0, \bar{l}) \tag{13}$$

[8]Where $\gamma \in (0,1)$ accounts for the findings by DiPasquale and Wheaton (1992) that construction and sustainment costs are increasing with the intensity of local construction.

with T_b as a tax on morphological capital (e.g. a real estate investment tax), T_l as a tax on urban land, and

$$\omega(l) = \beta(l)\bar{\omega} \tag{14}$$

as a spatially sensitive land rent which scales a fringe rent, $\bar{\omega}$, in proportion to the spatial bid rents, $\beta(l) > 1$. Since the F.O.C. w.r.t. $b(t)$ yields $\gamma p_a(l)a(l) = (1+T_b)p_b b(t)$, the WTP for inner urban land stabilizes at $p_a(l)a(l) - (1+T_b)p_b b(l) = (1-\gamma)p_a(l)a(l) = \beta(l)\bar{\omega}$. If $\beta(l)$ were higher, a real estate producer would have an incentive to produce at the fringe, whereas if $\beta(l)$ were lower, free markets would attract further bids from other providers until the break-even condition holds. That creates constant expenditure shares among morphological capital and land, with the consequence that

$$b(l) = \left(\frac{\gamma p_a \mathcal{B}}{\bar{p}_e(1+T_b)p_b l^{\theta}}\right)^{\frac{1}{1-\gamma}} \tag{15}$$

with $\bar{p}_e = \frac{(1-T_{\bar{c}c})(1+T_e)p_{e,b}}{\mathcal{I}}$ if $e = e_b$ and $\bar{p}_e = \frac{(1-T_{\bar{c}c})p_{e,g}}{\mathcal{I}}$ if $e = e_g$, whereas

$$\beta(l) = \frac{(1-\gamma)}{\gamma}\frac{(1+T_b)p_b}{(1+T_l)\bar{\omega}}b(l). \tag{16}$$

So the higher the land price, the higher the morphological capital intensity. Note that the willingness to pay for real estate affects real estate revenues. These revenues determine the budget for morphological capital and land. Such budget reflections scale the exponent of l. While regions at a greater distance from the CBD suffer from more severe commuting costs, as measured with θ, there is also a lower expenditure share to pay on land, which is represented by $1-\gamma$. As commuting taxes, commuting technology, housing production technology, and energy prices affect the net real estate price proportionally, they scale the marginal product of morphological capital in the same proportion as the land price. The real estate production function, therefore, turns linear homogeneous in b and $\bar{l}$. Hence,

$$a = b^{\gamma}\bar{l}^{1-\gamma}\Gamma_a\Lambda_a \ \ \textit{with} \ \ \Gamma_a := \left(\frac{1}{1+T_b}\right)^{\gamma}, \ \ \Lambda_a := \frac{\mathcal{B}(1-\gamma)^{1-\gamma}(1-\gamma-\theta)^{\gamma}}{(1-\gamma-\gamma\theta)}. \tag{17}$$

This expression is worth being discussed in more depth as it is a core determinant for the decentralized density profile. First, the denominator of the parameter conglomerate Λ_a accounts for that the decreasing returns in local real estate provision have to be sufficiently high to motivate expansion. There is consequently only real estate provision if $\frac{1}{1+\theta} > \gamma$, a condition which holds by assumption[9].

Then, for the use of morphological capital and land, expression (17) accounts for that the local morphological capital intensity $b(l)$ suffers from decreasing returns in horizontal construction, as measured via γ. A real estate producer generates more real estate when it spreads its morphological capital over a larger area and utilizes relatively low morphological capital at each location. However, since this reduces the local morphological capital expenditures, it increases the bids for land since the sector is perfectly competitive so that new providers will enter as long as there are profits to generate. This bidding effect is considered with $1 - \gamma$. As a result, the spatial commuting costs affect the willingness to pay for a housing unit at *each* location so that they scale the real estate sector's expenditure shares on morphological capital and land in the same proportion. Henceforth, not only the land use is affected by commuting costs but also the morphological capital intensity. The intensity of b and $\bar{l}$ is then subject to relative prices and addressed in depth in Section (3.6).

Hinterland production

The hinterland can either be interpreted as a sector that provides ecosystem services or as a land-intensive agricultural or manufacturing sector where technology, capital, energy, and labor are of little importance so that its provision is approximated by

$$x = (\hat{l} - \bar{l}). \tag{18}$$

Consequently, $(1 + T_x)p_x = \bar{\omega}$ because the hinterland price is identical to the real estate rent at the fringe.

[9]If this criterion were not met, it would be more efficient to vertically expand real estate. Hence, the higher the energy-cost elasticity of commuting, θ, the higher the degree of decreasing returns necessary to make expansion attractive.

Energy

Energy can either be ‘brown’, e_b, or ‘green’, e_g. The former relates to energy from non-renewable sources such as gas, oil, or coal, and its utilization causes harmful pollutants. The latter relates to renewable energy based on wind, geothermal sources, hydropower, solar radiation, and nuclear power[10]. Brown energy is exogenous and costs an exogenous price, $p_{e,b}$, which is differentiated later. Green energy requires installing power plants and costs $p_{e,g}$ per unit. In aggregate, the plants generate energy proportional to $\mathcal{E}$, thus $\mathcal{E} = e_g$.

3.4.3 Agglomeration economies

This theory will focus on how the quality of agglomeration economies affects the dynamic dimension of the economy as that dominates in the long term and represents the biggest challenge for spatially organized (growing) economies. The attention is on the *quality* of agglomeration economies, which combine density-related network effects with pollution-related environmental damage. In order to evaluate the network effects in the CBD, it is assumed that the higher the apartment density, the higher the construction density at the CBD[11]. Therefore, $D = \frac{a}{l}$, accordingly

$$D = \left(\frac{b}{\bar{l}}\right)^{\gamma} \Gamma_a \Lambda_a. \tag{19}$$

With this focus on average density, the relation reveals that a (convex) set of $\{b, \bar{l}\}$ pairs leads to the same density profile. In any case, pollution is described with

$$P = \frac{e_{y,b}}{\mathcal{F}_Y} + \frac{e_{\bar{c}c,b}}{\mathcal{F}_{\bar{c}c}} \tag{20}$$

[10]Nuclear power can also be interpreted as brown due to radioactive waste, see later.

[11]The challenge with this assumption is that the construction intensity at the CBD is not further specified. An increase in the apartment demand increases the apartment size for each citizen. Though, a previous version of this theory (available on request) also modeled the CBD and assumed that each citizen requires a fixed amount of office space. While the remaining specification stayed identical, it was possible to show that a perfectly competitive real estate sector will adjust the CBD density proportionally to the apartment density if it uses the same technology to construct offices and apartments. However, this model was not very tractable.

where $\mathcal{F}$ represents an abatement technology detailed later while $e_{\bar{c}c,b} = \int_0^{\bar{l}} \varsigma_{\bar{c}c} l^\theta dl = \varsigma_{\bar{c}c}\frac{\bar{l}^{1+\theta}}{1+\theta}$ represents the aggregate commuting energy use.

To specify the effects of pollution and density on the quality of agglomeration economies, it is necessary to find a functional relation, $\Phi = \Phi(D, P)$, which satisfies Assumptions (1) and (2) of Section (3.2), namely $\frac{\partial \Phi}{\partial \bar{P}} < 0$ so $\frac{\partial \Phi}{\partial F} > 0$, while $\frac{\partial \Phi}{\partial D} > 0|_{D<D_{peak}(P)}$, and $\frac{\partial \Phi}{\partial D} < 0|_{D>D_{peak}(P)}$, where D_{peak} refers to the agglomeration externality maximizing density[12]. $\Phi(D, P)$ must consequently be hump-shaped in D and reduce in P. A Ricker function, popular in ecology, is suitable to describe such attributes. The advantage of this function is its reference to exponential power-law relations that are relatively handy to solve. Therefore, it is assumed that

$$\Phi = \underline{\varsigma}_\Phi D \frac{e^{-\bar{\varsigma}_\Phi D}}{1 + \varsigma_P P} \quad \textit{with} \quad \underline{\varsigma}_\Phi, \bar{\varsigma}_\Phi, \varsigma_P > 0. \tag{21}$$

Figure (3.7) sketches this indicator, showing that Φ is hump-shaped in D and peaks with D_{peak} while pollution, P, pivots the relation.

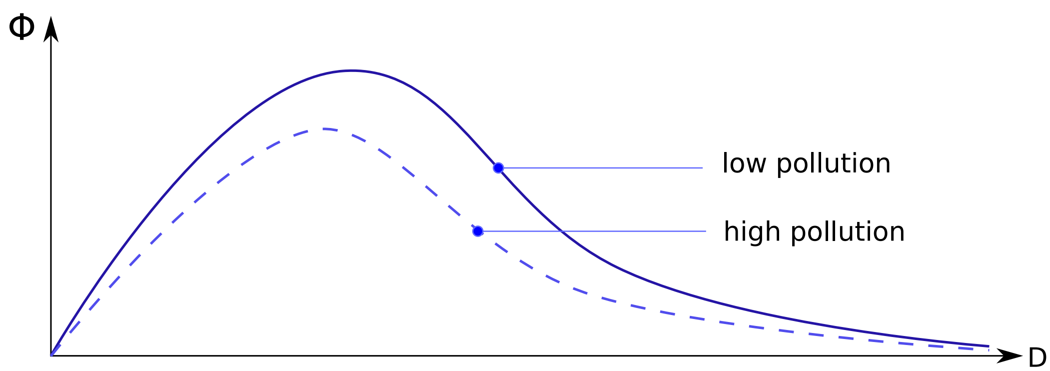

Figure 3.7: Density impact on the quality of agglomeration economies.

[12]Note that urban heat island (UHI) impacts are qualitatively comparable to these density effects, relatively cold locations can profit from heat islands up to a critical temperature $C^{\circ *}$.

3.4.4 Technologies

Table (3.1) summarize the four technologies of the model. In support of a well-

type	impact	static	dynamic
general technology	numéraire productivity	$\mathcal{A}$	$g_{\mathcal{A}} = \Phi g_A$
abatement technology	pollution reduction	$\mathcal{F}$	$g_{\mathcal{F}} = \Phi g_F$
renewable energy	renewable energy production	$\mathcal{E}$	$g_{\mathcal{E}} = \Phi g_E$
commuting technology	commuting energy reduction	$\mathcal{I}$	$g_{\mathcal{I}} = \Phi g_I$

Table 3.1: List of all technologies.

structured discussion, the technologies are initially considered exogenous but endogenized subsequently. Importantly, the technology growth rates are proportional to the quality of agglomeration economies.

There is a distinction between technologies $\mathcal{A}$ and $\mathcal{E}$ and infrastructure $\mathcal{F}$ and $\mathcal{I}$. The latter follow a long tradition in economic theory discussing the role of infrastructure for growth and welfare, see Aschauer (1990), Calderón, Moral-Benito, and Servén (2011), or Calderón and Servén (2014). Infrastructure can identify various publicly provided goods, including local law and access to education. In Quaas (2007), infrastructure refers to the stock of material and social capital in the public sector, while the author cites utilities, communication infrastructure, transport infrastructure, and land development measures as examples. This theory sets its focus on a subset of that interpretation, namely abatement technologies[13], $\mathcal{F}$, and commuting technologies, $\mathcal{I}$. The provision of these two types of infrastructure represents non-market-based policy.

3.4.5 Government

Table (3.2) summarizes the policy instruments of a local government. While commut-

[13]This comes close to the pollution reduction infrastructure discussed in Quaas (2007). While Chapter (2) has assessed decentralized incentives for abatement, this chapter relates abatement to the public infrastructure. The reason for this distinction is addressed in the discussion.

policy target	tool	static	dynamic
environmental quality	brown energy tax	T_e	g_{T_e}
	abatement technology	$\mathcal{F}$	$g_{\mathcal{F}}$
commuting	commuting infrastructure	$\mathcal{I}$	$g_{\mathcal{I}}$
	commuting subsidy	$T_{\bar{c}c}$	$g_{T_{\bar{c}c}}$
consumption	consumption tax	T_c	g_{T_c}
	apartment tax	T_b	g_{T_h}
	hinterland tax	T_x	g_{T_x}
rents and investments	tax on manufactured capital	T_r	g_{T_r}
	urban land tax	T_l	g_{T_l}
	tax on morphological capital	T_b	g_{T_b}
return on labor	numéraire labor	T_w	g_w

Table 3.2: List of all policy instruments.

ing infrastructure costs $p_{\mathcal{I}}$, abatement is implemented via environmental standards and hence a non-market-based command and control instrument. Given that, the governmental budget relation reads

$$\mathcal{T} = G \tag{22}$$

where G either serves as a lump sum tax or a tax income transfer to the households and $\mathcal{T}$ represents the tax income or expenditure channel related to by $\mathcal{T} := T_b p_b b + T_w(1-\lambda) + T_r r k_y + T_r r k_a + T_a p_a a + T_x p_x x + T_l \bar{\omega}\bar{\beta} + T_e e_b - T_{\bar{c}c} p_e e_{\bar{c}c} - p_{\mathcal{I}}\mathcal{I}$. With this, the model is fully described so that the next section can address a spatial equilibrium and the related spatial balanced growth path.

3.5 Balanced growth with exogenous technology

This section assesses an economy with exogenous technology. This arrangement helps focus on the fundamental spatial and environmental challenges for sustainable growth without being distracted by endogenous research decisions. After having derived some core results for sustainable growth, the chapter then considers endogenous research. It, therefore, first defines an equilibrium and then evaluates the corresponding balanced growth path.

Definition 3.1. *Given a sequence of state-initiated infrastructure* $\{\mathcal{I}(t), \mathcal{F}(t)\}_0^\infty$, *taxes* $\{T_e(t), T_{\bar{c}c}(t), T_c(t), T_a(t), T_x(t), T_l(t), T_r(t), T_b(t), T_w(t)\}_0^\infty$, *and technologies* $\{\mathcal{A}(t), \mathcal{E}(t)\}_0^\infty$, *a competitive spatial equilibrium refers to a sequence of allocations* $\{C(t), e(t), k(t), \bar{l}(t), \lambda(t)\}_0^\infty$ *with* $k(t) = k_y(t) + k_a(t)$, $C(t) = c(t) + a(t) + x(t)$, *and* $e(t) = e_y(t) + e_{\bar{c}c}(t) = \mathcal{E}(t) + e_b(t)$, *together with a sequence of prices* $\{p_a(t), p_x(t), p_{e,b}(t), p_{e,g}(t), p_{\mathcal{I}}, r(t), w(t), \bar{\omega}(t)\}_0^\infty$ *that occurs if*

(i) (utility maximization) households maximize their utility subject to their budget constraints,

(ii) (factor prices) factors are paid their marginal product and allocate via a no-arbitrage condition,

(iii) (government budget) the government budget clears,

(iv) (feasibility) all non consumed numéraire output is invested.

This theory is interested in a long-run perspective[14], which implies the impossibility of equipping a steadily increasing population with space. The vertical building expansion is limited by natural obstacles such as gravity and the atmosphere, whereas the horizontal expansion is bound to the earth's surface[15]. Therefore, a balanced growth path is defined as follows:

Definition 3.2. *A Spatial Balanced Growth Path (SBGP) is a trajectory along which urban expansion, population and the return on capital are constant while production, consumption, and manufactured capital are growing at a constant, positive rate.*

This definition is in line with the United Nations Population Report (Habitat, 2015) predicting that the world's population stabilizes in 2100. Assessing the related

[14]Stiglitz (1997), for example, argued that it is sufficient if a balanced growth path relates to a limited period of time. This is not the focus of this work.

[15]Note that the vertical construction constraints are accounted for by introducing decreasing returns in local construction by $\gamma \in [0, 1]$ while the horizontal constraints are implied by $l \in [0, \hat{l}]$.

growth path requires a clear understanding of energy price developments. Brown and green energies are perfect substitutes, so the energy source with the lower (gross) price will attract all demand. When ignoring hypothetical underpricing strategies, there remain three stylized scenarios for the energy market, differentiated in three alternative brown energy price paths (BEP):

BEP (1) *A competitive exogenous brown energy sector, no scarcity in the resource*: If brown energy is abundant, a competitive exogenous energy sector will refrain from increasing the energy price above the numéraire price. It is consequently assumed that the brown energy price is proportional to the numéraire price. For this reason, $g_{p_{e,b}}(t) = 0$, so along an SBGP $g_y = g_e$.

BEP (2) *A monopolistic exogenous brown energy sector, no scarcity in the resource, constant energy supply*: If there is no scarcity in the energy resource and the energy sector is monopolistic, it will choose a revenue-maximizing pricing strategy. Any price that grows at a higher rate than the numéraire good will cease long-run energy demand. Thereupon, the profit-maximizing pricing strategy is $g_{p_{e,b}} = g_y$, leading to $g_{e_b} = 0$.

BEP (3) *Resource scarcity*: If energy is scarce, it is charged a scarcity rent that follows Hotelling's rule (Hotelling, 1931), so $g_{p_{e,b}} = r$, see Chapter (2).

For a BEP (1) and (2) note that brown energy must not be related to a finite resource, as nuclear energy can be considered brown because of its contaminated waste problem. This theory is deliberately open along these lines and purposely considers a flexible use of the derived model. In addition, when the gross prices of brown and green energy are identical, the theory for simplicity suggests a 'green' glow, stating that agents prefer the green source for its environmental friendliness.

For assessing the choice among brown and green energy, it is further essential to consider the green energy price development, a feature addressed with the endogenization of the research sector. Proposition (3.1) subsequently presents the necessary conditions for an SBGP to exist, details its properties, and shows that four alternative regimes meet its definition. These regimes represent four alternative roads to sustainable urban growth.

Proposition 3.1. *An SBGP exists iff $g_D = g_P = 0$, hence $g_\Phi = 0$. If an SBGP exists, it is characterized by*

$$g_y = g_k = g_c = g_w = g_{p_a} = g_{p_b} = g_{p_x} = g_{\bar{\omega}} = g_L = r - \rho. \tag{23}$$

An SBGP is possible along a brown or a green energy scenario and differentiated among four alternative growth regimes:

(i) (Brown energy scenario) if $(1 + T_e)p_{e,b} < p_{e,g}$, then $g_\mathcal{E} = 0$ and $g_{e_D} > 0$. Three alternative regimes are possible:

- *(a) energy saving regime (ES regime), relevant for a BEP (1), (2) or (3), and characterized by $\frac{1-\varkappa-\varepsilon}{1-\varkappa} g_\mathcal{A} = g_{T_{\bar{c}c}} + g_\mathcal{I} = g_{p_{e_{y,b}}} + g_{T_e}$, $g_\mathcal{F} = g_{e_{y,b}} = g_P = 0$, resulting in $g_y = \frac{1-\varkappa-\varepsilon}{1-\varkappa} g_\mathcal{A}$.*
- *(b) pollution saving regime (PS regime), relevant for a BEP (1), and characterized by $g_{p_{e_{y,b}}} + g_{T_e} = g_P = g_{T_{\bar{c}c}} + g_\mathcal{I} = g_D = 0$, $g_\mathcal{F} = g_{e_{y,b}} = g_y$, resulting in $g_y = g_\mathcal{A}$.*
- *(c) energy and pollution saving regime (EPS regime), relevant for a BEP (1), and characterized by $g_y > g_{T_e} > 0$, $g_\mathcal{F} = g_{e_{y,b}} = g_\mathcal{A} - \frac{1-\varkappa}{1-\varkappa-\varepsilon} g_{T_e}$, $g_{T_e} = g_{T_{\bar{c}c}} + g_\mathcal{I}$, resulting in $g_y = g_\mathcal{A} - \frac{\varepsilon}{1-\varkappa-\varepsilon} g_{T_e}$.*

(ii) (Green energy scenario) if $(1+T_e)p_{e,b} \geq p_{e,g}$, then $g_\mathcal{E} > 0$ and $g_{e_D} = 0$ associated with a green energy regime (GE regime), characterized by $g_\mathcal{E} = g_y - g_{p_{e,g}} > 0$, $g_\mathcal{E} = g_{e_y}$ and $g_{p_{e,g}} = g_{T_{\bar{c}c}} + g_\mathcal{I}$, so that $g_y = g_\mathcal{A} - \frac{\varepsilon}{1-\varkappa-\varepsilon} g_{p_{e,g}} > 0$.

Irrespective of the regime, an SBGP path satisfies the Kaldor facts.

Proof: See Appendix (B).

The proposition emphasizes that an SBGP only exists if the quality of agglomeration economies, Φ, remains constant[16]. This constancy is fundamental as technology

[16]Beyond the parameter restrictions discussed when specifying the model, a constant ϕ is then sufficient for balanced growth.

growth rates are scaled proportionally to Φ, see Table (3.1). Since Φ decreases monotonously with pollution, P, and is hump-shaped in density, D, it is only constant if P and D are constant. This requirement is met along four alternative growth regimes, representing alternative directions of technical change. Three of them relate to a *brown energy scenario* where the economy uses brown energy sources, which causes harmful pollution. The fourth regime describes a *green energy scenario* which uses green energy sources, so no harmful pollutants emerge, see Figure (3.8).

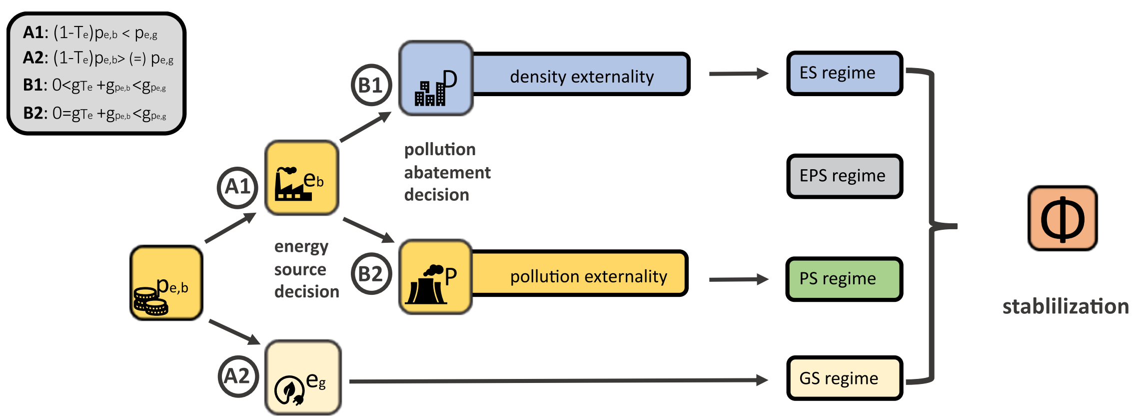

Figure 3.8: Summary of the alternative growth regimes for a SBGP.

For all that, as long as $(1 + T_e)p_{e,b} < p_{e,g}$, the economy is in a brown energy scenario and faces a pollution problem, a density problem, or both, depending on the evolution of brown energy prices. If $g_{T_e} + g_{p_{e,b}} = 0$, a growing economy will increase its energy consumption and thus pollution. If $g_{T_e} + g_{p_{e,b}} > 0$, the pollution problem can be mitigated or even eliminated as energy demand reduces. Regardless, an energy price rise increases commuting costs, pulling citizens to the center. This dynamic density externality is a serious threat to growth; high commuting prices make commuting hardly affordable, the concentration in the center can become so intense that any innovation potential subsides, see therefore also Section (3.2).

There are three alternative paths to address the externalities associated with a brown energy scenario. First, it is possible to stop harmful pollution growth by keeping brown energy consumption constant while the economy still grows due to sufficient improvements in the numéraire productivity, $\mathcal{A}$. This path is discussed as an *energy saving* (ES) regime and calls for the gross price of brown energy (net price with taxes) to grow at the numéraire growth rate. Yet, such a scenario requires stabilizing an energy price induced rise in commuting costs. If the increase in energy prices is purely tax-driven, one option is to exclude commuting from the tax. If that is not possible or if the net price of energy rises due to resource scarcity, then an increase in commuting technologies, $\mathcal{I}$, or commuting subsidies, $T_{\bar{c}c}$, is needed.

The second option with brown energy use is to improve abatement technologies, $\mathcal{F}$, sufficiently to completely abate pollution growth, a path which is discussed as a *pollution saving* (PS) regime. For a structured discussion, this scenario is related to a $BEP(1)$, thereupon the corresponding pollution abatement is only technology-based, not price-based. The third scenario for brown energy based growth consequently refers to a combination of price-based and abatement-based pollution reduction, discussed as an *energy and pollution saving* (EPS) regime.

Whenever $(1+T_e)p_{e,b} \geq p_{e,g}$, the economy shifts to a green energy scenario referred to as a *green energy* (GE) regime. In this case, green energy provision, $\mathcal{E}$, increases at a higher or equal rate as general productivity, $\mathcal{A}$, so that all energy demand (including commuting energy) is satisfied with green energy and no harmful pollution remains. If green energy prices increase, improvements in commuting technology or commuting subsidies are needed to stabilize commuting costs.

Note that the transition paths to the SBGP are not addressed for the arguments raised in Chapter (2). Technically, the economy is constantly on a transition path since the provision of apartments and hinterland remain constant while numéraire production increases. The numéraire consumption growth determines the utility growth rate and the growth rate of manufactured capital in the numeraire sector (as real estate capital demand remains constant). Similarly, if energy consumption increases, constant commuting energy demand leads to the numéraire sector driving the aggregate energy demand. Hence, the respective growth rates only approach the balanced growth rates cited in Proposition (3.1) in the limit[17]. This attribute, however, has little impact on the qualitative discussion (see appendix for details).

Finally, all regimes satisfy the Kaldor facts, a feature discussed in depth in the appendix[18]. Beyond, they all lead to sustainable growth, as detailed subsequently.

Sustainable urban growth

An SBGP requires positive production growth and hence needs to be sustainable per definition. For relating the discussion to the genuine investment concept and its interpretation of sustainability, remember that the value of the productive base has to remain non-decreasing. The introduction to this chapter has emphasized that sustainable growth requires the stability of the sink of the environmental quality. An identical requirement applies to density, which can be interpreted as a sink describing the potency of land for economic activity. It is impossible to compensate for a constant reduction in the environmental quality and the consequences of a continuous increase (or reduction) in central densities with other assets. Therefore, the environmental sink must stay non-decreasing, and densities need to stabilize. All four growth regimes satisfy both requirements.

For a more nuanced perspective on the determinants of sustainable growth, the subsequent section augments the model with an endogenous research sector.

3.6 Balanced growth with endogenous technology

The endogenization of research follows the principles of horizontal innovation theory in the tradition of Romer (1990), Gancia and Zilibotti (2005), and Ricci (2007), see also Romer (1994). A distinction is made between market technologies, $\mathcal{A}, \mathcal{E}$, and

[17] For a similar case see Kongsamut, Rebeleo and Xie (2001). While it would be possible to adapt the model to achieve the SBGP rather than approaching it, the chosen specification is preferred as it allows for more flexible discussion (and simplifies the numerical solution).

[18] Literature usually considers the satisfaction of the Kaldor facts a precondition for an empirically valid theory. As discussed in more depth in the appendix, this requires that technological growth is Harrod-neutral (labor augmenting) if there is population growth, a feature that is not as restrictive without population growth.

state-initiated infrastructure, $\mathcal{F}, \mathcal{I}$, which is subsequently detailed.

Market technologies

The formally exogenous $\mathcal{A}$ and $\mathcal{E}$ are now described by an elasticity weighted aggregate of alternative intermediates, $I_{\mathcal{A}}$, respectively, $I_{\mathcal{E}}$. This means that the larger the number of intermediates and the intensity of their provision, the greater the technological progress. Both types of intermediates are provided by an R&D sector and rented by the numéraire sector, costing $(1-T_{\wp_{\mathcal{A}}})\wp_{\mathcal{A}}$, respectively, $(1-T_{\wp_{\mathcal{E}}})\wp_{\mathcal{E}}$, with $T_{\wp_i}, i \in \{\mathcal{A}, \mathcal{E}\}$ as a price subsidy to address monopolistic markup pricing.

While numéraire technology intermediates are well discussed in standard literature, intermediate energy products are best understood as power plants for green energies. Examples are hydropower plants or wind turbines, but may include nuclear power plants if one ignores the challenges with contaminated fuel rods (so that nuclear energy is considered green). In any case, if an individual power plant gets installed, it can create a certain amount of green energy, so the more power plants, the more energy. An indicator variable, $i_{\mathcal{E}}$, will count 0 if the R&D sector does not provide green energy, 1 otherwise. Numeraire production is hence described with

$$y = \big(1-\sigma-\lambda\big)^{1-\varkappa-\varepsilon}\Big(\int_0^E I_{\mathcal{E}}(j)^{\varepsilon i_{\mathcal{E}}} dj\Big) + (1-i_{\mathcal{E}})e_{y,b}^{\varepsilon}\int_0^A I_{\mathcal{A}}(j)^{\varkappa} dj \tag{24}$$

whereby $(1-\sigma-\lambda) \in (0,1)$ measures the numéraire production labor fraction. Note that if $i_{\mathcal{E}} = 0$, there is a default stock of green energy[19], $E = E_0$. The intermediate prices are $(1-T_{\wp_{\mathcal{A}}})\wp_{\mathcal{A}}$ for $I_{\mathcal{A}}$ and $(1-T_{\wp_{\mathcal{E}}})\wp_{\mathcal{E}}$ for $I_{\mathcal{E}}$. Thereby, $T_{\wp_{\mathcal{A}}}$ and $T_{\wp_{\mathcal{E}}}$ denote subsidies that can be used to compensate the numéraire sector for price markups. With this, the return on labor is $w = (1-\varkappa-\varepsilon)\frac{y}{(1-\sigma-\lambda)}$ while the return on manufactured capital (due to symmetry in the individual intermediates demands) reads $(1-T_{\wp_{\mathcal{A}}})\wp_{\mathcal{A}} = \varkappa\big(1-\sigma-\lambda\big)^{1-\varkappa-\varepsilon}e_{y,b}^{\varepsilon}I_{\mathcal{A}}^{\varkappa-1}$ if $i_{\mathcal{E}} = 0$, $(1-T_{\wp_{\mathcal{A}}})\wp_{\mathcal{A}} = (1-\sigma-\lambda)^{1-\varkappa-\varepsilon}(EI_{\mathcal{E}}^{\varepsilon})I_{\mathcal{A}}^{\varkappa-1}$, $(1-T_{\wp_{\mathcal{E}}})\wp_{\mathcal{E}}(j) = \varepsilon(1-\sigma-\lambda)^{1-\varkappa-\varepsilon}I_{\mathcal{E}}^{\varepsilon-1}AI_{\mathcal{A}}^{\varkappa}$ if $i_{\mathcal{E}} = 1$.

[19]The impact of this stock on the long-run growth rates is negligible. However, the stock will be relevant for the social planner discussion because a scenario will be considered that avoids using any brown energy in production and does not improve green energy (an ES^+ regime).

State-initiated infrastructure

Section (3.4.4) introduced commuting and abatement technologies as specific types of public infrastructure. Commuting infrastructure, $\mathcal{I}$, can be interpreted as a public tender for any infrastructure that saves energy in transport and commuting. If these tenders offer a sufficiently high price for commuting technology, they initiate corresponding research and development. This may lead to new resource-efficient means of public transport, charging stations for electric cars, or progress in the organization of traffic flows. Each infrastructure type is associated with a new intermediate that improves the commuting technology stock according to

$$\mathcal{I} = \int_0^I I_{\mathcal{I}}(j)^{\Theta i_{\mathcal{I}}} dj \tag{25}$$

with $i_{\mathcal{I}}$ as an indicator variable which switches from 0 to 1 if there is innovation in commuting infrastructure while Θ accounts for that the effectiveness of a single technology decreases. Since innovators can also increase $\mathcal{A}$ or $\mathcal{E}$, they compare the research path-specific profitability of an innovation. With $v_{\mathcal{I}}$ as the governmental price offer for commuting intermediates, researchers interested in commuting consider

$$\max_{\mathcal{I}_c(j)} \int_0^I v_{\mathcal{I}} I_{\mathcal{I}}(j)^{\Theta i_{\mathcal{I}}} dj - \int_0^I \wp_{\mathcal{I}}(j) I_{\mathcal{I}}(j) dj \tag{26}$$

where $\wp_{\mathcal{I}}$ is the intermediate price the monopolistic competitive research sector charges (detailed later). A symmetry argument then leads to the infrastructure demand relation $\wp_{\mathcal{I}} = v_{\mathcal{I}} \Theta I_{\mathcal{I}}^{\Theta-1}$ if $i_{\mathcal{I}} = 1$. The government can further force the numéraire sector to introduce environmental standards which increase the supply of the abatement technology, $\mathcal{F}$. That leads to innovations in filter technologies and other pollution-reducing equipment, e.g., measures for decontamination of soils or recycling technologies for waste, in case pollution is interpreted more broadly[20]. The evolution of $\mathcal{F}$ is subject to environmental standards. These standards drag on the

[20] E.g., if nuclear power were considered brown due to the challenges with radioactive waste disposal.

innovation potential and will be detailed subsequently[21].

Research labor

A fraction $\sigma \in (0,1)$ of all citizens allocates to the research sector with the expectation to earn an individual innovation value, V_j, so the no-arbitrage condition in the labor market reads

$$(1 - T_w)w = (1 + T_v)V_j \tag{27}$$

with T_v as a general research subsidy and $\pi = (1 - \varkappa - \varepsilon)\frac{y}{(1-\lambda-\sigma)}$ as the wage determined by the marginal product of labor in numéraire production.

Innovations

Researchers have an effort contingent standardized to unity, which they allocate to different research directions. Thereby, $\chi \in (0,1]$, denotes the time used for research in the numéraire sector. Such efforts combine research for general technology, green energy technology, and abatement. From these efforts, a fraction of $\eta \in [0,1]$ goes into the improvement of general technologies and abatement, the remaining fraction into green energy production. Whether there is abatement depends on whether there are environmental standards. The government can determine the degree of abatement efforts with $(1 - \kappa) \in (0,1]$ which can be interpreted as the stringency of the environmental standards. Such standards are neither relevant to green energy innovations (since green technology does not cause pollution) nor raised on commuting technology innovations (since this type of innovation reduces the energy needs of commuting and thus pollution). They consequently only affect the numéraire sector. Therefore, while the two pollution sources (energy in numéraire production and commuting) described in (20) are both subject to the same initial technology stock, $\mathcal{F}_0$, only numéraire pollution-related abatement technologies can increase. Given this,

[21] Technically, assume that each innovation represents an intermediate that, in turn, suffers from decreasing returns in its efficiency, so $\mathcal{F} = \int_0^F I_{\mathcal{F}}(j)^{\varkappa i_{\mathcal{F}}} dj$ where the elasticity, $\varkappa$, is identical to elasticity characterizing general technology intermediates, emphasizing that abatement efforts present research efforts that are not used for general technology improvements. With an indicator variable, $i_{\mathcal{F}}$, switching from 0 to 1 if a government initiates environmental standards.

the innovation differential equations follow[22]:

$$\dot{A} = \sigma \Phi (\chi \eta)^{\psi} \kappa A \tag{28}$$

$$\dot{E} = \sigma \Phi (\chi (1-\eta))^{\psi} \varsigma_{\mathcal{E}} E \tag{29}$$

$$\dot{F} = \sigma \varsigma_F \chi^{\psi} \Phi (1-\kappa) F \tag{30}$$

$$\dot{I} = \sigma i_{\mathcal{I}} \Phi (1-\chi)^{\psi} \varsigma_{\mathcal{I}} I \tag{31}$$

where $0 < \varsigma_i$ for $i \in \{\mathcal{F}, \mathcal{I}, \mathcal{E}\}$ describe research efficiencies (whereby implicitly $\varsigma_{\mathcal{A}} = 1$). Further, while the benchmark specification will focus on a scenario where $\psi \in (0,1)$, it is worth also evaluating a case with $\psi = 1$. Since scenarios with $\psi > 1$ will lead to qualitatively comparable results as scenarios with $\psi = 1$, they are not additionally considered. Therefore, this theory distinguishes

$$\psi = \begin{cases} \in [0,1] & \textit{open access to technologies (GE regime correspondence)} \\ 1 & \textit{specialization in technologies (SK regime correspondence).} \end{cases}$$

With open access, researchers can improve all technologies in parallel. With specialization, research becomes a binary decision among alternative paths. This differentiation enables a comparison with a generalized knowledge (GK) regime and a specialized knowledge (SK) regime distinguished[23] in Chapter (2). However, this chapter will not continue this discussion in a comparable depth, but will briefly reveal how its model can address this fundamental aspect of innovations.

Research profits

An innovator can only improve the technologies once and then sticks to its intervention, but can carry out further innovations in any proceeding period. In the case of

[22]Note that κ is not subject to ψ since it is a stringency variable of the environmental standards.

[23]There are never the less differences in both chapters. In Chapter (2), specialization was not explained via $\psi = 1$ since it was possible to improve general technologies and at least one additional technology (adaptation or abatement). Here, optimization with $\psi = 1$ will result in a bang bang solution for the innovation direction (one direction only).

a successful innovation, a researcher becomes an intermediate producer who can use putty-clay technology to convert manufactured capital into intermediate goods, thus $I_i = k_i$ for $i \in \{\mathcal{A}, \mathcal{I}, \mathcal{E}\}$. Its profit function is therefore described with

$$\Pi_i = \big(\wp_i - r)(1 + T_{I,i}\big)I_i \;\; \textit{for} \;\; i \in \{\mathcal{A}, \mathcal{I}, \mathcal{E}\} \tag{32}$$

where $T_{I,i}$ for $i \in \{\mathcal{A}, \mathcal{I}, \mathcal{E}\}$, denotes a path-specific innovation subsidy to support path-specific research. The expected value of aggregate innovations, V, is subject to the effort allocation, η and χ, the number of intermediates in each research direction, A, I, and E, as well as the direction specific profits for each intermediate, $\Pi_{\mathcal{A}}$, $\Pi_{\mathcal{I}}$, and $\Pi_{\mathcal{E}}$. It follows

$$V = \frac{\Gamma_V}{r} \;\; \textit{whereby} \;\; \Gamma_V := \chi^{\psi}\eta^{\psi}\kappa A\Pi_{\mathcal{A}} + i_{\mathcal{I}}\eta^{\psi}(1-\chi)^{\psi}\varsigma_{\mathcal{I}}\kappa I\Pi_{\mathcal{I}} + (1-\eta)^{\psi}\varsigma_{\mathcal{E}}E\Pi_{\mathcal{E}}. \tag{33}$$

Spatial equilibrium with endogenous research

Before addressing endogenous balanced growth characteristics, it is necessary to adjust the spatial equilibrium definition (3.1) with endogenous innovations and, on this occasion, address how endogenous research affects the determinants of the density profile. This discussion is crucial because density impacts the quality of agglomeration economies and hence the innovation rate.

Definition 3.3. *Given a sequence of state-initiated infrastructure* $\{\mathcal{I}(t), \mathcal{F}(t)\}_0^\infty$, *taxes* $\{T_e(t), T_{\bar{c}c}(t), T_c(t), T_a(t), T_x(t), T_l(t), T_r(t), T_b(t), T_w(t), T_{\wp_{\mathcal{A}}}, T_{\wp_{\mathcal{E}}}, T_V\}_0^\infty$, *and technologies* $\{\mathcal{A}(t), \mathcal{E}(t)\}_0^\infty$, *a competitive spatial equilibrium with endogenous research refers to a sequence of allocations* $\{C(t), e(t), k(t), \bar{l}(t), \lambda(t), \sigma(t), \chi(t), \eta(t)\}_0^\infty$ *with* $e(t) = e_y(t) + e_{\bar{c}c}(t) = \mathcal{E}(t) + e_{y,b}(t)$, $k(t) = k_y(t) + k_a(t)$, *and* $C(t) = c(t) + a(t) + x(t)$, *together with a sequence of prices* $\{p_a(t), p_x(t), p_{e,b}(t), \wp_{\mathcal{A}}(t), \wp_{\mathcal{E}}(t), \wp_{\mathcal{I}}(t), r(t), w(t), \bar{\omega}(t)\}_0^\infty$ *that occurs if*

(i) (utility maximization) households maximize their utility subject to their budget constraints,

(ii) (factor prices) factors are paid their marginal product and allocate via a no-arbitrage condition while intermediates are charged monopolistic markup prices,

(iii) (government budget) the government budget clears,

(iv) (feasibility) all non consumed numéraire output is invested.

In general, an essential question of this theory is how decentralized factor allocations affect the static and dynamic conditions of a growing economy. Several allocation principles are subject to whether growth is exogenous or endogenous, and have therefore not yet been discussed. A core difference of factor allocation principles with endogenous research is their sensitivity to the fraction of research labor, σ. On the one hand, σ has a direct effect on the pace of innovation. On the other hand, it affects the general allocation of labor and hence the fraction of real estate labor. The real estate sector combines real estate labor and manufactured capital in fixed proportions. While manufactured capital is reproducible and consequently not affected by scarcity, the labor fraction is. Accordingly, it determines the morphological capital intensity and, therefore, the CBD density.

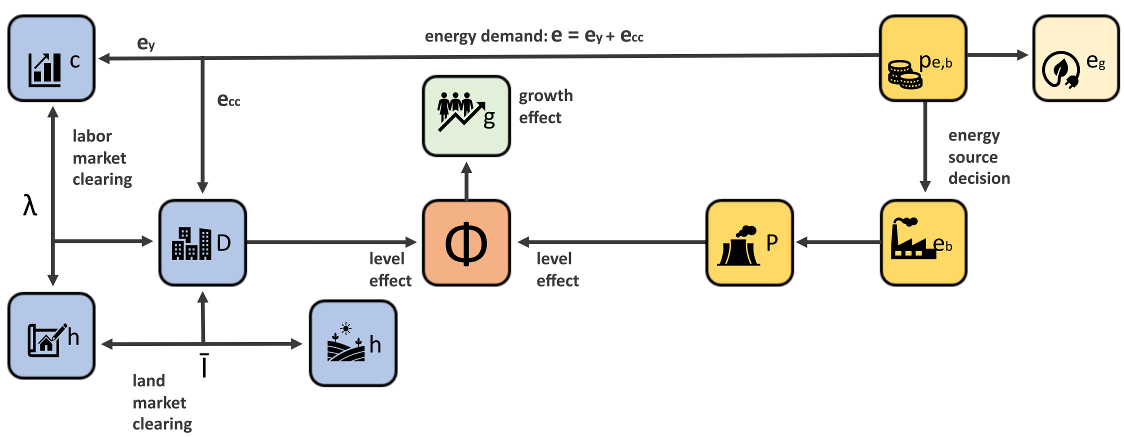

Figure 3.9: Static determinants of the quality of agglomeration economies.

The combination of density and pollution then determines the quality of agglomeration economies, which scales the pace of innovations. Understanding density hence requires comprehending the clearing of the markets for land and labor. On the land market, the real estate sector and the hinterland sector bid for land, on the labor market, the real estate sector and the numéraire sector bid for labor.

Figure (3.9) summarizes how static factor allocations determine the quality of agglomeration economies. They will be detailed subsequently. Lemma (3.1) initiates this discussion with a closer look at the land market.

Lemma 3.1. *Suppose an economy achieves an SBGP. Then, the land market clears with*

$$\frac{\Gamma_l \Lambda_l}{\bar{p}_e} = \frac{\bar{l}^{1-\theta}}{(\hat{l}-\bar{l})} \quad \textit{with } \bar{p}_e = \begin{cases} \frac{(1+T_e)(1-T_{\bar{c}c})p_{e,b}}{\mathcal{I}} & if \ \ e = e_b \\ \frac{(1-T_{\bar{c}c})p_{e,g}}{\mathcal{I}} & if \ \ e = e_g, \end{cases} \tag{34}$$

and $\Gamma_l := \frac{(1+T_x)}{(1+T_a)(1+T_l)}$, $\Lambda_l := \frac{(1-\gamma-\gamma\theta)\iota}{(1-\iota-\epsilon)}$.

Proof: See Appendix (B).

Expression (34) follows when equating the real estate and the hinterland sector bids for land. It is convenient to discuss this clearing relation graphically. Figure (3.10) therefore sketches the left-hand side (LHS) and the right-hand side (RHS) of (34) for increasing degrees of urban spread, $\bar{l}$, on the basis of a parameter calibration introduced later in Section (3.8). The LHS is independent of $\bar{l}$ and thus a horizontal line that intersects the RHS at the equilibrium of the land market. A higher land tax, T_l, and a higher apartment tax, T_a, reduce the real estate revenues and thus urban expansion. A higher net energy price, $\bar{p}_e$, increases commuting costs, reducing apartment demand at less central locations, and hence the degree of urban spread. Infrastructure investments, $\mathcal{I}$, mitigate this development. A tax on the hinterland, T_x, also reduces the relative price of urban land and, therefore, increases urban land use.

That being the case, the second density determinant is the morphological capital intensity, which follows with the labor market clearing described next.

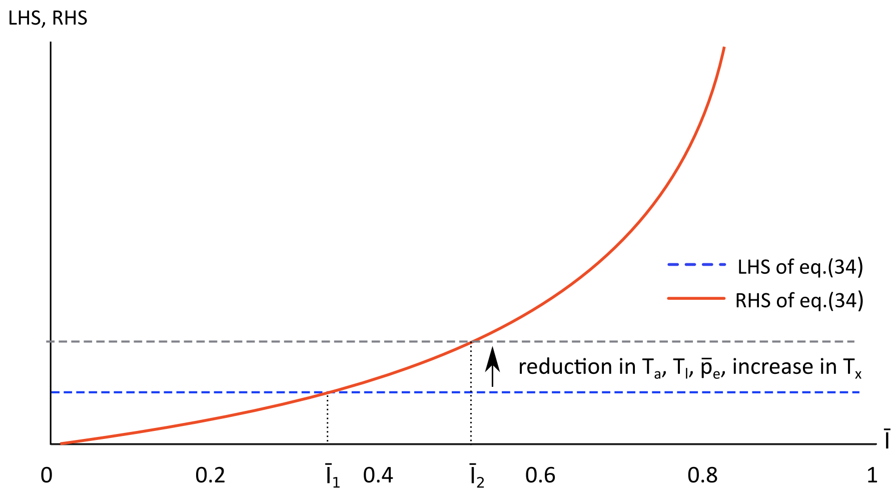

Figure 3.10: Land market clearing in a decentralized economy.

Lemma 3.2. *Suppose an economy achieves an SBGP. Then, the labor market clears with*

$$\lambda = \frac{1-\sigma}{1+\frac{\bar{p}_e \bar{l}^{\theta} \Gamma_\lambda \Lambda_\lambda}{\mathcal{C}}}, \tag{35}$$

with $\mathcal{C} := \frac{c}{y}$, $\Gamma_\lambda := \frac{(1+T_a)}{(1+T_c)(1+T_w)}$, $\Lambda_\lambda := \frac{(1-\varkappa-\varepsilon)\epsilon}{\iota}\frac{(1-\gamma-\theta)}{(1-\gamma-\gamma\theta)}$.

Proof: See Appendix (B).

Expression (35) follows when equating the marginal product of labor in the numéraire sector with the marginal product of labor in the real estate sector, revealing that the employment of real estate labor is conditional on both the degree of urban expansion, $\bar{l}$, and the consumption rate $\mathcal{C} = \frac{c}{y}$. Expression (34) evaluates the former, while the latter is assessed a little later in depth.

The discussion will show that the consumption rate is subject to the consumption growth rate and is, thence, affected by the quality of agglomeration economies, Φ. Since σ is also affected by Φ, any adjustment in λ causes complex feedback effects. These effects are hard to assess analytically. However, a numerical application of the theory reveals that for reasonable parameter values and variable ranges, such feedback effects only play a minor role and have no qualitative effect on the curvature of expression (35). In light of this, it is convenient to treat $\mathcal{C}$ and σ as exogenous when assessing the land market clearing graphically. Accordingly, Figure (3.11) sketches expression (35), assuming that $\mathcal{C} = 0.7$ and $\sigma = 0.75$. While $\mathcal{C} = 70\%$ is a reasonable consumption rate, $\sigma = 75\%$ is considerably high, emphasizing that this theory interprets σ as the fraction of the innovative labor force, including any worker occupied in innovative activities (see appendix for details).

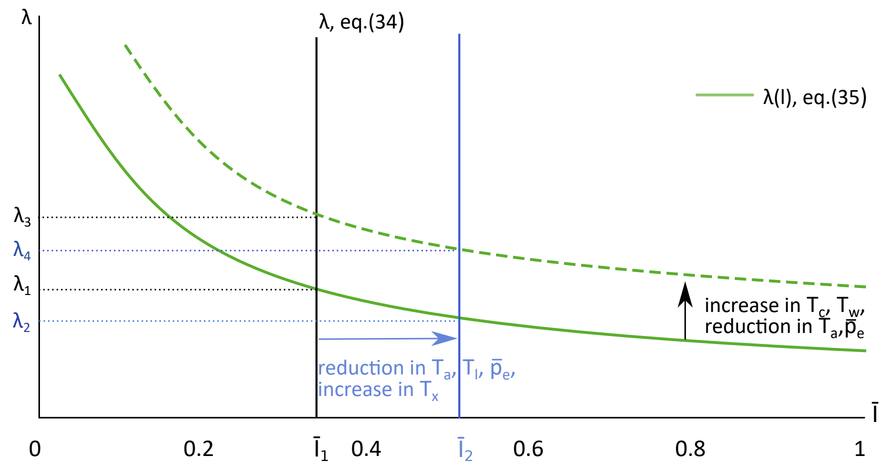

Figure 3.11: Labor market clearing as a function of urban expansion with λ_1 relating to an initial steady-state, λ_3 expressing a steady-state after an increase in T_c, T_w, or a reduction in T_a, $\bar{p}_e$), λ_2 expressing a steady-state after a reduction in T_a, T_l, $\bar{p}_e$, or an increase in T_x, λ_4 expressing a steady-state after reduction in T_a or $\bar{p}_e$.

The figure shows that the labor market clears when (35) intersects the vertical line representing (34). The above discussion has emphasized that the greater the area on which the real estate sector employs a specific amount of morphological capital to construct apartments, the lower the local decreasing returns in vertical construction at each spot. Hence, the larger $\bar{l}$, the lower λ.

A consumption tax, T_c, reduces the relative price of apartments, increasing apartment demand and thus real estate labor use. Similarly, a tax on numéraire labor,

T_w, reduces the real estate labor price and, accordingly, increases its demand. An apartment tax, T_a, and an increase in the net energy price, $\bar{p}_e$, reduce the willingness to pay for apartments and, therefore, real estate labor employment. Consequently, any environmental tax that does not exclude commuting affects the morphological capital intensity. Improvements in commuting infrastructure, $\mathcal{I}$, or commuting subsidies, $T_{\bar{c}c}$, can encounter these price effects. Yet, their net impacts on labor are ambiguous, as they affect labor and land markets in parallel. The same principles apply to an apartment tax, T_a, also affecting both markets. Finally, the real estate labor demand reduces with a hinterland tax, T_x, but increases with a land tax, T_l, via their land allocation impacts.

In light of this, Lemma (3.3) presents the combined effects of land and labor market clearing on density.

Lemma 3.3. *Suppose an economy achieves an SBGP, then the average construction density follows*

$$D = \left(\frac{1-\sigma}{\bar{l} + \frac{\bar{p}_E \bar{l}^{1+\theta} \Gamma_\lambda \Lambda_\lambda}{\mathcal{C}}} \right)^{\gamma} \Gamma_a \Lambda_a. \tag{36}$$

$\Gamma_a := \left(\frac{1}{1+T_b} \right)^{\gamma}$, $\Lambda_a := \frac{\mathcal{B}\gamma^\gamma (1-\gamma)^{1-\gamma} (1-\gamma-\theta)^\gamma}{(1-\gamma-\gamma\theta)}$.

Proof: See Appendix (B).

In principle, Expression (36) follows when including (35) in (19). It represents the CBD density as a function of the land market clearing only. Figure (3.12) sketches this relation. The vertical line depicts the solution to (34) so its intersection with the D curve describes the steady-state CBD density. All policies that increase λ also increase density, as long as they do not in parallel affect the land market.

An apartment tax, T_a, and policies affecting the energy price ($T_{\bar{c}c}$, $\mathcal{I}$, and T_e in a brown energy scenario) have an impact on the land and labor allocation[24]. Their net effect on density is therefor ambiguous without calibration. Table (3.3) summarizes these policy effects.

[24] Note that a tax on morphological capital T_b only affects density but has no direct impact on the real estate labor use. This holds since the tax burden is shared among morphological capital expenditures and land bids such that it has no relative price effects.

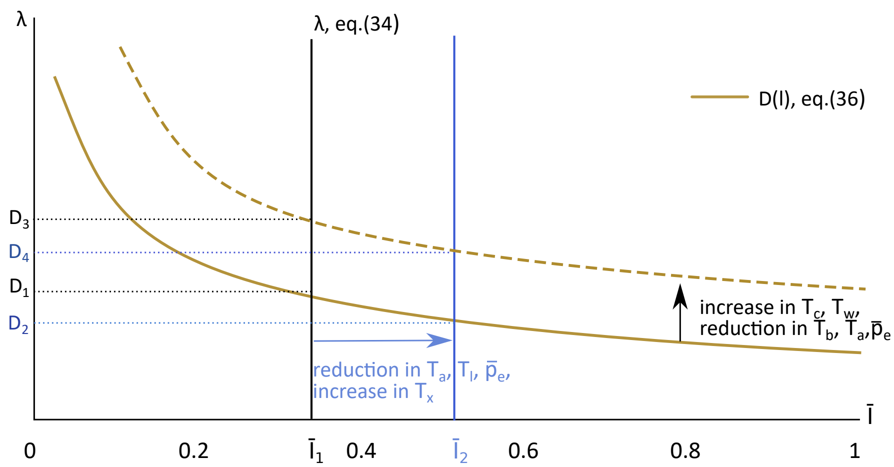

Figure 3.12: Density conditional on urban expansion with D_1 relating to an initial steady-state, D_3 expressing a steady-state after an increase in T_c, T_w, or a reduction in T_a, $\bar{p}_e$ and T_b, D_2 expressing a steady-state after a reduction in T_a, T_l, $\bar{p}_e$, or an increase in T_x, D_4 expressing a steady-state after a reduction in T_a or $\bar{p}_e$.

Policy effect on the spatial economy.

	T_a	T_l	T_x	T_c	T_b	T_w	T_r	$T_{\bar{c}c}$	$\mathcal{I}$	T_e
$\bar{l}$	$\downarrow$	$\downarrow$	$\uparrow$	$--$	$--$	$--$	$--$	$\uparrow$	$\uparrow$	$\downarrow$
λ	?	$\uparrow$	$--$	$\uparrow$	$--$	$\uparrow$	$\uparrow$	?	?	?
D	?	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$	?	?	?

Table 3.3: Net policy effects on land, labor, and density. With '$--$' for no impact and '?' indicating an ambiguous impact.

Given this, this discussion proceeds with the characteristics of a spatial balanced growth path with endogenous research.

Spatial balanced growth with endogenous research

Endogenous research has static and dynamic impacts on the spatial economy. To detail these effects, Proposition (3.2) first introduces the necessary and sufficient conditions for an SBGP with endogenous research. Lemma (3.4) then addresses path-specific research effort and research labor allocation principles for $i_{\mathcal{E}} \in \{0, 1\}$ and $i_{\mathcal{I}} \in \{0, 1\}$ while Lemma (3.5) describes the decision criteria for $i_{\mathcal{E}} \in \{0, 1\}$ and $i_{\mathcal{I}} \in \{0, 1\}$. Proposition (3.3) finally combines results to translate Proposition (3.1) into a setting with endogenous research.

Proposition 3.2. *An SBGP with endogenous research exists if the conditions cited in Proposition (3.1) are met and $\eta \in [0, 1]$, $\chi \in (0, 1]$, and $\sigma \in (0, 1)$ (all constant). If an SBGP exists, it is characterized by (23) and $g_w = g_{\Gamma_V} = g_y$.*

Proof: See Appendix (B).

Proposition (3.2) reveals that numéraire production focused research can either improve general technology and green energy in parallel, in which case $\eta \in (0, 1)$, or concentrate on general technology only ($\eta = 1$), or green energy production only ($\eta = 0$). An SBGP thereby requires that the research efforts are constant. Further, there is only positive numéraire production growth when $\chi > 0$. Similarly, to avoid an unsustainable scenario where all labor allocates to either production or research, it is necessary that $\sigma \in (0, 1)$, implicitly calling for $g_w = g_{\Gamma_V} = g_y$. The remaining necessary condition[25] for an SBGP is a constant Φ. The reasons for this condition are discussed in Proposition (3.1).

Taking that on board, if an SBGP exists, it follows Expression (23) and exhibits characteristics presented in Lemma (3.4).

Lemma 3.4. *Along an SBGP with endogenous research, if $i_{\mathcal{E}} = 1$, a rational government choses $i_{\mathcal{I}} = 0$, hence $\chi = 1$ while $e = e_y + e_{cc} = e_g = \mathcal{E}$. Further,*

[25] Appendix (B) details the conditional initial factor endowments necessary to be on a stable trajectory, these conditions are then also sufficient for an SBGP to exist.

	$i_{\mathcal{E}}=0$		$i_{\mathcal{E}}=1$
	$i_{\mathcal{I}}=0$	$i_{\mathcal{I}}=1$	
χ	1	$\frac{1}{1+\left(\frac{\varsigma_{\mathcal{I}} I\Pi_{\mathcal{I}}}{\kappa A\Pi_{\mathcal{A}}}\right)^{\frac{1}{1-\psi}}}$	1
η	1		$\frac{1}{1+\Lambda\Gamma}$
$\Pi_{\mathcal{A}}$	$\frac{(1-\varkappa)\varkappa^{\frac{1+\varkappa}{1-\varkappa}}(1+T_{I,\mathcal{A}})(1-\sigma-\lambda)^{\frac{1-\varkappa-\varepsilon}{1-\varkappa}}e_{y,b}^{\frac{\varepsilon}{1-\varkappa}}}{(1-T_{\wp_{\mathcal{A}}})^{\frac{1}{1-\varkappa}}r^{\frac{\varkappa}{1-\varkappa}}}$		$(1-\lambda-\sigma)\Lambda_A\Gamma_A\left(\frac{A^{\varepsilon}E^{1-\varepsilon}}{r^{\varkappa+\varepsilon}}\right)^{\frac{1}{1-\varkappa-\varepsilon}}$
$\Pi_{\mathcal{E}}$	0		$\Pi_{\mathcal{A}}\Gamma_E\Lambda_E\frac{A}{E}$
$\Pi_{\mathcal{I}}$	0	$\frac{v_{\mathcal{I}}^{\frac{1}{1-\Theta}}(1+T_{I,\mathcal{I}})(1-\Theta)\Theta^{\frac{2\Theta}{1-\Theta}}}{r^{\frac{\Theta}{1-\Theta}}}$	0
σ	$1-\lambda-\frac{r}{\kappa\Gamma_s\Lambda_s}$		$1-\lambda-\frac{r}{(1+\Lambda\Gamma)^{1-\psi}\Gamma_s\Lambda_s}$
y	$(1-\sigma-\lambda)^{1-\varkappa-\varepsilon}A^{1-\varkappa}k_y^{\varkappa}e_{y,b}^{\varepsilon}$		$(1-\sigma-\lambda)^{1-\varkappa-\varepsilon}A^{1-\varkappa}E^{1-\varepsilon}k_y^{\varkappa+\varepsilon}\Sigma_y$
g_A	$\sigma\Phi\kappa$	$\frac{\sigma\Phi\kappa}{\left(1+\left(\frac{\varsigma_{\mathcal{I}} I\Pi_{\mathcal{I}}}{\kappa A\Pi_{\mathcal{A}}}\right)^{\frac{1}{1-\psi}}\right)^{\psi}}$	$\frac{\sigma\Phi}{\left(1+\Lambda\Gamma\right)^{\psi}}$
$g_{\mathcal{F}}$	$\sigma\varsigma_F\Phi(1-\kappa)$	$\frac{\sigma\varsigma_F\Phi(1-\kappa)}{\left(1+\left(\frac{\varsigma_{\mathcal{I}} I\Pi_{\mathcal{I}}}{\kappa A\Pi_{\mathcal{A}}}\right)^{\frac{1}{1-\psi}}\right)^{\psi}}$	0
g_E	0	0	$\frac{\sigma\Phi\varsigma_{\mathcal{E}}}{\left(1+\frac{1}{\Lambda\Gamma}\right)^{\psi}}$
g_I	0	$\frac{\sigma\Phi\varsigma_{\mathcal{I}}}{\left(1+\left(\frac{\kappa A\Pi_{\mathcal{A}}}{\varsigma_{\mathcal{I}} I\Pi_{\mathcal{I}}}\right)^{\frac{1}{1-\psi}}\right)^{\psi}}$	0

with $\Lambda := \left(\frac{\varsigma_{\mathcal{E}}\varepsilon^2}{\varkappa^2}\right)^{\frac{1}{1-\psi}}$, $\Gamma = \left(\frac{(1+T_{I,\varepsilon})(1-T_{\wp_{\mathcal{A}}})}{(1+T_{I,\mathcal{A}})(1-T_{\wp_{\mathcal{E}}})}\right)^{\frac{1}{1-\psi}}$, $\Lambda_A := \left(\frac{1-\varkappa}{\varkappa}\right)\left(\varkappa^{2(1-\varepsilon)}\varepsilon^{2\varepsilon}\right)^{\frac{1}{1-\varkappa-\varepsilon}}$, $\Gamma_A := \left(\frac{(1+T_{I,\mathcal{A}})^{1-\varkappa-\varepsilon}}{(1-T_{\wp_{\mathcal{A}}})^{1-\varepsilon}(1-T_{\wp_{\mathcal{E}}})^{\varepsilon}}\right)^{\frac{1}{1-\varkappa-\varepsilon}}$, $\Gamma_E := \left(\frac{(1+T_{I,\varepsilon})(1-T_{\wp_{\mathcal{A}}})}{(1+T_{I,\mathcal{A}})(1-T_{\wp_{\mathcal{E}}})}\right)$, $\Lambda_E := \frac{\varepsilon(1-\varepsilon)}{\varkappa(1-\varkappa)}$, $\Sigma_y := \left(\frac{\varkappa^{2\varkappa}(1-T_{\wp_{\mathcal{A}}})^{\varepsilon}(1-T_{\wp_{\mathcal{E}}})^{\varkappa}}{\varkappa^2(1-T_{\wp_{\mathcal{E}}})+\varepsilon^2(1-T_{\wp_{\mathcal{A}}})}\right)^{\varkappa+\varepsilon}$, $\Gamma_s := \frac{(1+T_v)(1+T_{I,\mathcal{A}})}{(1-T_{\wp_{\mathcal{A}}})(1-T_w)}$, $\Lambda_s := \left(\frac{(1-\varkappa)\varkappa}{(1-\varkappa-\varepsilon)}\right)$.

Table 3.4: SBGP characteristics with endogenous research.

Proof: See Appendix (B).

The lemma presents the labor and research effort allocations and their consequences on technology growth, output, and innovation profits, distinguishing among alternative research directions. Several features stand out. First, the equation describing y

expresses intermediates by their capital intensity in order to establish comparability to the exogenous growth representation. Second, there is no commuting-based research if $i_{\mathcal{E}} = 1$ since, in this case, brown energy prices are constant and with this commuting costs. Hence, no rational government will initiate commute-based research as it has no productivity effect. This leads to the three alternative innovation scenarios described in Table (3.4): a scenario where $i_{\mathcal{E}} = 0$ and $i_{\mathcal{I}} = 0$, one with $i_{\mathcal{E}} = 0$ and $i_{\mathcal{I}} = 1$, and one with $i_{\mathcal{E}} = 1$ and $i_{\mathcal{I}} = 0$.

Table (3.4) reveals that research efforts increase with the respective research efficiencies, ς_i, and sector-specific profits, Π_i, $i = \mathcal{A}, \mathcal{E}, \mathcal{I}$. Note that for $i_{\mathcal{E}} = 1$, balanced growth would be possible with only improvements in either $\mathcal{E}$ or $\mathcal{A}$. Still, the decentralized economy improves both because the two technologies complement one another; improvements in one of both increases the marginal product of the other.

In view of this, Lemma (3.5) discusses the determinants for alternative research.

Lemma 3.5. *Along an SBGP with endogenous research,*

$$i_{\mathcal{E}} = \begin{cases} 0 \;\; \text{if} \;\; (1+T_e)p_{e,b} < \wp_{\mathcal{E}} = \frac{r}{\varepsilon} & \Rightarrow \;\; e = e_b \\ 1 \;\; \text{if} \;\; (1+T_e)p_{e,b} \geq \wp_{\mathcal{E}} = \frac{r}{\varepsilon} & \Rightarrow \;\; e = e_g = \mathcal{E} = \int_0^E I_{\mathcal{E}}(j)^{\varepsilon} dj, \end{cases}$$

$$i_{\mathcal{I}} = \begin{cases} 1 \;\; \text{with} \;\; \chi = \frac{1}{1+\frac{1}{\varsigma_I^{\frac{1}{\psi}}}} & \text{if} \;\; W_1 = W_2 \\ \text{not feasible} \;\; \chi \to 0 & \text{if} \;\; W_1 < W_2 \\ 0 \;\; \text{with} \;\; \chi \to 1 & \text{if} \;\; W_1 > W_2, \end{cases}$$

$$W_1 := \frac{v_{\mathcal{I}}^{\frac{1}{1-\Theta}}(1+T_{I,\mathcal{I}})(1-T_{\wp_{\mathcal{A}}})^{\frac{1}{1-\varkappa}}}{(1+T_{I,\mathcal{A}})}, \; W_2 := \kappa \frac{A}{I} \frac{(1-\varkappa)\varkappa^{\frac{1+\varkappa}{1-\varkappa}}(1-\sigma-\lambda)^{\frac{1-\varkappa-\varepsilon}{1-\varkappa}} e_{y,b}^{\frac{\varepsilon}{1-\varkappa}}}{r^{\frac{\varkappa}{1-\varkappa}-\frac{\Theta}{1-\Theta}} \varsigma_{\mathcal{I}}^{\frac{1}{\psi}}(1-\Theta)\Theta^{\frac{2\Theta}{1-\Theta}}}.$$

Proof: See Appendix (B).

The lemma summarizes the determinants for improvements in green energy and commuting technology. While Proposition (3.1) has established that $i_{\mathcal{E}} = 1$ if $(1 + T_e)p_{e,b} \geq \wp_{\mathcal{E}}$, Lemma (3.5) reveals that endogenous research only considers green innovation if it can charge an elasticity-weighted markup[26] on the interest rate, $\wp_{\mathcal{E}} = \frac{r}{\varepsilon}$. The lower the elasticity, the higher the markup, the higher the reservation price for green energy innovation.

Further, if a government intends to initiate commuting based research, it is necessary that such research generates the same return as if allocated to general technology improvements in the numéraire sector. This requirement is satisfied when the two weights W_1 and W_2 are equal. If $W_1 < W_2$, commuting technology focused research crowds out all other research, so there is no technology improvement in the numéraire sector. If $W_1 > W_2$, all R&D activity allocates to the numéraire sector what crowds out research in the commuting sector. Henceforth, only $W_1 > W_2$ and $W_1 = W_2$ are leading to an SBGP, see Proposition (3.2). In any case, the government can impact the weight relation, as explained in Corollary (3.1).

Corollary 3.1. *Along an SBGP with endogenous research, there is always a* $v_{\mathcal{I}}$ *achieving* $W_1 = W_2$*. Alternatively, if* $W_1 < (>)\ W_2$*,* $W_1 = W_2$ *is reached with either* $T_{I_{\mathcal{A}}} > (<)\ 0$ *or* $T_{I_{\mathcal{I}}} < (>)\ 0$*. Further,* χ *is determined by* $W_1 = W_2$ *and, therefore, cannot be adjusted.*

Proof: The results follow directly with Lemma (3.5). □

The corollary reveals that a government can always stabilize commuting related research via the price it offers for infrastructure intermediates, $v_{\mathcal{I}}$. Alternatively, there is also the option to use research subsidies $T_{I,\mathcal{A}}$ and $T_{I,\mathcal{I}}$. These policies can be interpreted as prize money for innovations or infrastructure for research, and could be of practical relevance if there were some restrictions for $v_{\mathcal{I}}$, for example if expensive infrastructure projects lack public support[27].

Notably, because a stable research effort allocation requires that the rates of research returns among numéraire production and commuting equalize, a government cannot adjust the research efforts allocated to commuting, χ, at will. Thereupon,

[26] Note that the numéraire sector pays a subsidized price, $\frac{(1-T_{p_{\mathcal{E}}})r}{\varepsilon}$, that is below the price the intermediate charges in case the government decides to offer an intermediate subsidy.

[27] For instance, from 2009 to 2020, the city of Berlin had a short (1,8 km) and expensive U55 subway line that had provoked great public protest. The line was ultimately closed.

whenever the government finances infrastructure projects, it cannot control the pace of idea creation. On that premise, Proposition (3.3) connects results to translate Proposition (3.1) into an endogenous growth setting.

Proposition 3.3. *With endogenous research, a decentralized economy can achieve an SBGP along the regimes introduced in Proposition (3.1) if the following conditions are satisfied:*

(i) (ES regime) $i_{\mathcal{E}} = i_{\mathcal{F}} = 0$ *and* $\eta = \kappa = 1$. *Hence,*

- *(a)* $g_P = 0$ *requires that* $g_y = g_{p_{e,b}} + g_{T_e}$ *for* $g_{e_{y,b}} = 0$,
- *(b)* $\eta = 1$ *requires that* $T_{\wp_e} < 0$ *&* $g_y < g_{T_{\wp_e}}$, $T_{I,\mathcal{E}} < 0$ *&* $0 < g_{T_{I,\mathcal{E}}}$, *or both,*
- *(c)* $g_D = 0$ *requires that* $g_y = g_{T_{\bar{c}c}}$ *if* $i_{\mathcal{I}} = 0$ *and* $g_y = g_{\mathcal{I}}$ *if* $i_{\mathcal{I}} = 1$.

(ii) (PS Regime) $i_{\mathcal{E}} = 0$, $\eta = 1$ *and* $i_{\mathcal{F}} = 1$ *with* $\kappa = \frac{\varsigma_F}{1+\varsigma_F}$. *Thereby,*

- *(a)* $g_P = 0$ *given if* $\kappa = \frac{\varsigma_F}{1+\varsigma_F}$,
- *(b)* $\eta = 1$ *given if* $i_{\mathcal{E}} = 0$,
- *(c)* $g_D = 0$ *requires that* $g_{p_{e,b}} + g_{T_e} = 0$ *and* $i_{\mathcal{I}} = 0$.

(iii) (EPS Regime) $i_{\mathcal{E}} = 0$, $i_{\mathcal{F}} = 1$, $\eta = 1$ *and* $\kappa = \frac{\varsigma_F + \frac{1-\varkappa}{1-\varkappa-\varepsilon}\frac{g_{T_e}}{\sigma\Phi}}{\left(\varsigma_F + \frac{1-\varkappa}{1-\varkappa-\varepsilon}\right)}$. *Thereby,*

- *(a)* $g_P = 0$ *requires that* $g_{\mathcal{F}} = g_{e,b} = g_y - g_{T_e} = \frac{1-\varkappa}{1-\varkappa-\varepsilon}(g_A - g_{T_e})$,
- *(b)* $\eta = 1$ *requires that* $T_{\wp_e} < 0$ *&* $g_y < g_{T_{\wp_e}}$, $T_{I,\mathcal{E}} < 0$ *&* $0 < g_{T_{I,\mathcal{E}}}$, *or both,*
- *(c)* $g_D = 0$ *requires that* $g_{T_e} + g_{p_{e,b}} = g_{T_{\bar{c}c}}$ *if* $i_{\mathcal{I}} = 0$, *while* $i_{\mathcal{I}} = 1$ *is not possible.*

(iv) (GE Regime) $i_{\mathcal{E}} = 1$, $i_{\mathcal{F}} = 1$ *hence* $\kappa = 1$, η *detailed in Lemma (3.4). Thereby,*

- *(a)* $g_P = 0$ *given if* $i_{\mathcal{E}} = 1$,
- *(b) any* $0 < \eta < 1$ *feasible, while the decentralized economy chooses* $\frac{1}{1+\Lambda\Gamma}$,
- *(c)* $g_D = 0$ *given if* $i_{\mathcal{I}} = 0$.

This leads to

$$\mathcal{C}_i = \frac{c_i}{y_i} = \begin{cases} 1 - \frac{\varkappa}{(1+T_r)(1+\frac{\rho}{g_{y_i}})} & if \;\; i \in \{ES, EPS, PS\} \\ 1 - \frac{\varepsilon+\varkappa}{(1+T_r)(1+\frac{\rho}{g_{y_i}})} & if \;\; i = GE, \end{cases} \tag{37}$$

$$g_{y_i} = \sigma_i \Phi_i \Omega_i, \quad with \;\; \Omega_i := \begin{cases} \frac{1-\varkappa-\varepsilon}{1-\varkappa} & if \;\; i = ES, \;\; i_{\mathcal{I}} = 0 \\ \frac{1-\varkappa-\varepsilon}{1-\varkappa} \frac{1}{\left(1+\frac{1}{\varsigma_I^{\frac{1}{\psi}}}\right)^{\psi}} & if \;\; i = ES, \;\; i_{\mathcal{I}} = 1 \\ \frac{\varsigma_{\mathcal{F}}}{1+\varsigma_{\mathcal{F}}} & if \;\; i = PS \\ \frac{(1-\varkappa)\varsigma_F + (1-\varkappa-\varepsilon\varsigma_F)\frac{g_{T_e}}{\sigma\Phi}}{\left((1-\varkappa-\varepsilon)\varsigma_F + 1 - \varkappa\right)} & if \;\; i = EPS \\ \frac{\left((1-\varkappa)+(1-\varepsilon)\varsigma_E(\Lambda\Gamma)^{\psi}\right)}{(1-\varepsilon-\varkappa)(1+\Lambda\Gamma)^{\psi}} & if \;\; i = GE, \;\; i_{\mathcal{I}} = 0, \end{cases} \tag{38}$$

$$P = \begin{cases} \frac{\epsilon(1-\sigma-\lambda)^{\frac{1-\varkappa-\varepsilon}{1-\varepsilon}} A^{\frac{1-\varkappa}{1-\varepsilon}}}{(1-T_e)p_{e_{y,b}}\mathcal{F}} \left(\frac{\varkappa}{(1+T_r)(g_{y_i}+\rho)}\right)^{\frac{\varkappa}{1-\varepsilon-\varkappa}} \left(\frac{\varepsilon}{p_{e_{y,b}}}\right)^{\frac{\varepsilon}{1-\varepsilon-\varkappa}} + \varsigma_{\bar{c}c} \frac{\bar{l}^{1+\theta}}{(1+\theta)\mathcal{F}_0} & if \;\; i \in \{ES, EPS, PS\} \\ 0 & if \;\; i = GE, \end{cases} \tag{39}$$

$$r_i = g_{y_i} + \rho = \begin{cases} \varkappa \frac{y_i}{k_{y,i}} & for \;\; i \in \{ES, PS, EPS\} \\ (\varepsilon + \varkappa)\frac{y_i}{k_{y,i}} & for \;\; i = GE. \end{cases} \tag{40}$$

Proof See Appendix (B).

The proposition demonstrates that the endogenization of formally exogenous growth has only minor implications on the balanced growth characteristics previously described in Proposition (3.1). However, two aspects are crucial to address. First, with (iii) (c), in an EPS regime, it is impossible to stabilize density externalities through commuting infrastructure. This result holds since commuting technologies would need to grow at a lower rate than numéraire output. Differences in these growth rates lead to different returns in research which destabilize the research sector and are hence not bearable, see Corollary (3.1) for further details. Second, the ES and EPS regimes described in (i) (b) respectively (iii) (b) both present strategies to avoid a green energy transition. Such plans can become relevant if brown energy is welfare superior to green energy. As will be discussed, this is the case if it generates a higher rate of productivity growth. In addition, there are arguments to avoid green energy employment not addressed so far. For example, if nuclear energy is considered green, the model does not discuss contamination risks that may motivate to remain in a brown energy scenario. If policymakers lack legal options to ban the construction of nuclear power plants, the above policies become relevant. Such a case may occur if a neighboring region or country plans the construction of a power plant[28]. A green energy transition can be avoided by turning the intermediate subsidy, $T_{\wp_e}$, into a tax which grows faster than the brown energy price. Alternatively, it is possible to implement an increasing tax on green energy innovation, $T_{I,\mathcal{E}}$, avoiding the development of green energy power plants. Technically, a combination of both instruments is also possible, but not further addressed to ease the discussion.

Expressions (37) to (40) describe fundamental balanced growth characteristics with endogenous research. Hereby, Expression (37) introduces the consumption rate, $\mathcal{C}$, as a function of the decentralized numéraire growth rate. The higher the numéraire growth rate, the lower the consumption rate, the higher the savings rate, a standard feature of any Ramsey-related growth model.

The numéraire growth rate is detailed with Expression (38) and illustrated in Figure (3.13). There are three determinants to consider: the proportion of researchers, σ, the quality of agglomeration economies, Φ, and the growth burden related to the mitigation of the dynamic externalities, Ω. This latter component can be associated with an environmental growth drag, e.g., discussed in Nordhaus (2002) and Bruvol et al. (1999). It measures the dynamic costs of environmental degradation, which in this theory represent regime-specific costs of avoiding the respective environmental damage. In an ES regime without commuting technology improvements, $\frac{\varepsilon}{1-\varkappa}$ of the potential output growth rate compensates for keeping energy demand constant so that the gross output growth rate, $\sigma\Phi$, is sized by $\frac{1-\varkappa-\varepsilon}{1-\varkappa}$. This rescaling is even stronger with commuting technology improvements, since fewer research efforts are allocated to general technology. Similarly, in a PS regime, $1-\kappa = 1-\frac{\varsigma\mathcal{F}}{1+\varsigma\mathcal{F}}$ of the in-

[28] As the case may be with the German plans to shut down nuclear power plants by 2022 while France still invests in nuclear energy.

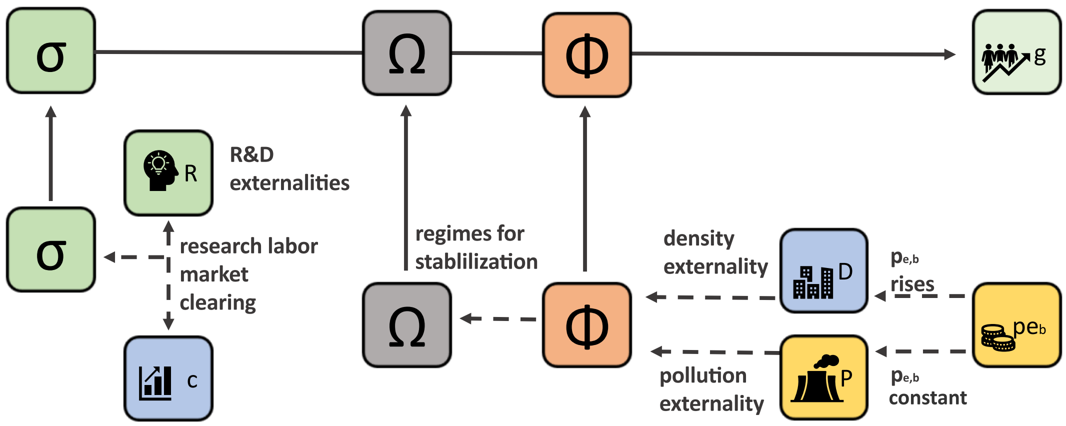

Figure 3.13: Determinants of the numéraire output growth rate.

novation potential is shifted to improving abatement technology and therefore cannot be used to improve productivity. For an EPS regime, the discussion is more complex since there is an additional component, $\frac{(1-\varkappa)\varsigma_F+(1-\varkappa-\varsigma_F\varepsilon)\frac{g_{T_e}}{\sigma\Phi}}{(1-\varkappa-\varepsilon)\varsigma_F+1-\varkappa}$, that weights the effects of environmental standards relatively to energy savings. Finally, a GE regime describes a scenario with no dynamic externalities, so its growth effect depends solely on the combination of research efforts with research efficiencies, $\frac{\left((1-\varkappa)+(1-\varepsilon)\varsigma_E(\Lambda\Gamma)^{\psi}\right)}{(1-\varepsilon-\varkappa)(1+\Lambda\Gamma)^{\psi}}$.

Expression (39) sheds light on the contributors to pollution. While green energy is free of any pollution, a brown energy scenario faces two sources of pollutants: numéraire production and commuting. Numeraire production related pollution is represented in the first summand of the expression. It reveals that the return on capital determines the capital intensity of numéraire production, so the greater the steady-state capital use, the higher the steady-state pollution concentration. The second summand of the expression illustrates that the greater the commuting distance, the more severe the pollution intensity.

Expression (40) concludes with the Euler equation for endogenous research, combined with the clearing condition of the savings market. This relation is standard for a Ramsey-related theory. A clearing of the savings market requires that the interest on consumption and the return on manufactured capital correspond. So, the lower the capital stock, the higher the interest rate[29]. As a result, manufactured capital use reduces with higher innovation rates.

With this knowledge at hand, it remains to assess the role of specialization for a sustainable spatial balanced growth path.

Specialization in research

In this theory, specialization restricts research options but has no fundamental effect on the general sustainability discussion. Since Chapter (2) has examined specialization in detail, this chapter will keep its assessment parsimonious and only evaluate how specialization narrows the alternative sustainable growth options. A more detailed specialization discourse is left to future research. The evaluation will afterward concentrate on the general case.

Lemma 3.6. *With endogenous research and $\psi = 1$, an R&D sector chooses one research direction based on $j^c = j \in \{\mathcal{A}, \mathcal{E}, \mathcal{I}\} \mid \max\{\kappa A \Pi_{\mathcal{A}}, i_{\mathcal{I}} \varsigma_{\mathcal{I}} \kappa I \Pi_{\mathcal{I}}, i_{\mathcal{E}} \varsigma_{\mathcal{E}} E \Pi_{\mathcal{E}}\}$.*

(i) If $j^c = \mathcal{A}$, $\chi = 1$, $\eta = 1$, $\Pi_{\mathcal{A}} = \frac{(1-\varkappa)\varkappa^{\frac{1+\varkappa}{1-\varkappa}}(1+T_{I,\mathcal{A}})(1-\lambda-\sigma)^{\frac{1-\varkappa-\varepsilon}{1-\varkappa}} e_{y,0}^{\frac{\varepsilon}{1-\varkappa}}}{(\kappa(1-T_{\wp_{\mathcal{A}}}))^{\frac{1}{1-\varkappa}} r^{\frac{\varkappa}{1-\varkappa}}}$.

(ii) If $j^c = \mathcal{E}$, $\chi = 1$, $\eta = 0$, $\Pi_{\mathcal{E}} = \frac{(1-\varepsilon)\varepsilon^{\frac{2}{1-\varepsilon}}(1+T_{I,\mathcal{E}})}{(1-T_{\wp_{\mathcal{E}}})^{\frac{1}{1-\varepsilon}}} \frac{(1-\lambda-\sigma)^{\frac{1-\varepsilon-\varkappa}{1-\varepsilon}}}{r^{\frac{\varepsilon}{1-\varepsilon}}} A_0^{\frac{\varepsilon}{1-\varepsilon}}$.

(iii) If $j^c = \mathcal{I}$, $\chi = 0$, $\eta = 0$, $\Pi_{\mathcal{I}} = \frac{v_{\mathcal{I}}^{\frac{1}{1-\Theta}}(1+T_{I,\mathcal{I}})(1-\Theta)\Theta^{\frac{2\Theta}{1-\Theta}} I_0^{\frac{\Theta}{1-\Theta}}}{r^{\frac{\Theta}{1-\Theta}}}$.

Thereby, σ follows with Table (3.4) whereby $\Gamma_V = \max\{\kappa A \Pi_{\mathcal{A}}, i_{\mathcal{I}} \varsigma_{\mathcal{I}} \kappa I \Pi_{\mathcal{I}}, i_{\mathcal{E}} \varsigma_{\mathcal{E}} E \Pi_{\mathcal{E}}\}$.

Proof: This result follows with Lemma (3.4) and (3.5) and Proposition (3.1) and (3.3). □

[29]Remember that Lemma (3.4) has shown that in a *GE* regime, the elasticity of manufactured capital in numéraire production is scaled by $\varkappa$ and ε. Hence, both parameters scale the marginal product of manufactured capital, an aspect considered in (40).

The lemma emphasizes that when researchers need to specialize, they focus their efforts in the direction with the greatest increase in profits. They consequently assess individual profits, $\Pi_{\mathcal{A}}$, $\Pi_{\mathcal{E}}$, or $\Pi_{\mathcal{I}}$, the number of intermediates, A, E, I, research efficiencies, $\varsigma_{\mathcal{E}}$, $\varsigma_{\mathcal{I}}$, and the stringency of environmental standards, κ. Proposition (3.4) describes how this effort focus affects numéraire production growth rates.

Proposition 3.4. *Along an SBGP with endogenous research and* $\psi = 1$*, either* $g_y = g_{\mathcal{A}} = \sigma\Phi_{\mathcal{A}}\kappa$ *or* $g_y = g_{\mathcal{E}} = \sigma\Phi_{\mathcal{E}}\varsigma_{\mathcal{E}}$*, while* $i_{\mathcal{I}} = 1$ *is not possible.*

Proof. See Appendix (B).

While specialization enables the economy to grow along one of the four growth regimes discussed in Proposition (3.1), it reduces the option space for policymakers as it introduces several restrictions for research and infrastructure provision. For example, it is not possible to address a potential density externality with improvements in commuting technology. Further, a GE regime is feasible, but only if all research is focused on green energy production. Note also that specialization may hinder research in abatement, an aspect not considered in the discussion. If this were the case, the consequence was that there are no PS or EPS regimes, and the economy can only grow in an ES or a GS regime. In any case, the next section will return to the benchmark scenario related to general access to technology and, based on this, assess the social optimal growth scenario.

3.7 Socially optimal growth

The fact that a growth path is sustainable does not say anything about whether it is socially optimal. Three factors are relevant for the latter: *First*, of the four innovation regimes introduced in Proposition (3.1), only one will be socially optimal. This regime is characterized by a specific combination of research efforts. *Second*, balanced growth rates depend on the quality of agglomeration economies, Φ, which is a public good determined by density and pollution, two public goods which lack a price valuing their social costs or benefits. *Third*, in decentralized economies, the research sector is monopolistic competitive, so that decentralized R&D decisions are likely not socially optimal. As discussed in Chapter (2), this concerns both the provision of intermediates and the allocation of research labor. This section subsequently describes these three factors for socially optimal growth in detail.

3.7.1 Socially optimal research efforts

Proposition (3.1) and (3.3) have introduced four regimes leading to sustainable growth. In all but an arbitrary case, only one of these regimes is socially optimal. For its identification, it is essential to first derive the social optimal effort allocation in each regime separately and then compare the regimes.

Since the social planner faces an exogenous brown energy price, it facilitates this discussion to associate an ES, EPS and PS regime with the brown energy price paths (BEP) introduced above. Therefore, it is assumed that in an *ES regime*, the brown energy price rises at the rate of numéraire output, relating to a BEP (2). A *PS regime* is associated with a BEP (1), where the brown energy price remains constant. Finally, an *EPS regime* relates to a peculiar scenario where the energy price path lies in-between a BEP (1) and a BEP (2).

Because all sustainable growth scenarios with brown energy have positive (and constant) pollution, this section will additionally consider two scenarios without production-induced pollution. One scenario completely abandons the use of brown energy in numéraire production, referred to as an *ES*$^{+}$ *regime*, the other follows a more ambitious pollution-saving strategy that not only keeps pollution constant but eliminates all production-related pollution, discussed as a *PS*$^{+}$ *regime*.

Table (3.5) presents the six regime variations that follow with this expanded evaluation. Note that an *ES*$^{+}$ *regime* only uses 'initial' energy, E_0, gained via 'initially' installed green energy power plants (e.g., wind, water, and solar energy). This initial green energy endowment follows directly with expression (24) when setting $i_{\mathcal{E}} = 0$. As a result, production increases due to improvements in general technologies without intensifying energy use. Distinctively, a *PS*$^{+}$ *regime* grows with an intensified energy use but requires an additional increase in abatement-focused research. Hence-

	g_y	P	e_y	$g_{e,y}$
ES	$\frac{1-\varkappa-\varepsilon}{1-\varkappa}\sigma_i\chi_i^{\psi}\Phi_i$	$\frac{e_{y,b}}{\mathcal{F}_0}+\varsigma_{\bar{c}c}\frac{\bar{l}^{1+\theta}}{1+\theta}$	$\mathcal{E}_0+e_{y,b,0}$	0
ES^+	$\frac{1-\varkappa-\varepsilon}{1-\varkappa}\sigma_i\chi_i^{\psi}\Phi_i$	$\varsigma_{\bar{c}c}\frac{\bar{l}^{1+\theta}}{1+\theta}$	$\mathcal{E}_0$	0
PS	$\frac{\varsigma_{\mathcal{F}}}{1+\varsigma_{\mathcal{F}}}\sigma_i\Phi_i$	$\frac{e_{y,b}}{\mathcal{F}_0}+\varsigma_{\bar{c}c}\frac{\bar{l}^{1+\theta}}{1+\theta}$	$e_{y,b}$	$g_{\mathcal{F}}$
PS^+	$\frac{\varsigma_{\mathcal{F}}}{1+\varsigma_{\mathcal{F}}}\sigma_i\Phi_i-\iota^+$	$\varsigma_{\bar{c}c}\frac{\bar{l}^{1+\theta}}{1+\theta}$	$e_{y,b}$	$g_{\mathcal{F}}$
EPS	$\frac{(1-\varkappa)\varsigma_F+(1-\varkappa-\varepsilon\varsigma_F)\frac{g_{T_e}}{\sigma\Phi}}{\left((1-\varkappa-\varepsilon)\varsigma_F+1-\varkappa\right)}$	$\frac{e_{D,y,0}}{\mathcal{F}_0}+\varsigma_{\bar{c}c}\frac{\bar{l}^{1+\theta}}{1+\theta}$	$e_{y,b}$	$g_{\mathcal{F}}$
GE	$\left(\frac{1-\varkappa-\varepsilon}{1-\varkappa}\eta_i^{\psi}+\frac{\varepsilon}{1-\varkappa}\varsigma_{\mathcal{E}}(1-\eta_i)^{\psi}\right)\sigma_i\Phi_i$	0	$\mathcal{E}$	$g_{\mathcal{E}}$

Table 3.5: Alternative social planner growth regimes and their key characteristics, with $\mathcal{F}_0$, $\mathcal{E}_0$ and $e_{y,b,0}$ referring to initial steady state stocks.

forth, the numéraire growth rate for a PS^+ regime differs from a PS regime by ι^+, summarizing additional research efforts necessary to eliminate the harmful pollution stock[30], see Appendix (B) for details.

Against this background, Lemma (3.7) prepares for the social planner results.

Lemma 3.7. *Suppose a social planner can choose among the alternative balanced growth regimes listed in Table (3.5), then*

(i) $\chi^*=1$ *in all regimes,* $\eta^*=\frac{1}{1+\Lambda^*}$*, and* $\eta^*=1$ *otherwise,*

(ii) $\Omega^*_{GE}=\frac{\left((1-\varkappa)+(1-\varepsilon)\varsigma_E(\Lambda^*)^{\psi}\right)}{(1-\varepsilon-\varkappa)(1+\Lambda*)^{\psi}}$ *with* $\Lambda^*=\left(\frac{(1-\varepsilon)\varsigma_{\mathcal{E}}}{1-\varkappa}\right)^{\frac{1}{1-\psi}}$ *while for the remaining scenarios* Ω^*_i *follows with (38). Thereby,* $\Omega^*_{ES}=\Omega_{ES^{*+}}$ *and* $\Omega_{PS}=\Omega_{PS^{*+}}+\iota^+$.

Proof: See Appendix (B).

The lemma emphasizes two salient features. *First*, the social planner will not improve commuting technologies because the corresponding efforts are only suitable for stabilizing commuting costs without having a growth effect. If a decentralized economy faces increasing commuting costs, it is socially optimal to shift all research

[30]That does not mean that the actual production growth rate in a PS^+ regime is smaller than in a PS regime, as both Φ_{PS^+} and σ_{PS^+} can be larger than Φ_{PS} and σ_{PS}.

to numéraire technologies and subsidize commuting, thence $\chi^* = 1$. *Second*, the socially efficient research efforts in a GE regime are different from those observed in a decentralized economy, given with $\eta = \frac{1}{1+\left(\frac{\varsigma_{\mathcal{E}}\varepsilon^2}{\varkappa^2}\right)^{\frac{1}{1-\psi}}}$, see Table (3.4) for details. While a planner allocates research efforts by contemplating aggregate growth rate effects, the decentralized economy only considers individual research profits. These private profits are subject to price markups, which weigh $\varsigma_{\mathcal{E}}$ quadratically. This weighing biases the decentralized efforts for green technology improvements.

Given the effort allocation described in Lemma (3.7), the planner compares the productivity growth rates of the alternative innovation regimes and chooses the one with the highest numéraire production growth rate, as detailed subsequently.

Proposition 3.5. *Suppose a social planner can choose among the alternative balanced growth regimes listed in Table (3.5), then the planner selects*

$$i^{**} = i^* \in \{ES, ES^+, PS, PS^+EPS, GE\} \mid g_{y_i^{**}} \geq g_{y_i^*}.$$

Proof: See Appendix (B).

The proposition underlines the long-term perspective of the model and the corresponding interpretation of social optimality. In the long run, the growth rate of numéraire consumption dominates the representative household utility. Hence, the planner chooses the regime with the highest rate of numéraire production growth. Significantly, before selecting the regime, the planner allocates all available production factors by weighing instant consumption-dependent utility effects with innovation-related utility effects. The proceeding section will detail that these social planner allocation decisions are strongly affected by density-related considerations.

3.7.2 Socially optimal density

The central density follows with the allocation of land, $\bar{l}$, and real estate labor, λ, since the two production factors determine the morphological capital intensity per location. The socially optimal allocation of both factors values instant utility effects and utility growth effects. Instant utility effects follow, since land and labor are used for the provision of numéraire production, apartment use, and the hinterland. Utility growth effects follow since the CBD density determines the quality of agglomeration economies and thus the pace of innovation. The subsequent evaluation will first describe the socially optimal allocation of land, then the socially optimal allocation of labor, and finally combine both to assess the socially optimal density.

Lemma 3.8. *Suppose a social planner can choose among the alternative balanced growth regimes listed in Table (3.5), then in each regime*

$$\bar{l}^* = \frac{\hat{l}}{1 + \frac{1}{\mathcal{W}_{l_i}}} \tag{42}$$

with

$$\mathcal{W}_{l_i} := \begin{cases} \frac{\iota(1-\gamma)}{(1-\iota-\epsilon)} - \frac{\epsilon\left(\frac{\varepsilon(1+\theta)e_{\bar{c}c_i}}{e_{y,b,i}} + (1-\varkappa)\sigma_i\Phi_i(1-\overline{\varsigma}_\Phi D_i)\frac{\gamma}{\rho}\right)}{\mathcal{C}_i^*(1-\iota-\epsilon)} & if \;\; i = ES, ES^+ \\ \frac{\iota(1-\gamma)}{(1-\iota-\epsilon)} - \frac{\epsilon\left(\frac{(1+\theta)\varepsilon e_{\bar{c}c_i}}{e_{y,b,0,i}} + \frac{(1-\varkappa)\sigma_i\Phi_i(1-\overline{\varsigma}_\Phi D_i)\gamma}{\rho}\left(\frac{1}{\frac{1+\varsigma_F}{\varsigma_F} - \frac{e_{y,b,0}\sigma_i\Phi_i\varsigma_P}{\rho(1+\varsigma_P P_i)\mathcal{F}_0}}\right)\right)}{\mathcal{C}_i^*(1-\iota-\epsilon)} & if \;\; i = PS \\ \frac{\iota(1-\gamma)}{(1-\iota-\epsilon)} - \frac{\epsilon\left(\frac{(1+\theta)\varepsilon e_{\bar{c}c_i}}{e_{y,b,0,i}} + \frac{(1-\varkappa)\sigma_i\Phi_i(1-\overline{\varsigma}_\Phi D_i)\gamma}{\rho}\left(\frac{1}{\kappa - \frac{(1-\kappa)\varsigma_F e_{y,b,i}\sigma_i\Phi_i\varsigma_P}{\kappa\rho(1+\varsigma_P P_i)F}}\right)\right)}{\mathcal{C}_i(1-\iota-\epsilon)} & if \;\; i = PS^+, EPS \\ \frac{\iota(1-\gamma)}{1-\iota-\epsilon} - \frac{\epsilon\sigma\Phi(1-\varkappa)\left(1+\left(\frac{(1-\varepsilon)\varsigma_{\mathcal{E}}}{1-\varkappa}\right)^{\frac{1}{1-\psi}}\right)^{1-\psi}(1-\overline{\varsigma}_\Phi D_i)\gamma}{\mathcal{C}_i\rho} & if \;\; i = GE, \end{cases}$$

$$\mathcal{C}_i^* := \begin{cases} 1 - \frac{\varkappa}{1+\frac{\rho}{g_{y_i}}} - \frac{p_{e_{b,i}}(e_{y,b,i}+e_{\bar{c}c_i})}{y_i} & if \;\; i = ES, PS, EPS \\ 1 - \frac{\varkappa}{1+\frac{\rho}{g_{y_i}}} - \frac{p_{e_{y,b,i}}e_{\bar{c}c_i}}{y_i} & if \;\; i = ES^+, PS^+ \\ 1 - \varkappa - \varepsilon + \frac{\varkappa+\varepsilon}{1+\frac{g_{y_i}}{\rho}} & if \;\; i = GE. \end{cases} \tag{43}$$

Proof: See Appendix (B).

The lemma presents the socially optimal allocation of land as a relation weighing the aggregate available land, $\hat{l}$, with $1+\frac{1}{\mathcal{W}_{l_i}}$. The higher $\mathcal{W}_{l_i}$, the higher real estate labor demand. The first factor in $\mathcal{W}_{l_i}$, namely $\frac{\iota(1-\gamma)}{(1-\iota-\epsilon)}$, presents a social weight of labor in real estate provision. It increases with the elasticity of apartment-based utility, ι, and the expenditure share of the real estate budget on land, $1-\gamma$, and reduces with $1-\iota-\epsilon$, describing the social benefit of labor for numéraire production. The second summand in $\mathcal{W}_{l_i}$ describes the net effect of expansion on the quality of agglomeration economies. It increases with the elasticity of hinterland-based utility, ϵ, accounting for the social value of land for hinterland consumption, and reduces with the socially optimal consumption rate, $\mathcal{C}^*$, representing the social benefit of consumption[31]. Expression (43) details the socially optimal consumption rate which reduces with both higher energy expenditures and the numéraire growth rate. Accordingly, the planner reduces consumption to generate savings, the greater the growth potential of the economy, and reduces consumption if the energy expenditures are high.

When comparing the social planner results with the decentralized economy, Expression (34) reveals that the decentralized valuation of land among hinterland and apartments is biased as it only contemplates direct consumption effects, ignoring repercussions of land use on the density profile. However, without defining the parameter values, it is unclear whether urban expansion is too low or too high, a question only a numerical application can answer. The subsequent section provides such an application, while this section proceeds with an assessment of the socially optimal real estate labor allocation.

Lemma 3.9. *Suppose a social planner can choose among the alternative balanced growth regimes listed in Table (3.5), then the planner selects*

$$\lambda_i = \frac{1-\sigma}{1+\frac{1}{\mathcal{W}_{\lambda_i}}}, \tag{44}$$

[31]The summands in the brackets in $\mathcal{W}_{l_i}$ consider that adjustments in the urban area affect the quality of agglomeration economies via density and pollution impacts. The first summand weighs commuting pollution with numéraire pollution. The second summand weighs the net effect of expansion on the quality of the agglomeration economies.

$$W_{\lambda_i} := \begin{cases} \frac{\iota\gamma\mathcal{C}_i}{(1-\varkappa-\varepsilon)\epsilon} + \frac{(1-\varkappa)\sigma_i\Phi_i(1-\bar{\varsigma}_\Phi D_i)\gamma}{(1-\varkappa-\varepsilon)\rho} & if \;\; i = ES, ES^+ \\ \frac{\iota\gamma\mathcal{C}_i}{(1-\varkappa-\varepsilon)\epsilon} + \frac{(1-\varkappa)\sigma_i\Phi_i^*(1-\bar{\varsigma}_\Phi D_i)\gamma}{(1-\varkappa-\varepsilon)}\left(\frac{1}{\frac{1+\varsigma_F}{\varsigma_F} - \frac{e_{y,b,i}\sigma_i\Phi_i\varsigma_P}{\rho(1+\varsigma_P P_i)F}}\right) & if \;\; i = PS \\ \frac{\iota\gamma\mathcal{C}_i}{(1-\varkappa-\varepsilon)\epsilon} + \frac{(1-\varkappa)\sigma_i\Phi_i(1-\bar{\varsigma}_\Phi D_i)\gamma}{(1-\varkappa-\varepsilon)\rho}\left(\frac{\kappa}{1-\frac{(1-\kappa)\varsigma_F e_{y,b,i}\sigma_i\Phi_i\varsigma_P}{\rho(1+\varsigma_P P_i)F}}\right) & if \;\; i = PS^+, EPS \\ \frac{\gamma\iota\mathcal{C}_i}{(1-\varkappa-\varepsilon)\epsilon} + \frac{(1-\varkappa)\sigma_i\Phi_i}{(1-\varkappa-\varepsilon)\rho}\left(1+\left(\frac{(1-\varepsilon)\varsigma_\mathcal{E}}{1-\varkappa}\right)^{\frac{1}{1-\psi}}\right)^{1-\psi}(1-\bar{\varsigma}_\Phi D_i)\gamma & if \;\; i = GE. \end{cases}$$

Proof: See Appendix (B).

The lemma reveals that the socially optimal real estate labor allocation principle described with Expression (44) and the decentralized research labor allocation principle described with Expression (35) follow similar patterns. Yet, the planner considers that a rise in real estate labor affects the quality of agglomeration economies, Φ, as it increases morphological capital use and hence density, plus pollution if the commuting energy source is brown. Similarly to the above-described land allocation, a decentralized economy ignores such elementary repercussions[32].

Combining the socially optimal land and labor allocations leads to the socially optimal density, as addressed with Proposition (3.6) next.

Proposition 3.6. *Suppose a social planner can choose among the alternative balanced growth regimes listed in Table (3.5), then the socially optimal density follows with*

$$D^* = \Lambda_D \left(\frac{\lambda^*}{\bar{l}^*}\right)^\gamma \tag{45}$$

with $\Lambda_D = \mathcal{B}\frac{(1-\gamma)^{1-\gamma}(1-\gamma-\theta)^\gamma}{1-\gamma-\gamma\theta}$. *Thereby,* $\bar{l}^*$ *is given with Expression (42) and* λ^* *with Expression (44).*

Proof: See Appendix (B).

[32]The real estate labor allocation principle of a decentralized economy can also be represented by a weighting factor $\mathcal{W}_D := \frac{\mathcal{C}}{\bar{p}_e\bar{l}^\theta}\frac{\gamma\iota}{(1-\varkappa-\varepsilon)\epsilon}\frac{(1-\gamma-\gamma\theta)}{(1-\gamma-\theta)}\frac{(1+T_c)(1+T_w)}{(1+T_a)}$. This factor illuminates the commuting cost sensitivity of the decentralized economy, measured with $\frac{\bar{p}_e\bar{l}^\theta(1-\gamma-\theta)}{(1-\gamma-\gamma\theta)}$, which weigh static energy costs. While commuting expenditures have no direct effect on the social planner weights, they affect the socially optimal consumption rate, a feature detailed in the appendix.

The CBD density follows with the apartment provision per area. The socially optimal density is subject to two aspects: the socially optimal weighing of land and labor within the apartment production process and the socially optimal allocation of land and labor among the sectors.

The proposition reveals that the social planner density Expression (45) follows the same patterns as the decentralized density described with Expression (19). This result states that the combination of the bid rent principle and a perfectly competitive real estate sector leads to a socially efficient weighing of land and labor in the decentralized production of apartments. Still, the difference between the social planner density and the decentralized density is the actual use of labor and land, respectively their actual allocation among the sectors. Both factors are subject to the complex valuation described above. Accordingly, while a socially optimal allocation of labor and land determines Expression (45), Expression (19) follows with land and labor allocations that are likely biased. It is not possible to qualify the direction of this bias without calibration, so it is assessed in the subsequent section. This section concludes with an assessment of a socially optimal R&D sector.

3.7.3 Socially optimal R&D

In this theory, the R&D sector delivers its invented intermediates as a monopolistic competitive provider. The monopolistic market conditions bias both the number of intermediates provided and the allocation of research efforts for new intermediates. The former occurs since the R&D sector charges its intermediates a markup in proportion to the elasticity of intermediate demand, which inflates the price and reduces demand. This static research market externality also affects the value of innovations. So it additionally impacts the decentralized research intensity and, therefore, creates a dynamic externality as well. The production of intermediate goods is based on a putty-clay technology that directly converts manufactured capital into intermediate goods. Accordingly, the socially optimal intermediate price is the marginal product of manufactured capital in numéraire production. In decentralized economies, this marginal product corresponds to the interest rate, r.

There are additional factors that bias the research intensity. While the previous discussion has revealed that the allocation of research efforts among alternative innovation pathways can be biased, there is also a general bias in the number of researchers. Chapter (2) has intensively discussed the reasons for such a bias, so this section will focus on its impacts on decentralized growth and welfare.

Proposition 3.7. *On an SBGP with endogenous research, the socially optimal fraction of researchers is given with*

$$\sigma_i^* = \begin{cases} 1-\lambda-\frac{(1-\varkappa-\varepsilon)\rho}{(1-\varkappa)\Phi} & \textit{if } \ i=ES, ES^+ \\ 1-\lambda-\frac{(1-\varkappa-\varepsilon)}{1-\varkappa}\left(\frac{\rho(1+\varsigma_F)}{\Phi\varsigma_F}-\frac{e_{y,b}\varsigma_P}{(1+\varsigma_P P)\mathcal{F}}\right) & \textit{if } \ i=PS \\ 1-\lambda-\frac{(1-\varkappa-\varepsilon)}{(1-\varkappa)\kappa}\left(\frac{\rho}{\Phi}-\frac{(1-\kappa)\varsigma_F e_{y,b}\varsigma_P}{(1+\varsigma_P P)\mathcal{F}}\right) & \textit{if } \ i=PS^+, EPS \\ 1-\lambda-\frac{\rho}{\Phi\left(1+\left(\frac{(1-\varepsilon)\varsigma_{\mathcal{E}}}{1-\varkappa}\right)^{\frac{1}{1-\psi}}\right)^{1-\psi}} & \textit{if } \ i=GE. \end{cases} \tag{46}$$

Proof. See Appendix (B).

While decentralized economies focus on private profits of innovation, a planner only considers their social benefits. The biased decentralized economy perspective affects the innovation rate (as it biases the fraction of researchers) and the instant production intensity (as it affects the numéraire and apartment production intensity). The proposition displays the sources of this bias in the last summand of the RHS of expression (46). For example, in an ES, PS, or EPS regime without any policy ($\Gamma_s = 1$), the decentralized research labor allocation only corresponds to the social planner allocation if $\frac{g_y+\rho}{\kappa\varkappa} = \frac{\rho}{\Phi^*}$, which is an arbitrary event.

In summary, this section has elaborated that several sources may bias the allocation of land and labor. These distortions affect the density profile and the number of researchers, and, therefore, have both an instant utility effect and a utility growth effect, as summarized in Figure (3.14) graphically.

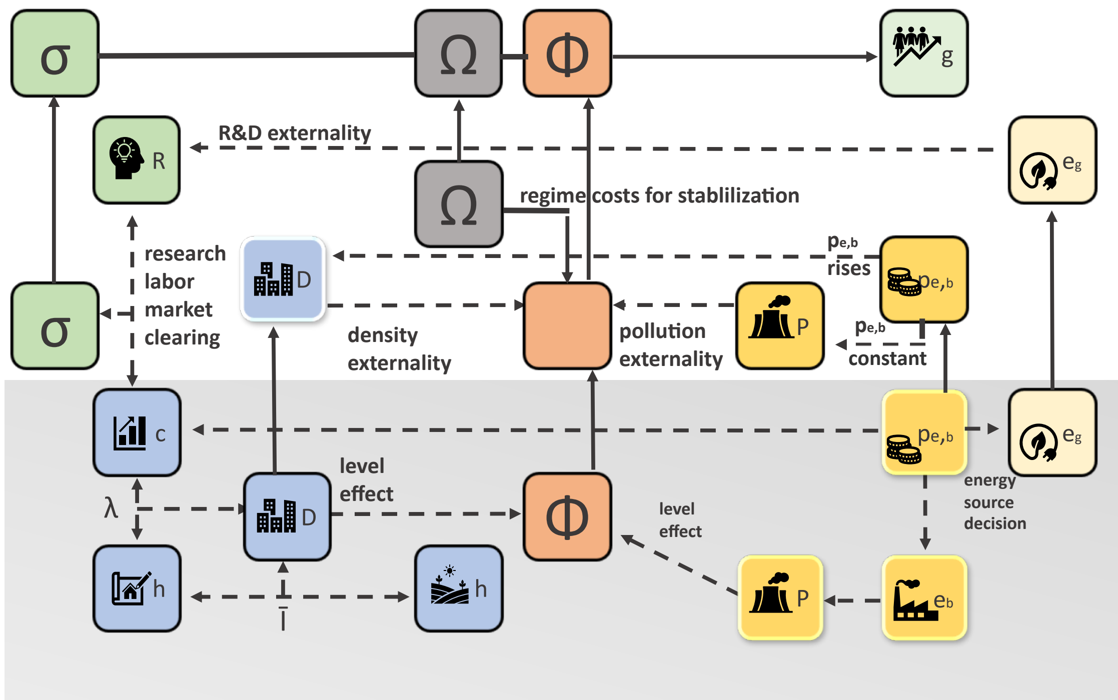

Figure 3.14: Overview of the determinants and the potential biases of the decentralized factor allocations and the corresponding production growth rate.

3.8 Numerical application

Several non-linearities characterize the first-order conditions of the land and labor market in this theory. While it is impossible to solve the model analytically, it is conceivable to discuss a numerical solution of a calibrated model. Such a numerical application is the focus of this section. For this purpose, the model is written in the General Algebraic Modeling System (GAMS) and solved as a nonlinear program with the CONOPT3 solver (Version 3.17K, see Drud, 1994). For a well-structured discussion of the solution, the section first introduces the calibration strategy, then evaluates the numerical results, and finally discusses the robustness of the solution to adjustments in critical parameters.

3.8.1 Calibration strategy

Several subject areas of this chapter's theory are new to the literature. Henceforth, it is inconceivable to calibrate all parameters based on references to other work. While it is possible to standardize most undefinable parameters to unity, a few require estimations. Given the rough data available, these estimates are ad hoc and only help to illustrate qualitative features of the theory and the qualitative direction of welfare-improving policy. Table (3.6) presents the parameters.

	Parameters based on literature reference	
$\varkappa$	elasticity of capital	0.250
ε	elasticity of energy	0.050
γ	elasticity of morph. capital	0.700
θ	elasticity of commuting costs	0.150
ϵ	consumption share	0.330
ι	apartment share	0.330
ρ	time preference	0.015
$\mathcal{B}$	real estate technology	1.000
	Parameters based on own calculation	
$\varsigma_{\mathcal{E}}, \varsigma_{\mathcal{F}}$	research efficiencies (I)	1.000
ψ	research efficiency (II)	0.500
$\bar{\varsigma}_{\Phi}$	quality of aggl. economies	0.600
$\underline{\varsigma}_{\Phi}$	quality of aggl. economies	0.100
ς_P, $\varsigma_{\bar{c}c}$	quality of aggl. economies	1.000
$\hat{l}$	available land	1.000
$\mathcal{A}_0, \mathcal{F}_0$	factor endowments	1.000
$\mathcal{E}_0$	factor endowment	1.000
$\wp_{E_0}$	initial brown energy price	1.000

Table 3.6: Parameter selection.

3.8.2 Results

Table (3.7) presents the key characteristics of steady-states in different growth regimes[33]. It reveals that the planner's GE^{*} regime is the socially optimal reference scenario, as it achieves the highest numéraire growth rate (5.51%). For this reason, it is indicated by **. The GE^{**} regime's high growth rate is reached since improvements in general technologies and green energies complement one another. Innovations in one technology improve the productivity of the other technology. Since a green energy innovation strategy addresses two technologies (general technology and green energy provision), it is more effective than the innovation strategies of the other regimes (which focus on general technology improvements and possibly abatement).

	λ	$\bar{l}$	D	P	σ	Φ	c	a	x	g_y
ES	0.25	0.12	1.58	0.09	0.58	0.056	0.21	0.19	0.88	3.04 %
PS	0.18	0.12	1.26	0.09	0.69	0.054	0.18	0.15	0.88	1.88 %
EPS	0.20	0.12	1.35	0.09	0.65	0.055	0.19	0.16	0.88	2.35 %
GE	0.27	0.12	1.69	0.00	0.54	0.060	0.30	0.20	0.88	3.54 %
ES^{*}	0.12	0.05	1.84	0.04	0.64	0.059	0.24	0.09	0.95	3.50 %
ES^{+*}	0.14	0.05	1.86	0.00	0.63	0.061	0.27	0.10	0.95	3.59 %
PS^{*}	0.27	0.15	1.41	0.34	0.28	0.045	0.48	0.22	0.85	0.62 %
PS^{+*}	0.22	0.12	1.46	0.08	0.41	0.057	0.35	0.18	0.88	1.17 %
EPS^{*}	0.20	0.10	1.50	0.19	0.47	0.051	0.28	0.16	0.9	1.74 %
GE^{**}	0.27	0.22	1.09	0.00	0.56	0.057	0.11	0.24	0.78	**5.51 %**

Table 3.7: Regime-specific solutions to the benchmark calibration with the social planner reference scenario indicated by $**$.

33 For the discussion of the results, note that the numerical solution of this section is sensitive to the initial guesses used in the solution procedure of the CONPOT solver. Technically, the non-linear equations of this theory may result in multiple solutions, which are sensitive to the starting values used for the solver. All results presented are based on starting values in the middle range of reasonable variable values. Yet, starting values closer to the lower or higher bound of credible variable values all lead to the same outcome (or do not result in a solution). For this reason, there is no further systematic attempt to find an alternative solution, and the outcome discussed in the following is treated as the reference steady-state of the economy.

Decentralized researchers are aware of the efficiency of combined research. Therefore, the GE regime grows faster than all alternative decentralized regimes. Anyhow, the corresponding growth rate (3.54%) is considerably lower than the planner's reference rate. This difference emerges because decentralized economies undervalue the benefits of innovation for growth and welfare, resulting in a lower fraction of researchers ($\sigma_{GE} = 0.54$ compared to $\sigma_{GE}^{**} = 0.56$).

Two allocation qualities are specifically worth emphasizing. *First*, the apartment provision in a GE^{**} regime is considerably higher than in any alternative regime, be it planner reference regimes or decentralized regimes. The explanation is that there is no commuting-induced pollution, and the social planner economy does not face excessive energy price markups. These conditions motivate the planner to intensify both morphological capital utilization and urban expansion. *Second*, the real estate sector's land and labor employments are relatively high in a GE^{**} regime. Thereby, the land allocation has a more dominant effect on density, so the corresponding city is relatively flat[34].

Interestingly, the GE^{*} regime density counts 1.09, which is below the density where the quality of agglomeration economies peaks. The latter is given with $D_{peak} = \frac{1}{\varsigma_{\Phi}} = 1.\bar{6}$ and obtained when maximizing Φ for D. To give this abstract density number a practical reference, remember the literature discussion which cited Carlino and Saiz (2019) finding that 2,200 jobs per square mile lead to the highest rates of innovation in the US. Assuming that this rate relates to the peak density $D_{peak} = 1.\bar{6}$, the social planner suggests to reduce this job count to $\frac{2200}{1.\bar{6}} \times 1.09 \approx 1440$ jobs per square mile. Translated into this theory, the result states that it is socially preferable to expand the city and reduce hinterland consumption to provide more but flatter apartment space for the citizens. The reason for this finding is that the planner's intention to find a utility-maximizing consumption savings decision shifts some labor that decentralized economies use for innovations to instant consumption generation[35].

[34]Interestingly, the quality of agglomeration economies of a GE^{**} regime is below the one of an ES^{*} and ES^{+*} regime. Yet, the combined productivity effects of abatement research and green energy-based research are so strong that it is even possible to allocate more labor from research to the numéraire and real estate sector and still achieve the highest growth rates.

[35]This allocation principle is comparable to the golden rule, describing the savings decision in Solow

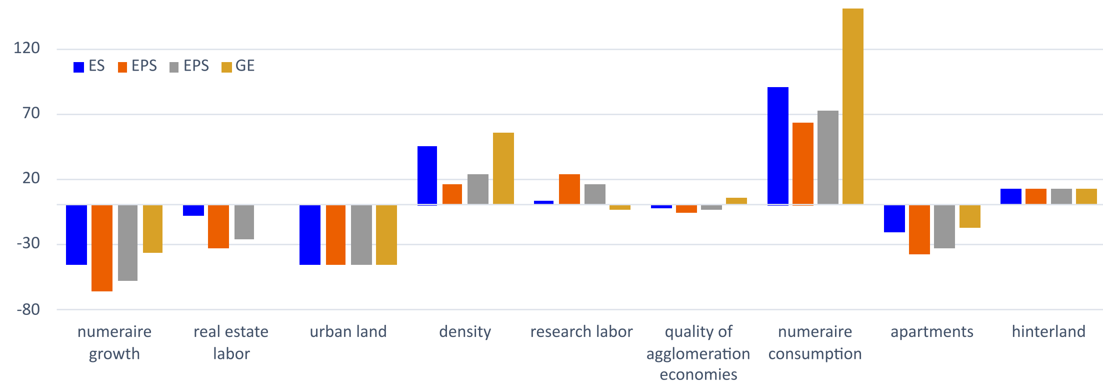

Figure 3.15: Percentage deviations of the economic core indicators in each decentralized growth regime from the social planner benchmark GE^{**} regime.

Figure (3.16) illustrates the bias in decentralized allocation principles by presenting their percentage deviations from the planner's GE^{**} reference regime.

For these results, note that a PS and EPS regime reveal lower technology improvements since some research efforts improve abatement. The no-arbitrage condition in the labor market, therefore, leads to more research labor than in other regimes. Accordingly, production-related labor employment (in the numéraire and real estate sector) is lower in regimes with abatement. The lower real estate labor use leads to a lower morphological capital intensity per location, and consequently to a lower construction density. Further, abatement efforts reduce the rate of innovation in general technologies. In combination, these effects have the consequence that decentralized PS and EPS regimes grow at lower rates than decentralized ES and GE regimes[36]. The GE regime profits by a comparably effective combination

growth theory. This rule proposes a saving rate that maximizes instant consumption rate and not the growth rate.

[36]Note that in decentralized and social planner economies, an ES regime leads to a higher growth rate than an EPS or PS regime. Since energy has a relatively low weight in numéraire production, it is more efficient to keep energy consumption constant than using research efforts to abate energy-induced pollution. Consequently, a PS regime has the lowest growth rate among the GE

of general technologies with green energies, creating the highest productivity growth rates. Adjusting the regime-specific factor allocations to their socially optimal level further improves welfare and the numéraire production growth rate.

In summary, the numerical application illuminates four fundamental findings. *First*, if the economy is not already operating in a green energy scenario (a question not answered), a green energy transition is socially optimal. *Second*, decentralized land and labor allocations are not socially optimal, so policies can considerably improve welfare. *Third*, the monopolistic research sector provides too little intermediates. *Forth*, there is not sufficient research labor in a decentralized GE regime. Therefore, land, labor, and innovation policy can have profound welfare effects. The subsequent section will address the optimal design of policy in depth. Before that, the section tests the robustness of the results to parameter adjustments.

Robustness of the consumption weights

As explained above, the benchmark calibration analyses a city. It is hence promising to examine whether a specification that assesses a metropolitan statistical area (MSA), representing an urban region, affects results. World Bank data on MSAs suggest that 80% of the global GDP gets produced in urban areas, a feature accounted for when setting $1 - \iota - \epsilon = 0.2$. If still assuming that the apartment expenditure share remains around 30%, this leads to $\iota = 0.3$ and $\epsilon = 0.5$ (whereby $\iota = 0.3$ instead of 1/3 was chosen to simplify notation with $\epsilon = 0.5$ instead of $0.4\bar{6}$).

Table (3.8) shows how the adjusted parameter weights affect the economic core indicators. Crucially, there is no solution for a social planner PS^{*+} regime since the relatively low CBD density in combination with a stark use of research efforts for abatement lowers the numéraire growth rate to such an extent that the regime is not stable[37]. This being the case, with the new parameter weights, the planner's GE^{**} regime remains the socially optimal reference scenario. Compared to the benchmark parameter selection, the GE^{**} regime has a lower numéraire growth rate of 4.87 %

alternatives, while the ES has the highest growth rate.

[37]The PS^{*} regime finds a solution as the commuting-induced pollution initiates a greater urban area use as in the PS^{*+} regime. As a result, Φ^{*} remains sufficiently high for a stable result.

	λ	$\bar{l}$	D	P	σ	Φ	c	a	x	g_y
ES	0.13	0.19	0.76	0.14	0.71	0.042	0.20	0.14	0.81	2.78 %
PS	0.11	0.19	0.65	0.13	0.78	0.039	0.17	0.12	0.81	1.51 %
EPS	0.12	0.19	0.71	0.14	0.75	0.041	0.18	0.13	0.81	2.05 %
GE	0.15	0.18	0.85	0.00	0.66	0.051	0.31	0.15	0.81	3.58 %
ES^*	0.07	0.03	1.78	0.03	0.69	0.060	0.24	0.05	0.93	3.86 %
ES^{+*}	0.08	0.03	1.79	0.00	0.69	0.061	0.26	0.06	0.92	3.95 %
PS^*	0.15	0.08	1.43	0.34	0.45	0.045	0.35	0.12	0.90	0.99 %
PS^{+*}	na	na	na	na	na	na	na	na	na	na
EPS^*	0.12	0.02	1.54	0.22	0.55	0.050	0.12	0.09	0.88	1.91 %
GE^{**}	0.28	0.31	0.88	0.00	0.54	0.052	0.12	0.27	0.69	**4.87 %**

Table 3.8: Regime-specific solutions to a calibration with adjusted consumption weights, representing an MSA, with the social planner reference scenario indicated by $**$.

(compared to 5.51 %), which explains a lower research labor fraction of 54 % (previously 56 %) and a lower average density with $D = 0.88$ (previously $D = 1.09$). The adjusted consumption weights reduce the density of decentralized economies and the social planner reference scenario. The consequence is a lower quality of agglomeration economies and a lower numéraire growth rate.

For a more intuitive interpretation of Table (3.8), Figure (3.16) follows Figure (3.15) in showing how the factor allocations in all relevant decentralized regimes differ from the social planner's GE^{**} benchmark. In most cases, the qualitative patterns of the city specification discussed in Figure (3.15) and the MSA specification discussed in Figure (3.16) are identical. A difference is observed for the decentralized GE regime whose density profile is below the social planer reference. In a GE regime in an MSA specification, the quality of agglomeration economies is also below its GE^{**} reference. This finding comes with the adjusted parameter weights, reducing the value of urban land for the apartment provision.

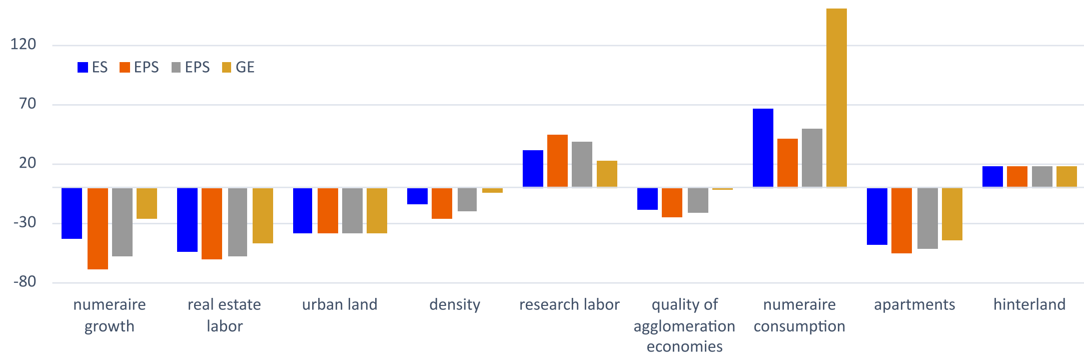

Figure 3.16: Percentage deviations of the economic core indicators in each decentralized growth regime from the social planner's GE^{**}regime benchmark scenario.

Adjusting research efficiencies

As a second robustness test, it is valuable to consider alternative research parameters. While the benchmark specification refers to identical research efficiencies, $\varsigma_{\mathcal{F}} = \varsigma_{\mathcal{E}} = 1$, and an intermediate degree of research returns given with $\psi = 0.5$, it is interesting to test whether an adjustment of these research parameters affects the socially optimal innovation regime. Table (3.9) lists how variations in ψ, $\varsigma_{\mathcal{F}}$, and $\varsigma_{\mathcal{E}}$ affect the numéraire growth rates. Thereby, $\psi = 0.25$, describes strong decreasing returns in research efforts, $\psi = 0.5$ represents medium decreases, while $\psi = 0.75$ describes low decreases. In parallel, $\varsigma_{\mathcal{F}} = 0.5$ and $\varsigma_{\mathcal{E}} = 0.5$ represent a low efficiency of abatement-focused research, respectively green energy focused research while $\varsigma_{\mathcal{F}} = 1.5$ and $\varsigma_{\mathcal{E}} = 1.5$ represent high research efficiencies.

For keeping the discussion parsimonious, Table (3.9) only exhibits how the different research efficiency parameters affect the numéraire production growth rates. The table reveals that a GE^{*} regime is a very robust reference scenario for a variety of efficiency parameters. Irrespective of the parameter choice, abatement-based research (related to a PS^{*} and a PS^{*+} regime) is never socially optimal. Distinctively, there are research parameter combinations where an ES^* and an ES^{*+} regime are welfare superior to a GE^* regime. This is the case if $\varsigma_{\mathcal{E}} \leq 0.5$ and $\psi \geq 0.5$, whereby an ES^{*+} regime is welfare superior to an ES^* scenario.

ψ	ES	PS	EPS	GE	ES*	ES*+	PS*	PS*+	EPS*	GE*
					$\varsigma_F = \varsigma_E = 1$					
0.25	3,04	1,88	2,35	3,98	3,5	3,59	0,62	1,17	1,91	**6,68**
0.5	3,04	1,88	2,35	3,54	3,5	3,59	0,62	1,17	1,91	**5,51**
0.75	3,04	1,88	2,35	3,53	3,5	3,59	0,62	1,17	1,91	**4,39**
					$\varsigma_F = \varsigma_E = 0.5$					
0.25	3,04	1,3	1,94	3,72	3,5	3,59	na	0,03	1,46	**4,45**
0.5	3,04	1,3	1,94	3,53	3,5	**3,59**	na	0,03	1,46	3,54
0.75	3,04	1,3	1,94	3,53	3,5	**3,59**	na	0,03	1,46	2,8
					$\varsigma_F = \varsigma_E = 1.5$					
0.25	3,04	2,2	2,58	4,28	3,5	3,59	1,28	1,74	2,29	**9,41**
0.5	3,04	2,2	2,58	3,55	3,5	3,59	1,28	1,74	2,29	**7,86**
0.75	3,04	2,2	2,58	3,53	3,5	3,59	1,28	1,74	2,29	**6,62**
					$\varsigma_F = 1.5$, $\varsigma_E = 0.5$					
0.25	3,04	2,2	2,58	3,72	3,5	3,59	1,28	1,74	2,29	**4,45**
0.5	3,04	2,2	2,58	3,53	3,5	**3,59**	1,28	1,74	2,29	3,54
0.75	3,04	2,2	2,58	3,53	3,5	**3,59**	1,28	1,74	2,29	2,8
					$\varsigma_F = 0.5$, $\varsigma_E = 1.5$					
0.25	3,04	1,3	1,94	4,28	3,5	3,59	na	0,03	1,46	**9,41**
0.5	3,04	1,3	1,94	3,55	3,5	3,59	na	0,03	1,46	**7,86**
0.75	3,04	1,3	1,94	3,53	3,5	3,59	na	0,03	1,46	**6,62**

Table 3.9: Regime-specific production growth rates for alternative research efficiency parameters. The bold numbers indicate the socially optimal innovation regime.

Accordingly, a reduction in pollution has a more severe impact on welfare than a reduction in numéraire energy use (although the latter reduces instant consumption, its pollution reduction increases consumption growth, causing a stronger welfare effect). While these results should not be interpreted as a literal suggestion to completely keep brown energy out of production, they emphasize that it is beneficial to use as much technology as possible to reduce brown energy employment. In any case, as soon as the green energy research efficiency is at least 50 % as innovative as general research, green energy production becomes welfare superior.

In all, the benchmark result and the robustness test have revealed that decentralized growth paths are characterized by biased factor allocations irrespective of whether they result in the socially optimal direction of technical change. Therefore, policies not only need to initiate a sustainable growth path, but they can also significantly improve welfare. The subsequent section addresses these qualities. It concentrates on the benchmark results and assesses the policies needed to correct market distortions in order to achieve a socially optimal green energy growth path.

3.9 Urban policy

The spatial growth model distinguishes two policy purposes: guaranteeing sustainable growth and improving welfare. Four alternative innovation regimes lead to sustainable growth. Thereby, only a green energy regime does not require policy actions to stabilize the growth path. According to the numerical application, this regime is also the social planner benchmark.

One essential question of urban policy is thus whether a decentralized economy grows on a green energy path. The answer is subject to decentralized energy demand and, thence, the relative price of brown and green energy. A second essential question is how to allocate labor, land, and research in this green innovation regime. The numerical application above indicated that the corresponding factor allocations are all biased. As will be detailed subsequently, achieving a socially optimal factor allocation requires combining policy measures in a holistic strategy. Divide and conquer approaches, where policy instruments tackle individual externalities independently of others are not feasible since policies interact. In view of this, this section will describe such a holistic policy recommendation and present its specific policies in an order that facilitates the assessment.

(A) Policies to avoid markup pricing: Intermediate producers charge a markup that inflates intermediate prices. It is hence necessary to subsidize intermediates with $T_{\wp\mathcal{A}} = 1 - \varkappa$, $T_{\wp\mathcal{E}} = 1 - \varepsilon$, calling for $T_{\wp\mathcal{A}} = 0.75$ and $T_{\wp\mathcal{E}} = 0.95$. Especially with green energy, the height of the subsidy underlines the intensity of the price distortions that occur when a monopolistically competitive intermediate sector faces inelastic intermediate demand.

(B) Policies for a GE regime: A green energy scenario requires that $(1+T_e)p_{e,b} \geq \wp_{\mathcal{E}}$. Therefore, if brown energy is cheaper than green energy, a brown energy tax following $T_e \geq \frac{r}{p_{e,b}\varepsilon} - 1$ is necessary. The calibration reveals that in a $BEP(1)$ with brown energy prices standardized to unity, there is a green energy transition if $T_e \geq \frac{0.08}{0.05} - 1 = 0.6$. This result states that if brown energy and numéraire output have the same price, a green energy transition requires the brown energy price to increase by an additional 60% (so that a brown energy unit costs 1.6 numéraire output units).

(C) Policies for socially optimal research efforts: If there are green innovations, the large price markups charged for green energy intermediates inflate the revenues of green energy innovations, which lead to excessive green research efforts[38]. Reformulations show that a socially optimal innovation effort allocation is achieved with $\frac{(1+T_{I,\mathcal{E}})}{(1+T_{I,\mathcal{A}})} = \frac{\kappa\varkappa^2(1-T_{\wp\mathcal{E}})}{\varsigma_{\mathcal{E}}^2\varepsilon^2(1-T_{\wp\mathcal{A}})} = \frac{\varkappa}{\varsigma_{\mathcal{E}}^2\varepsilon}$. Since with the above results, $\frac{(1+T_{I,\mathcal{E}})}{(1+T_{I,\mathcal{A}})} = 5$, this could be reached with $T_{I,\mathcal{A}} = 0$ and $T_{I,\mathcal{E}} = 4$. While it is possible to apply several alternative tax and subsidy schemes, internalizing the research effort externality re-

[38]While the appendix for Proposition (3) derives that $\frac{1}{(1+\frac{1}{\varsigma_{\mathcal{E}}^{\frac{1}{1-\psi}}})} = \eta^*$, we know from Lemma (1), that decentralized research chooses $\eta = \frac{1}{1+\Lambda\Gamma}$, so achieving $\eta = \eta^*$ calls for a combination of $T_{\wp\mathcal{E}}$ and $T_{\wp\mathcal{A}}$ with $\frac{(1+T_{I,\mathcal{E}})}{(1+T_{I,\mathcal{A}})} = \frac{\kappa\varkappa^2(1-T_{\wp\mathcal{E}})}{\varsigma_{\mathcal{E}}^2\varepsilon^2(1-T_{\wp\mathcal{A}})} = \frac{\varkappa}{\varsigma_{\mathcal{E}}^2\varepsilon}$.

quires that the value of a general innovation compared to innovations in green energy needs to be $(1 + 4 =)$ 5 times the value observed in decentralized markets. This research policy requirement again illuminates the intensity of the monopolistic market distortions.

(D) Policies for socially optimal density: The numerical application recommends a socially optimal density of $D^* \approx 1.09$, associated with $\lambda^* = 0.27$ and $\bar{l}^* = 0.22$. The numerical results further indicate that a decentralized GE regime allocates too little land and slightly too much labor to the real estate sector so that corrective policies improve welfare. For a full picture of the alternative policy options, it remains to complete Table (3.3) for the policy effects that were ambiguous without calibration, which leads to Table (3.10).

Policy effect on the spatial economy.

	T_a	T_l	T_x	T_c	T_b	T_w	T_r	$T_{\bar{c}c}$	$\mathcal{I}$	T_e
$\bar{l}$	$\downarrow$	$\downarrow$	$\uparrow$	$--$	$--$	$--$	$--$	$\uparrow$	$\uparrow$	$\downarrow$
λ	$\downarrow$	$\uparrow$	$--$	$\uparrow$	$--$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$
D	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$	$--$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$

Table 3.10: Net policy effects on land, labor, and density given calibration. With '$--$' to denote no impact.

The challenge with the listed policies is that most of the instruments affect both labor and land allocations, so only using one policy does not lead to a socially efficient allocation of both factors. In all these cases, a combination of at least two policies is needed, while different strategies can be e distinguished.

One strategy is to first achieve a socially optimal real estate labor allocation via T_a, T_l, T_c, T_w, T_r, $T_{\bar{c}c}$, T_e (but not $\mathcal{I}$ for the efficiency consideration discussed above) and then correct the resulting land allocation with a hinterland tax, T_x (which only affects the land market). An alternative is to first achieve a socially optimal land allocation via T_a, T_l, T_x,$T_{\bar{c}c}$, T_e and then adjust the labor allocation with T_c, T_w, T_r. Another path is to achieve a socially optimal land allocation without affecting the land market via T_c, T_w, T_r and in parallel install a hinterland tax, T_x, to adjust the land allocation. It is also possible to combine policies that affect both markets in such a intensity that they reach the socially optimal results.

In any case, an additional complexity is that the above policies can affect factor allocations beyond real estate land and labor. For example, a wage tax also affects research labor. Accordingly, a coordinated policy strategy is required. The individual policy choice is thereby subject to policy preferences. Plenty of alternative policy combinations are possible[39]. Because of this rich set of policy alternatives, it is not possible to segment this discussion into a few leading examples.

(E) Policies for socially optimal research labor: The numerical application shows that the proportion of researchers is below its socially optimal level. While the decentralized RE regime allocates 54 % of its citizens to creative activities, a planner would raise this number to 56 %. Such an increase can be reached with a research subsidy, T_v. Alternatively, a wage tax on numéraire labor, T_w, can be used. Such a tax would have to be coordinated with other land and labor market policies to neither affect the real estate labor provision nor the density profile. Given the above explanation that multiple policy measures affect the land and labor allocations, this discussion is left to future research.

Summarizing the above results, policymakers have considerable flexibility in their choice of instruments to address the quality of agglomeration economies and some flexibility in the use of tax and subsidy policies in the innovation sector. Since there is at least one policy instrument to internalize each externality, a first best result is possible. It is then a question of policy preferences how to exactly combine policies, while a wide range of alternative policy combinations are possible. In light of this, the next section critically discusses the model assumptions backing these results.

[39] Which strategy to implement is, therefore, a question of preferences of policymakers and not addressed in this discussion.

3.10 Chapter discussion

This chapter illuminates how spatial and environmental externalities affect sustainable and socially optimal urban growth. Its findings are subject to several simplifying assumptions that require a critical assessment. This section provides such a debate and further considers alternative uses of the presented theory framework.

3.10.1 Theory assumptions

The chapter's model follows neoclassical standards so that most of its central assumptions have sufficiently been discussed in the literature. Yet, one peculiarity is its assessment of density in an environmental context. The related modeling is unprecedented and technically more complex than other theory features. Therefore, the sustainability evaluation is kept tractable by concentrating on a local commons problem in a closed city that gets confronted with an exogenously provided pollution-intensive energy source. These aspects require a critical review.

Sustainability as a local commons challenge

In principle, two simplifications characterize the sustainability discussion of this chapter. First, a flow variable represents the damages of resource depletion. Second, the focus is on a closed economy. One consequence of this framing is that environmental damage portrays a local rather than a global commons problem what raises the question of whether the theory sufficiently evaluates the fundamental challenges of sustainable urban growth.

To begin with, Chapter (2) has demonstrated that the distinction of flow and stock effects is often subject to the interpretation of the framework and not relevant for the qualitative results of the theory. These arguments are also valid for the third chapter and are consequently not further detailed for parsimony.

Still, the role of climate change in the context of this theory requires a closer look. Regional emissions have a global impact. One attribute of the climate discussion is its global commons nature that requires coordinating environmental policies worldwide.

Any assessment in this direction demands an open city theory, discussed a little later.

Nevertheless, it is possible to interpret the urban economy as a representative global city and environmental pollution as greenhouse gas emissions. Alternatively, the theory can address the local consequences of climate change, e.g., by considering how exogenous carbon emissions affect the quality of agglomeration economies. Therefore, $\underline{\varsigma}_\Phi$ and $\overline{\varsigma}_\Phi$ could describe an implicit damage function of a local temperature (or carbon stock) indicator, e.g., T, thus $\underline{\varsigma}_\Phi = \underline{\varsigma}_\Phi(T)$ and $\overline{\varsigma}_\Phi = \overline{\varsigma}_\Phi(T)$. In such a specification, steadily rising temperatures would lead to a collapse, while stable temperatures affect the degree of Marshallian agglomeration economies[40].

Yet, this theory's closed city focus not only affects the environmental discourse but impacts the general theoretical basis of the discussion, as addressed next.

Sustainability in a closed city

Although this chapter introduces an exogenous energy sector to analyze the role of external brown energy supply, its closed city perspective ignores the mobility of labor and capital, so it is questionable whether its results remain valid in an open city setting. Similar thoughts are relevant for the monocentric focus of the theory. As detailed in the data section, only 100 among the 160 OECD FUA city representations in the US are monocentric, so it is crucial to understand whether polycentric cities would lead to similar qualitative findings.

Interestingly, a polycentric theory, e.g., discussed in Schneider (1981), Griffith (1981), or Kloosterman and Musterd (2001) and an open city theory, e.g., discussed in Turnbull (1988), Brueckner (1990) or Redding and Rossi-Hansberg (2017), would not affect this chapter's qualitative density discussion. This peculiarity is best explained when noting that worldwide, the national distribution of cities is stable and follows Zipfs' law (Zipf, 1935), which classifies the distribution of cities according to a Pareto law, see, e.g., P. Krugman (1996), Fujita, P. R. Krugman, and A. Venables (1999),

[40] This setting could evaluate the Representative Concentration Pathways (RCP) discussed by the Intergovernmental Panel on Climate Change (IPCC), see, e.g., Van Vuuren et al. (2011), and could assess how urban heat islands (UHI) affect the quality of agglomeration economies, Φ, and hence growth and welfare. Peng et al. (2012).

or Gabaix (2009) for details. Hence, multiple cities of different sizes are growing within a stable distribution. If cities were identical and the factor allocation was not restricted, all economic activity would concentrate in the city with the highest technology growth rate[41]. Such a concentration is not observed.

One explanation is that cities are endowed with different immobile natural resources. An example is land that, as an input in production, affects the marginal product of other factors such as manufactured capital. No arbitrage leads to a reallocation of these factors until their returns adjust in all regions. Henceforth, different initial endowments with immobile production factors will lead to differently sized cities but equivalent returns on production factors, resulting in equal and stable growth rates. An alternative explanation is that the allocation of manufactured capital is affected by city-specific risk premiums, e.g., due to environmental sensitivities[42]. If the Euler equation adjusts for risk, differently sized cities would then grow at the same rate. Finally, a less traditional explanation is that cities have different types of citizens with diverse preferences and different rates of innovation.

Decisively, none of these explanations has a qualitative impact on this chapter's core discussion. Irrespective of the number of centers or the interaction with other cities, density remains determined by the relative price of labor and land, the environmental degradation remains subject to the energy source. Crucially, some of the arguments listed for differently sized cities could initiate a transition to a distinct long-run city distribution which is a promising area for research (e.g., a constant energy price but different per capita productivity rates could motivate some cities to remain in a brown energy scenario while others turn green what affects the city

[41] This can be illustrated on the basis of the directed technical change literature discussed in Chapter (2). If the final output gets generated with a CES production function combining the inputs of two sectors, and if sector 1 has a higher technology growth than sector 2, then there is capital deepening in sector 2 when the elasticity of substitution is below one and capital deepening in sector 1 if the elasticity of substitution is greater than one. National GDP allocates thousands of individual urban production functions. If there were no heterogeneities or constraints in the allocation of production factors, the elasticity of substitution of urban production to national GDP would approach infinity so that, in the long run, production would concentrate in one city.

[42] For instance, the risk of earthquakes, coastal flooding, droughts, or heat waves, to name a few. Such a scenario could explain differences in the interest rates which lead to different factor endowments.

distribution). However, this theory has its focus on the very long run, so that an assessment of such transition phases is left to other research.

Polycentric theory can further be helpful to compare the local effects of differently organized urban structures (e.g., whether the number of centers affects the pace of innovation) while open city theory can follow the footsteps of Ricardo (1817) and explain how environmental conditions affect the comparative advantages of individual cities. Both qualities present promising paths for future research.

Population and agglomeration economies

Although the above assessment has discussed their choice in detail, two further assumptions require a brief consideration. First, the model concentrates on a long-run scenario where the population stabilizes. This feature is necessary for long-run sustainability but has the consequence that understanding short-term density effects requires a different model. Second, since there is no reference theory for comparison and no data for empirical estimations, the chapter only presents a qualitative idea on the interaction of density and the environment. Thereby, the specification of the quality of agglomeration economies, Φ, serves as the first step for the evaluation and needs some critical reassessment as soon as more quality data is available.

In view of this, it is promising to consider other areas to apply the theory.

3.10.2 Alternative theory applications

The fact that this model is unprecedented in its inclusion of density into growth theory raises the question of whether it is helpful for alternative research. Technically, the model could address almost any field where population or construction density is relevant. However, the theory is not very instrumental in assessing temporary shocks. For instance, the impacts of a virus or an unexpected heatwave will have fundamental level effects but negligible growth effects[43] and are thus better assessed with other models. Other qualities have permanent effects. A good example is digitalization,

[43] At least as long as they do not alter deep structural parameters.

which is a core element of the Smart City concept discussed in the introduction (see Angelidou et al., 2018, for an overview) and provides several intriguing areas for discussions. For instance, the potential to work from home makes location choices less significant. Digitalization hence has a great potential to mitigate real estate scarcity in the city centers, increasing the resilience of cities to commuting cost increases. In addition, digitalization presents a new technology affecting the production and innovation conditions of an economy profoundly. So while digitalization may increase energy demand, it can also increase the efficiency of production and research. These two channels could present competing forces challenging the sustainability of urban growth and the social optimality of a decentralized growth path.

Another avenue for applying the theory is an assessment of urban heat island (UHI) effects. The UHI deal with the positive correlation between temperatures and building density which increasingly challenges urban planners around the globe, see, e.g., Oke (1973), Rizwan, Dennis, and Chunho (2008) and Peng et al. (2012). An assessment of these effects requires focusing theory on climate change while the quality index, Φ, could assess their intensity and impact. Henceforth, there is a promising area for future research. With these prospects, it remains to conclude.

3.11 Chapter conclusion

This chapter develops a new model that provides unprecedented insights into how density and environmental pollution in interaction affect sustainable urban growth. The challenge of cities is that production and commuting require energy which is either brown or green. Brown energy pollutes. Green energy requires the development of new energy power plants, which involves R&D activity that is monopolistically competitive and consequently leads to biased research efforts. Further, urban land is limited while growth is subject to the quality of agglomeration economies, which scales how the quality of the environment and density-related network effects determine urban productivity growth rates.

If the brown energy price is lower than the green energy price, brown energy demand will increase with intensified production. This development deteriorates the environmental quality until the economy collapses. A brown energy tax can avoid such a scenario but needs to exclude commuting energy since it would raise commuting costs, elevating CBD density to a growth-threatening extent. While commuting infrastructure improvements could reduce the energy demand of commuting (and encounter these destabilizing tendencies), it is more efficient to use commuting subsidies. Green energies do not cause such problems. If they attract research, the urban economy produces sufficient power plants for a green energy transition and achieves a sustainable growth path.

A numerical application reveals that a green energy scenario leads to the highest productivity growth rates and is, therefore, the socially optimal scenario. However, socially optimal growth further requires correcting distortions in the research sector, namely price markups with a subsidy and research efforts with either a tax on green energy innovations or a subsidy on general research. Beyond, the quality of agglomeration economies is below its potential, as the CBD density is too low. Several alternative market-based policies affect the relative price of labor and land and, accordingly, the average construction density. Political decision-makers thus have great flexibility in terms of choosing their policy measures, while a holistic policy approach can be first best and lead to a socially optimal growth scenario.

There are several areas for future research. Some core concerns of urban planners are urban heat islands, which require optimally organized urban structures. These are not simply physical, but given by the entire organization of the cities. This theory could address these effects and further consider a second concept dominating urban planning, Smart Cities. Such cities are strongly affected by digitalization, which presents another crucial area for future research. Expanding this chapter's theory in those research directions is promising and will profoundly broaden our understanding of sustainable growth and the impact of urban structures on urban welfare.

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Chapter 4

Synthesis

Anyone who believes in indefinite growth on a finite planet is either mad, or an economist.

Kenneth Boulding, n.d.

Thinking about the sustainability of urban growth enters too many fascinating subject areas for one thesis, regardless of its length and the research time invested. It is, therefore, necessary to focus on some aspects of the discussion at the cost of ignoring others. This work concentrates on the driving force behind urban innovations and their role for sustainable growth and welfare if production requires scarce natural resources that lack clearly assigned property rights. Two fundamental questions dominate the thesis: 1. Which are the conditions that an urban economy finds a sustainable growth path? 2. What defines a socially optimal urban growth scenario?

In order to shed light on different aspects of the urban sustainability discourse, these questions are addressed in two chapters, each with its own theory. Chapter (2) gives a non-spatial answer with a focus on endogenous innovation supply decisions. Chapter (3) answers the question in a spatial context with a focus on density. While the second chapter serves as an introduction to the sustainable growth debate, the third chapter presents the core analysis of the work. So, it is crucial to illuminate the concepts and reasons for this theory segmentation in more depth.

This chapter explains the decision criteria for the theoretical foundations of Chapter (2) and (3), compares their core results, concludes with a synthesis of both models to a joint policy recommendation, and closes the discussion with an outlook.

4.1 Chapter intentions

Chapter (2), therefore, initiates the analysis of sustainable urban growth with a focus on the determinants for innovations if alternative technologies are available. There is a great need for such a discourse, since the established environmental growth literature usually addresses the demand for technology but hardly illuminates the determinants for their supply. As this thesis reveals, such handling leads to an underestimation of intrinsic incentives for socially optimal research decisions.

For avoiding distractions by an overwhelming theory, the chapter ignores spatial aspects and simplifies the natural capital assessment. The latter relates to a homogeneous conglomerate without clearly assigned property rights. It exhausts with production activities what causes damages that reduce production. Innovators can mitigate or eliminate the damage through adaptation or abatement, and combine related technologies at will. In a theory augmentation, the final production sector must specialize in either adaptation or abatement.

This simplified framework is sufficient to show that environmental externalities in research do not follow due to a lack of property rights to natural capital goods such as clean air and a favorable climate, but due to a lack of information and difficulties in accessing and combining technologies. For internalizing these externalities, non-market-based policies related to education and knowledge transfers are similarly effective as market-based policies related to taxes or cap-and-trade concepts. While these policies are no remedy to hinder monopolistic research market distortions, they lead to a socially optimal direction of technical change.

When translating the chapter's non-spatial theory in an urban context, they strongly advocate the Smart Cities concept. While there are many interpretations for this notion, it, in principle, emphasizes the importance of an intelligently organized city where agents interact and use all available technology to improve urban welfare.

Core ingredients for this purpose are access to technology and information.

Since the second chapter does not consider the spatial dimension of the discussion, space is a core element of the third chapter's assessment. The chapter, therefore, evaluates two types of natural capital goods: (1.) nature's capability to absorb pollution and (2.) nature's capability to absorb economic activity on a specific location. The latter is better known as density.

Compared to the relatively abstract specification of the second chapter, the third chapter's simplified growth model is rather practically oriented and addresses how exogenous forces affect endogenous innovation decisions. Environmental pollution emerges with exogenously provided brown energy use in numéraire production and commuting. Innovators can avoid pollution if they improve renewable energy power plants. Alternatively, the government can force innovators to abate. However, there is no intrinsic incentive for pollution reduction and the innovators compete with exogenous brown energy suppliers, so energy provision depends on the exogenous brown energy price development. Beyond, innovations lead to research-specific crowding-out effects within sectors and among sectors. Although the second chapter also addresses crowding-out effects, the third chapter gives a more distinguished perspective on their variety along an urban growth path.

Hence, while the second chapter emphasizes the fundamental role of a Smart City for sustainable growth, the third chapter illustrates the practical challenges that encounter such a Smart City if exogenous energy providers and endogenous innovators interact in a spatially organized economy. Thereby, two fundamental results stand out. First, environmental externalities emerge due to exogenous forces and non-anticipating energy consumers. Second, in an urban context, there is a risk of a density externality. These externalities not only have a welfare effect, but can ultimately endanger balanced growth. The complexity in a city is that any movement in the energy price destabilizes the density profile and either drags citizens to the fringe (if prices reduce) or to the urban center (if prices increase). An environmental tax on brown energy can therefore jeopardize growth as it increases commuting costs, so a carefully crafted policy strategy is needed.

4.2 Chapter comparison

Chapter (2) and (3) assess environmental externalities with a different theoretical basis. Thereby, their findings contrast or complement each other, depending on the perspective. It is consequently essential to compare both approaches.

While Chapter (2) gives innovators full responsibility for natural resource depletion, Chapter (3) lays this responsibility in the hands of the energy consumers. These are price-sensitive and cannot anticipate how the dynamic costs of pollution affect their entire consumption options in the future. Accordingly, they choose the cheaper energy product without considering the environmental consequences of their actions. Technically, they lack information, as they do not know how their consumption decision affects future consumption possibilities. Henceforth, the explanation for environmental externalities of Chapters (2) and (3) are identical, both relating to a lack of information.

However, in the two theories, information affects different groups of decision-makers, researchers in the second chapter, consumers in the third chapter. This distinction emphasizes two distinguished aspects of technology supply decisions. While the research sector in the second chapter has complete control and responsibility for the sustainability of growth, the third chapter restricts its influence by introducing an exogenous brown energy sector. This step initiates a moral hazard problem because the exogenous energy sector does not care about the environmental footprint of its energy.

Although the two chapters shed light on different aspects of the sustainability discussion, the evaluation demonstrates that policies should assist the Smart City concept as access to information and technology support socially optimal research decisions. This view is interesting as it initiates an alternative interpretation of the chapter's results: the more industries produce locally, the higher the likelihood that self-enforcing mechanisms lead to sustainable and socially optimal growth.

In light of this, the subsequent section summarizes the chapter's core findings via a combined policy recommendation.

4.3 Conclusion

When collecting Chapter (2) and (3) results, the first key lesson to learn is that a sustainable urban future requires access to information and access to technologies. The former helps strengthen environmental awareness, the latter to improve the creative potential of the citizens to promote and combine new technologies. Supportive policies are investments in education infrastructure like schools, universities, libraries, media, or Internet access, to name a few. In addition, meaningful patent protection is pivotal. While the proceeds of knowledge need to be privatized via patents to foster innovation, this protection must be carefully designed to not restrict innovations.

Yet, these policies may not be sufficient for achieving sustainable and socially optimal growth. The third chapter emphasizes that policies need to support a shift to green energy technologies whenever there is no initial green energy provision. This shift, however, could not yet lead to socially optimal growth as there are further distortions to consider. One is the inelastic energy demand. Energy is an essential factor in production, and substitution options are rare. Constructing green energy power plants yields high gains as soon as the green energy price exceeds a critical level. Since this quality attracts too many research efforts, innovation subsidies for alternative research are needed. In parallel, innovators do not consider the social value of innovations, so an optimal amount of researchers requires innovation subsidies.

The pleasant aspect of green energy is that it is devoid of any dynamic density externality. Consequently, urban planning policies can focus on welfare improvements. Thereby, a core challenge is finding the socially optimal density at the urban center, requiring to identify a land and labor allocation that leads to a high instant consumption and a high quality of agglomeration economies. Any policy that affects the relative price of labor and land (e.g., taxes on consumption and production factors) serves this purpose. In all but an arbitrary case, at least two policies are required to shift land and labor. Still, there are several alternative policy paths, so policymakers have great flexibility in choosing their specific instruments.

4.4 Outlook

In all, this dissertation gives an optimistic perspective on the social optimality of decentralized research decisions in urban areas. It demonstrates that a combination of innovation policies, intermediate market policies, and policies affecting the land and labor allocation can be first best. Carefully designed policies not only support sustainable growth, but can thoroughly improve urban welfare.

Still, there are various subject areas this dissertation has not addressed. From digitalization to open economies, from a specific consideration of the coordination of policies to address climate change to a more detailed assessment of the optimal design of patent policies, several fields deserve further discussions.

This being the case, this dissertation demonstrates that cities have great potential to succeed in tackling the severe sustainability challenges of our economic development. While the environmental economic literature often associates urban structures with smog, heat islands, and environmental degradation, they build unique entities that support network effects, labor matching, value chain effects, capital concentration, and create considerable social benefits due to vivid social interactions. In combination, these forces define a Smart City and lead to alluring developments whenever citizens are given the best infrastructure possible for pursuing their rich everyday endeavors. Hopefully, this potential will inspire sustainable action and initiate a promising urban future. According to this dissertation, such a bright future is definitely within reach.

Appendix A

Lemma (2.1) Following the appendix of Grimaud and Ricci (1999) but applying their methodology to three rather than one innovation direction, consider the cumulative distribution function (CDF) for a productivity parameter A_j within the $j \in [0,1]$ dimension along a BGP. Denote $\bar{A}$ as the leading edge technology at an arbitrary date $t = s$, then trivially $F(\bar{A}, s) = 1$, whereas given the replacement rate, it holds that $\frac{dF(\bar{A},t)}{dt} = -\lambda n F(\bar{A}, t)$ so that for $t \geq s$, $F(\bar{A}, t) = e^{-\lambda n(t-s)}$. Now since (12) also has to hold for $A(t) = A(s) = \bar{A}$, the leading edge technology improves according to $A(t) = \bar{A} e^{\lambda n \varsigma_A(\eta)^\theta (t-s)}$. The relation among the leading edge technology in $t = s$ and in $t > s$ thus follows $\frac{\bar{A}}{A(t)} = e^{-\lambda n \varsigma_A(t)(\eta)^\theta (t-s)}$. This gives the CBD: $F(\bar{A}, t) = \left(\frac{\bar{A}}{A(t)}\right)^{\frac{1}{\varsigma_A \eta)^\theta}}$. Now use $a_j(t) := \frac{A_j}{\bar{A}(t)} \in (0,1]$ to relate a sector technology to the leading edge productivity in t, then along a BGP, the CBD needs to exhibit $F(a) = a^{\frac{1}{\varsigma_A \eta^\theta}}$. The probability density function (PDF) thus follows $f(a) = \frac{a^{\frac{1}{\varsigma_A \eta^\theta} - 1}}{\varsigma_A(\eta)^\theta}$. If denoting the average technology with $\bar{a}(t)$, then $\bar{a}(t) := \int_0^1 A_j(t) dj = A(t) \int_0^1 a_j(t) dj = A(t) \int_0^1 a f(a) da = A(t) \int_0^1 \frac{a^{\frac{1}{\varsigma_A \eta^\theta}}}{\varsigma_A \eta^\theta} da = \frac{A(t)}{1 + \varsigma_A \eta^\theta}$. Thus

$$\bar{a}(t) = \frac{A(t)}{1 + \varsigma_A \eta^\theta}.$$

In this theory, a BGP will follow basic Ramsey properties, i.e. $i(t) = \dot{K}(t) = y(t) - c(t) - P(t)$, so that $g_k(t) = g_k = g_c = g_y = g_P$ if $P(t) > 0$, respectively $g_k(t) = g_k = g_c = g_y$ if $P(t) = 0$ (see later). Now consider $\mathcal{I}(t) := A(t)R(t)G(t)$, which grows with $g_\mathcal{I}(t) = g_A(t) + g_R(t) + g_G(t)$, thus $g_\mathcal{I}(t) = g_A(t) + g_R(t) + g_G(t) = \lambda n(t) \mathcal{J}(t)$ with $\mathcal{J}(t) := \varsigma_A \eta(t)^\theta + \varsigma_R \kappa(t)^\theta + \varsigma_G (1 - \kappa(t) - \eta(t))^\theta$. Given the proportional technology relation, on a BGP the average representation of this technology bundle must satisfy

$$\bar{\imath}(t) = \frac{\mathcal{I}(t)}{1 + \mathcal{J}(t)}$$

where $\bar{\imath}(t)$ denotes the average technological intensity. Since $\int_0^1 \mathcal{I}_j(t)x_j(t)dj = K(t)$, a BGP requires that $\bar{\imath}(t)$ grows at a constant rate, so $g_{\mathcal{I}} = g_{\bar{\imath}}$, while κ and η need to be constant. Including $x_j(t) = \frac{K(t)}{\mathcal{I}_j(t)}$ in (1) leads to $y(t) = (1-n)^{1-\alpha}K(t)^{\alpha}\int_0^1 \frac{A_j(t)^{1-\alpha}R_j(t)^{1-\alpha}}{\bar{E}(t)^{\iota\omega}G_j(t)^{\alpha}}dj$. Similarly,

$$\bar{E}(t) = (1-n)^{1-\alpha}K(t)^{\alpha}\Big(\int_0^1 \frac{A_j(t)^{1-\alpha}R_j(t)^{1-\alpha}}{G_j(t)^{1+\alpha}}dj\Big).$$

Next, distinguish $g_{\bar{E}}(t) > 0$ from $g_{\bar{E}}(t) = 0$. If $g_{\bar{E}}(t) > 0$, including $\bar{E}(t)$ in $y(t)$ gives

$$y(t) = (1-n)^{(1-\alpha)(1-\phi)}K(t)^{\alpha(1-\phi)}\frac{\int_0^1 \frac{(A_j(t)R_j(t))^{1-\alpha}}{G_j(t)^{\alpha}}dj}{\Big(\int_0^1 \frac{(A_j(t)R_j(t))^{1-\alpha}}{G_j(t)^{1+\alpha}}dj\Big)^{\phi}}.$$

Defining $H_j(t) := \frac{A_j^{1-\alpha}R_j^{1-\alpha}}{G_j^{\alpha}}$, along a BGP, the average technology bundle must follow $\bar{h}(t) = \frac{H(t)}{1+\mathcal{H}}$, whereby $\mathcal{H} := (1-\alpha)\varsigma_A\eta^{\theta} + (1-\alpha)\varsigma_R\kappa^{\theta} - \alpha\varsigma_G(1-\kappa-\eta)^{\theta}$. Similarly, defining $P_i := \int_0^1 \frac{A_j^{1-\alpha}R_j^{1-\alpha}}{G_j^{1+\alpha}}dj$, then along a BGP $\bar{p}(t) = \frac{P(t)}{1+\mathcal{P}}$ with $\mathcal{P} := (1-\alpha)\varsigma_A\eta^{\theta} + (1-\alpha)\varsigma_R\kappa^{\theta} - (1+\alpha)\varsigma_G(1-\kappa-\eta)^{\theta}$. In combination, $y(t) = (1-n)^{(1-\alpha)(1-\phi)}K(t)^{\alpha(1-\phi)}\frac{\bar{h}(t)}{\bar{p}(t)^{\phi}}$. Along a BGP, $\bar{h}(t)$ and $\bar{p}(t)$ both grow with the same sequence of innovations. So using standardization, it must be possible to express the above results with $y(t) = (1-n)^{(1-\alpha)(1-\phi)}K(t)^{\alpha(1-\phi)}\bar{b}(t)$ with $\bar{b}(t) = \frac{B(t)}{1+\mathcal{B}}$, where

$$\bar{b}(t) := \bar{r}(t)^{(1-\alpha)(1-\phi)}\bar{a}(t)^{(1-\alpha)(1-\phi)}\bar{g}(t)^{\phi-\alpha(1-\phi)} = \frac{R(t)^{(1-\alpha)(1-\phi)}A(t)^{(1-\alpha)(1-\phi)}G(t)^{\phi-\alpha(1-\phi)}}{1+\mathcal{B}}$$

with $\bar{r}(t), \bar{a}(t), \bar{g}(t)$ as average adaptation knowledge, average general knowledge, and average abatement knowledge respectively. Thus

$$B(t) := R(t)^{(1-\alpha)(1-\phi)}A(t)^{(1-\alpha)(1-\phi)}G(t)^{\phi-\alpha(1-\phi)},$$

$$\mathcal{B} := (1-\phi)(1-\alpha)[\varsigma_A \eta^\theta + \varsigma_R \kappa^\theta] + [\phi - \alpha(1-\phi)]\varsigma_G(1-\kappa-\eta)^\theta.$$

Also note that $g_B = \lambda n \mathcal{B}$. Yet, along a BGP, $g_y = g_K$, thus

$$g_y = \frac{(1-\alpha)(1-\phi)\big(g_R + g_A\big) + (\phi - \alpha(1-\phi))g_G}{1-\alpha(1-\phi)} = \frac{\lambda n \mathcal{B}}{1-\alpha(1-\phi)}.$$

As detailed with Proposition (2.1), there is then only positive growth if $1 > \phi$. Note that $g_{\bar{E}} \leq 0$ ultimately leads to $\phi = 0$, thus $\kappa = 0$, so that with standardization, $\bar{E}(t) = \bar{E}_0 = R_0 = 1$. In this case, $x_j = \frac{k_j}{A_j G_j}$, so that expression (1) reads

$$y(t) = (1-n)^{1-\alpha} K(t)^\alpha \int_0^1 \frac{A_j(t)^{1-\alpha}}{G_j(t)^\alpha} dj$$

which accounts for that investments in G do not have a direct productivity effect, but an indirect productivity effect, as they make the production of intermediates more capital intense. Thereby, $g_y = \alpha g_K + (1-\alpha) g_A - \alpha g_G$. Since $g_y = g_K$, this gives $g_{\bar{E}} = g_y - g_G$, thus $g_y = g_A - \frac{\alpha}{1-\alpha} g_G = \lambda n \big(\varsigma_A \eta^\theta - \frac{\alpha}{1-\alpha}\varsigma_G (1-\eta)^\theta\big)$. Since $g_{\bar{E}} = g_y - g_G$, $g_{\bar{E}} = 0$ requires that $g_A = \frac{1}{1-\alpha} g_G$, $\bar{\eta} = \frac{1}{1+\left(\frac{(1-\alpha)\varsigma_A}{\varsigma_G}\right)^{\frac{1}{\theta}}}$. Therefore, $\hat{\mathcal{B}} := \frac{(1-\alpha)\varsigma_A \varsigma_G}{\left(((1-\alpha)\varsigma_A)^{\frac{1}{\theta}} + \varsigma_G^{\frac{1}{\theta}}\right)^\theta}$ and $g_y = \lambda n \hat{\mathcal{B}}$, whilst $\hat{\bar{b}}$ and $\hat{B}$ follow with the above principles. $\square$

Proposition (2.1) First, for (ii), Lemma (1) has derived that along a BGP $g_K = g_c = g_y$. Further, $g_K = g_x + g_{\bar{\iota}}$, with $g_{\bar{\iota}} = \lambda n \mathcal{J}$, where $\mathcal{J} := \varsigma_A \eta^\theta + \varsigma_R \kappa^\theta + \varsigma_G (1-\kappa-\eta)^\theta$. With (9), a BGP requires $g_p = g_\varrho + g_{\bar{\iota}}$. If this is the case, then along a BGP: $g_p = g_A + g_R + g_G + g_\varrho = g_{\bar{\iota}} + g_\varrho$, so with (6), $g_x + g_{\bar{\iota}} = g_K$, where g_x refers to the (average) intermediate goods growth rate and g_K to the average capital growth rate. Along a BGP, we thus find that $g_x + g_{\bar{\iota}} = g_x + g_p - g_\varrho = g_K = g_y$. Further (9) states that $g_w = g_p + g_x$. Therefore, $g_w = g_p + g_x = g_\varrho + g_K$. However, for a BGP, (11) must follow

$$V(t) = \Big(\frac{1-\alpha}{\alpha}\Big)\frac{\pi(t)}{r + \lambda n - g_{\bar{\pi}}}$$

with $g_{\bar{\pi}}$ denoting the profit growth rate after innovation. This rate must remain constant because a constant n in the no arbitrage conditions (17) and (21) requires that $V(t)$ grows at a constant rate. Since g_π is constant, $g_{\bar{\pi}}$ needs to be constant as well. Now with (10) and (4), a constant n requires that $g_w = g_V$, so $g_y = g_w = g_V = g_\pi$. Yet, with (9) and (6), along a BGP, it is necessary that $g_\pi = g_p - g_{\bar{\iota}} + g_K$. Noting that $g_\pi = g_y = g_K$, this states that $g_p = g_{\bar{\iota}}$. Since $g_p = g_\varrho + g_{\bar{\iota}}$, balanced growth can only be reached with $g_\varrho = 0$. Since vintage providers cannot adjust their technologies, any continuous change in ϱ will skew the profit distribution among vintage providers. As discussed with (10) and (4), this is not feasible along a BGP. As a consequence, ϱ needs to remain constant. Now with (9)

$$g_{\bar{\pi}} = \frac{\iota\omega}{1-\alpha} g_{\bar{E}}.$$

Then, with Lemma (1), the marginal product of capital in final production follows

$$r = \alpha(1-\phi)\frac{y}{K}.$$

Note that $g_x = \frac{g_B}{1-\alpha(1-\phi)} - g_{\bar{\iota}}$ yields $g_x = -\frac{(1-\phi)g_G + \phi(g_R+g_A)}{1-\alpha(1-\phi)}$, showing that the physical dimension of intermediate production steadily decreases but is proportionally replaced by technologies. This can be interpreted in response to the fact that environmental impacts are constantly causing damage that motivates individuals to increasingly rely on technology. The environmental effect thus results in an implicit capital depreciation, which is compensated for by technology. The net effect is that capital and output still grow at the same rate. Note that $\Pi(t) = \frac{\Lambda(1-n)}{r^{\frac{\alpha}{1-\alpha}} \bar{E}^{\frac{\iota\omega}{1-\alpha}}} \int_0^j \frac{A_j R_j}{\varrho_j^{\frac{\alpha}{1-\alpha}} G_j^{\frac{\alpha}{1-\alpha}}} dj$. Similarly, (1) and (8) give $y = \frac{\alpha^{\frac{2\alpha}{1-\alpha}}(1-n)}{r^{\frac{\alpha}{1-\alpha}} \bar{E}^{\frac{\iota\omega}{1-\alpha}}} \int_0^j \frac{A_j R_j}{\varrho_j^{\frac{\alpha}{1-\alpha}} G_j^{\frac{\alpha}{1-\alpha}}} dj$, so $\Pi(t) = (1-\alpha)\alpha(1-n)^{(1-\alpha)(1-\phi)} K(t)^{\alpha(1-\phi)} \frac{B(t)}{1+\mathcal{B}}$. Therefore, $\Pi(t) = \frac{\pi(t)}{1+\mathcal{B}}$, so $\pi(t) = (1-\alpha)\alpha(1-n)^{(1-\alpha)(1-\phi)} K(t)^{\alpha(1-\phi)} B(t) = (1-\alpha)\alpha y(t)$, yielding $V(t) = \frac{(1-\alpha)^2 y(t)}{(r+\lambda n - g_{\bar{\pi}})}$. If $1 > \phi$, a BGP is therefore described by

$$\hat{g} = g_y = g_\pi = g_V = g_w = g_{\bar{\iota}} + g_x = g_k = \frac{\mathcal{B}}{1-\alpha(1-\phi)}$$

with $\mathcal{B}$ introduced in Lemma (2.1). The corresponding BGP is then characterized by $g_{\bar{E}} = \alpha g_K + (1-\alpha)(g_A + g_R) - (1+\alpha) g_G$. Thus using $g_y = \frac{1}{1-\alpha(1-\phi)}\big[(1-\alpha)(1-\phi)\big(g_R +$

$g_A\big) + (\phi - \alpha(1-\phi))g_G\big]$ and $g_y = g_K$ leads to $g_{\bar{E}} = \frac{(1-\alpha)(g_R+g_A)-(\alpha+(1-\phi)(1-\alpha^2))g_G}{(1-\alpha(1-\phi))}$, respectively $g_y = (1-\phi)g_E - (\phi - \alpha(1-\phi)g_G)$.

If $\phi \geq 1$, a BGP is only possible if there is no exhaustion of the natural capital stock. Sufficient abatement following the conditions defined with Lemma (2.1) is needed, what completes (ii). For (i), proofing the existence of a BGP follows standard Ramsey theory. Denoting variables in efficiency units with a hat, the two differential equations $\dot{\hat{c}} = \frac{(r-\rho)}{\epsilon}\hat{c}$ and $\dot{\hat{k}} = \hat{y} - \hat{c} - \hat{g}\hat{k}$ describe the entire dynamic framework so that the Jacobian matrix evaluated at a steady state reads

$$J(\hat{k},\hat{c}) = \begin{bmatrix} \partial\dot{\hat{k}}/\partial\hat{k} & \partial\dot{\hat{k}}/\partial\hat{c} \\ \partial\dot{\hat{c}}/\partial\hat{k} & \partial\dot{\hat{c}}/\partial\hat{c} \end{bmatrix} = \begin{bmatrix} r-\hat{g} & -1 \\ 0 & \frac{r-\rho}{\epsilon} \end{bmatrix}$$

where the determinant of the Jacobian matrix proofs local saddle point stability. As $r = \alpha(1-\phi)(1-n)^{(1-\alpha)(1-\phi)}K_0^{\alpha(1-\phi)-1}\frac{(A_0R_0)^{(1-\alpha)(1-\phi)}G_0^{\phi-\alpha(1-\phi)}}{1+\mathcal{B}}$, there is always a bundle $\{A_0, R_0, G_0, K_0\}$ that clears the savings investment market with

$$\epsilon\frac{\lambda n\mathcal{B}}{1-\alpha(1-\phi)} + \rho = \alpha(1-\phi)(1-n)^{(1-\alpha)(1-\phi)}K_0^{\alpha(1-\phi)-1}\frac{(A_0R_0)^{(1-\alpha)(1-\phi)}G_0^{\phi-\alpha(1-\phi)}}{1+\mathcal{B}}$$

which satisfies $r > \rho$, so local saddle point stability exists. The Hamiltonian is jointly concave in control and states, hence Mangasarian's sufficiency theorem applies. So given initial state and transversality conditions, the Maximum Principle yields first order conditions which complete the set of necessary conditions that are sufficient for local stability. It can also be shown that the saddle point is globally stable (for a proof see Aghion et al., 1998). Finally, (iii) directly follows since the innovation and production equations are monotonous in their arguments. □

Proposition (2.2) First, for $g_{\bar{E}} > 0$ and $1 > \phi$, the social planner intends to find the κ, η, n combination that maximizes $c = c(t)$ based on the Hamiltonian

$$H : \frac{c^{1-\epsilon}}{1-\epsilon} + \psi_K(y-c) + \psi_b(\lambda n\mathcal{B}\bar{b}(1+\mathcal{B}))$$

s.t. $\lim_{t\to\infty} e^{-\rho t}\psi_k k(t) = 0$ and $\lim_{t\to\infty} e^{-\rho t}\psi_g \bar{b}(t) = 0$ with predetermined $\bar{b}_0, K_0 > 0$ and $\bar{b} = \bar{b}(t) = \frac{B(t)}{1+\mathcal{B}}$ with $\dot{B}(t) = \lambda n\mathcal{B}B(t)$. Given this, if $1 > \phi$,

$$H_c = c^{-\epsilon} = \psi_k \tag{A1}$$

thus $-g_{\psi_k} = \epsilon g$. Then, $\dot{\psi}_k = -H_k + \rho\psi_k = -\frac{(\alpha-\phi))y}{K}\psi_k + \rho\psi_k$, so

$$-\frac{\dot{\psi}_K}{\psi_k} = -g_{\psi_k} = \frac{\alpha(1-\phi)y}{K} - \rho \tag{A2}$$

what with (A1) results in the social planner Euler equation,

$$\frac{\alpha(1-\phi)y}{K} = \epsilon g_c + \rho = \epsilon g + \rho. \tag{A3}$$

Next, $\dot{\psi}_{\bar{b}} = -H_{\bar{b}} + \rho\psi_{\bar{b}}$ yields

$$g_{\psi_{\bar{b}}} = -\frac{\psi_k}{\psi_{\bar{b}}}\frac{y}{\bar{b}} - \lambda n\mathcal{B}(1+\mathcal{B}) + \rho. \tag{A4}$$

Further, H_n gives

$$\psi_k(1-\alpha)(1-\phi)\frac{y}{1-n} = \psi_{\bar{b}}\lambda\mathcal{B}\bar{b}(1+\mathcal{B}). \tag{A5}$$

Considering the dynamics of (A5), along a BGP $g_{\psi_k} + g_y = g_{\psi_{\bar{b}}} + g_B$. Further, (A1) gives $-g_{\psi_k} = \epsilon g_y$, thus $(1-\epsilon)g_y - g_B = g_{\psi_{\bar{b}}}$. Along a BGP, $g_y = \frac{\lambda n\mathcal{B}}{1-\alpha(1-\phi)}$ and $g_B = \lambda n\mathcal{B}$. This gives $\left(\frac{(1-\epsilon)}{1-\alpha(1-\phi)} - 1\right)\lambda n\mathcal{B} = g_{\psi_{\bar{b}}}$. Including this in (A4) gives $\left(\frac{(1-\epsilon)}{1-\alpha(1-\phi)} + \mathcal{B}\right)\lambda n\mathcal{B} - \rho = -\frac{\psi_k}{\psi_{\bar{b}}}\frac{y}{\bar{b}}$, so with (A5)

$$n = \frac{\left(1+\mathcal{B} - \frac{\rho}{\lambda\mathcal{B}}(1-\alpha)(1-\phi)\right)}{\frac{\phi+\epsilon(1-\alpha)(1-\phi)}{1-\alpha(1-\phi)} + \mathcal{B}(\phi+\alpha(1-\phi))}.$$

For $n > 0$, it is necessary that $1+\mathcal{B} > \frac{\rho}{\lambda\mathcal{B}}$ thus $(1+\mathcal{B})\frac{\lambda\mathcal{B}}{(1-\alpha)(1-\phi)} > \rho$, so that there is a critical $\underline{\mathcal{B}}$ where $(1+\underline{\mathcal{B}})\frac{\lambda\mathcal{B}}{(1-\alpha)(1-\phi)} = \rho$. Since the corresponding quadratic equation has a unique (reasonable) solution, this states that $\mathcal{B} > \frac{(1+\frac{4\rho(1-\alpha)(1-\phi)}{\lambda})^{\frac{1}{2}}-1}{2}$. Further,

$1 > n$ yields $\rho > \lambda\mathcal{B}\big(\mathcal{B} + \frac{(1-\epsilon)}{1-\alpha(1-\phi)}\big)$, requiring that $\mathcal{B}$ is below a critical $\overline{\mathcal{B}}$ where $\rho = \lambda\overline{\mathcal{B}}\big(\overline{\mathcal{B}} + \frac{(1-\epsilon)}{1-\alpha(1-\phi)}\big)$, thus $\frac{\left(\frac{(1-\epsilon)^2}{1-\alpha(1-\phi)^2}+4\frac{\rho}{\lambda}\right)^{\frac{1}{2}} - \frac{(1-\epsilon)}{1-\alpha(1-\phi)}}{2} > \mathcal{B}$. Combining both critical intensities gives $\frac{\left(\frac{(1-\epsilon)^2}{1-\alpha(1-\phi)^2}+4\frac{\rho}{\lambda}\right)^{\frac{1}{2}} - \frac{(1-\epsilon)}{1-\alpha(1-\phi)}}{2} > \mathcal{B} > \frac{(1+\frac{4\rho(1-\alpha)(1-\phi)}{\lambda})^{\frac{1}{2}}-1}{2}$.

Direction of technical change: H_η gives $\psi_b\frac{\partial\mathcal{B}}{\partial\eta}(1+2\mathcal{B}) = 0$, what simplifies to $\psi_b\frac{\partial\mathcal{B}}{\partial\eta} = 0$ and yields $\left(\frac{\varsigma_A}{\Gamma\varsigma_G}\right)^{\frac{1}{1-\theta}} = \frac{\eta}{1-\kappa-\eta}$, so $\frac{1}{1+\left(\frac{\Gamma\varsigma_G}{\varsigma_A}\right)^{\frac{1}{1-\theta}}}(1-\kappa) = \eta$ with $\Gamma := \big(\frac{\phi}{(1-\phi)(1-\alpha)} - \frac{\alpha}{1-\alpha}\big)$, while H_κ gives $\psi_b\mathcal{B}_\kappa(1+2\mathcal{B})) = 0$, thus $\psi_b\mathcal{B}_\kappa = 0$, so $\left(\frac{\varsigma_R}{\Gamma\varsigma_G}\right)^{\frac{1}{1-\theta}} = \frac{\kappa}{1-\kappa-\eta}$ yielding $1-\kappa\Big(1+\left(\frac{\Gamma\varsigma_G}{\varsigma_R}\right)^{\frac{1}{1-\theta}}\Big) = \eta$. Therefore, $\kappa^* = \frac{\varsigma_R^{\frac{1}{1-\theta}}}{\left(\varsigma_A^{\frac{1}{1-\theta}}+\varsigma_R^{\frac{1}{1-\theta}}+\left(\Gamma\varsigma_G\right)^{\frac{1}{1-\theta}}\right)}$, $\eta^* = \frac{\varsigma_A^{\frac{1}{1-\theta}}}{\left(\varsigma_A^{\frac{1}{1-\theta}}+\varsigma_R^{\frac{1}{1-\theta}}+\left(\Gamma\varsigma_G\right)^{\frac{1}{1-\theta}}\right)}$ and $1-\kappa^*-\eta^* = \frac{\left(\Gamma\varsigma_G\right)^{\frac{1}{1-\theta}}}{\left(\varsigma_A^{\frac{1}{1-\theta}}+\varsigma_R^{\frac{1}{1-\theta}}+\left(\Gamma\varsigma_G\right)^{\frac{1}{1-\theta}}\right)}$. For $1 > \phi$ with $g_{\bar{E}} > 0$, there is only abatement if $\Gamma > 0$. As there are improvements in abatement and adaptation, this is a *gray* direction of technical change. Reformulations yield

$$g_{gray} = \frac{\lambda n}{1-\alpha(1-\phi)}\mathcal{B}_{gray} \;\; with \;\; \mathcal{B}_{gray} := (1-\phi)(1-\alpha)\Big(\varsigma_A^{\frac{1}{1-\theta}}+\varsigma_R^{\frac{1}{1-\theta}}+\Big(\Gamma\varsigma_G\Big)^{\frac{1}{1-\theta}}\Big)^{1-\theta}.$$

Yet, other alternatives are possible. Consider a complete concentration on general technologies without abatement or adaptation, then $\eta = 1$. Referring to this scenario as a *black* direction of technical change, this leads to

$$g_{black} = \frac{\lambda n}{1-\alpha(1-\phi)}\mathcal{B}_{black} \;\; with \;\; \mathcal{B}_{black} := (1-\phi)(1-\alpha)\varsigma_A.$$

Further, consider a *brown* direction of technical change in which there is no abatement but both adaptation and general innovation. Therefore, $\eta^* + \kappa^* = 1$, leading to

$\eta^* = \frac{1}{1+\left[\frac{\varsigma_R}{\varsigma_A}\right]^{\frac{1}{1-\theta}}}$ and $\kappa^* = \frac{1}{1+\left[\frac{\varsigma_A}{\varsigma_R}\right]^{\frac{1}{1-\theta}}}$, thus

$$g_{brown} = \frac{\lambda n}{1-\alpha(1-\phi)}\mathcal{B}_{brown} \ \ with \ \ \mathcal{B}_{brown} := (1-\phi)(1-\alpha)(\varsigma_A^{\frac{1}{1-\theta}} + \varsigma_R^{\frac{1}{1-\theta}})^{1-\theta}.$$

In addition, consider a no adaptation case, $\kappa = 0$, with abatement, but $g_{\bar{E}} > 0$, called a *yellow* direction of technical change. In this case, efficient research leads to $\eta^* = \frac{1}{1+\left(\frac{\Gamma\varsigma_G}{\varsigma_A}\right)^{\frac{1}{1-\theta}}}$, again with $\Gamma := \left(\frac{\phi}{(1-\phi)(1-\alpha)} - \frac{\alpha}{1-\alpha}\right)$, so that

$$g_{yellow} = \frac{\lambda n}{1-\alpha(1-\phi)}\mathcal{B}_{yellow} \ \ with \ \ \mathcal{B}_{yellow} := (1-\phi)(1-\alpha)\left(\varsigma_A^{\frac{1}{1-\theta}} + (\Gamma\varsigma_G)^{\frac{1}{1-\theta}}\right)^{1-\theta}.$$

Since $N = \frac{1}{\bar{E}^{\iota\omega}} \leq 1$, $g_{\bar{E}} < 0$ is not possible. Since $g_{\bar{E}} = \frac{(1-\alpha)(g_R+g_A)-(\alpha+(1-\phi)(1-\alpha^2))g_G}{(1-\alpha(1-\phi))}$, $(1-\alpha)(g_R+g_A) < (\alpha+(1-\phi)(1-\alpha^2))g_G$. Therefore, $(\varsigma_A\eta^\theta+\varsigma_R\kappa^\theta) > (\frac{\alpha}{(1-\alpha)}+(1-\phi)(1-\alpha))\varsigma_G(1-\kappa-\eta)^\theta$ which translates into $\eta > \frac{1}{1+\left(\frac{\varsigma_A}{\Lambda\varsigma_G}\right)^{\frac{1}{\theta}}}$ with $\Lambda := \left(\frac{\alpha}{(1-\alpha)}+(1-\phi)(1-\alpha)\right)$. Hence, $\left(\frac{\varsigma_A}{\varsigma_G}\right)^{\frac{1}{1-\theta}} > \Gamma^{\frac{\theta}{1-\theta}}\Lambda$. Whenever this is not the case, then $g_{\bar{E}} = 0$. For a yellow direction, this condition reads $\frac{\varsigma_A^{\frac{1}{1-\theta}}}{\varsigma_G^{\frac{1}{1-\theta}}} > \Gamma^{\frac{\theta}{1-\theta}}\Lambda$, for a gray direction, $\frac{\varsigma_A^{\frac{1}{1-\theta}}+\varsigma_R^{\frac{1}{1-\theta}}}{\varsigma_G^{\frac{1}{1-\theta}}} > \Gamma^{\frac{\theta}{1-\theta}}\Lambda$. Finally, $g_{\bar{E}} = 0$ relates to a *green* direction of technical change, characterized by $\kappa = 0$ (thus no adaptation). As derived with Lemma (2.1), then $\eta^* = \bar{\eta} = \frac{1}{1+\left[\frac{\varsigma_A}{\varsigma_G}\right]^{\frac{1}{\theta}}}$,

$$g_{green} = \lambda n\mathcal{B}_{green} \ \ with \ \ \mathcal{B}^*_{green} = \hat{\mathcal{B}} := \frac{(1-\alpha)\varsigma_A\varsigma_G}{\left(((1-\alpha)\varsigma_A)^{\frac{1}{\theta}} + \varsigma_G^{\frac{1}{\theta}}\right)^\theta}.$$

Importantly, the allocation principle for n is distinct. Thereby, $g_{\psi_{\bar{b}}} = -\frac{\psi_k}{\psi_{\bar{b}}}\frac{y}{\bar{b}} - \lambda n\mathcal{B}(1+\mathcal{B}) + \rho$, $\psi_k(1-\alpha)\frac{y}{1-n} = \psi_{\bar{b}}\lambda\mathcal{B}\bar{b}(1+\mathcal{B})$, so $-g_{\psi_{\bar{b}}} = \lambda n\mathcal{B}(1+\mathcal{B})\left(\frac{1-n\alpha}{n(1-\alpha)}\right) - \rho$. Further, (A1) gives $-g_{\psi_k} = \epsilon g_y$, thus $(1-\epsilon)g_y - g_B = g_{\psi_{\bar{b}}}$. However, along a green direction $g_y = g_B = \lambda n\mathcal{B}$ so $-\epsilon\lambda n\mathcal{B} = g_{\psi_{\bar{b}}}$. Therefore,

$$n_{green} = \left(\frac{(1-\alpha)}{\epsilon(1-\alpha)+\alpha}\right)\left(\frac{(1+\mathcal{B}(1-\alpha))}{(1-\alpha)} - \frac{\rho}{\lambda\mathcal{B}(1-\alpha)}\right).$$

So for $1 > n$, it is necessary that $\frac{\left(\frac{(1-\epsilon)^2}{(1-\alpha)^2}+4\frac{\rho}{\lambda}\right)^{\frac{1}{2}}-\frac{(1-\epsilon)}{(1-\alpha)}}{2} > \mathcal{B}_{green}$, so $\frac{\left(\frac{(1-\epsilon)^2}{(1-\alpha)^2}+4\frac{\rho}{\lambda}\right)^{\frac{1}{2}}-\frac{(1-\epsilon)}{(1-\alpha)}}{2} > \mathcal{B}_{green} > \frac{(1+\frac{4\rho(1-\alpha)(1-\phi)}{\lambda})^{\frac{1}{2}}-1}{2}$.

Now, for $1 > \phi$, any combination of research efforts is sustainable. The planner chooses the path with the highest consumption growth rate because utility is purely consumption based while growth effects dominate level effects. Since n and $\mathcal{B}$ are positively correlated and $\mathcal{B}_{brown} > \mathcal{B}_{black}$, respectively $\mathcal{B}_{gray} > \mathcal{B}_{yellow}$, a planner will never choose a black and a yellow direction of technical change. Hence, $(\varsigma_A^{\frac{1}{1-\theta}} + \varsigma_R^{\frac{1}{1-\theta}})^{1-\theta} > \varsigma_A$, which is always satisfied when $\varsigma_R > 0$. The same argument holds for $\mathcal{B}_{gray} > \mathcal{B}_{brown}$ as long as $\Gamma > 0$. $\square$

Proposition (2.2) First, $\pi(t) = (1-\alpha)\alpha(1-n)^{(1-\alpha)(1-\phi)}K(t)^{\alpha(1-\phi)}B(t) = (1-\alpha)\alpha y(t)$, see Proposition (2.1), so if $1 > \phi$, (15) and (16) lead to

$$\max_{\kappa,\eta}\mathcal{B} = \max_{\kappa,\eta}(1-\phi)(1-\alpha)[\varsigma_A(\kappa\eta)^\theta + \varsigma_R((1-\kappa)\eta)^\theta] + [\phi-\alpha(1-\phi)]\varsigma_G(1-\eta)^\theta$$

yielding $\eta = \eta^{**}$ and $\kappa = \kappa^{**}$. Further, whenever $\phi \geq 1$, a rational agent will anticipate that there are only profits if there is no exhaustion of the natural capital stock. Therefore, $\kappa = 0$, $\eta = \eta^{**} = \bar{\eta}$, see Proposition (1) and (2). Next, for the labor allocation, Proposition (2.1) has shown that $(1-\alpha)\frac{y}{(1-n)} = w$ and $V(t) = \frac{(1-\alpha)^2 y(t)}{(r+\lambda n-g_{\bar{\pi}})}$. With this, (23) can be reformulated to $\frac{1}{1-n} = (1+T_V)\frac{(1-\alpha)}{(r+\lambda n-g_{\bar{\pi}})}$. If $1 > \phi$, $\frac{(g_y-(\phi-\alpha(1-\phi))g_G)}{(1-\phi)} = g_{\bar{E}}$, thus with $g_{\bar{\pi}} = \frac{\phi}{1-\alpha}g_{\bar{E}}$, we find $g_{\bar{\pi}} = \frac{\phi(g_y-(\phi-\alpha(1-\phi)g_G)}{(1-\phi)(1-\alpha)}$. Now with (23), it also holds that $\epsilon\lambda n\frac{\mathcal{B}}{1-\alpha(1-\phi)} + \rho = r$, so

$$\frac{\lambda n\mathcal{W}+\rho}{1-n} = \lambda(1+T_V)(1-\alpha)$$

with $\mathcal{W} = \frac{\mathcal{B}}{1-\alpha(1-\phi)}\left(\epsilon + \frac{\phi}{(1-\phi)(1-\alpha)}\right) + 1 - \frac{\phi(\phi-\alpha(1-\phi)\varsigma_G(1-\eta)^\theta))}{(1-\phi)(1-\alpha)}$. Therefore, reformulating results gives $n = \frac{\lambda(1+T_V)(1-\alpha)-\rho}{(\lambda\mathcal{W}+(1+T_V)(1-\alpha))}$, so since $\partial n/\partial T_V > 0$ (as $\partial n/\partial T_V = \frac{(1-\alpha)(1-n)}{(\lambda\mathcal{W}+(1+T_V)(1-\alpha))}$), a research subsidy $T_V > 0$ increases n. Whenever $j^{**} = green$, then decentralized agents choose to fully abate any increase in the environmental effect. Therefore, $g_{\bar{\pi}} = 0$, so $\frac{1}{1-n} = (1+T_V)\frac{(1-\alpha)}{(r+\lambda n)}$ what with $\epsilon\lambda n\hat{\mathcal{B}} + \rho = r$ again yields $n = \frac{(1+T_V)(1-\alpha)-\rho}{(\lambda\mathcal{W}+(1+T_V)(1-\alpha))}$, this time with $\mathcal{W} = \epsilon\hat{\mathcal{B}} + 1$. $\square$

Calibration: The calibration is best related to an early stage of environmental degradation where the elasticity of environmental damage is relatively low, so research is following a brown direction of technical change per assumption. The economy is in the process of updating its information on the environmental impact and is starting to consider a gray, or green direction of technical change. Current literature usually sets α between 0.3 and $\frac{1}{3}$. Therefore, $\alpha = \frac{1}{3}$ and $\alpha(1-\phi) = 0.3$, so $\phi = 0.1$ for simplicity. Proposition (2.3) then gives $\lambda n \mathcal{W}_{brown} = \frac{2}{3}(\lambda - n) - \rho$ and $\mathcal{W}_{brown} = \frac{\mathcal{B}\left(\epsilon + \frac{\phi}{(1-\phi)(1-\alpha)}\right)}{1-\alpha(1-\phi)} + 1 \approx 2.96\mathcal{B} + 1$, so $\frac{\mathcal{W}_{brown}-1}{2.96} = \mathcal{B}$.

Therefore, $g_y = \lambda n \frac{\mathcal{B}}{1-\alpha(1-\phi)}$, so $0.0432 \approx \lambda n(\mathcal{W}_{brown} - 1)$. Combining results and using $\rho = 0.015$ (Nordhaus, 2007) leads to $0.0873 \approx \lambda - n - \frac{3}{2}\lambda n$ which with $\lambda = 0.5$ (Ricci, 2007) gives $n \approx 0.236$. Note that this fraction of researchers can be interpreted quite broadly as it simply describes individuals engaged in innovative activities. In any case, $g_y = \lambda n \frac{\mathcal{B}}{1-\alpha(1-\phi)}$ gives $0.124 \approx \mathcal{B}$.

Now since $\mathcal{B}_{brown} = (1-\phi)(1-\alpha)(\varsigma_A^{\frac{1}{1-\theta}} + \varsigma_R^{\frac{1}{1-\theta}})^{1-\theta}$, $0.2 \approx (\varsigma_A^{\frac{1}{1-\theta}} + \varsigma_R^{\frac{1}{1-\theta}})^{1-\theta}$. Further, with $x\varsigma_A = \varsigma_R$, $\frac{0.2}{(1+x^{\frac{1}{1-\theta}})^{1-\theta}} \approx \varsigma_A$, it is worth to consider two scenarios. Scenario (1) relating to a case where adaptation is 100% as efficient as general research, so $x = 1$ which serves as a benchmark. Scenario (2) serves as a robustness check and considers $x = 0.5$, stating that adaptation research is only 50% as effective as general research. For direct comparability of adaptation and abatement, $\varsigma_R = \varsigma_G$. With this, Scenario (1) is associated with $\varsigma_A \approx 0.14$, so $\varsigma_A := 0.14$ and $\varsigma_R := 0.14$. Scenario (2) relates to $\varsigma_A \approx 0.18$, so $\varsigma_A := 0.18$ and $\varsigma_R := 0.09$.

Corollary (2.1) Reformulating the research labor allocation principle for T_V and setting $n = n^{**}$ leads to $\frac{\lambda n^{**}\mathcal{W}+\rho}{(1-n^{**})(1-\alpha)} - 1 = T_V$. Since the markup is determined by $\frac{1}{\alpha}$, it is necessary to choose $1 - T_p = \frac{1}{\alpha}$, so $T_p = \frac{1-\alpha}{\alpha}$. $\square$

Proposition (2.4) Agents that have no information about the environmental implications of an innovation consider $\phi = 0$. Therefore, they face $y = \frac{(1-n)^{1-\alpha}}{E^{\iota\omega}} K^{\alpha} \underline{b}$

with $\underline{b} = \frac{\underline{\mathcal{B}}}{1+\underline{\mathcal{B}}}$ and $\underline{B} := A(t)^{1-\alpha}D(t)^{1-\alpha}$, whereas the intermediate profits in period t, described with (9), now follow

$$\pi(t) = \frac{\Lambda(1+T_p)^{\frac{\alpha}{1-\alpha}}A(t)R(t)(1-n)}{(r(1+T_E))^{\frac{\alpha}{1-\alpha}}G(t)^{\frac{\alpha}{1-\alpha}}\bar{E}^{\frac{\iota\omega}{1-\alpha}}}.$$

The agents choose $\kappa + \eta = 1$. They therefore address current damages but do not anticipate the future. Evidently, if $\phi \geq 1$, this innovation strategy is not sustainable, whereas if $1 > \phi$ it is.

For the labor allocation, first note that if an environmental effect is ignored, then (4) gives $w = (1-\alpha)\frac{(1-n)^{-\alpha}}{\bar{E}^{\iota\omega}}K^{\alpha}\underline{b}$, so with $V(t) = \frac{(1-\alpha)^2}{\lambda n(r+\lambda n - g_{\bar{\pi}})}\frac{(1-n)^{1-\alpha}}{\bar{E}^{\iota\omega}}K^{\alpha}\underline{b}$, (17) yields $\frac{1}{1-n} = (1+T_V)\frac{(1-\alpha)}{(r+\lambda n - g_{\bar{\pi}})}$. However, the agents do not anticipate environmental effects, so $g_{\bar{\pi}} = 0$ whilst (23) gives $\epsilon\lambda n\frac{\mathcal{B}_{anticipated}}{1-\alpha} + \rho = r_{anticipated}$ as an anticipated discount rate with $\mathcal{B}_{anticipated} := \frac{\mathcal{B}_{brown}}{1-\phi} = (1-\alpha)[\varsigma_A(\kappa)^{\theta} + \varsigma_R((1-\kappa))^{\theta}]$. Hence,

$$n = \frac{(1+T_V)(1-\alpha)-\rho}{(\lambda\mathcal{W}_{anticipated} + (1+T_V)(1-\alpha))}$$

with $\mathcal{W}_{anticipated} = \epsilon\frac{\mathcal{B}_{anticipated}}{(1-\alpha)(1-\phi)} + 1$. There is thus a twofold bias. Firstly, $\mathcal{B}_{brown} < \mathcal{B}_{anticipated}$. Secondly, $\epsilon\lambda n\frac{\mathcal{B}_{brown}}{1-\alpha(1-\phi)} + \rho = r_{brown} < \epsilon\lambda n\frac{\mathcal{B}_{anticipated}}{1-\alpha} + \rho = r_{anticipated}$ (which is beyond the effort effect). The agents therefore apply the same labor allocation rule as agents with complete information, however, with a focus on $j = brown$. $\square$

Corollary (2.2) Expression (9) gives $\pi(t) = \frac{\Lambda(1+T_p)^{\frac{\alpha}{1-\alpha}}A(t)R(t)(1-n)}{(r(1+T_E))^{\frac{\alpha}{1-\alpha}}G(t)^{\frac{\alpha}{1-\alpha}}\bar{E}^{\frac{\iota\omega}{1-\alpha}}}$. Lemma (2.1) has derived that $\bar{E}(t) = (1-n)^{1-\alpha}K(t)^{\alpha}\Big(\int_0^1 \frac{A_j(t)^{1-\alpha}R_j(t)^{1-\alpha}}{G_j(t)^{1+\alpha}}dj\Big)$, so the contribution of the newest innovator to the environmental effect is

$$E(t) = (1-n)^{1-\alpha}K(t)^{\alpha}\Big(\frac{A(t)^{1-\alpha}R(t)^{1-\alpha}}{G(t)^{1+\alpha}}\Big).$$

For the sake of the argument, first assume that the government simply taxes the intensity of the environmental effect with $T_E := \Gamma_E(t)(1-n)^{1-\alpha}K(t)^{\alpha}\Big(\frac{A(t)^{1-\alpha}R(t)^{1-\alpha}}{G(t)^{1+\alpha}}\Big) - 1$, then the intermediate profit function follows $\pi(t) = \big(A(t)R(t)\big)^{1-\alpha}G(t)^{\frac{1}{1-\alpha}}\mathcal{C}(t)$ with

$\mathcal{C}(t) = \frac{\Lambda(1+T_p)^{\frac{\alpha}{1-\alpha}}}{\Gamma_E(t)^{\frac{\alpha}{1-\alpha}}} \frac{(1-n)^{1-\alpha}K(t)^{\alpha}}{(r)^{\frac{\alpha}{1-\alpha}}\bar{E}^{\frac{\iota\omega}{1-\alpha}}}$. Along a BGP, $K(t)$ is proportional to $\pi(t)$, so the marginal contribution of efforts on capital is proportional to $\big(A(t)R(t)\big)^{1-\alpha}G(t)^{\frac{1}{1-\alpha}}$. Therefore, for the following argument, the capital related effects (i.e. repercussions of a tax on the capital endowment) can be ignored since they do not affect the relative weight among technologies. Given this, an innovator evaluates $\max_{\kappa,\eta} \underline{\hat{\mathcal{B}}} = (1-\alpha)\big(\varsigma_A\eta^{\theta} + \varsigma_R(\kappa)^{\theta}\big)\eta^{\theta} + \frac{\varsigma_G(1-\kappa-\eta)^{\theta}}{(1-\alpha)}$ what results in $\kappa = \frac{1}{1+\left(\frac{\varsigma_A}{\varsigma_R}\right)^{\frac{1}{1-\theta}}}$ and $\eta = \frac{1}{1+\left(\frac{\varsigma_G}{(1-\alpha)^2\left(\varsigma_A\kappa^{\theta}+\varsigma_R(1-\kappa)^{\theta}\right)}\right)^{\frac{1}{1-\theta}}}$. This example reveals that an unweighted environmental tax simply incentivizes to reduce the tax burden that affects profits.

In order to impact the abatement intensity to the socially optimal level, the tax has to weight the environmental effect according to its social costs. This calls for a dynamic Pigouvian tax which considers the externality of innovations on damages in the production growth rate. Hence,

$$(1+T_E) = \Gamma_E(t)\Big((1-n)^{1-\alpha}K(t)^{\alpha}\Big(\frac{A(t)^{1-\alpha}R(t)^{1-\alpha}}{G(t)^{1+\alpha}}\Big)\Big)^{\Omega}.$$

Therefore, $\pi(t) = \big(A(t)R(t)\big)^{1-\alpha\Omega}G(t)^{\frac{\Omega}{1-\alpha}}\mathcal{C}(t)$ with $\mathcal{C}(t) = \frac{\Lambda(1+T_p)^{\frac{\Omega\alpha}{1-\alpha}}}{\Gamma_E(t)^{\frac{\alpha}{1-\alpha}}} \frac{(1-n)^{1-\Omega\alpha}K(t)^{\Omega\alpha}}{(r)^{\frac{\alpha}{1-\alpha}}\bar{E}^{\frac{\iota\omega}{1-\alpha}}}$, so an innovator evaluates $\max_{\kappa,\eta} \underline{\hat{\mathcal{B}}} = (1-\alpha\Omega)\big(\varsigma_A\eta^{\theta} + \varsigma_R\kappa^{\theta}\big) + \big(\frac{\Omega}{1-\alpha}\big)\varsigma_G(1-\kappa-\eta)^{\theta}$ what results in $\kappa = \frac{1}{1+\left(\frac{\varsigma_A}{\varsigma_R}\right)^{\frac{1}{1-\theta}}}$ and $\eta = \frac{1}{1+\left(\frac{\left(\frac{\Omega}{1-\alpha}\right)\varsigma_G}{\left(1-\Omega\alpha\right)\left(\varsigma_A\kappa^{\theta}+\varsigma_R(1-\kappa)^{\theta}\right)}\right)^{\frac{1}{1-\theta}}}$.

A *gray* direction of technical change is then characterized by

$$\eta = \eta^* = \frac{1}{1+\left(\frac{\left(\frac{\phi}{1-\phi}-\alpha\right)\varsigma_G}{(1-\alpha)\left(\varsigma_R(1-\kappa)^{\theta}+\varsigma_A\kappa^{\theta}\right)}\right)^{\frac{1}{1-\theta}}},$$

what is achieved with $\frac{\frac{\Omega}{1-\alpha}}{\left(1-\Omega\alpha\right)} = \frac{\left(\frac{\phi}{1-\phi}-\alpha\right)}{(1-\alpha)}$, which simplifies to $\Omega = \frac{1}{\alpha+\left(\frac{1-\phi}{\phi-\alpha(1-\phi)}\right)}$.

Finally, the *green* direction requires

$$\eta^* = \bar{\eta} = \frac{1}{1 + \left(\frac{\varsigma_A}{\varsigma_G}\right)^{\frac{1}{\theta}}},$$

so $\bar{\eta} = \frac{1}{1+\left(\frac{\varsigma_G}{\varsigma_A}\right)^{\frac{1}{\theta}}}$, thus $\Omega := \frac{1}{1+\alpha} + \alpha$. Further, since $(1 + T_E)$ needs to be constant in order to not endanger the stability of the vintage technology distribution (see Proposition (2.1)), it is necessary that

$$g_{\Gamma_E} := \Omega\big((1 + \alpha)g_G - \alpha g_k - (1 - \alpha)(g_A + g_R)\big).$$

Since along a BGP $g_k = g_y$, we thus find that $g_{\Gamma_E} = \frac{\Omega\phi}{(1-\alpha)}\big(\frac{1}{1-\alpha(1-\phi)} g_G - (g_A + g_R)\big)$. This, however, is only required for a gray direction since it is associated with an increasing environmental effect and thus a potentially increasing environmental tax burden that skews the vintage cost distribution. In a green direction, the natural capital stock remains constant, so there is no increasing tax burden on its exhaustion. Further, since $\Gamma_E(t) = \Gamma_{E,0}e^{g_{\Gamma_E} t}$, it holds that $\Gamma_{E,0} = \frac{(1+T_E)}{\bar{E}_0^{\Omega}}$. Setting $n = \frac{(1+T_V)(1-\alpha)-\rho}{(\lambda \mathcal{W}_{anticipated}+(1+T_V)(1-\alpha))}$ equal to n^{**} and reformulating results yields $T_V = \frac{n^* \lambda \mathcal{W}_{anticipated}+\rho}{(1-n^*)(1-\alpha)} - 1$, so including the n^{**} findings of Proposition (2.2) leads to the stated results. For the markup prices, it is necessary to adjust for the environmental tax, so $T_p = 1 - \frac{1+T_E}{\alpha}$. Now since $\Gamma_{E,0}\bar{E}_0^{\Omega} = (1+T_E)$, it thus holds that $T_p = \frac{\Gamma_0 \bar{E}_0^{\Omega}}{\alpha} - 1$, so $\Gamma_{E,0} = \alpha \frac{1+T_p}{\bar{E}_0^{\Omega}}$. $\square$

Proposition (2.5) If $\phi \geq 1$, an agent with information on the environmental effect will correctly anticipate the required research strategy and thus select socially optimal research efforts. With specialization, research spillover effects are path specific, so an innovator compares alternative innovation values. Aggregate technologies would evolve according to

$$B(t) = \gamma\big(A(t)R(t)\big)^{(1-\alpha)(1-\phi)} + (1 - \gamma)A(t)^{(1-\alpha)(1-\phi)}G(t)^{\phi-\alpha(1-\phi)}.$$

In the long run, three growth paths for this bundle are possible

(1.) (indifference) $(1-\alpha)(1-\phi)g_R = \phi - \alpha(1-\phi)g_G$, so $0 < \gamma < 1$ and $g_B = (1-\alpha)(1-\phi)(g_A + g_R) = (1-\alpha)(1-\phi)(g_A + g_G)$,

(2.) (brown growth) $(1-\alpha)(1-\phi)g_R > \phi - \alpha(1-\phi)g_G$, so $\gamma = 1$ and $g_B = (1-\alpha)(1-\phi)(g_A + g_R)$,

(3.) (green growth) $(1-\alpha)(1-\phi)g_R < \phi - \alpha(1-\phi)g_G$, so $\gamma = 0$ and $g_B = (1-\alpha)(1-\phi)g_A + \big(\phi - \alpha(1-\phi)\big)g_G$.

In any case, for γ, an innovator compares $V_{brown}(t) \lesseqgtr V_{green}(t)$. With Proposition (2.1), $V(t) = \left(\frac{1-\alpha}{\alpha}\right)\frac{(1-\alpha)\alpha(1-n)^{(1-\alpha)(1-\phi)}K(t)^{\alpha(1-\phi)}B(t)}{r+\lambda n-g_{\bar{\pi}}}$. Therefore, an innovator is indifferent if

$$\frac{K_{brown}(t)^{\alpha(1-\phi)}R(t)^{(1-\alpha)(1-\phi)}}{r+\lambda n-g_{\bar{\pi}_{brown}}} = \frac{K_{green}(t)^{\alpha(1-\phi)}(1+T_{V,R})G(t)^{\phi-\alpha(1-\phi)}}{r+\lambda n-g_{\bar{\pi}_{green}}}.$$

In this case, we are in Scenario (1.) were both paths face the same $g_{\bar{\pi}}$, what leads to $K_{brown}(t)^{\alpha(1-\phi)}R(t)^{(1-\alpha)(1-\phi)} = K_{green}(t)^{\alpha(1-\phi)}(1+T_{V,R})G(t)^{\phi-\alpha(1-\phi)}$. Yet, since capital is priced with r, a standard marginal product consideration states that the path specific capital intensity is proportional to technology. It is consequently not possible to have $K_{brown} > K_{green}$ and $R(t)^{(1-\alpha)(1-\phi)} < (1+T_{V,R})G(t)^{\phi-\alpha(1-\phi)}$. Hence, there is only indifference among the research direction if $R(t)^{(1-\alpha)(1-\phi)} = (1+T_{V,R})G(t)^{\phi-\alpha(1-\phi)}$. If $R(t)^{(1-\alpha)(1-\phi)} > (1+T_{V,R})G(t)^{\phi-\alpha(1-\phi)}$, the economy enters brown research ($\gamma = 1$). If $R(t)^{(1-\alpha)(1-\phi)} < (1+T_{V,R})G(t)^{\phi-\alpha(1-\phi)}$, the economy enters green research ($\gamma = 0$). In Scenario (1.),

$$g_B = \lambda n\Big((1-\alpha)(1-\phi)(\varsigma_A\eta^\theta + \varsigma_R\kappa^\theta\Big) = \lambda n\Big(1-\alpha)(1-\phi)(\varsigma_A + \varsigma_G(1-\kappa-\eta)^\theta)\Big).$$

Stability among this path requires that both innovators spend the same amount of research on general innovations, so it is necessary that $\kappa = \eta$. Scenario (1.) can thus only occur if $\varsigma_R = \varsigma_G$, which requires that $(1-\alpha)(1-\phi)g_R = \phi - \alpha(1-\phi)g_G$, thus $(1-\alpha)(1-\phi)\varsigma_R(1-\kappa)^\theta = \phi - \alpha(1-\phi)\varsigma_G(1-\eta)^\theta$, so $\kappa = \eta$ is necessary and hence $\varsigma_G = \phi$. Therefore, there is only indifference among the research direction if

$\varsigma_R = \varsigma_G = \phi$ and $R(t)^{(1-\alpha)(1-\phi)} = (1 + T_{V,R})G(t)^{\phi-\alpha(1-\phi)}$, so $\gamma \in (0, 1)$. In any other case, $\gamma = 0$ or $\gamma = 1$. $\square$

Corollary (2.3) For agents with complete information, no environmental tax is required since the innovators correctly anticipate the social costs that come along with the alternative innovation directions. Proposition (2.5) has emphasized that it is possible to face environmental lock-in effects. Reformulating the equality condition for research indifference leads to $\frac{R(t)^{(1-\alpha)(1-\phi)}}{G(t)^{\phi-\alpha(1-\phi)}} - 1 = T_{V,R}$. $\square$

Proposition (2.6) Since agents without the information on the environmental effect do not anticipate any profits of green innovations, they set $\eta + \kappa = 1$ and maximize the innovation related profit growth rate for η. This rate is proportional to $\mathcal{B}$ and in their view given with $\mathcal{B} = \varsigma_A \eta^\theta + (1-\eta)^\theta \varsigma_R$, so they choose $\gamma = 1$ and allocate the research efforts as in a brown direction. As with general access to technologies, the general no arbitrage condition for labor described with (21) follows the above described logic, resulting in $n = \gamma n + (1-\gamma)n = \frac{1}{1+\frac{\lambda \mathcal{W}+\rho}{(1+T_V)(1-\alpha)}}$. $\square$

Corollary (2.4) Any tax that intends to incentivize green research needs to consider η and γ. Without information about the environmental impact of innovations, $\gamma = 1$, $\eta = 1-\kappa$, and $\eta = \eta^{**}$. To achieve green innovations, it is necessary that $\kappa = 0$ and $\eta = \frac{1}{1+\left(\frac{[\frac{\phi}{(1-\phi)}-\alpha]\varsigma_G}{(1-\alpha)\varsigma_A}\right)^{\frac{1}{1-\theta}}}$, what requires the environmental tax introduced in Corollary (2.3). Since a gray direction is not possible, agents have to decide among adaptation and abatement. So $B(t) = \gamma\big(A(t)R(t)\big)^{(1-\alpha)(1-\phi)} + A(t)^{(1-\alpha)(1-\phi)}G(t)^{\phi-\alpha(1-\phi)}$ with $g_A = \gamma\varsigma_A + (1-\gamma)\varsigma_A(1 - \frac{1}{1+\left(\frac{\left(\frac{\Omega}{1-\alpha}\right)\varsigma_G}{\left(1-\Omega\alpha\right)\left(\varsigma_A\kappa^\theta+\varsigma_R(1-\kappa)^\theta\right)}\right)^{\frac{1}{1-\theta}}})$. $\square$

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Appendix B

Expression (16): The F.O.C. of (13) w.r.t. $b(l)$ gives $\gamma\frac{p_a\mathcal{B}b(l)^{\gamma-1}}{\bar{c}c(l)} = (1+T_b)p_b$. This efficiency condition is met at each location, hence $\gamma\frac{p_a\mathcal{B}b(l)^{\gamma-1}}{\bar{c}c(l)} = (1+T_b)p_b$. Therefore,

$$b(l) = \Big(\frac{\gamma p_a\mathcal{B}}{\bar{p}_e(1+T_b)p_b l^\theta}\Big)^{\frac{1}{1-\gamma}} \tag{B1}$$

with $\bar{p}_e = \frac{(1+T_e)(1-T_{\bar{c}c})p_e}{\mathcal{I}}$ if $e = e_b$ and $\bar{p}_e = \frac{(1-T_{\bar{c}c})\wp_e}{\mathcal{I}}$ if $e = e_g$. Consequently,

$$a(l) = \mathcal{B}b(l)^\gamma = \mathcal{B}^{\frac{1+\gamma}{1-\gamma}}\Big(\frac{\gamma p_a}{\bar{p}_e(1+T_b)p_b l^\theta}\Big)^{\frac{\gamma}{1-\gamma}}. \tag{B2}$$

The local production budget is $B(l) := \frac{p_a\mathcal{B}b(l)^\gamma}{\bar{c}c(l)}$. Thereby, $\gamma B(l)$ of this budget is spend on morphological capital. The remaining budget, $(1-\gamma)B(l)$, is used to bid for land. As the sector is perfectly competitive, this bids up the land price until the sector breaks even, thus $(1-\gamma)\frac{p_a\mathcal{B}b(l)^\gamma}{\bar{c}c(l)} = \beta(l)\bar{\omega}$. The bid rent condition must hold at each location $l \in (0,\bar{l}]$. Therefore, in aggregate, it holds that

$$\int_0^{\bar{l}} B(l)dl = (1+T_b)p_b\bar{b} + (1+T_l)\bar{\beta}\bar{\omega} = \Big(\frac{\gamma^\gamma p_a\mathcal{B}}{(1+T_b)^\gamma p_b^\gamma \bar{p}_E}\Big)^{\frac{1}{1-\gamma}}\frac{(1-\gamma)}{(1-\gamma-\theta)}\bar{l}^{\frac{1-\gamma-\theta}{1-\gamma}}, \tag{B3}$$

$$b = \int_0^{\bar{l}} b(l)dl = \Big(\frac{\gamma p_a\mathcal{B}}{(1+T_b)p_b\bar{p}_E}\Big)^{\frac{1}{1-\gamma}}\frac{(1-\gamma)}{(1-\gamma-\theta)}\bar{l}^{\frac{1-\gamma-\theta}{1-\gamma}}, \tag{B4}$$

$$a = \int_0^{\bar{l}} a(l)dl = \mathcal{B}b(l)^\gamma = \mathcal{B}^{\frac{1+\gamma}{1-\gamma}}\Big(\frac{\gamma p_a}{(1+T_b)p_b\bar{p}_E}\Big)^{\frac{\gamma}{1-\gamma}}\frac{(1-\gamma)}{(1-\gamma-\gamma\theta)}\bar{l}^{\frac{1-\gamma-\gamma\theta}{1-\gamma}}. \tag{B5}$$

Since with the bid rent principle $\gamma B(l) = (1+T_b)p_b b(l)$ and $(1-\gamma)B(l)$,

$$\beta(l) = \frac{(1-\gamma)}{\gamma}\frac{(1+T_b)p_b}{(1+T_l)\bar{\omega}}b(l) = \frac{\gamma^{\frac{\gamma}{1-\gamma}}(1-\gamma)}{\bar{\omega}(1+T_l)(1+T_b)^{\frac{\gamma}{1-\gamma}}l^{\frac{\theta}{1-\gamma}}}\left(\frac{p_a\mathcal{B}}{\bar{p}_E p_b^{\gamma}}\right)^{\frac{1}{1-\gamma}}. \tag{B6}$$

Hence,

$$\bar{\beta} = \int_0^{\bar{l}}\beta(l)dl = \frac{(1-\gamma)}{\gamma}\frac{(1+T_b)p_b}{(1+T_l)\bar{\omega}}b = \frac{\gamma^{\frac{\gamma}{1-\gamma}}(1-\gamma)^2\bar{l}^{\frac{1-\gamma-\theta}{1-\gamma}}}{\bar{\omega}(1+T_l)(1+T_b)^{\frac{\gamma}{1-\gamma}}(1-\gamma-\theta)}\left(\frac{p_a\mathcal{B}}{\bar{p}_E p_b^{\gamma}}\right)^{\frac{1}{1-\gamma}}. \tag{B7}$$

For the land market clearing, first note that at the fringe $\beta(l) = 1$, so with (B6)

$$\bar{\omega} = \frac{\gamma^{\frac{\gamma}{1-\gamma}}(1-\gamma)}{(1+T_l)(1+T_b)^{\frac{\gamma}{1-\gamma}}}\left(\frac{p_a\mathcal{B}}{\bar{p}_E p_b^{\gamma}}\right)^{\frac{1}{1-\gamma}\bar{l}^{\frac{\theta}{1-\gamma}}}.$$

With (7),

$$p_a = \frac{(1+T_c)\iota}{(1+T_a)\epsilon}\frac{c}{a}. \tag{B8}$$

So households pay a gross price p_a and expect to receive a apartment units while they spend some of their expenditures on commuting. The net price the real estate sector earns is $\frac{p_a}{\bar{c}c}$, which was the basis of the real estate housing provision calculation described above. Hence, (B5) leads to

$$a = \left(\frac{c}{\bar{p}_E p_b}\right)^{\gamma}\Gamma_a\Lambda_a\bar{l}^{1-\gamma-\gamma\theta} \tag{B9}$$

with $\Gamma_a := \left(\frac{(1+T_c)}{(1+T_a)(1+T_b)}\right)^{\gamma}$, $\Lambda_a := \frac{\iota^{\gamma}\mathcal{B}\gamma^{\gamma}(1-\gamma)^{1-\gamma}}{\epsilon^{\gamma}(1-\gamma-\gamma\theta)^{1-\gamma}}$. Including (B9) in (B8) leads to

$$p_a = \frac{(\bar{p}_E p_b)^{\gamma}c^{1-\gamma}}{\Gamma_{p_a}\Lambda_{p_a}\bar{l}^{1-\gamma-\gamma\theta}} \tag{B10}$$

with $\Gamma_{p_a} := \frac{(1+T_a)^{1-\gamma}}{(1+T_c)^{1-\gamma}(1+T_b)^{\gamma}}$, $\Lambda_{p_a} := \frac{\epsilon^{1-\gamma}}{\iota^{1-\gamma}}\frac{\mathcal{B}\gamma^{\gamma}(1-\gamma)^{1-\gamma}}{(1-\gamma-\gamma\theta)^{1-\gamma}}$. With this,

$$b = \frac{c}{\bar{p}_E p_{b_R}\bar{l}^{\theta}}\frac{1}{\Gamma_b\Lambda_b}, \tag{B11}$$

$\Lambda_b := \frac{(1-\gamma-\theta)\epsilon}{(1-\gamma-\gamma\theta)\gamma\iota}$, $\Gamma_b := \frac{(1+T_a)(1+T_b)}{(1+T_c)}$. Beyond,

$$\beta(l) = \frac{c}{\bar{\omega}\bar{p}_E}\frac{1}{\Gamma_\beta\Lambda_\beta\bar{l}^{\frac{-1-\gamma-\gamma\theta}{1-\gamma}}l^{\frac{\theta}{1-\gamma}}}, \tag{B12}$$

$\Gamma_\beta := \frac{(1+T_a)(1+T_l)}{(1+T_c)}$ $\Lambda_\beta := \frac{\epsilon}{(1-\gamma-\gamma\theta)\iota}$. At the fringe, there are no bid rents, so $\beta(l) = 1$ and

$$\bar{\omega} = \frac{c}{\bar{p}_E}\frac{1}{\Gamma_\beta\Lambda_\beta\bar{l}^{1-\theta}}. \tag{B13}$$

Lemma (3.1) With (7), $\frac{(1+T_c)}{(1+T_x)}\frac{(1-\iota-\epsilon)}{\epsilon}c = p_x x$. Further, $p_x x = \bar{\omega}(\hat{l}-\bar{l})$. Therefore, $\frac{(1+T_c)}{(1+T_x)}\frac{(1-\iota-\epsilon)}{\epsilon}\frac{c}{(\hat{l}-\bar{l})} = \bar{\omega}$. With (B13), land market clearing is consequently given with

$$\frac{\Gamma_l\Lambda_l}{\bar{p}_E} = \frac{\bar{l}^{1-\theta}}{(\hat{l}-\bar{l})} \tag{B14}$$

with $\Gamma_l := \frac{(1+T_x)}{(1+T_a)(1+T_l)}$, $\Lambda_l := \frac{(1-\gamma-\gamma\theta)\iota}{(1-\iota-\epsilon)}$. $\square$

Lemma (3.2) Including (B10) in (B3) gives

$$\int_0^{\bar{l}} B(l)dl = \frac{c}{\bar{p}_E\bar{l}^\theta}\frac{1}{\Gamma_B\Lambda_B} \tag{B15}$$

with $\Lambda_B := \frac{\epsilon}{\iota}\frac{(1-\gamma-\theta)}{(1-\gamma-\gamma\theta)}$, $\Gamma_B := \frac{(1+T_a)}{(1+T_c)}$. So since $\int_0^{\bar{l}}\frac{p_a\mathcal{B}b(l)^\gamma}{\bar{c}c(l)}dl = \frac{w}{\gamma}\lambda$,

$$\frac{c}{\bar{p}_E\bar{l}^\theta}\frac{\gamma}{\lambda\Gamma_B\Lambda_B} = w. \tag{B16}$$

Further, $w = (1-T_w)(1-\varkappa-\varepsilon)\frac{y}{1-\lambda-\sigma}$. Therefore, the labor market clears with

$$\lambda = \frac{1-\sigma}{1+\frac{\bar{p}_E\bar{l}^\theta\Gamma_\lambda\Lambda_\lambda}{\mathcal{C}}} \tag{B17}$$

with $\mathcal{C} := \frac{c}{y}$, $\Gamma_\lambda := \frac{(1+T_a)(1-T_w)}{(1+T_c)}$ $\Lambda_\lambda := \frac{(1-\varkappa-\varepsilon)}{\gamma}\frac{\epsilon}{\iota}\frac{(1-\gamma-\theta)}{(1-\gamma-\gamma\theta)}$. $\square$

Lemma (3.3) Since $D = \frac{a}{l} = \left(\frac{c}{\bar{p}_E \pi}\right)^{\gamma} \frac{\Gamma_a \Lambda_a}{\bar{l}^{\gamma(1+\theta)}}$, see (B9), including (B20) leads to

$$D = \left(\frac{\lambda}{\bar{l}}\right)^{\gamma} \Gamma_D \Lambda_D. \tag{B18}$$

$\Gamma_D := \left(\frac{1}{1+T_b}\right)^{\gamma}$, $\Lambda_D := \frac{\mathcal{B}(1-\gamma)^{1-\gamma}(1-\gamma-\theta)^{\gamma}}{(1-\gamma-\gamma\theta)}$. Further,

$$a = \lambda^{\gamma} \bar{l}^{1-\gamma} \Gamma_D \Lambda_D. \tag{B19}$$

Including (B17) in (B18) gives

$$D = \left(\frac{1-\sigma}{\bar{l} + \frac{\bar{p}_E \bar{l}^{1+\theta} \Gamma_\lambda \Lambda_\lambda}{\mathcal{C}}}\right)^{\gamma} \Gamma_D \Lambda_D. \tag{B20}$$

□

Proposition (3.1) Since $\frac{\partial \Phi}{\partial P} < 0$, $\frac{\partial \Phi}{\partial D} > 0|_{D<D^*}$ and $\frac{\partial \Phi}{\partial D} < 0|_{D>D^*}$, a SBGP requires that $g_P = g_D = 0$. If this condition is not met, $\Phi \to 0$, hence $g_j \to 0$ for $j \in \{\mathcal{A}, \mathcal{E}, \mathcal{I}, \mathcal{F}\}$. Yet, $g_y > 0$ requires that at least $g_{\mathcal{A}} > 0$ or $g_{\mathcal{E}} > 0$, hence a constant positive Φ is necessary for a SBGP to exist. □

SBGP characteristics: While a potential need for increasing taxes will be discussed later, first assume that $g_T = 0$ for all taxes listed in (3.2). With (4), $\dot{S}(t) = i(t) = \hat{I}(t) - C(t)$. Since $S(t) = k(t) \Rightarrow \dot{S}(t) = \dot{k}(t)$. By definition, g_k is constant along a SBGP. Therefore, $g_S = g_k = \frac{\hat{I}(t)}{a(t)} - \frac{C(t)}{a(t)} \Rightarrow g_{\hat{I}} = g_c = g_k$, which are all constant. For further constancies, first note that with (6), $g_{\bar{c}} + \rho = r$ which is constant. The saving market clears when the interest on consumption corresponds to the return on manufactured capital. The latter is given by $(1+T_r)r = \varkappa \frac{y(t)}{k_y(t)} \Rightarrow g_y = g_{k_y}$, which is constant as well. Further, (7) leads to $g_{\bar{c}} = g_c = g_{p_a}(t) + g_a(t) = g_{p_x}(t)$. As $(1+Tx)p_x = \bar{\omega} \Rightarrow g_{p_x} = g_{\bar{\omega}}$, which is constant. Since D must be constant, b and $\bar{l}$ must remain constant, hence also $a \Rightarrow g_a = 0$, $g_{k_a} = 0$. Further, $\bar{\beta}(t) = \bar{\beta}$ so $\bar{p_E}$ needs to remain constant. Hence, $g_{\mathcal{I}} = g_{T_{\bar{c}c}} + g_{p_e}$ (whereby $g_{T_{\bar{c}c}}$ denotes subsidies).

With (14), $(1+T_l)\bar{\beta}\bar{\omega}(t) = (1-\gamma)p_a(t)a(t) \Rightarrow g_{p_a} = g_{\bar{\omega}}$, which is constant. Since

$k(t) = k_y(t) + k_a(t)$, for $t \to \infty$, $k_y(t) \to k(t)$, so an SBGP is a path which is approached in the long run. Similarly, whenever $g_{e_y} > 0$, $g_e \to g_{e_y}$ for $t \to \infty$ (since $e_{\bar{c}c}$ is constant along this SBGP). If in this case, $e = e_b$, then $g_e = g_P$ and $g_P \to g_{P_y}$ for $t \to \infty$ (since $g_{\bar{c}c} = 0$ if $\bar{l}$ is constant). Yet, $g_P > 0$ is not possible, so strategies to stop pollution growth are required. Those strategies are addressed below. Further, $w = (1 - \varkappa - \varepsilon)\frac{y(t)}{(1-\lambda)} \Rightarrow g_w = g_y$ (also constant). Beyond, $g_{\bar{\omega}} = g_{p_b}$, so in the long run $g_w = g_{p_b} = g_{\bar{\omega}} = g_y$. Then, with (22), either $g_G = g_{\mathcal{T}} = g_y$ or $g_G = g_{\mathcal{T}} = 0$, depending on the policy in place, as each component of $\mathcal{T}$ either remains constant or grows with g_y. Therefore, in (3), $g_L = g_{\bar{\omega}}$. With the Euler equation (6), $g_y + \rho = r$.

Kaldor facts: Neoclassical (endogenous) growth theory usually satisfies two conditions: 1. constant expenditure shares among the factor inputs, 2. labor augmenting (Harrod-neutral) technological progress. These are necessary conditions for the Kaldor facts to hold (see Uzawa, 1961, Sato and Beckmann, 1968 or Jones et al., 2003). Therefore, the literature commonly uses linear homogeneous production functions. Such functions satisfy the Euler theorem, stating that output exactly compensates for the factor expenditures if factors are paid their marginal product. Cobb-Douglas-type functions are linear homogeneous if their exponents add up to unity. A difficulty is then increasing population, as this can bias the marginal product of all factors and the corresponding expenditure shares. Any theory that refers to labor augmenting technological progress eases these challenges.

The endogenization of this chapter does not consider labor augmenting technology for tractability. While it would be possible to adjust the theory, this is not necessary since there is no population growth. A constant population enables alternative technology channels without impacting the expenditure shares, so the Kaldor facts remain valid. To see this, note that the energy factor share is $\mathcal{FS}_e = \frac{p_e e_y}{y}$, the labor factor share is $\mathcal{FS}_{(1-\lambda-\sigma)} = \frac{w(1-\lambda-\sigma)}{y}$ and the capital factor share is $\mathcal{FS}_k = \frac{r k_Y}{y}$. Therefore, $\mathcal{FS}_{(1-\lambda-\sigma)} + \mathcal{FS}_k + \mathcal{FS}_e = 1$. Since $w = \frac{(1-\varepsilon-\varkappa)y}{(1-\lambda-\sigma)}$ and $\mathcal{FS}_{(1-\lambda-\sigma)} = 1 - \varepsilon - \varkappa$, $r = \frac{\varkappa y}{k}$ leads to $\mathcal{F}_k = \varkappa$, while $p_e = \varepsilon \frac{e_y}{y}$, so $\mathcal{FS}_e = \epsilon$. The energy factor share requires further discussion. For $BEP(1)$, energy demand increases at the output growth rate, for $BEP(2)$, energy demand remains constant, while for $BEP(3)$, the energy demand reduces at $-g_e = r - g_y = \rho$, with the last equality occurring with the Euler equation. In the long run, there is hence no energy demand. Thereby, $\mathcal{F}_e = \varepsilon$ remains valid as the increase in energy prices and the reduction in energy demand adjust the relation. Against this background, there are four alternative regimes that keep pollution and densities constant and achieve sustainable growth, three are associated with brown energy, one with green energy. These regimes are detailed subsequently:

Brown energy regimes: If $(1+T_e)p_{e,b} < p_{e,g}$, then $e = e_y + e_{\bar{c}c} = e_{y,b} + e_{\bar{c}c,b} = e_b$. Here, three regimes lead to $g_P = g_D = 0$. In an **ES regime**, $e = e_b$ and $g_y = g_{p_{e,b}} + g_{T_e}$, so $g_{e_y} = g_P = 0$. This scenario can be achieved in BEP (1), (2) or (3). Since along a SBGP $g_y = g_k$, it follows that $g_y = \frac{1-\varkappa-\varepsilon}{1-\varkappa} g_{\mathcal{A}}$. In a **PS regime**, $e = e_b$ and $g_{P_{e,b}} + g_{T_e} = 0$, which is only relevant in BEP (1). With $P_{e,b} = \varepsilon \frac{y}{e_{y,b}}$, so $g_y = g_{e_{y,b}}$, so $g_P = 0$ requires that $g_{\mathcal{A}} = g_{\mathcal{F}} = g_{e_b}$. With $g_{p_{e_{y,b}}} + g_{T_e} = 0$, so $g_D = 0$. In an **EPS regime**, an ES and PS regime are combined, which is only relevant in a $BEP(1)$. Hence, $g_y > g_{p_{e_{y,b}}} + g_{T_E}$, thus $g_{e_{y,b}} > 0$ without further action, so $g_P = 0$ requires that $g_{e_y} = g_y - g_{p_{e_{y,b}}} - g_{T_e} = g_{\mathcal{F}}$. Since $y^{1-\varepsilon} = (1-\lambda)^{1-\varkappa-\varepsilon} \mathcal{A}^{1-\varkappa-\varepsilon} \big(k_y\big)^{\varkappa} \big(\frac{\epsilon}{(1+T_e)p_{e_{y,b}}}\big)^{\varepsilon} \Rightarrow g_y = g_{\mathcal{A}} - \frac{\varepsilon}{1-\varkappa-\varepsilon}(g_{T_e}) \Rightarrow g_{\mathcal{F}} = g_{\mathcal{A}} - \frac{1-\varkappa}{1-\varkappa-\varepsilon} g_{T_e}$ while $g_D = 0$ requires that $g_{T_e} = g_{T_{\bar{c}c}} + g_{\mathcal{I}}$. Note that $g_{\mathcal{A}} - \frac{\varepsilon}{1-\varkappa-\varepsilon} g_{T_e} \geq \frac{1-\varkappa-\varepsilon}{1-\varkappa} g_{\mathcal{A}}$ reduces to $g_{\mathcal{A}} \geq g_{T_e}$.

Green energy regime: In a **GE regime**, $e = e_g = \mathcal{E} \Rightarrow g_{e_b} = 0 \Rightarrow g_P = 0$. If $g_{p_{e,g}} > 0$, it is necessary that $g_{p_{e,g}} = g_{T_{\bar{c}c}} + g_{\mathcal{I}}$. Further, as $y^{1-\varepsilon} = (1-\lambda)^{1-\varkappa-\varepsilon} \mathcal{A}^{1-\varkappa-\varepsilon} \big(k_y\big)^{\varkappa} \big(\frac{\epsilon}{p_{e,g}}\big)^{\varepsilon} \Rightarrow g_y = g_{\mathcal{A}} - \frac{\varepsilon}{1-\varkappa-\varepsilon} g_{p_{e,g}}$.

Existence of an SBGP: As long as Φ remains constant, there are two differential equations: $\dot{\hat{c}} = (r-\rho)\hat{c}$ and $\dot{\hat{k}} = \hat{y} - \hat{c} - \hat{g}\hat{k}$ (with the hat referring to variables in efficiency units). Therefore, the Jacobian matrix evaluated at a steady state reads:

$$J(\hat{k}, \hat{c}) = \begin{bmatrix} \partial \dot{\hat{k}} / \partial \hat{k} & \partial \dot{\hat{k}} / \partial \hat{c} \\ \partial \dot{\hat{c}} / \partial \hat{k} & \partial \dot{\hat{c}} / \partial \hat{c} \end{bmatrix} = \begin{bmatrix} r - \hat{g} & -1 \\ -\frac{r}{\varkappa \hat{k}} & 0 \end{bmatrix}.$$

The determinant of the Jacobian matrix proofs local saddle point stability. Since in this theory a representative household Hamiltonian is jointly concave in control and states, Mangasarian's sufficiency theorem applies and the Maximum Principle gives the first order conditions which in combination with initial state values and transversality conditions provide a set of necessary conditions which are also sufficient for the existence of an SBGP.

Uniqueness: The above conditions for the existence of a balanced growth path technically lead to a locally unique *dynamic* result. However, combining (20) and (21) brings about that for $e = e_b$, $\Phi = \underline{\varsigma}_\Phi D \frac{e^{-\overline{\varsigma}_\Phi D}}{\left(1+\frac{e_{y,b}}{\mathcal{F}}+\frac{\varsigma_{\bar{c}\bar{c}}\bar{l}^{1+\theta}}{1+\theta}\right)}$ while for $e = e_g$, $\Phi = \underline{\varsigma}_\Phi D e^{-\overline{\varsigma}_\Phi D}$. Hence, there are possibly two D values associated with the same Φ. So while the dynamic conditions characterizing a SBGP are locally unique, they may be achieved with different factor allocations, a feature addressed later.

Transition paths and stability: As in most Ramsey based models, it is possible to identify an initial capital stock, k_0, for a stable saddle point. Therefore, note that with the Euler equation, an SBGP is associated with an initial endowment bundle $\{\mathcal{A}_0, e_{y,0}, k_0\}$ that satisfies $\hat{g} = (1+T_r)\varkappa(1-\lambda)^{1-\varkappa-\varepsilon}\frac{\mathcal{A}_0^{1-\varkappa-\varepsilon}e_{y,0}^{\varepsilon}}{k_0^{1-\varkappa}} - \rho$. There is then always a $\hat{k}$ associated with $\dot{\hat{k}} = \hat{y} - \hat{\bar{c}} - \hat{g}\hat{k}$, so an SBGP is technically feasible as the increase in manufactured capital keeps the marginal product of capital constant. For example, in an ES regime, there is always a k_0 that leads to a $\hat{k} = \frac{k_0}{\mathcal{A}_0}$ which satisfies this condition. It is further necessary to have $\dot{\hat{\bar{c}}} = (r-\rho)\hat{\bar{c}}$ for the Keynes-Ramsey-rule to hold. However, in $\hat{\bar{c}} = \frac{\bar{c}_0}{A_0}$, $\bar{c}_0$ can be considered as endogenously formed by the expectations of the rational citizens on the transversality conditions for k. So given a k_0, a rational agent will select an initial consumption level, $\bar{c}_0$, that satisfies the second condition. If a household were to select $\bar{c}_H > \bar{c}_0$, there were too few savings, so the capital stock deviates and thus the growth rate of the economy. Similarly, if $\bar{c}_L < \bar{c}_0$, there was too many savings while consumption deceases. □

Proposition (3.2) With endogenous innovations, the SBGP properties discussed with Proposition (3.1) need to be adjusted to the conditions for endogenous innovations. With exogenous growth, $g_{\mathcal{A}}$ and $g_{\mathcal{E}}$ are constant. With the endogenization, the technology growth rates need to remain constant, requiring that η, χ and σ are constant. For positive numéraire growth, it is necessary that $\chi > 0$ and $\sigma > 0$, while positive production requires that $\sigma < 1$. Henceforth, with (32), $\sigma \in (0,1)$ requires that $g_\pi = g_{\Gamma_V} = g_y$. $\square$

Lemma (3.4) If $i_{\mathcal{E}} = 1$, $g_{T_{\bar{c}c}} = 0$, and $g_{\mathcal{I}} = 0$, then $p_e = \wp_e = \frac{r}{\varepsilon}$, so $g_{p_{\bar{c}c}} = 0$. Commuting technologies and commuting subsidies have no productivity effects. Their purpose is to save commuting energy and thus to reduce commuting costs. There is consequently no rational argument for a government to improve commuting technologies if $i_{\mathcal{E}} = 1$. Therefore, if $i_{\mathcal{E}} = 1$, $i_{\mathcal{I}} = 0$. Further, if $i_{\mathcal{E}} = 1$, $g_e = g_{\mathcal{E}} = g_E + \varepsilon g_{I_{\mathcal{E}}} > 0$. Hence, since $e_{\bar{c}c}$ is constant on a SBGP, the economy will approach a state where $e = e_y + e_{\bar{c}c} = e_g = \mathcal{E}$. There are then three innovation scenarios to differentiate: (1.) $i_{\mathcal{E}} = 0$ and $i_{\mathcal{I}} = 0$, (2.) $i_{\mathcal{E}} = 0$ and $i_{\mathcal{I}} = 1$, (3.) $i_{\mathcal{E}} = 1$ and $i_{\mathcal{I}} = 0$, each associated with different research efforts, research labor allocations, innovation profits, innovation values and numéraire production, as detailed next.

Individual Profits along an SBGP:

(1.) If $i_{\mathcal{E}} = 0$ and $i_{\mathcal{I}} = 0$, then $y = \big(1-\sigma-\lambda\big)^{1-\varkappa-\varepsilon} A I_{\mathcal{A}}^{\varkappa} e_{y,b}^{\varepsilon}$. With (32), $\frac{(1-T_{\wp_{\mathcal{A}}})r}{\varkappa} = \wp_{\mathcal{A}}$, so $I_{\mathcal{A}} = \big(\frac{\varkappa^2(1-\sigma-\lambda)^{1-\varkappa-\varepsilon} e_{y,b}^{\varepsilon}}{(1-T_{\wp_{\mathcal{A}}})r}\big)^{\frac{1}{1-\varkappa}}$. Further, since the intermediate price is subsidized with $T_{\wp_{\mathcal{A}}}$, the intermediate producer earns $\wp_{\mathcal{A}} = \frac{r}{\varkappa}$. Therefore, $\Pi_{\mathcal{A}} = \big(\frac{1-\varkappa}{\varkappa}\big) r(1+T_{I,\mathcal{A}}) I_{\mathcal{A}}$, so $\Pi_{\mathcal{A}} = \frac{(1-\varkappa)\varkappa^{\frac{1+\varkappa}{1-\varkappa}}(1+T_{I,\mathcal{A}})(1-\sigma-\lambda)^{\frac{1-\varkappa-\varepsilon}{1-\varkappa}} e_{y,b}^{\frac{\varepsilon}{1-\varkappa}}}{(1-T_{\wp_{\mathcal{A}}})^{\frac{1}{1-\varkappa}} r^{\frac{\varkappa}{1-\varkappa}}}$.

(2.) If $i_{\mathcal{E}} = 0$ and $i_{\mathcal{I}} = 1$, then $\Pi_{\mathcal{A}}$ is identical to $i_{\mathcal{E}} = 0$ and $i_{\mathcal{I}} = 0$. For $\Pi_{\mathcal{I}}$, the marginal product of commuting intermediates is $\wp_{\mathcal{I}} = v_{\mathcal{I}} \Theta I_{\mathcal{I}}^{\Theta-1}$, so $\big(\frac{v_{\mathcal{I}}\Theta}{\wp_{\mathcal{I}}}\big)^{\frac{1}{1-\Theta}} = I_{\mathcal{I}}$. The profit for each intermediate is then $\Pi_{\mathcal{I}} = (\wp_{\mathcal{I}} - r)(1+T_{I,\mathcal{I}})\big(\frac{v_{\mathcal{I}}\Theta}{\wp_{\mathcal{I}}}\big)^{\frac{1}{1-\Theta}}$. Therefore, $\wp_{\mathcal{I}} = \frac{r}{\Theta}$ leads to $Pi_{\mathcal{I}} = \frac{v_{\mathcal{I}}^{\frac{1}{1-\Theta}}(1+T_{I,\mathcal{I}})(1-\Theta)\Theta^{\frac{2\Theta}{1-\Theta}}}{r^{\frac{\Theta}{1-\Theta}}}$.

(3.) If $i_{\mathcal{E}} = 1$ and $i_{\mathcal{I}} = 0$, then $\frac{(1-T_{\wp_{\mathcal{E}}})r}{\varepsilon} = \varepsilon(1-\sigma-\lambda)^{1-\varkappa-\varepsilon} I_{\mathcal{E}}^{\varepsilon-1} A I_{\mathcal{A}}^{\varkappa}$. Therefore, $I_{\mathcal{E}} = \big(\frac{\varepsilon^2(1-\sigma-\lambda)^{1-\varkappa-\varepsilon} A I_{\mathcal{A}}^{\varkappa}}{(1-T_{\wp_{\mathcal{E}}})r}\big)^{\frac{1}{1-\varepsilon}}$ and $I_{\mathcal{A}} = \big(\frac{\varkappa^2(1-\sigma-\lambda)^{1-\varkappa-\varepsilon} E I_{\mathcal{E}}^{\varepsilon}}{(1-T_{\wp_{\mathcal{A}}})r}\big)^{\frac{1}{1-\varkappa}}$ lead to $I_{\mathcal{A}} = \left(\frac{\varkappa^{2(1-\varepsilon)}\varepsilon^{2\varepsilon} A^{\varepsilon} E^{1-\varepsilon}}{r(1-T_{\wp_{\mathcal{A}}})^{1-\varepsilon}(1-T_{\wp_{\mathcal{E}}})^{\varepsilon}}\right)^{\frac{1}{1-\varkappa-\varepsilon}}(1-\lambda-\sigma)$ and $I_{\mathcal{E}} = \left(\frac{\varkappa^{2\varkappa}\varepsilon^{2(1-\varkappa)} A^{1-\varkappa} E^{\varkappa}}{r(1-T_{\wp_{\mathcal{E}}})^{1-\varkappa}(1-T_{\wp_{\mathcal{A}}})^{\varkappa}}\right)^{\frac{1}{1-\varkappa-\varepsilon}}(1-\lambda-$

σ). With (32), $\Pi_{\mathcal{A}} = \left(\frac{1-\varkappa}{\varkappa}\right)(1+T_{I,\mathcal{A}})rI_{\mathcal{A}}$ and $\Pi_{\mathcal{E}} = \left(\frac{1-\varepsilon}{\varepsilon}\right)(1+T_{I,\mathcal{E}})rI_{\mathcal{E}}$ it follows that $\Pi_{\mathcal{A}} = (1-\lambda-\sigma)\Gamma_A\Lambda_A\left(\frac{A^{\varepsilon}E^{1-\varepsilon}}{r^{\varkappa+\varepsilon}}\right)^{\frac{1}{1-\varkappa-\varepsilon}}$ whereby $\Gamma_A := \left(\frac{(1+T_{I,\mathcal{A}})^{1-\varkappa-\varepsilon}}{(1-T_{\wp_{\mathcal{A}}})^{1-\varepsilon}(1-T_{\wp_{\mathcal{E}}})^{\varepsilon}}\right)^{\frac{1}{1-\varkappa-\varepsilon}}$, $\Lambda_A := \left(\frac{1-\varkappa}{\varkappa}\right)\left(\varkappa^{2(1-\varepsilon)}\varepsilon^{2\varepsilon}\right)^{\frac{1}{1-\varkappa-\varepsilon}}$, while $\Pi_{\mathcal{E}} = \left(\frac{1-\varepsilon}{\varepsilon}\right)(1+T_{I,\mathcal{E}})\left(\frac{\varkappa^{2\varkappa}\varepsilon^{2(1-\varkappa)}A^{1-\varkappa}E^{\varkappa}}{r^{\varkappa+\varepsilon}(1-T_{\wp_{\mathcal{E}}})^{1-\varkappa}(1-T_{\wp_{\mathcal{A}}})^{\varkappa}}\right)^{\frac{1}{1-\varkappa-\varepsilon}}(1-\lambda-\sigma)$ can be rewritten as $\Pi_{\mathcal{E}} = \Pi_{\mathcal{A}}\Gamma_E\Lambda_E\frac{A}{E}$ with $\Gamma_E := \left(\frac{(1+T_{I,\mathcal{E}})(1-T_{\wp_{\mathcal{A}}})}{(1+T_{I,\mathcal{A}})(1-T_{\wp_{\mathcal{E}}})}\right)$, $\Lambda_E := \left(\frac{\varepsilon(1-\varepsilon)}{\varkappa(1-\varkappa)}\right)$.

Capital along an SBGP:

(1.) If $i_{\mathcal{E}} = 0$ and $i_{\mathcal{I}} = 0$, then $y = \left(1-\sigma-\lambda\right)^{1-\varkappa-\varepsilon}AI_{\mathcal{A}}^{\varkappa}e_{y,b}^{\varepsilon}$. With the putty-clay technology for intermediate producers, $I_{\mathcal{A}_i} = k_{y,i}$ leads to $AI_{\mathcal{A}} = k_y$. Therefore, $y = \left(1-\sigma-\lambda\right)^{1-\varkappa-\varepsilon}A^{1-\varkappa}k_y^{\varkappa}e_{y,b}^{\varepsilon}$ and $r = \varkappa\frac{y}{k}$.

(2.) If $i_{\mathcal{E}} = 0$ and $i_{\mathcal{I}} = 1$, the same allocation principles as in (1.) apply (note that as the growth rates may be distinct, r may be distinct and hence the steady state manufactured capital stock).

(3.) If $i_{\mathcal{E}} = 1$ and $i_{\mathcal{I}} = 0$, $y = \left(1-\sigma-\lambda\right)^{1-\varkappa-\varepsilon}AI_{\mathcal{A}}^{\varkappa}EI_{\mathcal{E}}^{\varepsilon}$. Generally, $\Pi_{\mathcal{E}}\frac{\varepsilon}{(1-\varepsilon)(1+T_{I_{\mathcal{E}}})r} = I_{\mathcal{E}}$, further $\Pi_{\mathcal{E}} = \Pi_{\mathcal{A}}\left(\frac{\varepsilon(1-\varepsilon)(1+T_{I,\mathcal{E}})(1-T_{\wp_{\mathcal{A}}})A}{\varkappa(1-\varkappa)(1+T_{I,\mathcal{A}})(1-T_{\wp_{\mathcal{E}}})E}\right)$ yields $\Pi_{\mathcal{A}}\left(\frac{\varepsilon^2(1-T_{\wp_{\mathcal{A}}})A}{r\varkappa(1-\varkappa)(1+T_{I,\mathcal{A}})(1-T_{\wp_{\mathcal{E}}})E}\right) = I_{\mathcal{E}}$. Using $\Pi_{\mathcal{A}} = \left(\frac{1-\varkappa}{\varkappa}\right)r(1+T_{I,\mathcal{A}})I_{\mathcal{A}}$ then gives $I_{\mathcal{A}}\left(\frac{\varepsilon^2(1-T_{\wp_{\mathcal{A}}})A}{\varkappa^2(1-T_{\wp_{\mathcal{E}}})E}\right) = I_{\mathcal{E}}$, so $y = \left(1-\sigma-\lambda\right)^{1-\varkappa-\varepsilon}AI_{\mathcal{A}}^{\varepsilon+\varkappa}E\left(\frac{\varepsilon^2(1-T_{\wp_{\mathcal{A}}})A}{\varkappa^2(1-T_{\wp_{\mathcal{E}}})E}\right)^{\varepsilon}$. Beyond, the capital demand for numéraire production is $AI_{\mathcal{A}} + EI_{\mathcal{E}} = k_y$. Therefore, $AI_{\mathcal{A}}\left(1+\frac{\varepsilon^2(1-T_{\wp_{\mathcal{A}}})}{\varkappa^2(1-T_{\wp_{\mathcal{E}}})}\right) = k_y$, so

$$y = \left(1-\sigma-\lambda\right)^{1-\varkappa-\varepsilon}A^{1-\varkappa}E^{1-\varepsilon}k_y^{\varkappa+\varepsilon}\left(\frac{\varkappa^{2\varkappa}(1-T_{\wp_{\mathcal{A}}})^{\varepsilon}(1-T_{\wp_{\mathcal{E}}})^{\varkappa}}{\varkappa^2(1-T_{\wp_{\mathcal{E}}})+\varepsilon^2(1-T_{\wp_{\mathcal{A}}})}\right)^{\varkappa+\varepsilon}$$

while $r = (\varepsilon+\varkappa)\frac{y}{k_y}$.

Efforts along an SBGP: The value of an intermediate, j, is $V_j = \int_0^{\infty} e^{-r(t)}\Pi_j(t)dt = \frac{\Pi_j}{r}$. The profits increase with σ, Φ, and the individual efforts χ and η. While σ and Φ follow with the labor and land allocations, χ and η determine how much profit a successful researcher achieves in the next period. In aggregate, this amounts to $(1+T_v)V = \frac{(1+T_v)\Gamma_V}{r}$ with $\Gamma_V = \chi^\psi \eta^\psi \kappa A \Pi_{\mathcal{A}} + \chi^\psi (1-\eta)^\psi \varsigma_{\mathcal{E}} E \Pi_{\mathcal{E}} + (1-\chi)^\psi \varsigma_{\mathcal{I}} I \Pi_{\mathcal{I}}$.

(1.) If $i_{\mathcal{E}} = 0$ and $i_{\mathcal{I}} = 0$, then $\chi = 1$ and $\eta = 1$. Therefore, $(1+T_v)V = \frac{(1+T_v)\Gamma_V}{r}$ with $\Gamma_V = \chi^\psi \eta^\psi \kappa A \Pi_{\mathcal{A}} + \chi^\psi (1-\eta)^\psi \varsigma_{\mathcal{E}} E \Pi_{\mathcal{E}} + (1-\chi)^\psi \varsigma_{\mathcal{I}} I \Pi_{\mathcal{I}}$.

(2.) If $i_E = 0$ and $i_{\mathcal{I}} = 1$, $\max_{\{\chi\}} V$ leads to $\chi = \frac{1}{1+\left(\frac{\varsigma_{\mathcal{I}} I \Pi_{\mathcal{I}}}{\kappa A \Pi_{\mathcal{A}}}\right)^{\frac{1}{1-\psi}}}$. Therefore, $(1+T_v)V = \frac{(1+T_v)}{r}\left(\frac{\kappa A \Pi_{\mathcal{A}}}{\left(1+\left(\frac{\varsigma_{\mathcal{I}} I \Pi_{\mathcal{I}}}{\kappa A \Pi_{\mathcal{A}}}\right)^{\frac{1}{1-\psi}}\right)^\psi} + \frac{\varsigma_{\mathcal{I}} I \Pi_{\mathcal{I}}}{\left(1+\left(\frac{\kappa A \Pi_{\mathcal{A}}}{\varsigma_{\mathcal{I}} I \Pi_{\mathcal{I}}}\right)^{\frac{1}{1-\psi}}\right)^\psi}\right) = \frac{(1+T_v)}{r}\kappa A \Pi_{\mathcal{A}}\left(1 + \left(\frac{\varsigma_{\mathcal{I}} I \Pi_{\mathcal{I}}}{\kappa A \Pi_{\mathcal{A}}}\right)^{\frac{1}{1-\psi}}\right)^{1-\psi}$.

(3.) If $i_{\mathcal{E}} = 1$ and $i_{\mathcal{I}} = 0$, $\chi = 1$. Further, $\max_{\{\eta\}} V$ gives $\eta = \frac{\left(A\Pi_{\mathcal{A}}\right)^{\frac{1}{1-\psi}}}{\left(A\Pi_{\mathcal{A}}\right)^{\frac{1}{1-\psi}} + \left(\varsigma_{\mathcal{E}} E \Pi_{\mathcal{E}}\right)^{\frac{1}{1-\psi}}}$, thus

$$\eta = \frac{1}{1+\Lambda\Gamma} \quad \textit{with} \quad \Lambda := \left(\frac{\varsigma_{\mathcal{E}}\varepsilon^2}{\varkappa^2}\right)^{\frac{1}{1-\psi}}, \quad \Gamma := \left(\frac{(1+T_{I,\mathcal{E}})(1-T_{\wp\mathcal{A}})}{(1+T_{I,\mathcal{A}})(1-T_{\wp\mathcal{E}})}\right)^{\frac{1}{1-\psi}}.$$

Therefore, $(1+T_v)V = \frac{(1+T_v)}{r}\left(\frac{A\Pi_{\mathcal{A}}}{(1+\Lambda)^\psi} + \frac{\varsigma_E E \Pi_{\mathcal{E}}}{(1+\frac{1}{\Lambda})^\psi}\right) = \frac{(1+T_s)}{r} A \Pi_{\mathcal{A}} (1+\Lambda)^{1-\psi}$.

Growth Rates along an SBGP:

(1.) If $i_{\mathcal{E}} = 0$ and $i_{\mathcal{I}} = 0$, then $g_A = \kappa\Phi\sigma$. Thereby, $g_y = \varepsilon g_{e_{y,b}} + g_A = g_{k_y}$, so with $g_{e_{y,b}} \geq 0$, $g_V = g_y$, whereas since $w = (1-\varkappa-\varepsilon)\frac{y}{(1-\sigma-\lambda)}$, $g_V = g_w = g_y$.

(2.) If $i_{\mathcal{E}} = 0$ and $i_{\mathcal{I}} = 1$, then $g_A = \frac{\kappa\sigma\Phi}{\left(1+\left(\frac{\varsigma_{\mathcal{I}} I \Pi_{\mathcal{I}}}{\kappa A \Pi_{\mathcal{A}}}\right)^{\frac{1}{1-\psi}}\right)^\psi}$ $g_I = \frac{\varsigma_{\mathcal{I}}\sigma\Phi\left(\frac{\varsigma_{\mathcal{I}} I \Pi_{\mathcal{I}}}{\kappa A \Pi_{\mathcal{A}}}\right)^{\frac{\psi}{1-\psi}}}{\left(1+\left(\frac{\varsigma_{\mathcal{I}} I \Pi_{\mathcal{I}}}{\kappa A \Pi_{\mathcal{A}}}\right)^{\frac{1}{1-\psi}}\right)^\psi} = \frac{\varsigma_I\sigma\Phi}{\left(1+\left(\frac{\kappa A \Pi_{\mathcal{A}}}{\varsigma_{\mathcal{I}} I \Pi_{\mathcal{I}}}\right)^{\frac{1}{1-\psi}}\right)^\psi}$. However, a constant χ requires that $g_{\Pi_{\mathcal{I}}} + g_I = g_{\Pi_{\mathcal{A}}} + g_A$. Since r is constant, $g_I = 0$, so $g_{\Pi_{\mathcal{I}}} = g_{\Pi_{\mathcal{A}}} + g_A$. Therefore, $g_y = g_V = g_w = \varepsilon g_{e_{y,b}} + g_A$.

(3.) If $i_{\mathcal{E}} = 1$ and $i_{\mathcal{I}} = 0$, then $g_A = \frac{\sigma\Phi}{(1+\Lambda)^\psi}$, $g_E = \frac{\sigma\varsigma_E\Phi\Lambda^\psi}{(1+\Lambda)^\psi} = \frac{\varsigma_E\Phi}{(1+\frac{1}{\Lambda})^\psi}$. With constant r, $g_{\Pi_{\mathcal{A}}} = g_{I_{\mathcal{A}}}$, and $g_{\Pi_{\mathcal{E}}} = g_{I_{\mathcal{E}}}$. Reformulating results leads to $g_{\Pi_{\mathcal{A}}} = \frac{1-\epsilon}{1-\varkappa-\varepsilon} g_E + \frac{\varepsilon}{1-\varkappa-\varepsilon} g_A$, $g_{\Pi_{\mathcal{E}}} = \frac{1-\varkappa}{1-\varkappa-\varepsilon} g_A + \frac{\varkappa}{1-\varkappa-\varepsilon} g_E$. Note that $g_A + g_{\Pi_{\mathcal{A}}} = \frac{1-\varkappa}{1-\varkappa-\varepsilon} g_A + \frac{1-\epsilon}{1-\varkappa-\varepsilon} g_E$.

Similarly, $g_E + g_{\Pi_{\mathcal{E}}} = \frac{1-\varkappa}{1-\varkappa-\varepsilon} g_A + \frac{1-\varepsilon}{1-\varkappa-\varepsilon} g_E$. Further, along a $SBGP$ $y = (1-\sigma-\lambda)^{1-\varkappa-\varepsilon} E I_E^{\varepsilon} A I_A^{\varkappa} \Rightarrow g_y = g_{k_y} = g_E + \varepsilon g_{\Pi_{\mathcal{E}}} + g_A + \varkappa g_{\Pi_{\mathcal{A}}} = \frac{1-\varkappa}{1-\varkappa-\varepsilon} g_A + \frac{1-\varepsilon}{1-\varkappa-\varepsilon} g_E = \frac{\left((1-\varkappa)+(1-\varepsilon)\varsigma_E \Lambda^{\psi}\right)}{(1-\varepsilon-\varkappa)(1+\Lambda)^{\psi}} \sigma\Phi$. Therefore, $g_y = g_V$.

Research Labor along an SBGP: With (32), each individual researcher considers $(1+T_v)V = (1-T_w)w$.

(1.) If $i_{\mathcal{E}} = 0$ and $i_{\mathcal{I}} = 0$, then for an individual researcher: $(1+T_v)\frac{\kappa A \Pi_A}{r} = (1-T_w)(1-\varkappa-\varepsilon)\frac{y}{1-\sigma-\lambda}$. Including $\Pi_{\mathcal{A}} = \left(\frac{1-\varkappa}{\varkappa}\right) r (1+T_{I,\mathcal{A}}) I_{\mathcal{A}}$ gives $(1+T_v)\frac{\kappa A \Pi_A}{r} = (1-T_w)(1-\varkappa-\varepsilon)\big(1-\sigma-\lambda\big)^{-\varkappa-\varepsilon} A I_{\mathcal{A}}^{\varkappa} e_{y,b}^{\varepsilon}$. Including $I_{\mathcal{A}} = \left(\frac{\varkappa^2 (1-\sigma-\lambda)^{1-\varkappa-\varepsilon} e_{y,b}^{\varepsilon}}{(1-T_{\wp_{\mathcal{A}}}) r}\right)^{\frac{1}{1-\varkappa}}$ leads to $\Gamma_s \kappa \Phi \Lambda_s (1-\sigma-\lambda) = r$ with $\Gamma_s := \frac{(1+T_v)(1+T_{I,\mathcal{A}})}{(1-T_{\wp_{\mathcal{A}}})(1-T_w)}$, $\Lambda_s := \left(\frac{(1-\varkappa)\varkappa}{(1-\varkappa-\varepsilon)}\right)$. Therefore,

$$\sigma = 1 - \lambda - \frac{r}{\kappa \Gamma_s \Lambda_s}.$$

(2.) If $i_{\mathcal{E}} = 0$ and $i_{\mathcal{I}} = 1$, following the procedures of (1.) leads to the same results as with $i_{\mathcal{E}} = 0$ and $i_{\mathcal{I}} = 0$.

(3.) If $i_{\mathcal{E}} = 1$ and $i_{\mathcal{I}} = 0$, then $(1+T_v)\frac{A\Pi_{\mathcal{A}}}{r}(1+\Lambda)^{1-\psi} = (1-T_w)(1-\varkappa-\varepsilon)\big(1-\sigma-\lambda\big)^{-\varkappa-\varepsilon} A I_{\mathcal{A}}^{\varkappa} E I_{\mathcal{A}}^{\varepsilon}$. With $\Pi_{\mathcal{E}}\left(\frac{\varkappa}{(1-\varkappa)(1+T_{I,\mathcal{E}})r}\right) = I_{\mathcal{E}}$ and $\Pi_{\mathcal{E}} = \Pi_{\mathcal{A}}\left(\frac{\varepsilon^2 (1+T_{I,\mathcal{E}})(1-T_{\wp_{\mathcal{A}}})A}{\varkappa^2 (1+T_{I,\mathcal{A}})(1-T_{\wp_{\mathcal{E}}})E}\right)$, $\Pi_{\mathcal{A}}\left(\frac{\varepsilon^2 (1+T_{I,\mathcal{E}})(1-T_{\wp_{\mathcal{A}}})A}{\varkappa^2 (1+T_{I,\mathcal{A}})(1-T_{\wp_{\mathcal{E}}})E}\right)\left(\frac{\varkappa}{(1-\varkappa)(1+T_{I,\mathcal{E}})r}\right) = I_{\mathcal{E}}$. Further, $I_{\mathcal{A}} = \Pi_{\mathcal{A}}\left(\frac{\varkappa}{(1-\varkappa)(1+T_{I,\mathcal{A}})r}\right)$. Therefore, $(1+T_v)\frac{\Phi}{r}\Pi_{\mathcal{A}}^{1-\varepsilon-\varkappa}(1+\Lambda)^{1-\psi} = \big(1-\sigma-\lambda\big)^{-\varkappa-\varepsilon} A^{\varepsilon} E^{1-\varepsilon} \Gamma_{SA} \Lambda_{SA} \left(\frac{1}{r}\right)^{\varepsilon+\varkappa}$ with $\Gamma_{SA} := \left(\frac{(1-T_w)(1-T_{\wp_{\mathcal{A}}})^{\varepsilon}}{(1+T_{I,\mathcal{A}})^{\varepsilon}(1-T_{\wp_{\mathcal{E}}})^{\varepsilon}(1+T_{I,\mathcal{A}})^{\varkappa}}\right)$, $\Lambda_{SA} := (1-\varkappa-\varepsilon)\left(\frac{\varkappa}{(1-\varkappa)}\right)^{\varepsilon+\varkappa}\left(\frac{\varepsilon}{\varkappa}\right)^{2\varepsilon}$. Including $\Pi_{\mathcal{A}} = \left(\frac{1-\varkappa}{\varkappa}\right)(1+T_{I,\mathcal{A}})\left(\frac{\varkappa^{2(1-\varepsilon)}\varepsilon^{2\varepsilon} A^{\varepsilon} E^{1-\varepsilon}}{r^{\varkappa+\varepsilon}(1-T_{\wp_{\mathcal{A}}})^{1-\varepsilon}(1-T_{\wp_{\mathcal{E}}})^{\varepsilon}}\right)^{\frac{1}{1-\varkappa-\varepsilon}}(1-\lambda-\sigma)$ gives

$$\sigma = 1 - \lambda - \frac{r}{(1+\Lambda)^{1-\psi}\Gamma_s \Lambda_s}. \quad \square$$

Lemma (3.5) If $(1+T_e)p_{e_{y,b}} \geq \wp_{\mathcal{E}} = p_{e,g}$, $i_{\mathcal{E}} = 1$. However, $e_b = e_{\bar{c}c,b} + e_{y,b} > 0$ if $e - e_g > 0$. Since $g_{e_{\bar{c}c}} = 0$, it follows that $\left(\varepsilon(1-\sigma-\lambda)^{1-\varkappa-\varepsilon}\frac{A I_{\mathcal{A}}^{\varkappa}}{p_{e_{y,b}}}\right)^{\frac{1}{1-\varepsilon}} - E I_{\mathcal{E}} > 0$, respectively $\left(\varepsilon(1-\sigma-\lambda)^{1-\varkappa-\varepsilon}\frac{A I_{\mathcal{A}}^{\varkappa}}{p_{e_{y,b}}}\right)^{\frac{1}{1-\varepsilon}} > \left(\varepsilon(1-\sigma-\lambda)^{1-\varkappa-\varepsilon} A I_{\mathcal{A}}^{\varkappa}\right)^{\frac{1}{1-\varepsilon}}$, thus $\frac{1}{p_{e_{y,b}}^{\frac{1}{1-\varepsilon}}} >$

E. Hence, a complete green energy transition requires that $|\frac{1}{1-\varepsilon} g_{p_{e_{y,b}}}| > g_E$. This condition is satisfied if $(1+T_e)p_{e_{y,b}} \geq \wp_{\mathcal{E}}$. In this case, if $t \to \infty$, then $e \to \mathcal{E}$, hence a green energy transition occurs and the energy sector in the long run provides energy for commuting and production, with only the latter growing. As $t \to \infty$, $k \to k_y$. If $i_{\mathcal{E}} = 0$, then $\eta = 1$ and

$$\chi = \frac{1}{1 + \left(\frac{\varsigma_{\mathcal{I}} I \Pi_{\mathcal{I}}}{\kappa A \Pi_{\mathcal{A}}}\right)^{\frac{1}{1-\psi}}}.$$

Since $g_{\Pi_{\mathcal{I}}} = 0$, these efforts are only constant if $g_A + g_{\Pi_{\mathcal{A}}} = g_I$. Thereby, $e_{y,b} = \varepsilon \frac{y}{(1+T_e)p_{e_{y,b}}} \Rightarrow g_{e_{y,b}} = g_y - g_{T_e} - g_{p_{e_{y,b}}}$. Further, $g_y = \frac{\varepsilon}{(1-\varkappa)} g_{e_{y,b}} + g_A$ leads to $g_y = \frac{(1-\varkappa)}{(1-\varepsilon-\varkappa)} g_A - \frac{\varepsilon}{(1-\varepsilon-\varkappa)}(g_{T_e} + g_{p_{e_{y,b}}})$, so $g_{e_{y,b}} = \frac{(1-\varkappa)}{(1-\varepsilon-\varkappa)}(g_A - g_{T_e} - g_{p_{e_{y,b}}})$. A government will only initiate commuting focused innovation if there is a need to abate energy price induced commuting increases since in any other case, commuting cost reduction is destabilizing. If there is a need for commuting technology improvements, $g_{T_e} + g_{p_{e_{y,b}}} = g_I$, so $g_{e_{y,b}} = \frac{(1-\varkappa)}{(1-\varepsilon-\varkappa)}(g_A - g_I)$ whilst $g_{e_{y,b}} = g_y - g_I$. A stable χ requires that $(1-\varkappa)g_A + \varepsilon \frac{(1-\varkappa)}{(1-\varepsilon-\varkappa)}(g_A - g_I) = (1-\varkappa)g_I$, which simplifies to $g_A = g_I$. Hence, $\sigma\Phi\chi^{\psi} = \varsigma_I \sigma \Phi (1-\chi)^{\psi}$, so

$$\chi = \frac{1}{1 + \frac{1}{\varsigma_I^{\frac{1}{\psi}}}}.$$

This result requires that $\left(\frac{\varsigma_{\mathcal{I}} I \Pi_{\mathcal{I}}}{\kappa A \Pi_{\mathcal{A}}}\right)^{\frac{1}{1-\psi}} = \frac{1}{\varsigma_I^{\frac{1}{\psi}}}$. Therefore, $\varsigma_{\mathcal{I}}^{\frac{1}{\psi}} I \Pi_{\mathcal{I}} = \kappa A \Pi_{\mathcal{A}}$. Implementing $\Pi_{\mathcal{A}}$ and $\Pi_{\mathcal{I}}$ leads to

$$W_1 = W_2$$

with $W_1 := \frac{v_{\mathcal{I}}^{\frac{1}{1-\Theta}}(1+T_{I,\mathcal{I}})(1-T_{\wp_{\mathcal{A}}})^{\frac{1}{1-\varkappa}}}{(1+T_{I,\mathcal{A}})}$ and $W_2 := \kappa \frac{A}{I} \frac{(1-\varkappa)\varkappa^{\frac{1+\varkappa}{1-\varkappa}}(1-\sigma-\lambda)^{\frac{1-\varkappa-\varepsilon}{1-\varkappa}} e_{y,b}^{\frac{\varepsilon}{1-\varkappa}}}{r^{\frac{\varkappa}{1-\varkappa} - \frac{\Theta}{1-\Theta}} \varsigma_{\mathcal{I}}^{\frac{1}{\psi}} (1-\Theta)\Theta^{\frac{2\Theta}{1-\Theta}}}$. So if $W_1 > W_2$, all research goes into commuting, if $W_1 < W_2$, all to general innovations, while stable research requires that $W_1 = W_2$. $\square$

Proposition (3.3) If Proposition (3.2) is satisfied, the SBGP properties with endogenous innovations are close to the ones observed with exogenous growth detailed in Proposition (3.1). Yet, there are minor adjustments.

In an **ES regime**, $e = e_b$ and $g_y = g_{p_{e,b}} + g_{T_e}$ leads to $g_{e_y} = g_P = 0$, which can be achieved in a BEP (1), (2) or (3). With endogenous innovation, it is assumed that $y = \big(1-\sigma-\lambda\big)^{1-\varkappa-\varepsilon} A^{1-\varkappa} k_y^{\varkappa} e_{y,b}^{\varepsilon}$. Using subscripts to refer to the exogenous and endogenous technology stocks, define $A_{endo} := A_{exo}^{\frac{1-\varepsilon-\varkappa}{1-\varkappa}}$, so $g_{A_{endo}} = \frac{1-\varepsilon-\varkappa}{1-\varkappa} g_{A_{exo}}$. The net growth effects are hence identical in an exogenous and an endogenous growth scenario. Therefore, $g_y = g_k = g_A$.

In a **PS regime**, $e = e_b$ and $g_{P_{e,b}} + g_{T_e} = 0$, relevant in BEP (1). In this case, $g_{p_{e_{y,b}}} + g_{T_e} = 0$, so $g_D = 0$. As with exogenous innovations, $g_y = g_{e_{y,b}}$ while a constant interest rate requires that $g_y = g_k$. Since $y = \big(1-\sigma-\lambda\big)^{1-\varkappa-\varepsilon} A^{1-\varkappa} k_y^{\varkappa} e_{y,b}^{\varepsilon}$, $g_y = g_{k_y} = g_{e_{y,b}}$ leads to $g_y = g_A$. For $g_P = 0$, it is necessary that $g_{\mathcal{F}} = g_y = g_{e_b} = g_A$. Since $\kappa\sigma\Phi = g_A$ and $(1-\kappa)\varsigma_F \sigma\Phi = g_{\mathcal{F}}$, it follows that $\kappa = \frac{\varsigma_F}{1+\varsigma_F}$, so $g_A = \frac{\varsigma_F}{1+\varsigma_F}\sigma\Phi$.

In an **EPS regime**, $g_y > g_{p_{e_{y,b}}} + g_{T_E}$. Therefore, $g_{e_{y,b}} > 0$, so $g_P = 0$ requires that $g_{e_y} = g_y - g_{p_{e_{y,b}}} - g_{T_e} = g_{\mathcal{F}}$ while $g_{T_e} = g_{T_{\bar{c}c}} + g_{\mathcal{I}}$. However, for the reasons explained with Lemma (3.5), it is not possible to adjust $g_{\mathcal{I}}$ to the rate necessary for stabilizing commuting costs in an EPS regime. Therefore, $g_{T_e} = g_{T_{\bar{c}c}}$ while $\chi = 1$. The regime is only relevant in an BEP (1) scenario where $g_{p_{e_{y,b}}} = 0$. Since $\varepsilon\frac{y}{(1+T_e)p_{e,b}} = e_b$, it follows that $y = \big(1-\sigma-\lambda\big)^{1-\varkappa-\varepsilon} A^{1-\varkappa} k_y^{\varkappa} \big(\frac{\varepsilon y}{(1+T_e)p_{e,b}}\big)^{\varepsilon}$. Importantly, $g_y = g_k = \frac{1-\varkappa}{1-\varkappa-\varepsilon} g_A - \frac{\varepsilon}{1-\varkappa-\varepsilon} g_{T_e}$ while $g_{\mathcal{F}} = g_{e,b} = g_y - g_{T_e} = \frac{1-\varkappa}{1-\varkappa-\varepsilon}(g_A - g_{T_e})$. It is hence necessary that $\sigma\varsigma_F(1-\kappa)\Phi = \frac{1-\varkappa}{1-\varkappa-\varepsilon}(\sigma\kappa\Phi - g_{T_e})$. Therefore, $\kappa = \frac{\varsigma_F + \frac{1-\varkappa}{1-\varkappa-\varepsilon}\frac{g_{T_e}}{\sigma\Phi}}{\left(\varsigma_F + \frac{1-\varkappa}{1-\varkappa-\varepsilon}\right)}$ while since $g_y = \frac{1-\varkappa}{1-\varkappa-\varepsilon} g_A - \frac{\varepsilon}{1-\varkappa-\varepsilon} g_{T_e}$, we find that

$$g_y = \frac{(1-\varkappa)\varsigma_F + (1-\varkappa-\varepsilon\varsigma_F)\frac{g_{T_e}}{\sigma\Phi}}{\Big((1-\varkappa-\varepsilon)\varsigma_F + 1 - \varkappa\Big)}\sigma\Phi.$$

Note that whenever $g_{Te} = 0$ we are in a PS regime while when $g_{T_e} = g_y$ we are in an ES regime.

In a **GE regime**, $e = e_g = \mathcal{E}$. Therefore, $g_{e_b} = 0$, so $g_P = 0$. Since $r = (\varkappa + \varepsilon)\frac{y}{k_y}$ and $w = (1-\varkappa-\varepsilon)\frac{y}{(1-\sigma-\lambda)}$, the expenditure share on capital is $\varkappa + \varepsilon$ while the expenditure share on labor is $1-\varkappa-\varepsilon$. As they add up to one, the Kaldor facts hold and the growth path is stable (see discussion of Proposition (3.1) for details).

Thereby,

$$g_y = \frac{\big((1-\varkappa) + (1-\varepsilon)\varsigma_E(\Lambda\Gamma)^{\psi}\big)}{(1-\varepsilon-\varkappa)(1+\Lambda\Gamma)^{\psi}}\sigma\Phi.$$

This being the case, the specific Ω values directly follow with the above elaborations while r follows directly with the results related to Lemma (3.4).

There are some peculiarities to note. First, in an ES and EPS regime, $i_{\mathcal{E}} = 0$ while in an ES regime, $g_{P_{e,b}} + g_{T_e} > 0$ and in an EPS regime $g_{T_e} > 0$ and $g_{\wp_e} = 0$. Therefore, it is necessary to use additional policies to avoid innovations in the green energy sector. One policy is to set $T_{\wp\mathcal{E}} < 0$, so that intermediates are subsidized instead of taxed, requiring that $g_{T_e} + g_{P_{e,b}} < g_{T_{\wp\mathcal{E}}}$ (the growth rate must be positive to have a growing tax while the subsidy now denotes a tax). The alternative is to set $T_{I,\mathcal{E}} < 0$, thus to tax green energy innovation. In this case, innovation profits in green energy will decrease what shifts labor to general technologies until $\eta = 1$. Again, while the subsidy turns into a tax, $g_{T_{I,\mathcal{E}}} > 0$.

Further, in an ES regime, Corollary (3.1) demands that $W_1 = W_2$. Therefore, $g_I + g_{\Pi_{\mathcal{I}}} = g_A + g_{\Pi_{\mathcal{A}}}$ which corresponds to $g_y = g_I$. So commuting technology can be used to address the density externality. In an EPS regime, it is similarly necessary that $W_1 = W_2$, so $g_{T_e} + g_{P_{e,b}} = g_I < g_y$ is not possible, hence $\chi = 1$.

Then, if $i_{\mathcal{E}} = 1$, $e_b = 0$, so $P = 0$. If $i_{\mathcal{E}} = 0$, then $P_y = \epsilon\frac{y}{(1-T_e)p_{e_{y,b}}\mathcal{F}}$ and $P_{\bar{c}\bar{c}} = e_{\bar{c}\bar{c}} = \varsigma_{\bar{c}\bar{c}}\frac{\bar{l}^{1+\theta}}{(1+\theta)\mathcal{F}_0}$. Thereby, $y = (1-\sigma-\lambda)^{\frac{1-\varkappa-\varepsilon}{1-\varepsilon}} A^{\frac{1-\varkappa}{1-\varepsilon}} (k_y)^{\frac{\varkappa}{1-\varepsilon}} \big(\frac{\varepsilon}{p_{e_{y,b}}}\big)^{\frac{\varepsilon}{1-\varepsilon}}$, so $k_y = \varkappa\frac{y}{(1+T_r)r}$ leads to $y = (1-\sigma-\lambda)^{\frac{1-\varkappa-\varepsilon}{1-\varepsilon}} A^{\frac{1-\varkappa}{1-\varepsilon}} \big(\frac{\varkappa}{(1+T_r)r}\big)^{\frac{\varkappa}{1-\varepsilon-\varkappa}} \big(\frac{\varepsilon}{p_{e_{y,b}}}\big)^{\frac{\varepsilon}{1-\varepsilon-\varkappa}}$. Therefore, $P_y = \frac{\epsilon(1-\sigma-\lambda)^{\frac{1-\varkappa-\varepsilon}{1-\varepsilon}}}{(1-T_e)p_{e_{y,b}}\mathcal{F}} A^{\frac{1-\varkappa}{1-\varepsilon}} \big(\frac{\varkappa}{(1+T_r)(g_{y_i}+\rho)}\big)^{\frac{\varkappa}{1-\varepsilon-\varkappa}} \big(\frac{\varepsilon}{p_{e_{y,b}}}\big)^{\frac{\varepsilon}{1-\varepsilon-\varkappa}}$, so aggregate pollution follows

$$P = \frac{\epsilon(1-\sigma-\lambda)^{\frac{1-\varkappa-\varepsilon}{1-\varepsilon}} A^{\frac{1-\varkappa}{1-\varepsilon}}}{(1-T_e)p_{e_{y,b}}\mathcal{F}} \big(\frac{\varkappa}{(1+T_r)(g_{y_i}+\rho)}\big)^{\frac{\varkappa}{1-\varepsilon-\varkappa}} \big(\frac{\varepsilon}{p_{e_{y,b}}}\big)^{\frac{\varepsilon}{1-\varepsilon-\varkappa}} + \varsigma_{\bar{c}\bar{c}}\frac{\bar{l}^{1+\theta}}{(1+\theta)\mathcal{F}_0}.$$

For $t \to \infty$, $k_y \to k$, further $\dot{k} = y - c$ leads to $g_y = \frac{y}{k} - \frac{c}{k}$, respectively $g_y k = y - c$ yields $\frac{c}{y} = 1 - g_y\frac{k}{y}$. In an ES, EPS, PS, regime, for $t \to \infty$, $\varkappa\frac{y}{k_y} \to \varkappa\frac{y}{k}$. Therefore, $(1+T_r)r = \varkappa\frac{y}{k}$, so $\frac{c}{y} = 1 - g_y\frac{\varkappa}{(1+T_r)r}$. In a GE regime, for $t \to \infty$, $(\varkappa+\varepsilon)\frac{y}{k_y} \to (\varepsilon+\varkappa)\frac{y}{k}$.

In all,

$$\mathcal{C}_i = \frac{c_i}{y_i} = \begin{cases} 1 - \frac{\varkappa}{(1+T_r)(1+\frac{\rho}{g_{y_i}})} & if \;\; i \in \{ES, EPS, PS\} \\ 1 - \frac{\varepsilon+\varkappa}{(1+T_r)(1+\frac{\rho}{g_{y_i}})} & if \;\; i = GE. \end{cases}$$

Yet, $g_y = \sigma\Omega\Phi$. When the discussion sets its focus on construction density, $\Phi_i = \underline{\varsigma}_{\Phi} D \frac{e^{-\bar{\varsigma}_{\Phi} D}}{\left(1+\varsigma_P\left(\frac{e_{y,b}}{\mathcal{F}}+\frac{\varsigma_{\bar{c}\bar{c}}\bar{l}^{1+\theta}}{(1+\theta)\mathcal{F}_0}\right)\right)}$ if $i \in \{ES, EPS, PS\}$ and $\Phi_i = \underline{\varsigma}_{\Phi} D e^{-\bar{\varsigma}_{\Phi} D}$ if $i = GE$. Finally, r_i and σ_i straightforwardly follow with Lemma (3.4). $\square$

Proposition (3.5) Since $U = \int ln(\bar{c}(t))dt$ is continuously growing, the socially optimal growth regime is characterized by the highest utility growth rate, which relates to the direction with the highest numéraire growth rate. Crucially, the planner first maximizes the factor allocations and then compares the growth rates.

Lemma (3.8), Lemma (3.9) and Proposition (3.6) Itt facilitates the discussion to derive Lemma (3.8), Lemma (3.9), and Proposition (3.6) together and only separate them in the text for better guidance through the solution. To reduce the complexity of the calculation steps, it is further helpful to solve some optimality features before settling the Hamiltonian.

Density and real estate: The socially optimal real estate production is characterized by $\int_0^{\bar{l}} \frac{\mathcal{B}b(l)^\gamma}{l^\theta} dl = \int_0^{\bar{l}} b(l)dl + \int_0^{\bar{l}} dl$ and $\int_0^{\bar{l}} \frac{\mathcal{B}b(l)^\gamma}{l^\theta} dl = a$. Spatial efficiency requires that at any location, $\max_{b(l)} \int_0^{\bar{l}} \frac{\mathcal{B}b(l)^\gamma}{l^\theta} dl - \psi_b(\bar{l}) \int_0^{\bar{l}} b(l)dl$ with $\psi_b(\bar{l})$ as a shadow value of horizontal construction. This requirement guarantees that a household is indifferent in its location choice. Spatial efficiency demands that at each spatial distance, l, the marginal contribution of morphological capital to real estate construction is identical. Denoting $\lambda(l) = b(l)$, with $\lambda(l)$ as the local use of morphological capital, local maximization leads to $\left(\frac{\gamma\mathcal{B}}{\psi_b(\bar{l})l^\theta}\right)^{\frac{1}{1-\gamma}} = \lambda(l)$, so that $\int_0^{\bar{l}} (\gamma \frac{\mathcal{B}}{\psi_b(\bar{l})l^\theta})^{\frac{1}{1-\gamma}} dl = \frac{1-\gamma}{1-\gamma-\theta}\left(\frac{\gamma\mathcal{B}}{\psi_b(\bar{l})}\right)^{\frac{1}{1-\gamma}} \bar{l}^{1-\frac{\theta}{1-\gamma}} = \lambda$, thus $\frac{(1-\gamma)^{1-\gamma}}{(1-\gamma-\theta)^{1-\gamma}} \left(\gamma\mathcal{B}\right) \frac{\bar{l}^{1-\gamma-\theta}}{\lambda^{1-\gamma}} = \psi_b(\bar{l})$. In aggregate, $a = \int_0^{\bar{l}} \mathcal{B}b(l)^\gamma dl = \int_0^{\bar{l}} \left(\frac{\gamma\mathcal{B}}{\psi_b(\bar{l})l^\theta}\right)^{\frac{\gamma}{1-\gamma}} dl = \left(\frac{1-\gamma}{1-\gamma-\gamma\theta}\right)\left(\frac{\gamma\mathcal{B}}{\psi_b(\bar{l})}\right)^{\frac{\gamma}{1-\gamma}} \bar{l}^{1-\frac{\theta\gamma}{1-\gamma}}$, so including $\psi_b(\bar{l})$ gives $a = \Lambda_D \lambda^\gamma \bar{l}^{1-\gamma}$ with Λ_D given in (17). The social planner provision weight is thus identical to the one observed in decentralized economies. Hence, $D = \Lambda_D\left(\frac{\lambda}{l}\right)^{\gamma}$, which presents the efficient labor and land employment in the real estate sector but is silent on how the planner weights land and labor in other sectors.

Commuting focused research: Commuting costs increase in proportion to the brown energy price. Distinct from the decentralized economy, the planner can keep commuting distances constant. Commuting expenditures consume a fraction of the production value. These expenditures reduce with improvements in commuting technologies, requiring $\chi < 1$. Hence, some research is used for non-productivity-enhancing activities. However, since growth effects dominate level effects, it is more efficient to sacrifice a fraction of production for commuting cost compensation than production growth rates. Therefore, the planner will select $\chi = 1$ in all regimes. In light of this, the Hamiltonian can be derived.

Hamiltonian: Along a SBGP, a and x have to remain constant, so for $t \to \infty$, $g_{\bar{c}} \to \epsilon g_c$, $k_y \to k$ and $e = e_y + e_{\bar{c}c} \to e_y$ if $g_{e_y} > 0$. Therefore, in a brown energy scenario, $\Phi = \varsigma_\Phi D \frac{e^{-\bar{\varsigma}_\Phi D}}{1+\varsigma_P P}$ and $y = A^{1-\varkappa} e_{y,b}^{\varepsilon} k^{\varkappa} (1-\sigma-\lambda)^{1-\varkappa-\varepsilon}$. In a green energy scenario, $\Phi = \varsigma_\Phi D e^{-\bar{\varsigma}_\Phi D}$ and $y = A^{1-\varkappa} E^{1-\varepsilon} k^{\varkappa+\varepsilon} (1-\sigma-\lambda)^{1-\varkappa-\varepsilon}$, with $\mathcal{E} = E\mathcal{I}_E = e_g = e_{y,b}$. Since $g_{e_{\bar{c}c}} = 0$, for $t \to \infty$, $e_y + e_{\bar{c}c} \to e_y = \mathcal{E}$.

To simplify the discussion, it is assumed that in an ES regime, the exogenous brown energy price grows at the output growth rate, thus $g_{p_{e_{y,b}}} = g_y$, relating to an $BEP(2)$ scenario. In an EPS regime, $g_y > g_{p_{e_{y,b}}} > 0$. In a PS and PS^+ regime, $g_{p_{e_{y,b}}} = 0$, which relates to a $BEP(1)$ scenario. Further, for a PS^+ regime, it is assumed that $1 - \kappa_{PS^+} = 1 - \kappa_{PS} + \iota^+$. For $t \to \infty$, any $\iota^+ > 0$ will lead to $P \to 0$. Hence, it simplifies the evaluation to approximate with $\iota^+ = 0$ as in the long run an incremental $\iota^+ > 0$ is sufficient to remove all harmful pollution.

Then, to simplify the discussion of a GE regime, for the sake of the argument assume that the social value of manufactured capital can be represented by $\hat{r}$, which describes the social costs of producing an intermediate. Hence, the social planner considers $\hat{r} = \varepsilon(1-\sigma-\lambda)^{1-\varkappa-\varepsilon} I_{\mathcal{E}}^{\varepsilon-1} A I_{\mathcal{A}}^{\varkappa}$ and $\hat{r} = \varkappa(1-\sigma-\lambda)^{1-\varkappa-\varepsilon} E I_{\mathcal{E}}^{\varepsilon} I_{\mathcal{A}}^{\varkappa-1}$. Therefore, $I_{\mathcal{E}} = \left(\frac{\varepsilon(1-\sigma-\lambda)^{1-\varkappa-\varepsilon} A I_{\mathcal{A}}^{\varkappa}}{\hat{r}}\right)^{\frac{1}{1-\varepsilon}}$ and $I_{\mathcal{A}} = \left(\frac{\varkappa(1-\sigma-\lambda)^{1-\varkappa-\varepsilon} E I_{\mathcal{E}}^{\varepsilon}}{\hat{r}}\right)^{\frac{1}{1-\varkappa}}$, so $I_{\mathcal{A}} =$

$\left(\frac{\varkappa^{(1-\varepsilon)}\varepsilon^{\varepsilon}A^{\varepsilon}E^{1-\varepsilon}}{\hat{r}}\right)^{\frac{1}{1-\varkappa-\varepsilon}}(1-\lambda-\sigma)$ and $I_{\mathcal{E}} = \left(\frac{\varkappa^{\varkappa}\varepsilon^{(1-\varkappa)}A^{1-\varkappa}E^{\varkappa}}{\hat{r}}\right)^{\frac{1}{1-\varkappa-\varepsilon}}(1-\lambda-\sigma)$, respectively $\varepsilon AI_{\mathcal{A}} = \varkappa EI_{\mathcal{E}}$. In view of this, $y = (1-\sigma-\lambda)^{1-\varkappa-\varepsilon}AI_{\mathcal{A}}^{\varkappa}EI_{\mathcal{E}}^{\varepsilon}$ can be represented by

$$y = (1-\sigma-\lambda)^{1-\varkappa-\varepsilon}A^{1+\varepsilon}I_{\mathcal{A}}^{\varepsilon+\varkappa}E^{1-\varepsilon}\Big(\frac{\varepsilon}{\varkappa}\Big)^{\varepsilon}.$$

Finally, the capital demand for intermediates is $AI_{\mathcal{A}} + EI_{\mathcal{E}} = k_y$, so $I_{\mathcal{A}} = \frac{\varkappa k_y}{(\varkappa+\varepsilon)A}$. Therefore,

$$y = (1-\sigma-\lambda)^{1-\varkappa-\varepsilon}A^{1-\varkappa}E^{1-\varepsilon}k_y^{\varepsilon+\varkappa}\Big(\frac{\varkappa^{\varkappa}\varepsilon^{\varepsilon}}{(\varkappa+\varepsilon)^{\varepsilon+\varkappa}}\Big).$$

Since $g_{e_y} = g_{\mathcal{E}}$ while $g_{e_{\bar{c}c}} = 0$, the social value of commuting energy can be ignored. Therefore, note that numéraire energy demand determines the long-run green energy growth rate. Consequently, the relative weight of commuting becomes negligible. Since there is further no pollution, the social planner Hamiltonian can ignore green energy utilization in commuting.

In view of the foregoing, the social planner Hamiltonian, H, follows

$$H_i : ln\Big((\mathcal{T}_i\hat{c})^{\epsilon}a^{\iota}x^{1-\iota-\epsilon}\Big) + \psi_x(\hat{l}-\bar{l}-x) + \psi_a\big(\Lambda_D\lambda^{\gamma}\bar{l}^{1-\gamma}-a\big) + \mathfrak{R}_i,$$

$$\mathfrak{R}_i := \begin{cases} \psi_{e_{\bar{c}c}}\Big(e_{\bar{c}c} - \varsigma_{\bar{c}c}\frac{\bar{l}^{1-\theta}}{1-\theta}\Big) + \psi_y\mathcal{T}\big(\hat{y}-\hat{c}-\hat{p}_{e_b}(e_{y,b}+e_{\bar{c}c})\big) + \psi_A A\sigma\Phi & if \;\; i = ES \\ \psi_{e_{\bar{c}c}}\Big(e_{\bar{c}c} - \varsigma_{\bar{c}c}\frac{\bar{l}^{1-\theta}}{1-\theta}\Big) + \psi_y\mathcal{T}(\hat{y}-\hat{c}-\hat{p}_{e,b}e_{\bar{c}c}) + \psi_A A\sigma\Phi & if \;\; i = ES^{+} \\ \psi_{e_{\bar{c}c}}\Big(e_{\bar{c}c} - \varsigma_{\bar{c}c}\frac{\bar{l}^{1-\theta}}{1-\theta}\Big) + \psi_y\mathcal{T}(\hat{y}-\hat{c}-p_{e,b}\hat{e}_{y,b}) & if \;\; i = PS, PS^{+} \\ +\psi_A\sigma\kappa\Phi A + \psi_F(1-\kappa)\sigma\Phi\varsigma_F\mathcal{F} & \\ \psi_{e_{\bar{c}c}}\Big(e_{\bar{c}c} - \varsigma_{\bar{c}c}\frac{\bar{l}^{1-\theta}}{1-\theta}\Big) + \psi_y\mathcal{T}\big(\hat{y}-\hat{c}-\big(\frac{p_{e_{y,b}}e_{y,b}-p_e e_{\bar{c}c}}{\mathcal{T}}\big)\big) & if \;\; i = EPS \\ +\psi_A A\sigma\kappa\Phi + \psi_F(1-\kappa)\sigma\Phi\varsigma_F\mathcal{F} & \\ \psi_y\mathcal{T}(\hat{y}-\hat{c}) + \psi_A A\eta^{\psi}\sigma\Phi & if \;\; i = GE \\ +\psi_{\mathcal{E}}\big(1-\eta\big)^{\psi}\varsigma_{\mathcal{E}}E\sigma\Phi. & \end{cases}$$

Thereby, $\mathcal{T}$ supports an efficiency unit representation, $\hat{y} = \frac{y}{\mathcal{T}} \Rightarrow g_{\mathcal{T}} = g_y$. So, $\mathcal{T} = A$ if $i \in \{ES, ES^{+}, EPS, PS, PS^{+}\}$ and $\mathcal{T} = A^{1-\varkappa}E^{1-\varepsilon}$ if $i = GE$. Note further that $\big(\frac{p_{e_{y,b}}e_{y,b}-p_e e_{\bar{c}c}}{\mathcal{T}}\big)$ since in an EPS regime, $g_{p_{e_{y,b}}} < g_y$. Beyond, $P_y = \frac{e_{y,b}}{\mathcal{F}}$ and $P_{\bar{c}c} = \frac{e_{\bar{c}c}}{\mathcal{F}_0}$

lead to $P_{ES} = \frac{e_{y,b_0}}{\mathcal{F}_0} + \frac{\varsigma_{\bar{c}c}\bar{l}^{1+\theta}}{(1+\theta)\mathcal{F}_0}$, $P_{ES^+} = \frac{\varsigma_{\bar{c}c}\bar{l}^{1+\theta}}{(1+\theta)\mathcal{F}_0}$, $P_{PS} = P_{EPS} = \frac{e_{y,b_{PS}}}{\mathcal{F}_{PS}} + \frac{\varsigma_{\bar{c}c}\bar{l}^{1+\theta}}{(1+\theta)\mathcal{F}_0}$, $P_{PS^+} = \frac{\varsigma_{\bar{c}c}\bar{l}^{1+\theta}}{(1+\theta)\mathcal{F}_0}$, $P_{GE} = 0$. Thereby, $\mathcal{F}_0$ denotes the initial abatement technology. In an PS and $PS+$ regime, this technology improves for numéraire production but remains constant for commuting. Finally, for the capital formation equation note that, e.g., in an ES regime, the planner considers $\dot{k} = \mathcal{T}(\hat{y} - \hat{c} - \hat{p}_{e_b}(e_{y,b} + e_{\bar{c}c}))$.

Denoting the specific derivative in the subscripts, $H_c = \partial H/\partial c$ gives

$$\frac{\epsilon}{c} = \psi_y, \tag{B21}$$

thus $g_c = -g_{\psi_y}$. $H_k = \partial H/\partial k$ yields $-\dot{\psi}_y + \rho\psi_y = \psi_y \varkappa \frac{y}{k}$ if $i \in \{ES, ES^+, EPS, PS, PS^+\}$ and $-\dot{\psi}_y + \rho\psi_y = \psi_y(\varepsilon + \varkappa)\frac{y}{k}$ if $i = GE$. Along an SBGP,

$$g_c + \rho = g_y + \rho = \begin{cases} \varkappa \frac{y_i}{k_i} & if \;\; i \in \{ES, ES^+, EPS, PS, PS^+\} \\ (\varepsilon + \varkappa)\frac{y_i}{k_i} & if \;\; i = GE. \end{cases} \tag{B22}$$

H_h gives

$$\psi_a = \frac{\iota}{a}, \tag{B23}$$

H_x gives

$$\psi_x = \frac{(1 - \iota - \epsilon)}{x}. \tag{B24}$$

All further steps are then regime specific.

ES Regime: $H_A = -\dot{\psi}_A + \rho\psi_A = \psi_y(1-\varkappa)\frac{y}{A} + \psi_A\sigma\Phi \Rightarrow$

$$-g_{\psi_A} + \rho = (1-\varkappa)\frac{\psi_y}{\psi_A}\frac{y}{A} + \Phi\sigma. \tag{B25}$$

Hence, along an SBGP, $g_{\psi_y} + g_y = g_{\psi_A} + g_A$. Further, since $\frac{\epsilon}{c} = \psi_y$, an SBGP is characterized by $-g_{\psi_y} = g_y \Rightarrow -g_{\psi_A} = g_A$. With $\sigma\Phi = g_A$, it follows that

$$\psi_A = (1-\varkappa)\frac{\psi_y}{\rho}\frac{y}{A}. \tag{B26}$$

$H_{e_{y,b}}$ gives (since e.g. $\mathcal{T}\hat{p}_{e,b} = p_{e,b}$ and $P = \frac{e_{y,b,0}+e_{\bar{c}c}}{\mathcal{F}_0}$)

$$\psi_y(\varepsilon\frac{y}{e_{y,b,0}} - p_{e,b,0}) = \psi_A\frac{\varsigma_P\sigma\Phi A}{(1+\varsigma_P P)\mathcal{F}_0} \tag{B27}$$

while $H_{e_{\bar{c}c}}$ yields

$$\psi_{e_{\bar{c}c}} = \psi_y p_{e,b,0} + \psi_A\frac{\varsigma_P\sigma\Phi A}{(1+\varsigma_P P)\mathcal{F}_0}. \tag{B28}$$

In combination,

$$\psi_y\varepsilon\frac{y}{e_{y,b,0}} = \psi_{e_{\bar{c}c}}. \tag{B29}$$

Using (B27) and (B26) gives

$$e_{y,b} = \frac{\varepsilon}{\frac{\varsigma_P(1-\varkappa)}{\rho}\frac{\sigma\Phi}{(1+\varsigma_P P)\mathcal{F}_0} + \frac{p_{e,b}}{y}}. \tag{B30}$$

However, since $y = y(e_b)$ and $P = P(e_b)$, this cannot be solved analytically for e_b. Next, $g_k = \frac{y}{k} - \frac{c}{k} - \frac{p_{e_b}(e_{y,b}+e_{\bar{c}c})}{k}$, whilst (B22) states that along a BGP, $g_k + \rho = \varkappa\frac{y}{k}$. Therefore, $(1-\varkappa)y + \rho k = c + p_{e_b}(e_{y,b} + e_{\bar{c}c})$, so

$$\frac{c}{y} = \mathcal{C}_{ES} = 1 - \frac{\varkappa}{1+\frac{\rho}{g_y}} - \frac{p_{e_b}(e_{y,b} + e_{\bar{c}c})}{y}.$$

$H_{\bar{l}}$ results in

$$-\psi_x = -\psi_a(1-\gamma)\frac{a}{\bar{l}} + \psi_{e_{\bar{c}c}}(1-\theta)\frac{e_{\bar{c}c}}{\bar{l}} - \psi_A\sigma A\Phi_l. \tag{B31}$$

Further, $D_l = -\gamma\frac{D}{\bar{l}}$, $\Phi_D = \Phi(\frac{1}{D} - \bar{\varsigma}_\Phi)$, $\Rightarrow \Phi_l = \Phi_D D_l = -\Phi(\frac{1}{D} - \bar{\varsigma}_\Phi)\gamma\frac{D}{\bar{l}}$. Note that in the Hamiltonian specification used here, both density representatives are functions of $\bar{l}$ while pollution is a function of $e_{\bar{c}c}$. Hence, land related pollution effects on Φ are calculated via the $e_{\bar{c}c}$ related derivative. In any case, with (B23), (B24), (B26) and (B29),

$$\bar{l} = \frac{\hat{l}}{1+\frac{1}{\mathcal{W}_l}} \tag{B32}$$

with $\mathcal{W}_l := \frac{\iota(1-\gamma)}{(1-\iota-\epsilon)} - \frac{\epsilon}{\mathcal{C}_{ES}(1-\iota-\epsilon)}\big(\frac{\varepsilon(1-\theta)e_{\bar{c}c}}{e_{y,b}} + (1-\varkappa)\sigma\Phi(1-\overline{\varsigma}_\Phi D)\frac{\gamma}{\rho}\big)$. H_λ gives

$$\psi_a\frac{\gamma a}{\lambda} - \psi_y\frac{(1-\varkappa-\varepsilon)y}{(1-\lambda-\sigma)} = -\psi_A\sigma A\Phi_\lambda. \tag{B33}$$

With $D_\lambda = \gamma\frac{D}{\lambda}$, $\Phi_D = \Phi(\frac{1}{D} - \overline{\varsigma}_\Phi)$, $\Phi_\lambda = \Phi(\frac{1}{D} - \overline{\varsigma}_\Phi)\gamma\frac{D}{\lambda}$, as well as (B21), (B23) and (B26), so reformulations yield

$$\frac{\lambda}{(1-\lambda-\sigma)} = \frac{\iota\gamma\mathcal{C}_{ES}}{(1-\varkappa-\varepsilon)\epsilon} + \frac{(1-\varkappa)\sigma\Phi(1-\overline{\varsigma}_\Phi D)\gamma}{(1-\varkappa-\varepsilon)\rho} \tag{B34}$$

so that

$$\lambda = \frac{(1-\sigma)}{1+\frac{1}{W_\lambda}} \tag{B35}$$

with $W_\lambda := \frac{\iota\gamma\mathcal{C}_{ES}}{(1-\varkappa-\varepsilon)\epsilon} + \frac{(1-\varkappa)\sigma\Phi(1-\overline{\varsigma}_\Phi D)\gamma}{(1-\varkappa-\varepsilon)\rho}$. Finally, H_σ gives $\psi_A\Phi A = \psi_y(1-\varkappa-\varepsilon)\frac{y}{1-\lambda-\sigma}$. Hence with (B26),

$$\sigma = 1 - \lambda - \frac{(1-\varkappa-\varepsilon)\rho}{(1-\varkappa)\Phi}.$$

ES$^+$ Regime: Following the procedure used to derive the ES path and noting that for a $ES+$ regime, $e_{y,b} = \mathcal{E}_0$, the steps introduced for an ES regime also apply here. Therefore,

$$\frac{c}{y} = \mathcal{C}_{ES^+} = 1 - \frac{\varkappa}{1+\frac{\rho}{g_y}} - \frac{p_{e_{y,b}}e_{\bar{c}c}}{y}.$$

PS Regime: $H_A = -\dot{\psi}_A + \rho\psi_A = \psi_y(1-\varkappa)\frac{y}{A} + \psi_A\kappa\sigma\Phi$ and $H_\mathcal{F} = -\dot{\psi}_\mathcal{F} + \rho\psi_\mathcal{F} = \psi_\mathcal{F}(1-\kappa)\varsigma_\mathcal{F}\sigma\Phi + \psi_\mathcal{F}(1-\kappa)\sigma\varsigma_\mathcal{F}\frac{\varsigma_P\Phi}{(1+P)}\frac{e_{y,b}}{\mathcal{F}} + \psi_A\kappa\sigma A\frac{\varsigma_P\Phi}{(1+P)}\frac{e_{y,b}}{\mathcal{F}^2}$ (whereby the last two derivatives account for that abatement efforts reduce the pollution intensity of numéraire production what improves Φ and hence the innovation rates of $\mathcal{F}$ and A). Therefore,

$$-g_{\psi_A} + \rho = \frac{\psi_y}{\psi_A}(1-\varkappa)\frac{y}{A} + g_A, \tag{B36}$$

$$-g_{\psi_F} + \rho = (1-\kappa)\sigma\varsigma_\mathcal{F}\Phi\Big(1 + \frac{\varsigma_P}{(1+\varsigma_P P)}\frac{e_{y,b}}{\mathcal{F}}\Big) + \frac{\psi_A}{\psi_\mathcal{F}}\kappa\sigma A\frac{\varsigma_P\Phi}{(1+\varsigma_P P)}\frac{e_{y,b}}{\mathcal{F}^2}. \tag{B37}$$

Expression (B36) gives $g_{\psi_y} + g_y = g_{\psi_A} + g_A$. Since $\frac{\epsilon}{c} = \psi_y$, it follows that $-g_{\psi_y} = g_y$, so $-g_{\psi_A} = g_A$. Therefore,

$$\psi_A = (1-\varkappa)\frac{\psi_y}{\rho}\frac{y}{A}. \tag{B38}$$

Then, (B37) gives $g_{\psi_A} + g_A = g_{\psi_\mathcal{F}} + g_\mathcal{F}$, so $-g_{\psi_\mathcal{F}} = g_\mathcal{F}$. Thus with $\kappa = \frac{\varsigma_F}{1+\varsigma_F}$, (B37) can be rewritten as

$$\psi_\mathcal{F} = \frac{\psi_A \frac{A}{\mathcal{F}} g_A}{\rho(1+\varsigma_P P)\frac{\mathcal{F}}{e_{y,b}\varsigma_P} - g_\mathcal{F}} = \frac{\psi_A \frac{A}{\mathcal{F}}}{\frac{(1+\varsigma_F)\rho(1+\varsigma_P P)\mathcal{F}}{\varsigma_\mathcal{F}\sigma\Phi e_{y,b}\varsigma_P} - 1}. \tag{B39}$$

$H_{e_{y,b}}$ gives

$$\psi_y(\varepsilon\frac{y}{e_{y,b}} - p_{e,b}) = \psi_A\varsigma_P\frac{g_A A}{(1+\varsigma_P P)\mathcal{F}} + \psi_F\varsigma_P\frac{g_F}{(1+\varsigma_P P)} = \psi_A\Big(\frac{\varsigma_P g_A A}{(1+\varsigma_P P)\mathcal{F} - \frac{e_{y,b}\varsigma_P}{\rho}g_\mathcal{F}}\Big) \tag{B40}$$

where the last equality follows with (B40). Then, $H_{e_{\bar{c}c}}$ yields

$$\psi_{e_{\bar{c}c}} = \psi_y p_{e,b} + = \psi_A\varsigma_P\frac{g_A A}{(1+\varsigma_P P)\mathcal{F}_0} + \psi_F\varsigma_P\frac{g_F\mathcal{F}}{(1+\varsigma_P P)\mathcal{F}_0}$$

so that combining both gives $\psi_y\varepsilon\frac{y\mathcal{F}}{e_{y,b}\mathcal{F}_0} = \psi_{e_{\bar{c}c}}$. Since $\frac{\mathcal{F}}{e_{y,b}} = \frac{\mathcal{F}_0}{e_{y,b,0}}$, $\psi_y\varepsilon\frac{y}{e_{y,b,0}} = \psi_{e_{\bar{c}c}}$. Further, combining (B40) with (B38) gives

$$e_{y,b} = \frac{\varepsilon y}{p_{e,b} + \frac{(1-\varkappa)y}{\frac{(1+\varsigma_F)\rho(1+\varsigma_P P)\mathcal{F}}{\varsigma_F\sigma\Phi\varsigma_P} - e_{y,b}}}.$$

Again, since $y = y(e_{y,b})$, $P = P(e_{y,b})$, it follows that $\Phi = \Phi(e_{y,b})$, hence there is no analytical solution. Then, $g_k = \frac{y}{k} - \frac{c}{k} - \frac{p_{e_{y,b}}(e_y + e_{\bar{c}c})}{k}$, whilst (B22) states that along a BGP, $g_k + \rho = \varkappa\frac{y}{k}$, so $(1-\varkappa)y + \rho k = c + p_{e_{y,b}}(e_y + e_{\bar{c}c})$. Therefore,

$$\frac{c}{y} = \mathcal{C}_{PS} = 1 - \frac{\varkappa}{1+\frac{\rho}{g_y}} - \frac{p_{e_{y,b}}(e_y + e_{\bar{c}c})}{y}.$$

$H_{\bar{l}}$ with $\big(\psi_A \kappa A + (1-\kappa)\varsigma_{\mathcal{F}}\psi_F \mathcal{F}\big) = \psi_A A\big(\frac{1}{\frac{1+\varsigma_F}{\varsigma_F} - \frac{e_{y,b}\sigma\Phi\varsigma_P}{\rho(1+\varsigma_P P)\mathcal{F}}}\big)$, it follows that

$$\psi_x = \psi_a(1-\gamma)\frac{h}{\bar{l}} - \psi_{e_{\bar{c}c}}(1-\theta)\varsigma_{\bar{c}c}\frac{e_{\bar{c}c}}{\bar{l}} + \sigma\psi_A A\Big(\frac{1}{\frac{1+\varsigma_F}{\varsigma_F} - \frac{e_{y,b}\sigma\Phi\varsigma_P}{\rho(1+\varsigma_P P)F}}\Big)\Phi_l. \tag{B41}$$

With $\Phi = \underline{\varsigma}_\Phi D\frac{e^{-\bar{\varsigma}_\Phi D}}{1+\varsigma_P P}$, $D_l = -\gamma\frac{D}{\bar{l}}$, $\Phi_D = \Phi(\frac{1}{D} - \bar{\varsigma}_\Phi)$, $\Phi_l = \Phi_D D_l = -\Phi(\frac{1}{D} - \bar{\varsigma}_\Phi)\gamma\frac{D}{\bar{l}}$. Hence, (B23), (B24), (B26) and (B29) give

$$\bar{l} = \frac{\hat{l}}{1+\frac{1}{\mathcal{W}_l}} \tag{B42}$$

with $\mathcal{W}_l := \frac{\iota(1-\gamma)}{(1-\iota-\epsilon)} - \frac{\epsilon}{\mathcal{C}_{PS}(1-\iota-\epsilon)}\Big(\frac{(1-\theta)\varepsilon e_{\bar{c}c}}{e_{y,b,0}} + \frac{(1-\varkappa)\sigma\Phi(1-\bar{\varsigma}_\Phi D)\gamma}{\rho}\Big(\frac{1}{\frac{1+\varsigma_F}{\varsigma_F} - \frac{e_{y,b,0}\sigma\Phi\varsigma_P}{\rho(1+\varsigma_P P)\mathcal{F}_0}}\Big)\Big)$ where it has been used that $\frac{\mathcal{F}}{e_{y,b}} = \frac{\mathcal{F}_0}{e_{y,b,0}}$. H_λ yields

$$\psi_a\frac{\gamma a}{\lambda} = \psi_y\frac{(1-\varkappa-\varepsilon)y}{(1-\lambda-\sigma)} - \psi_A\sigma A\Big(\frac{1}{\frac{1+\varsigma_F}{\varsigma_F} - \frac{e_{y,b}\sigma\Phi\varsigma_P}{\rho(1+\varsigma_P P)\mathcal{F}}}\Big)\Phi_\lambda. \tag{B43}$$

With $D_\lambda = \gamma\frac{D}{\lambda}$, $\Phi_D = \Phi(\frac{1}{D} - \bar{\varsigma}_\Phi)$, $\Phi_\lambda = \Phi(\frac{1}{D} - \bar{\varsigma}_\Phi)\gamma\frac{D}{\lambda}$, (B21), (B23) and (B26), this can be reformulated to

$$\lambda = \frac{(1-\sigma)}{1+\frac{1}{W_\lambda}}, \tag{B44}$$

$W_\lambda := \frac{\iota\gamma\mathcal{C}_{SP}}{(1-\varkappa-\varepsilon)\epsilon} + \frac{(1-\varkappa)\sigma\Phi(1-\bar{\varsigma}_\Phi D)\gamma}{(1-\varkappa-\varepsilon)}\Big(\frac{1}{\frac{1+\varsigma_F}{\varsigma_F} - \frac{e_{y,b}\sigma\Phi\varsigma_P}{\rho(1+\varsigma_P P)\mathcal{F}}}\Big)$. Finally, H_σ leads $\big(\psi_A\chi^\psi A\kappa + \psi_F\chi^\psi(1-\kappa)\varsigma_F\mathcal{F}\big)\Phi = \psi_y(1-\varkappa-\varepsilon)\frac{y}{1-\sigma-\lambda}$. Hence, with (B39) , $\psi_A A\Big(\frac{1}{\frac{1+\varsigma_F}{\varsigma_F} - \frac{e_{y,b}\Phi\varsigma_P}{\rho(1+\varsigma_P P)\mathcal{F}}}\Big)\Phi = (1-\varkappa-\varepsilon)\frac{y}{1-\sigma-\lambda}$, so with (B38),

$$\sigma = 1 - \lambda - \frac{(1-\varkappa-\varepsilon)}{1-\varkappa}\Big(\frac{\rho(1+\varsigma_F)}{\Phi\varsigma_F} - \frac{e_{y,b}\varsigma_P}{(1+\varsigma_P P)\mathcal{F}}\Big).$$

PS$^+$ Regime: The derivation is closely comparable with the one of a PS regime, however, κ is distinct. Again, $H_A = -\dot{\psi}_A + \rho\psi_A = \psi_y(1-\varkappa)\frac{y}{A} + \psi_A\kappa\sigma\Phi$ and $H_{\mathcal{F}} = -\dot{\psi}_{\mathcal{F}} + \rho\psi_{\mathcal{F}} = \psi_{\mathcal{F}}(1-\kappa)\varsigma_{\mathcal{F}}\sigma\Phi + \psi_{\mathcal{F}}(1-\kappa)\sigma\varsigma_{\mathcal{F}}\frac{\varsigma_P\Phi}{(1+P)}\frac{e_{y,b}}{\mathcal{F}} + \psi_A\kappa\sigma A\frac{\varsigma_P\Phi}{(1+P)}\frac{e_{y,b}}{\mathcal{F}^2}$

leads to $\psi_A = (1-\varkappa)\frac{\psi_y}{\rho}\frac{y}{A}$ and $\psi_{\mathcal{F}} = \frac{\psi_A \frac{A}{\mathcal{F}} g_A}{\rho(1+\varsigma_P P)\frac{\mathcal{F}}{e_{y,b}\varsigma_P} - g_F}$. $H_{e_{y,b}}$ gives $\psi_y(\varepsilon\frac{y}{e_{y,b}} - p_{e,b}) = \psi_A\big(\frac{\varsigma_P g_A A}{(1+\varsigma_P P)\mathcal{F} - \frac{e_{y,b}\varsigma_P}{\rho} g_F}\big)$. $H_{e_{\bar{c}c}}$ yields $\psi_{e_{\bar{c}c}} = \psi_y p_{e,b} + = \psi_A \varsigma_P \frac{g_A A}{(1+\varsigma_P P)\mathcal{F}_0} + \psi_F \varsigma_P \frac{g_{\mathcal{F}}\mathcal{F}}{(1+\varsigma_P P)\mathcal{F}_0}$. Combining both gives $\psi_y \varepsilon \frac{y\mathcal{F}}{e_{y,b}\mathcal{F}_0} = \psi_{e_{\bar{c}c}}$. Again, along a SBGP it is necessary that $\frac{y\mathcal{F}}{e_{y,b}\mathcal{F}_0}$ remains constant. Since κ in $(0,1)$, any adjustment in κ will meet this requirement. Hence, it is possible to simplify with $\psi_y \varepsilon \frac{y}{e_{y,b,0}} = \psi_{e_{\bar{c}c}}$. Further,

$$e_{y,b} = \frac{\varepsilon y}{p_{e,b} + \frac{(1-\varkappa)\kappa y}{\frac{\rho(1+\varsigma_P P)\mathcal{F}}{\sigma\Phi\varsigma_P} - e_{y,b}(1-\kappa)\varsigma_F}}$$

and

$$\frac{c}{y} = \mathcal{C}_{PS^+} = 1 - \frac{\varkappa}{1+\frac{\rho}{g_y}} - \frac{p_{e_{y,b}} e_{\bar{c}c}}{y}.$$

Then,

$$\bar{l} = \frac{\hat{l}}{1+\frac{1}{\mathcal{W}_l}}$$

with $\mathcal{W}_l := \frac{\iota(1-\gamma)}{(1-\iota-\epsilon)} - \frac{\epsilon}{\mathcal{C}_{PS^+}(1-\iota-\epsilon)}\Big(\frac{(1-\theta)\varepsilon e_{\bar{c}c}}{e_{y,b,0}} + \frac{(1-\varkappa)\sigma\Phi(1-\bar{\varsigma}_\Phi D)\gamma}{\rho}\Big(\frac{\kappa}{1-\frac{(1-\kappa)\varsigma_F e_{y,b}\sigma\Phi\varsigma_P}{\rho(1+\varsigma_P P)F}}\Big)\Big)$ and

$$\lambda = \frac{(1-\sigma)}{1+\frac{1}{W_\lambda}} \tag{B45}$$

with $W_\lambda := \frac{\iota\gamma\mathcal{C}_{PS^+}}{(1-\varkappa-\varepsilon)\epsilon} + \frac{(1-\varkappa)\sigma\Phi(1-\bar{\varsigma}_\Phi D)\gamma}{(1-\varkappa-\varepsilon)\rho}\Big(\frac{\kappa}{1-\frac{(1-\kappa)\varsigma_F e_{y,b}\sigma\Phi\varsigma_P}{\rho(1+\varsigma_P P)\mathcal{F}}}\Big)$. Finally, H_σ, gives $\big(\psi_A\kappa A + \psi_F(1-\kappa)\varsigma_F\mathcal{F}\big)\Phi = \psi_y(1-\varkappa-\varepsilon)\frac{y}{1-\sigma-\lambda}$, thus

$$\sigma = 1 - \lambda - \Bigg(\frac{\frac{\rho}{\Phi} - \frac{(1-\kappa)\varsigma_F e_{y,b}\varsigma_P}{\rho(1+\varsigma_P P)\mathcal{F}}}{\frac{(1-\varkappa)}{(1-\varkappa-\varepsilon)}\kappa}\Bigg).$$

EPS Regime: The derivation of an EPS regime is almost identical to a PS^+ regime (while pollution has to account for production induced sources as well). Thereby,

$$\kappa = \frac{\varsigma_F(1-\varepsilon-\varkappa) + \frac{g_{p_{e,b}}}{\sigma\Phi}}{\varsigma_F(1-\varepsilon-\varkappa) + 1 - \varkappa}$$

and

$$\frac{c}{y} = \mathcal{C}_{EPS} = 1 - \frac{\varkappa}{1+\frac{\rho}{g_y}} - -\frac{p_{e_b}(e_{y,b} + e_{\bar{c}c})}{y}.$$

GE Regime: $H_E = -\dot{\psi}_{\mathcal{S}} + \rho\psi_{\mathcal{E}} = \psi_y(1-\varepsilon)\frac{y}{E} + \psi_{\mathcal{E}}\varsigma_{\mathcal{E}}\sigma\Phi$ thus

$$-g_{\psi_{\mathcal{E}}} + \rho = (1-\varepsilon)\frac{\psi_y}{\psi_{\mathcal{E}}}\frac{y}{E} + \varsigma_{\mathcal{E}}(1-\eta)^{\psi}\sigma\Phi, \tag{B46}$$

so that differentiating equation (B46) gives $g_{\psi_y} + g_y = g_E + g_{\psi_{\mathcal{E}}}$. Further, since $\frac{\epsilon}{c} = \psi_y$, any SBGP is characterized by $-g_{\psi_y} = g_y$, so $g_E = -g_{\psi_{\mathcal{E}}}$. Since $\varsigma_{\mathcal{E}}(1-\eta)^{\psi}\sigma\Phi = g_E$,

$$\psi_{\mathcal{E}} = (1-\varepsilon)\frac{\psi_y}{\rho}\frac{y}{E}. \tag{B47}$$

Similarly, $H_A = -\dot{\psi}_{\mathcal{A}} + \rho\psi_{\mathcal{A}} = \psi_y(1-\varkappa)\frac{y}{A} + \psi_{\mathcal{A}}\eta^{\psi}\sigma\Phi$ leads to

$$-g_{\psi_{\mathcal{E}}} + \rho = (1-\varkappa)\frac{\psi_y}{\psi_{\mathcal{E}}}\frac{y}{A} + \eta^{\psi}\sigma\Phi, \tag{B48}$$

so $g_{\psi_y} + g_y = g_A + g_{\psi_A}$. Since $\frac{\epsilon}{c} = \psi_y$, $-g_{\psi_y} = g_y$, it follows that $g_A = -g_{\psi_A}$, so

$$\psi_{\mathcal{A}} = (1-\varkappa)\frac{\psi_y}{\rho}\frac{y}{A}. \tag{B49}$$

Then, $g_k = \frac{y}{k} - \frac{c}{k}$, whilst with (B22), $g_k + \rho = (\varkappa + \varepsilon)\frac{y}{k}$ leads to $(1-\varkappa-\varepsilon)y + \rho k = c$, so

$$\frac{c}{y} = \mathcal{C}_{GE} = 1 - \varkappa - \varepsilon + \rho\frac{k}{y} = 1 - \varkappa - \varepsilon + \frac{\varkappa + \varepsilon}{1 + \frac{g_y}{\rho}}.$$

H_η leads to

$$\eta = \frac{1}{1 + \left(\frac{(1-\varepsilon)\varsigma_{\mathcal{E}}}{1-\varkappa}\right)^{\frac{1}{1-\psi}}}. \tag{B50}$$

$H_{\bar{l}}$ leads to

$$\psi_x = \psi_a\frac{h}{\bar{l}} + \big(\psi_A\sigma\eta^{\psi}A + \psi_{\mathcal{E}}\sigma(1-\eta)^{\psi}\varsigma_{\mathcal{E}}E\big)\Phi_l.$$

With $D_l = -\gamma\frac{D}{l}$, $\Phi_D = \Phi(\frac{1}{D} - \overline{\varsigma}_\Phi)$, $\Phi_l = \Phi_D D_l = -\Phi(\frac{1}{D} - \overline{\varsigma}_\Phi)\gamma\frac{D}{l}$, $\big(\psi_A\sigma\eta^\psi A + \psi_\mathcal{E}\sigma(1-\eta)^\psi\varsigma_\mathcal{E}E\big) = \epsilon\frac{y}{c\rho}\sigma(1-\varkappa)\big(1 + \big(\frac{(1-\varepsilon)\varsigma_\mathcal{E}}{1-\varkappa}\big)^{\frac{1}{1-\psi}}\big)^{1-\psi}$, (B21), and (B23),

$$\overline{l} = \frac{\hat{l}}{1 + \frac{1}{W_{\overline{l}}}} \tag{B51}$$

with $W_{\overline{l}} := \frac{\iota(1-\gamma)}{1-\iota-\epsilon} - \frac{\epsilon\sigma\Phi(1-\varkappa)}{\mathcal{C}_{GE}\rho}\big(1 + \big(\frac{(1-\varepsilon)\varsigma_\mathcal{E}}{1-\varkappa}\big)^{\frac{1}{1-\psi}}\big)^{1-\psi}(1 - \overline{\varsigma}_\Phi D)\gamma$. H_λ yields

$$\psi_a\frac{\gamma a}{\lambda} = \psi_y\frac{(1-\varkappa-\varepsilon)y}{(1-\lambda-\sigma)} - \big(\psi_A\sigma\eta^\psi A + \psi_\mathcal{E}\sigma(1-\eta)^\psi\varsigma_\mathcal{E}E\big)\Phi_\lambda. \tag{B52}$$

With $D_\lambda = \gamma\frac{D}{\lambda}$, $\Phi_D = \Phi(\frac{1}{D} - \overline{\varsigma}_\Phi)$, $\Phi_\lambda = \Phi_D D_\lambda = \Phi(\frac{1}{D} - \overline{\varsigma}_\Phi)\gamma\frac{D}{\lambda}$ this leads to

$$\lambda = \frac{(1-\sigma)}{1 + \frac{1}{W_\lambda}} \tag{B53}$$

with $W_\lambda := \frac{\gamma\iota\mathcal{C}_{GE}}{(1-\varkappa-\varepsilon)\epsilon} + \frac{(1-\varkappa)\sigma\Phi}{(1-\varkappa-\varepsilon)\rho}\big(1 + \big(\frac{(1-\varepsilon)\varsigma_\mathcal{E}}{1-\varkappa}\big)^{\frac{1}{1-\psi}}\big)^{1-\psi}(1 - \overline{\varsigma}_\Phi D)\gamma$. Finally, H_σ leads to $\big(\psi_A\eta^\psi A + \psi_\mathcal{E}(1-\eta)^\psi\varsigma_\mathcal{E}E\big)\Phi = \psi_y(1-\varkappa-\varepsilon)\frac{y}{1-\lambda-\sigma}$. Hence, with (B47), (B49), it follows that $\big(\eta^\psi + \frac{(1-\varepsilon)\varsigma_\mathcal{E}}{(1-\varkappa)}(1-\eta)^\psi\big) = \frac{\rho}{(1-\lambda-\sigma)\Phi}$. With the planer solution for η,

$$\sigma = 1 - \lambda - \frac{\rho}{\Phi\Big(1 + \big(\frac{(1-\varepsilon)\varsigma_\mathcal{E}}{1-\varkappa}\big)^{\frac{1}{1-\psi}}\Big)^{1-\psi}}. \quad \square \tag{B54}$$

Calibration strategy

While literature can identify some parameters, the remaining parameters need to be calculated, as discussed subsequently.

Production: The numéraire production parameters are calibrated in reference to Lindenberger and Kummel (2002) with $\varepsilon = 0.05$ and $\varkappa = 0.25$. Then, Combes and Gobillon (2015) consider $\gamma = 0.8$ and $0 \leq \theta \leq 0.25$ which is based on time related opportunity costs. Compared to energy related commuting costs addressed in this theory, their interpretation will likely reduce the commuting elasticity since some commuters can work while commuting. Delloye et al. (2018) discuss a comparable framework which can be translated into $\frac{\theta}{(1-\gamma)} = 1/3$. Calculating this relation for Combes and Gobillon (2015) parameters leads to $\frac{\theta}{(1-\gamma)} = 0.625$. To have the parameters in an intermediate range, thus $0.5 = \frac{\theta}{(1-\gamma)}$, the theory selects $\gamma = 0.7$ and $\theta = 0.15$. Further, with Epple, Gordon, and Sieg (2010) and Combes and Gobillon (2015), $\mathcal{B} = 1$.

Consumption preferences, energy price and aggregate land: As a rule of thumb, households spend 1/3 of the disposable income on rents, see e.g. Davis and Ortalo-Magné (2011). For not having to detail the interpretation of what hinterland contains, the benchmark identification simplifies the case with $\iota = \epsilon = 0.33$, so the expenditure shares on the three consumption goods are identical. Further, Nordhaus (2007) gives $\rho = 0.015$, while $\hat{l} = 1$ and $p_{e,b,0} = 1$ follow as a standardization.

Quality index: There is no reference for Φ and no adequate data for its estimation. To simplify matters, ς_P and $\varsigma_{\bar{c}c}$ are normalized to unity. For $\underline{\varsigma}_\Phi$ and $\overline{\varsigma}_\Phi$, the identification considers New York City (NY), Boston (BO), and Washington D.C.(WA). These regions are the most appropriate for an ad hoc parameter calculation as regional effects are less of a concern since these areas are relatively homogeneous in terms of regional characteristics such as geography, culture, climate, and general environmental quality, to name a few. In addition, GDP data is needed to quantify $g_y = \sigma\Omega\Phi$. The OECD only has data on GDP from 2000 to 2019, which is too short for solid estimates. Therefore, US Bureau of Economic Analysis (BEA) personal income (PI) is used. The data is on an MSA level, collected since 1969, while averaging leads to $g_{NY} = 0.058$, $g_{BO} = 0.06$, and $g_{WA} = 0.056$.

Next, the theory understands researchers as any creative workers, hence any employees who create additive value. The BEA provides CAEMP25N statistics on employment in the NAICS industries (sum of wage and salary employment and nonfarm proprietors employment, excluding farm and government). Adding up the employees in the information industry, finance & insurance, professional, scientific & technical services, and business & corporate management, and relating this workforce number to all private non-agricultural workers then gives a proportional indicator for the creative labor force. The latest data with complete information is for 2012 and leads to $\sigma_{NY} \approx X0.24$, $\sigma_{BO} \approx X0.24$ and $\sigma_{WA} \approx X0.28$. Thereby, X scales the proportionality with which the NAIC statistics relate to the actual fraction of innovative labor in the theory.

Beyond, the OECD provides information on the average per capita exposure of the population to fine particles (counted as PM_{25} in $ug/m3$), which serve as a proxy for pollution, see Chapter (3) for details. With the 2012 data, $P_{NY} = 9.1ug/m3$, $P_{BO} = 7.1ug/m3$ and $P_{WA} = 9.6ug/m3$.

Density estimates are then found by using OECD data for the mean population concentration (in km^2) in the metropolitan area of an FUA. An adjustment for construction density is not possible. The density numbers are divided by 1000 to have an adequate scaling (e.g. consider that this theory discusses a one-dimensional rather than a two-dimensional domain). In view of this, $D_{NY} = 1.43$, $D_{BO} = 0.77$ and $D_{WA} = 0.78$. Finally, since the current data are not subject to the dynamic environmental policies discussed in this work, $\Omega_i = 1$. Therefore, the calibration does not differentiate among the innovation regimes what simplifies the identification. Crucially, the pollution numbers in NY and Washington approach each other even closer in 2018. They reach $P_{NY} = 7.5ug/m3$ and $P_{WA} = 7.7ug/m3$. Hence, they are considered as identical what supports the identification considerably.

Therefore, $\sigma_{WA}\Phi_{WA}/g_{WA} = \sigma_{NY}\Phi_{NY}/g_{NY}$, thus $\frac{ln\left(\frac{\sigma_{WA}*g_{NY}*D_{W}}{\sigma_{NY}*g_{WA}*D_{NY}}\right)}{(D_{WA}-D_{NY})} = \overline{\varsigma}_{\Phi}$ gives $\frac{ln\left(\frac{0.28*0.058*0.78}{0.24*0.056*1.43)}\right)}{(0.78-1.43)} = \overline{\varsigma}_{\Phi}$, what leads to $\overline{\varsigma}_{\Phi} \approx 0.6$. Standardizing $\varsigma_P = \varsigma_{\bar{c}c} = 1$ then enables to select the remaining parameter $\underline{\varsigma}_{\Phi}$ in order to achieve a growth rate predicted by Nordhaus (2007) for 2100 of around 4.25%, so $\underline{\varsigma}_{\Phi} = 0.1$.

The parameters describing the quality index, Φ, are challenging since a lack of data makes their identification difficult. In parallel, their choice has a considerable impact on whether the numerical application finds a solution. All the other standardizations and the requirement for a reasonable numéraire growth rate restrict the feasible range for the parameters characterizing Φ. Any intension to evaluate the robustness of the results to parameter adjustments can thus only adjust a relatively narrow range of the parameter values. Too strong adjustments in $\underline{\varsigma}_\Phi$ destabilize the numerical solution. However, parameter adjustments within a feasible (stable) range do affect the qualitative results. Adjustments in $\overline{\varsigma}_\Phi$ are less sensitive but do not affect the qualitative results either. Henceforth, the discussion will focus on the parameter selection presented, so no further robustness tests are applied.

Standardization of further parameters: Since ς_A has been implicitly standardizes to 1, the other research efficiencies are also standardized to unity to have a benchmark with identical research efficiencies. Similarly, since the discussion is not interested in further detailing specialization, $\psi \geq 1$ is not considered. Therefore, the benchmark sets $\psi = 0.5$ to have an intermediate range of research efficiencies. All remaining parameters are standardized to unity. Hence, $\hat{l} = A_0 = E_0 = F_0 = p_{e,b} = 1$. For an EPS regime (and EPS^* regime), $g_{T_e} = 1\%$ (respectively $g_{p_{e,b}} = 1\%$) as an initial reference, which is adjusted when deriving numerical results.

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Statement of Contribution

All chapters are written by the author of this thesis.

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Resources

The entire dissertation was written with LaTex (Kopka, 1991), the reference management was organized with Biber. Figures (3.1) to (3.4) have been generated using R Studio, version 1.3.1093 (Gandrud, 2018), Figures (3.6) to (3.14) have been drawn by the author using Inkskape, version 0.92.4, and Microsoft Power Point, Figures (3.15) and (3.16) have been generated using Microsoft Excel contained within the Microsoft Office 365 package in combination with Inkskape, version 0.92.4. For data processing and modeling in Chapter (2), I utilized Python using Spyder 4 (Anaconda 3). In Chapter (3), the numerical applications presented in Table (3.7), (3.8) and (3.9) and Figures (3.15) and (3.16) were written in the General Algebraic Modeling System (GAMS) and were solved with the CONOPT3 solver, version 3.14S, for non-linear programs (Stolbjerg Drud, 1994).

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