Universität Potsdam

Finanzwissenschaftliche Diskussionsbeiträge

Income Taxation and the Choice of the Tax Rate Schedule: Sacrifice Principles and "Just" Tax Rates

Hans-Georg Petersen

Diskussionsbeitrag 62
Potsdam 2011

Hans-Georg Petersen
University of Potsdam and
German Institute for Economic Research (DIW Berlin)
E-mail: *hgpeter@uni-potsdam.de*

Publisher:
Prof. Dr. Hans-Georg Petersen
University of Potsdam
Faculty of Economics and Social Sciences
Chair of Public Finance
August-Bebel-Str. 89
D - 14482 Potsdam

Homepage: *http://lsfiwi.wiso.uni-potsdam.de/start/index.html*

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Juni 2011

ISSN 0948 - 7549

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Hans-Georg Petersen
University of Potsdam and DIW Berlin

Version June 2011

Abstract:

In the history of economic thoughts the problem of a “just” tax rate structure has played an important role. The paper reconsiders the discussions of the last two centuries and sheds additional light on the concrete tax schedules using the more recent methods of tax theory. Even if the substitution effects which play an important role in the theory of optimal taxation are neglected, the slope in the diminishing marginal utility of income causes tax rate structures reaching from accelerated progression to delayed regression. Interestingly the principle of equal relative sacrifice combined with a Bernoulli utility function yields a delayed progression, which is connected with a negative income tax.

JEL codes: H21, H24, D31, B13

Keywords: income tax, sacrifice principle, tax rate schedule, cardinal utility function.

I. Introduction

In the theory of taxation beside the question on the optimal tax base the tax rate structure has always played an important role. While recently the tax base problems have been intensively discussed (e.g., *Banks/Diamond* 2008 and *Gordon/Kopcuk* 2010), in history of economic thoughts the slope of the tax schedules has been in the focus.[1] In his article on optimal income tax rates *Saez* (2001) has identified the key relevant parameters for optimal tax formulas. Similar parameters have already been discussed in the context of the sacrifice theory being one of the fundamental pillars in the debates on progressive tax schedules. The purpose of this paper is to shed some light on these early discussions in estimating the tax rate structures for different utility functions as well as sacrifice principles. Some of the findings are highly relevant even for modern tax policy.

In his famous essay "On Progressive Taxation" *Cohen Stuart* (1958) has disproved the at that time dominating hypothesis that due to falling marginal utility a just and fair income taxation has necessarily to be progressive.[2] "As soon as we leave the well-trodden path of taxation proportional to income, we find ourselves in a wide open space where we can follow any number of paths, ranging of those which deviate very little from the road of proportionality to those which lead to confiscation of all higher portions of income and equalization of all incomes" (*Cohen Stuart* 1958, p. 67). *Frisch* (1932) has clearly demonstrated the far reaching assumptions of the utility oriented deduction of just tax schedules and tax redistribution. In connection with his empirical studies on the marginal utility of income he has analysed different concepts of a just distribution of tax burdens, in which the principle of an equal relative sacrifice as mentioned by *Cohen Stuart* (1958) is only one special case.

For a deduction based on utility theory principally the following assumptions are necessary:

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[1] An early version of this article has been published already in German; see *Hinterberger/-Mueller/Petersen* (1987).

[2] *Haller* (1960, pp. 35.57 and 1981, pp. 82-86) deserves the merit to commemorate this discussion for the academic profession. *Cohen Stuart* refers in his article to *Wagner* (1880, p. 350).

(1) It has to be defined what a “*just” distribution of tax sacrifice* (loss in utility) means (e. g. equal absolute, relative or marginal sacrifice).

(2) A cardinal *utility function* has to be defined, which is valid for all taxpayers and describes the utility *U*, which is derived from the current income *x*.[3]

(3) The utility function is based on the “law of diminishing marginal utility of income”[4], which is a very particular value judgement and not generally accepted by all members of a society.[5]

If these definitions are given, the derivation of a “just” tax scale can be reduced to a purely mathematical problem. However, Frisch has only derived the general conditions and Cohen Stuart merely determined the tax scales for some utility functions and specific income brackets. Modern simulation methods allow calculating the different resulting tax scales if all sacrifice principles which can be found in the classical literature are applied and combined with the alternative utility functions over the whole income range. Then the respective tax yield functions are derived and analysed using the usual methods for the determination of the type of the tax scale (*Bloecker/Petersen* 1975). Then the spectrum ranges from a differentiating lump sum tax to a totally equalizing progression, which is much broader as was mentioned in the proposition of *Cohen Stuart* (1958) above.

II. Alternative Sacrifice Principles and Utility Functions

Sacrifice based justifications, which evaluate the utility withdrawal as a “sacrifice” of the taxpayer for the advantage of the state, have been formulated in different specifications. The specific sacrifice theory delivers justice hypotheses; combined with concrete utility functions, “just” tax rate schedules can be derived from such an approach.

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[3] If the method of *Frisch* (1932) is applied, the function must be twice differentiable.

[4] For more details see *Marx* (1949), *Blum/Kalven* (1952) and *Clark* (1973).

[5] While the diminishing marginal utility regarding single consumption goods predominantly meets a broad acceptance, a monotonous decreasing marginal utility of income has intensively been discussed in the literature (see e.g., *Easterlin* 2004 and *Layard/Mayraz/Nickell* 2008).

II.1. Possible Characteristics of Sacrifice Theories

A whole set of justice hypotheses are to be found in the literature; beside the principles already mentioned above, additionally the postulations for an *equal absolute nominal burden* and an *equal relative nominal burden* have to be added. The former corresponds to a lump sum tax, which in practice of today's tax policy does not play any remarkable role but is of relevance in the optimal taxation literature. From the latter postulate a proportional tax schedule can be derived, which has been the base for the distribution of tax burdens in the classical theories. Hence, *Mill* (1965, p. 395) has stated that proportionality can be derived from *Smith* (1776/1981) tax principles, in spite of the fact that other authors have been of the opinion that in Smith's writings first attempts to justify progressive tax scales can be found. Be that as it may, in this analysis the lump sum tax and proportionality are taken as *possible results* of such a utility based approach but not as *independent concepts* of a just tax burden distribution.

Table 1 taken from *Frisch* (1932, p. 135) shows the well known justice principles based on the sacrifice theory;[6] also the names of the respective authors are mentioned in the sources cited there. For the justice hypothesis of an equal change of the marginal utility an author is missing so that this hypothesis might have been developed by Frisch himself.[7] As already mentioned above, *Frisch* (1932) does not postulate a given utility function but analyses under which conditions for given utility functions a progressive tax rate structure will result. As an example the derivation for the justice postulate "equal relative sacrifice" is presented where as measure of progression the average tax rate elasticity $E_{\bar{t},x}$ has been used (with $\bar{t}$ as average tax rate and x as income):

$$E_{\bar{t},x} = \frac{d\bar{t}}{dx} \cdot \frac{x}{\bar{t}} \quad \begin{cases} > 0 : progression \\ = 0 : proportionality \\ < 0 : regression \end{cases} \qquad (2.1)$$

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[6] See also *Pigou* (1956, p. 89).

[7] For the sake of completeness it should be mentioned that also postulates for an equal absolute change of average utility or equal relative change of average utility are possible justice hypotheses, respectively. In the following these hypotheses are neglected.

For “small” tax rates approximately the following equation holds true[8]:

$$U(x) - U(x - t) = U' \cdot \Delta x = U' \cdot t \, , \tag{2.2}$$

where *t* presents the tax yield. Using this simplification the postulate for the equal absolute sacrifice is defined as:

$$g = U' \cdot t = const \, . \tag{2.3}$$

or

$$t = g / U' \tag{2.4}$$

or

$$\bar{t} = \frac{g}{x \cdot U'} \, . \tag{2.5}$$

The total differential $d\bar{t}$ is:

$$d\bar{t} = -\frac{g}{x^2 \cdot U'} \cdot dx - \frac{g}{x \cdot U'^2} \cdot dU' \tag{2.6}$$

and after some transformations:

$$E_{\bar{t},x} = -1 - \frac{dU'}{dx} \cdot \frac{x}{U'} \tag{2.7}$$

or

$$E_{\bar{t},x} = -1 - E_{U',x} \, , \tag{2.8}$$

where $E_{U',x}$ is the elasticity of marginal utility related to income; as criteria for the type of tax scale in case of the absolute equal sacrifice it results with:

$$-E_{U',x} \quad \left\{ \begin{array}{l} > 1: \mathit{progression} \\ = 1: \mathit{proportionality} \\ < 1: \mathit{regression} \end{array} \right. \tag{2.9}$$

---

[8] The approximation by integration and power series development is substituted by the simpler and equally effective depiction by differentials.

In the last column of table 1 the identically derived progression conditions are shown. These progression conditions clearly reveal that for each course of the tax scale the utility functions and their derivatives are of relevance.

For the derivation of the tax rate schedule for the whole income range, in case of the ERS principle full information on the total utility function (and derivative) is necessary. For the other sacrifice principles the information about the marginal utility function is sufficient.[9]

Table 1
Justice Functions

Author	Justice Hypothesis		Condition for Progression
	verbal	formal	
E. SAX [a]	Equal absolute sacrifice (EAS)	$U(x) - U(x-t) = g$	$-E_{U',x} > 1$
A. J. COHEN STUART et al.	Equal relative sacrifice (ERS)	$\frac{U(x) - U(x-t)}{U(x)} = g$	$E_{U,x} - E_{U',x} > 1$
F. Y. EDGEWORTH [b]	Equal marginal utility or minimal sacrifice (MS)	$U'(x-t) = g$	$U''(x) < 0$
R. FRISCH	Equal absolute change in marginal utility (EACMU)	$U'(x-t) - U'(x) = g$	$-E_{U',x} - E_{(E_{U',x}),x} > 0$
K. SCHÖNHEYDER R. MEYER [c]	Equal relative change in marginal utility (ERCMU)	$\frac{U'(x-t) - U'(x)}{U'(x)} = g$	$-E_{(E_{U',x}),x} > 0$

[a] *Sax* (1887, 1924).
[b] *Edgeworth* (n. d.).
[c] *Meyer* (1884).
Source: Frisch (1932, p. 135).

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[9] In case of the ERS principle information on the marginal utility function is sufficient because for the disintegration to *t* the constant of integration is omitted.

II.2. Applied Utility Functions

In the following for three specified utility functions often used in the literature the possible tax schedules will be derived presumed that alternative justice hypotheses are applied.

The first of these functions is

$$U = a \cdot \ln x + b, \tag{2.10}$$

often denoted as “general BERNOULLI utility function”. The most simple special case in which the marginal utility is reciprocally proportional to income is the “BERNOULLI utility function” with $a = 1$ and $b = 0$ (see figure 1).[10] Next the function

$$U = c \cdot \sqrt{x} + d \tag{2.11}$$

is used which is called “general COHEN STUART utility function”.[11] A special case is the function used by Cohen Stuart with $c = 2$ and $d = 0$. Finally a quadratic function

$$U = e + f \cdot x - 0{,}5 \cdot h \cdot x^2 \tag{2.12}$$

is implemented and usually denoted as “general GOSSEN utility function”. The special case with $e = 0$ is named “GOSSEN utility function”.[12] It has a linear average and marginal utility function which are zero for $x = f/h$. Contrary to the Bernoulli and Cohen Stuart functions the Gossen function has a point of saturation. For the assurance of comparability and the avoidance of otherwise necessary differentiations, this function is only analysed left from the point of saturation.

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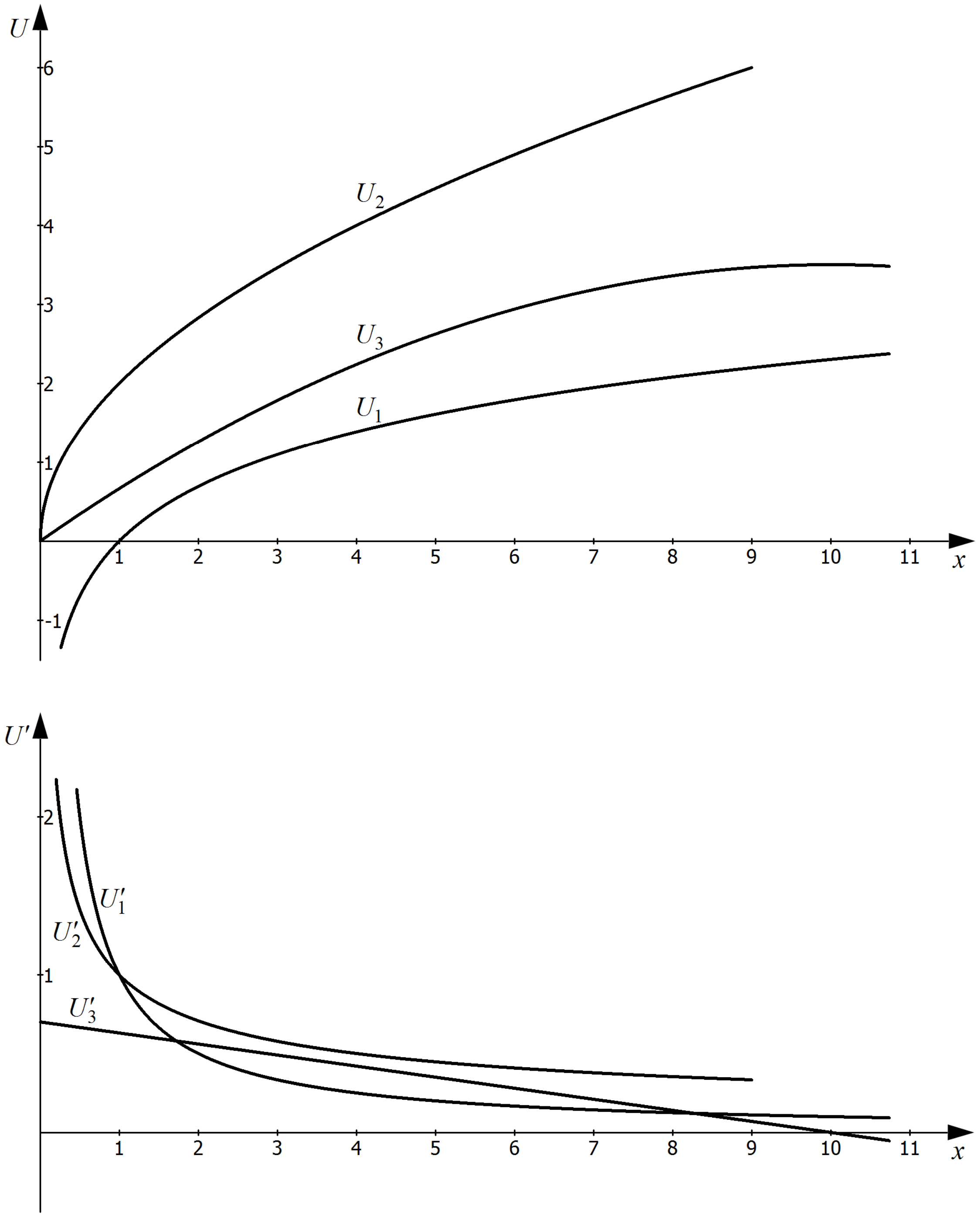
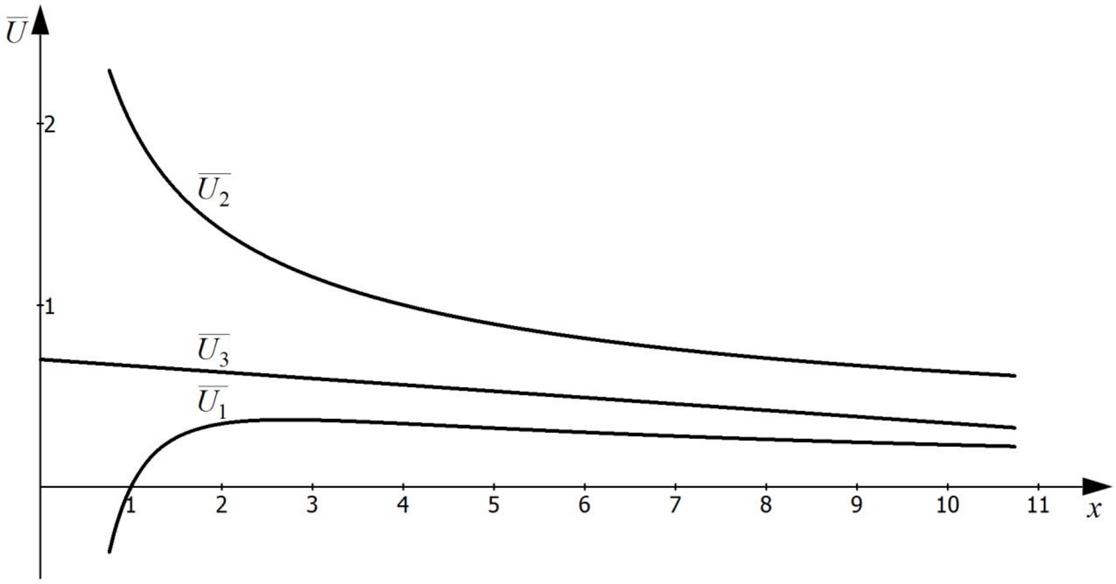
[10] See *Bernoulli* (1896, p. 30).

[11] The Cohen Stuart function corresponds to a Cobb-Douglas function with a degree of homogeneity of 0.5.

[12] See *Samuelson* (1947, p. 93).

Figure 1:
BERNOULLI $(U_1 = \ln x)$, COHEN STUART $(U_2 = 2 \cdot \sqrt{x})$ *and* GOSSEN $(U_3 = 0{,}7 \cdot x - 0{,}5 \cdot 0{,}07 \cdot x^2)$ Utility Functions and the Corresponding *Marginal* $(U_1', U_2' \text{ and } U_3')$ and *Average Utility Functions* $(\overline{U}_1, \overline{U}_2 \text{ and } \overline{U}_3)$





Source: Own calculations.

In figure 1 the courses of these three types of functions as well as the average and marginal utility functions are illustrated. The slopes of the utility functions are quite different. At the same income level x the Bernoulli function has utility levels which are clearly less than in case of the Cohen Stuart function, and the parameters of the Gossen function have been fixed so that U_3 is always located between U_2 and U_1. For income levels $x \leq 1$ the marginal utility U'_1 is larger than U'_2, if $x \geq 1$ the marginal utility of the Bernoulli function is below the values for the Cohen Stuart function and more strongly decreasing. For $x \geq 3$ and $x \leq 8$ the values for the Gossen function again are between the two other functions. The Gossen function has a linearly decreasing marginal utility which is zero for $x = 10$. Then the utility function reaches its maximum, which simultaneously is the point of saturation mentioned above. Beyond that the marginal utility is negative. Only the Bernoulli function has an increasing branch of the average utility function, while both other utility functions only have decreasing average utilities. As already mentioned above in (2.9), the slope of the marginal utility function has a dominating influence on the tax schedule.

In contrast to the theory of optimal taxation, in case of the sacrifice principle utility is only dependant on the income levels. The sacrifice theories neglect the fact that income also depends on the hours of work and possible substitution effects between working and leisure time. But even if substitution effects and disincentive effects for the supply of effort are excluded, the law of diminishing marginal utility of income is not a sufficient condition for progressive taxation as will be demonstrated in the following.

III. Tax Rate Schedules for Alternative Sacrifice Principles and Utility Functions

As mentioned above a concrete tax schedule cannot be simply derived from a specific justice hypothesis, although prima vista the concept of an equal relative utility withdrawal seems to correspond to a progressive schedule. Not even a tax yield being higher for the "rich" than the "poor" can be based on each of the five sacrifice concepts. In the following several tax scales are shown and analysed by way of example. Then a systematic overview on the results for alternative justice hypotheses and utility functions is presented. In a first step the constant term in the utility functions is set to zero *(b, d* and *e).* Afterwards the so-called "initial value problem" will be discussed (*Cohen Stuart* 1889, p. 57).

III.1. "Typical" Tax Schedules over the Whole Income Range for Functions without Constant Term

We start with the equal absolute sacrifice concept (EAS). The justice function G results with

$$G(U,x,t) = U(x) - U(x-t) = g = const \, . \qquad (3.1)$$

Here the problem arises that meaningful solutions are only possible as long as $U(x) > g$. For the income where $U(x) = g\,(which\ is\ called\ x_U)$ the income is equal to the tax yield $(x = t)$.

If the utility function (2.10) is inserted into this justice function, the following expression results

$$a \cdot \ln x - a \cdot \ln(x-t) = g \qquad (3.2)$$

or

$$\ln\left(\frac{x}{x-t}\right) = \frac{g}{a} \qquad (3.3)$$

and

$$t = x \cdot (1 - e^{(-g/a)}) \tag{3.4}$$

with the marginal tax rate

$$t' = 1 - e^{(-g/a)} = \bar{t}, \tag{3.4a}$$

which also coincides with the average tax rate. The average rate progression $\bar{t}'$ as well as the second derivative of the average tax function $\bar{t}''$ are zero:

$$\bar{t}' = \bar{t}'' = 0. \tag{3.4b}$$

The derived tax scale corresponds to a *proportional schedule*. Figure 2 shows the course of the tax scale for the Bernoulli function and the EAS principle. Left from point *A,* where $x = x_u$, the proportional schedule cannot be justified with the EAS principle; this is represented in figure 2 by the hatched area.

In case of the ERS principle the justice function results with

$$\frac{\ln x - \ln(x - t)}{\ln x} = g \tag{3.5}$$

(where reasonably g must be $(0 < g < 1)$[13] or

$$e^{(\ln x - g \cdot \ln x)} = x - t. \tag{3.6}$$

Resolved for *t* the tax yield function results with

$$t = x \cdot (1 - x^{-g}). \tag{3.7}$$

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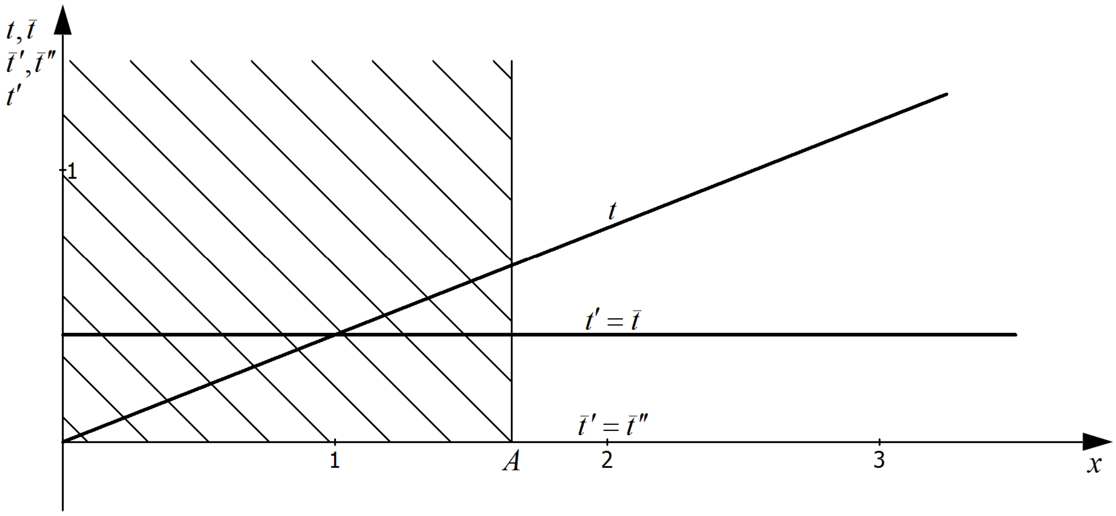
[13] The absolute utility loss has to be larger than zero but smaller than the total utility. The choice of the extent of g determines the volume of the tax revenue. For *g* > 1 a regression results, however a meaningful interpretation is no longer possible because the taxpayers are already in the negative utility area.

Figure 2
Tax Scale for a BERNOULLI Utility Function
and the ERS Principle (Proportionality)



Source: Own calculations.

The marginal tax rate[14] results with

$$t' = 1 - (1 - g) / x^g \tag{3.7a}$$

and the average tax rate with

$$\bar{t} = 1 - 1 / x^g \, . \tag{3.7b}$$

The average rate progression

$$\bar{t}' = g \cdot x^{-(g+1)} \tag{3.7c}$$

is larger than zero and the second derivative of the average tax rate function

$$\bar{t}'' = -(g^2 + g) \cdot x^{-(g+2)} \tag{3.7d}$$

is smaller than zero, therefore a *persistently delayed progression* results. The courses of the functions are represented in figure 3 for a Bernoulli utility function.

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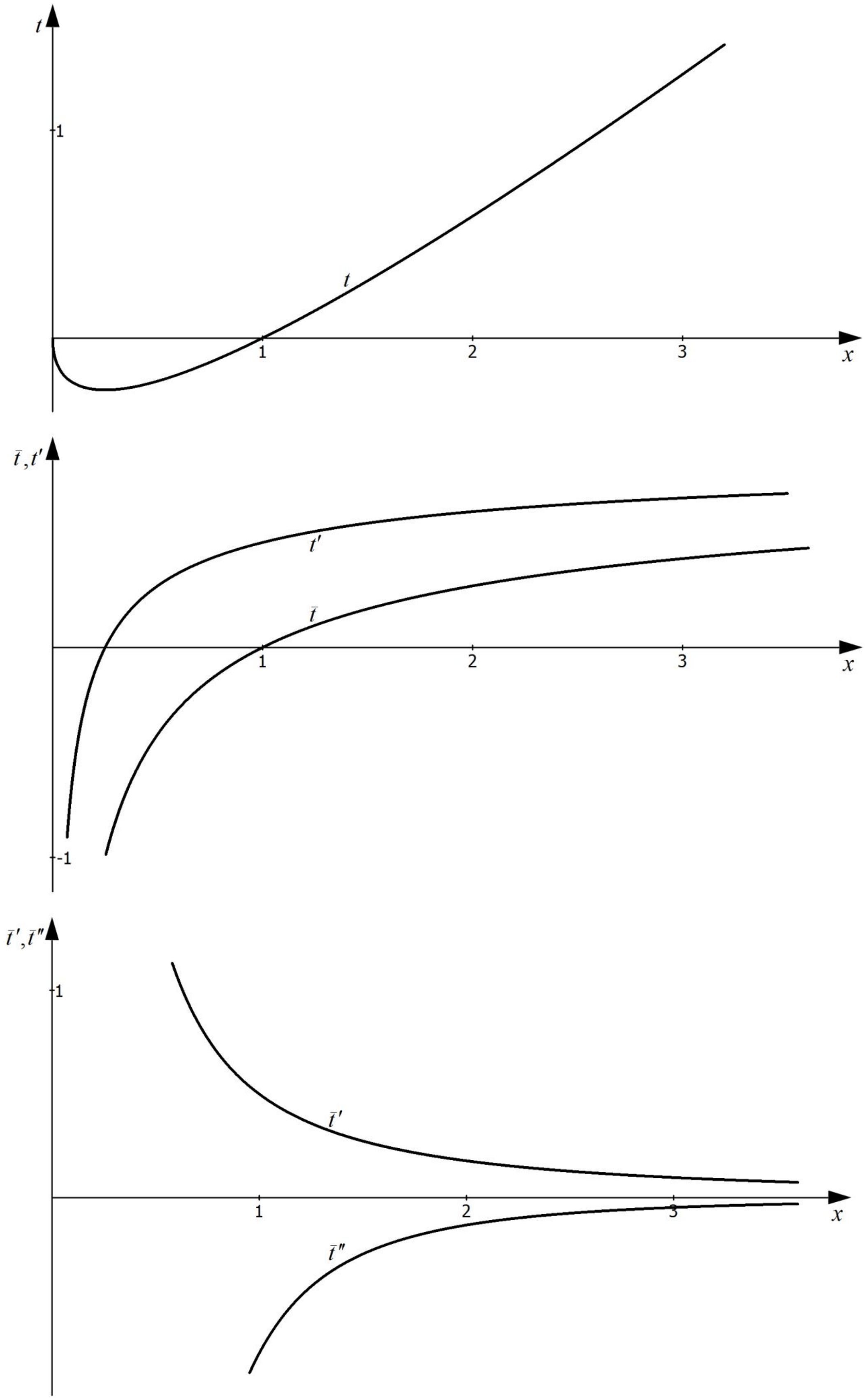
[14] The marginal tax rate asymptotically approaches one (100%): $\lim\limits_{x \to \infty} dt / dx = 1$.

Figure 3

Tax Scale for a BERNOULLI Utility Function and the ERS Principle



Source: Own calculations.

Cohen Stuart has also incorporated the area beyond $x = 1$ in his analysis and regarded $x = 1$ to a certain extent as minimum of subsistence[15], which has to be tax free and only beyond that amount the tax burden has to start. If $x = 1$ is taken as basic income of a negative income tax and one includes the area $0 < x \leq 1$ in the analysis, it becomes apparent that the ERS principle combined with a Bernoulli utility function requires a negative income tax.[16]

For the MS principle the justice function is as follows:

$$U'(x - t) = g \tag{3.8}$$

For each utility function this condition leads to a marginal tax rate which is equal to 100 % and to a tax exemption which represents the net income on which all the incomes (above and below) are levelled (totally equal income distribution). In this case not all values of g are meaningful. This is demonstrated in the example of the Gossen function; from (3.8) and (2.11) results

$$t = g / h + x - f / h \tag{3.9}$$

with

$$t' = 1 \tag{3.9a}$$

and

$$\bar{t} = (g / h) \cdot x^{-1} + 1 - (f / h) \cdot x^{-1}, \tag{3.9b}$$

being only meaningful if *g* is smaller than *f*, the latter determining the intercept point with the ordinate.[17]

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[15] The determination of $x = 1$ as minimum of subsistence by Cohen Stuart is more or less arbitrary; economic arguments are not mentioned.

[16] The minimum in the total tax yield function results because in the negative area the Bernoulli utility function asymptotically approaches the U-axis; the area left of the minimum does not allow for a meaningful interpretation.

[17] For $f = g$ proportionality results while the whole income is withdrawn by taxation.

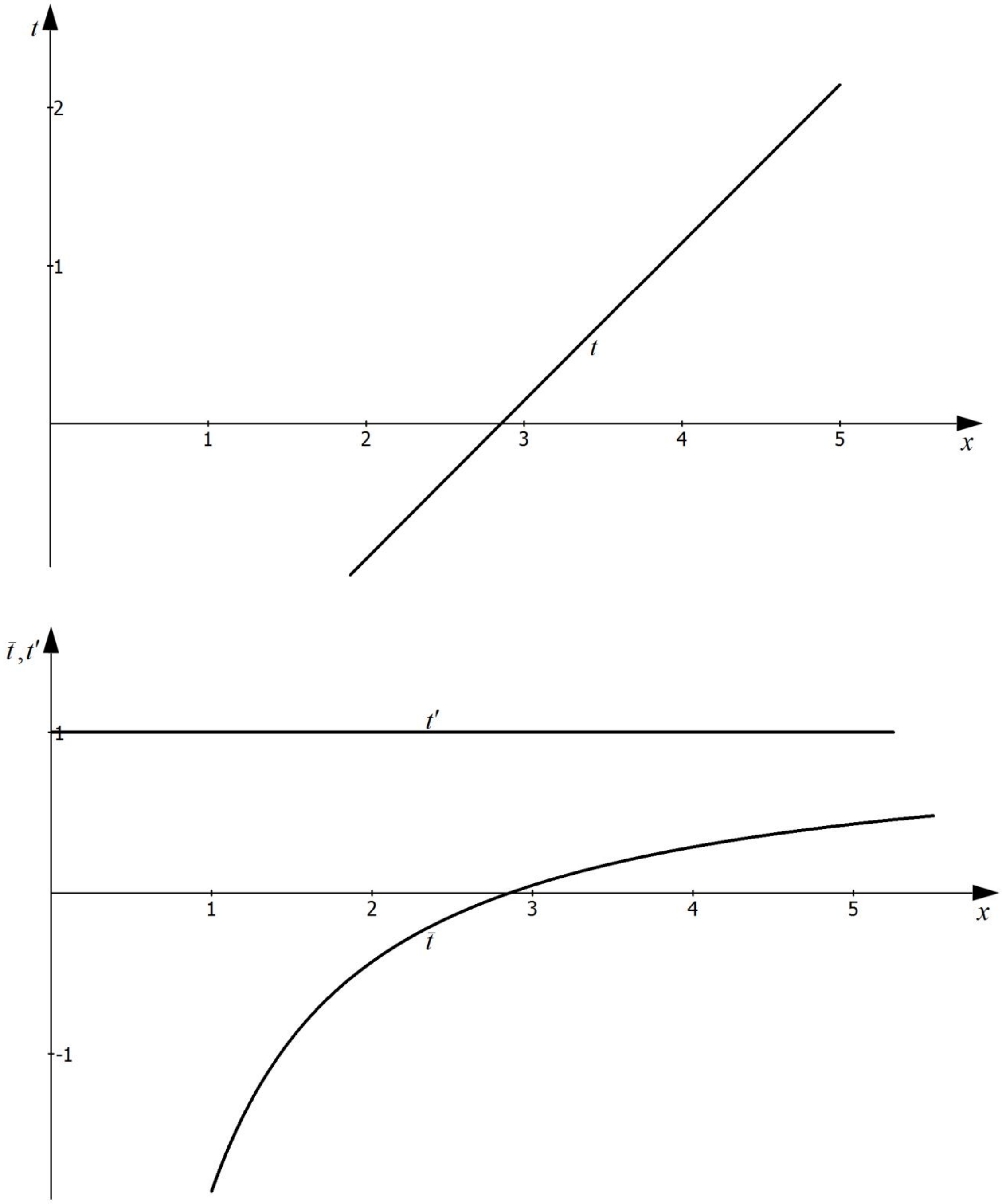
The average rate progression and its derivative are

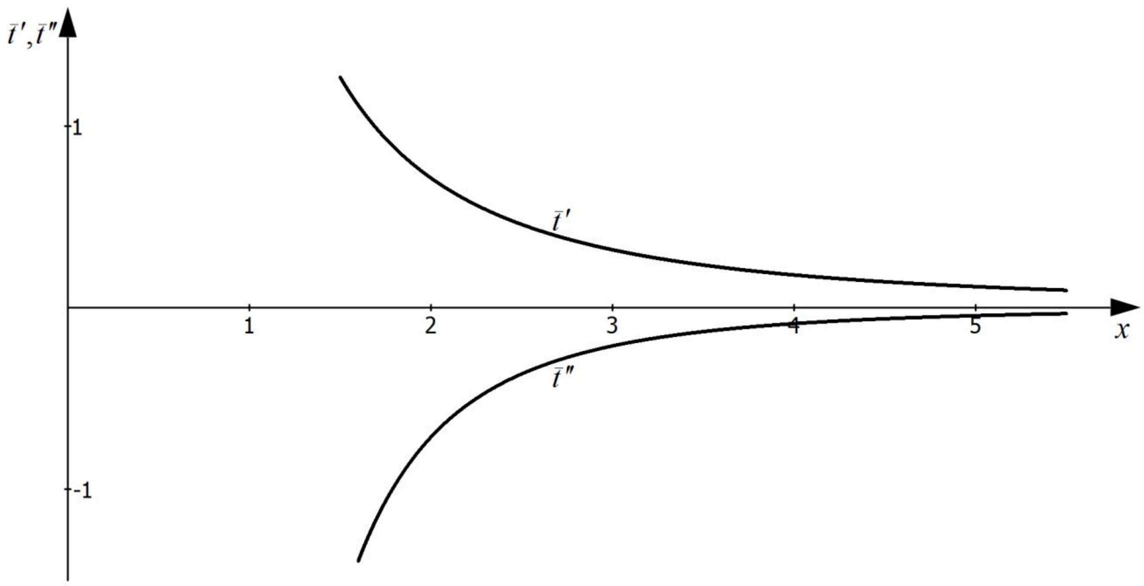
$$\bar{t}' = (-g/h)\cdot x^{-2} + (f/h)\cdot x^{-2} > 0 \text{ and} \qquad (3.9c)$$

$$\bar{t}'' = (2\cdot g/h)\cdot x^{-3} - (2\cdot f/h)\cdot x^{-3} < 0, \qquad (3.9d)$$

so that a delayed (here: indirect) progression follows.

Figure 4
Tax Scale for a GOSSEN Utility Function and the MS Principle





Source: Own calculations.

Figure 4 shows the single functional forms. The average rate is growing and approximates the constant marginal rate which is 100%. The progression is indirect. Until an income of

$$x = f/h - g/h \tag{3.10}$$

in these examples transfers are paid, which increase all lower incomes to the average income level. Beyond this level all parts of any incomes are withdrawn by tax so that all citizen do get the same amount of net income; the income distribution is totally equalized.

Another interesting tax scale, which will be described in more detail, follows from the EAS principle combined with a general Cohen Stuart utility function of the type (2.11). The corresponding justice function (for $g > 0$) is:

$$c \cdot \sqrt{x} - c \cdot \sqrt{x - t} = g\,. \tag{3.11}$$

The dissolution to t is:

$$t = 2 \cdot g/c \cdot \sqrt{x} - (g/c)^2\,. \tag{3.12}$$

The marginal tax rate follows with

$$t' = (g/c)/\sqrt{x} \tag{3.12a}$$

and the average tax rate with

$$\bar{t} = (2 \cdot g / c) / \sqrt{x} - (g / c)^2 / x \, . \tag{3.12b}$$

The prefixes of the average rate progression

$$\bar{t}' = (-g / c) \cdot x^{-1,5} + (g / c)^2 \cdot x^{-2} \tag{3.12c}$$

and of the second derivative of the average tax rate function

$$\bar{t}'' = (3 \cdot g) / (2 \cdot c) \cdot x^{-2,5} - 2 \cdot (g / c)^2 \cdot x^{-3} \tag{3.12d}$$

are not any longer definitely determinable. The resulting functions are represented in figure 5.

The tax yield function has again a negative bracket. The type of tax scale is not any longer clearly determined but is changing with increasing income. The average rate $\bar{t}$ at first increases (coming from the negative bracket) and in the positive bracket it reaches a maximum at $x = (g / c)^2$ (point A); the average rate progression $\bar{t}'$ points to a *progression* $(\bar{t}' > 0)$ which is of a *delayed* form $(\bar{t}'' < 0)$. Subsequently the average rate declines so that a *regression* follows $(\bar{t}' < 0)$ which is at first *accelerated* $(\bar{t}'' < 0)$ and then *delayed* $(\bar{t}'' > 0)$.[18]

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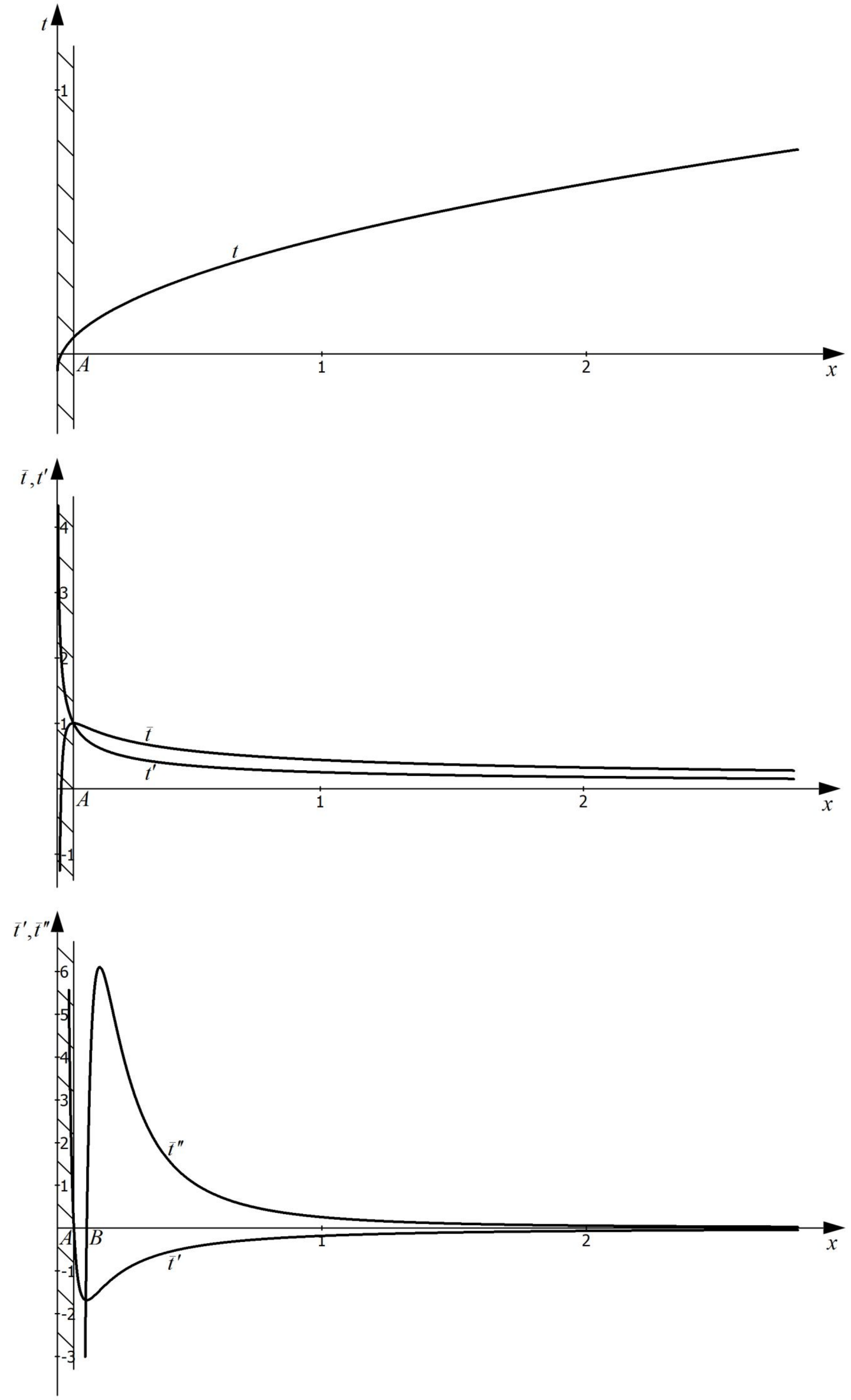
[18] This is right from point B, in which the income is $x = ((4 \cdot g) / (3 \cdot c))^2$.

Figure 5

Tax Scales for a COHEN STUART Utility Function and the EAS Principle



Source: Own calculations.

Although one and the same utility function results in a concrete tax scale, the scale type is changing with increasing income. Here it has to be mentioned again that the tax scale can only be interpreted above an income x_u for $U(x_u) = g$. If tax values left from $t = x$ are put into the formula of the EAS principle, it has to be mentioned that no solution does exist.

Therefore, progression and the negative tax branch are not in a range which can be justified with the EAS principle. The change from progression to regression is exactly at x_u (point *A*).

III.2. Overview on the Tax Scales

The tax yield function has been determined for all justice hypotheses mentioned in table 1 and the three types of utility functions. The resulting types of tax scales are analysed, so far neglecting the “initial value problem”. The results are presented in table 2. If a Bernoulli utility function is taken into consideration and combined with the EACMU principle then a *delayed progression* results, while a lump sum element (fixed amount or minimum tax) is included. Likewise in case of the EAS principle a constraint has to be mentioned for $U'(x) > g$, which leads to an upper income limit of x_0. In case of the ERCMU principle a *proportional tax scale* follows, while for $U'(x) = g$ the tax scale is not defined.

If the Cohen Stuart utility function is taken into consideration, with the exception of the EAS principle, a definite tax scale for the different sacrifice principles results, which is in between *proportionality* and *delayed (direct) progression.*[19] In case of the Gossen utility function the results are much more complex.[20] The EAS principle is the only one where within the relevant bracket a change in the average rate progression takes place, which is the case in between x_u and the point of saturation. The ERS principle yields a *delayed and accelerated progression* while

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[19] Table 2 is valid for all relevant values of a, also for $a = 1$. Relevant values for *g* are in case of the EACMU principle $g > 0$ and in case of the ERCMU principle $0 < g < 1$.

[20] Here the parameters have been fixed with $f = 0{,}7$ and $h = 0{,}07$. Then the marginal utility functions have similar slopes in the relevant income brackets and functional values as in case of the two other utility functions. For this function the parameter values are influencing the tax scale, e.g., the types of tax scales are differently combined but the change from progression to regression remains unaltered. Additionally the tax scale is dependent on *g* (see table 2).

the MS principle is connected with an *indirect progression*. For the EACMU principle a *lump sum tax (indirect regression)* results where the upper income limit *x₀* coincides with the point of saturation. The ERCMU principle causes a *(direct and indirect) delayed regression*. Here a per capita tax (poll tax) exists where the tax yield is reduced with increasing income.

Table 2
Justice Functions, Utility Functions, and Tax Schedules

Justice Hypothesis \ Utility Function	$U = a \cdot \ln(x)$ (BERNUOULLI)	$U = c \cdot \sqrt{x}$ (COHEN STUART)	$U = 0{,}7 \cdot x - 0{,}5 \cdot 0{,}07 \cdot x^2$ [a] (GOSSEN) (in the relevant range)
$g = U(x) - U(x-t)$ (EAS)	proportional scale	(1) accelerated regression (2) delayed regression	(1) delayed regression (2) accelerated progression
$g = \frac{U(x) - U(x-t)}{U(x)}$ (ERS)	delayed progression (NT)	proportional scale	(1) delayed progression (2) accelerated progression
$g = U'(x-t)$ (MS)	delayed (indirect) progression (NT)	delayed (indirect) progression (NT)	delayed (indirect) progression (NT)
$g = U'(x-t) - U'(x)$ (EACMU)	delayed progression	delayed progression	delayed (indirect) regression (LST)
$g = \frac{U'(x-t) - U'(x)}{U'(x)}$ (ERCMU)	proportional scale	proportional scale	delayed regression (LST)

LST = Lump Sum Tax
NT = Negative Tax
[a] see footnote 19
Source: Own calculations.

If the different sacrifice principles are taken into consideration for the three analysed types of utility functions, it is obvious that only for the MS principle a uniform result can be observed. Independent from the utility function an indirect progression results and the marginal tax rate is 100 %.

III.3. Initial Value Problem

As can be seen from table 1, the utility function (anti-derivative) is only an argument in case of the ERS justice principle; for all other principles the information about the marginal utility function is sufficient. If the marginal utility function is given, the total utility function is also known without the value of the constant of integration. Even in case of a given slope for the marginal utility curve the ERS principle does not answer the question if a “just” tax scale should be progressive, proportional or regressive. Only the determination of the constant of integration by a meaningful initial value decides about the type of the “just” tax scale – a fact which especially has been stressed by Cohen Stuart.

In the following the Bernoulli utility function can be neglected because this function does not run through the point of origin (zero) and approaches the *U*-axis in the negative utility area. The constant of integration only alters the intercept point with the *x*-axis, an initial value problem does not exist.[21]

The initial value problem occurs in case of the Cohen Stuart and the Gossen utility functions. The Cohen Stuart utility function is generally given with

$$U = c \cdot \sqrt{x} + d \, , \tag{2.11}$$

where d is the constant of integration. The criteria for progression analogous to Frisch (see table 1) is:

$$E_{U,x} - E_{U',x} = \frac{1}{d/\sqrt{x}+2} + 1/2 \quad \begin{cases} >1: \textit{progression} \\ =1: \textit{proportionality} \\ <1: \textit{regression} \end{cases}$$

The following three initial values cause different types of tax scales, respectively:

$$U(x=0) = 0 \qquad \text{and} \qquad U(x=1) = 2 \, , \tag{3.14a}$$

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[21] The general Bernoulli utility function is as follows:

$$U = a \cdot \ln x + b \, .$$

Only in the marginal case – if b is infinite – this function runs through zero. If the tax scale is derived from the ERS principle, it results with

$$\bar{t} = 1 - e^{(-b \cdot g)/a} \cdot x^{-g} \, .$$

The average rate progression is

$$\bar{t}' = g \cdot e^{(-b \cdot g)/a} \cdot x^{-g-1} ,$$

and the second derivation of the average tax rate function is

$$\bar{t}'' = -(g+1) \cdot g \cdot e^{(-b \cdot g)/a} \cdot x^{-g-2} \, .$$

Howsoever b is fixed, the average rate progression is larger than zero and progression is delayed. In case of the other justice hypotheses b does not show up in the total tax yield functions.

$$U(x=0) = -1 \quad \text{and} \quad U(x=1)=1, \tag{3.14b}$$

$$U(x=0) = 1 \quad \text{and} \quad U(x=1)=3. \tag{3.14c}$$

From these it follows that:

$$U = 2\cdot\sqrt{x}, \tag{3.14a'}$$

$$U = 2\cdot\sqrt{x}-1, \tag{3.14b'}$$

$$U = 2\cdot\sqrt{x}+1. \tag{3.14c'}$$

As is well known in *Frisch* (1932) the initial value (3.14a) leads to proportionality (see above). If the initial value is fixed according to (3.14b) with $U(x=1)=1$, the following criteria results:

$$E_{U,x} - E_{U',x} = \frac{1}{2-1/\sqrt{x}} + 1/2 > 1 \quad . \tag{3.15}$$

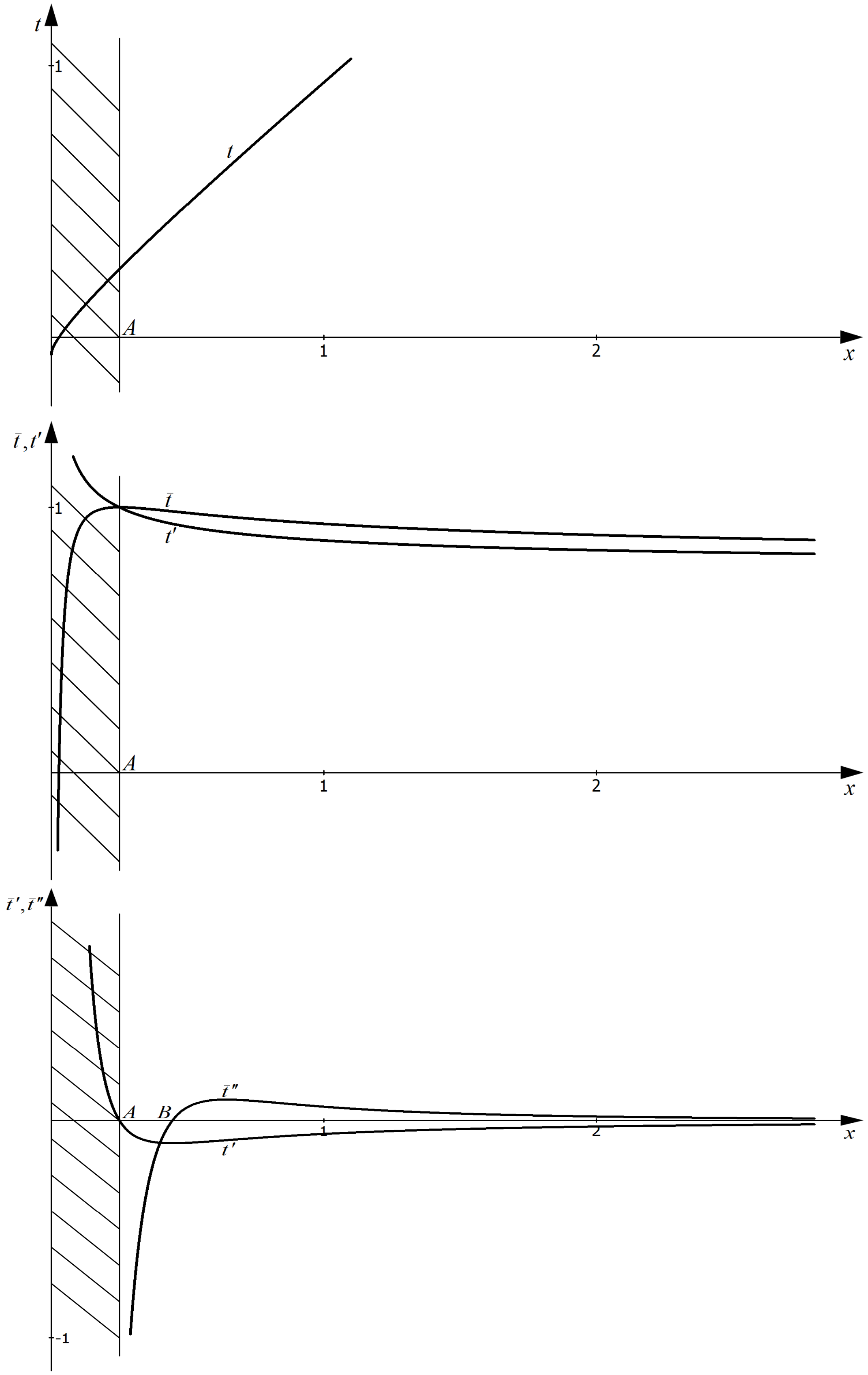
The tax scale is progressive; for the initial value (3.14c) the criteria is:

$$E_{U,x} - E_{U',x} = \frac{1}{2+1/\sqrt{x}} + 1/2 < 1 \quad . \tag{3.16}$$

The resulting tax scale is regressive.

In the latter case the results in this paper differ from the Frisch analyses (see figure 6) for $x < [2\cdot d\cdot g/(1{,}5\cdot(1-g)\cdot c)]^2$ (left from point *B* in figure 6); for this area at first a delayed progression, secondly from point *A* at $x = [d\cdot g/(c\cdot(1-g))]^2$ an accelerated, and thirdly from point *B* a delayed regression can be observed. The lower margin x_u is given by $U(x) = g+1$. This coincides with point *A*.

Figure 6
Tax Scales for a General COHEN STUART Utility Function
$(U = 2 \cdot \sqrt{x} + 1)$ and the ERS Principle



Source: Own calculations.

Therefore, also in this case progression and negative tax are justified by the ERS principle. For the initial value of (3.14b) the negative tax branch is omitted because the utility function is not defined left from $x_u = 0{,}25$.

If the general Gossen utility function

$$U = e + f \cdot x - 0{,}5 \cdot h \cdot x^2 \tag{2.11}$$

is taken into consideration, then tax scales are derived as shown in table 3.

IV. Relevance of the Results

(1) The derivation of "just" tax scales on the basis of utility oriented arguments has an extremely speculative character. Besides the well known facts that there is no agreement on (1) the definition what justice is and (2) the course and slopes of a marginal utility curve being valid for all individuals, Cohen Stuart and (later and in a more general form) Frisch have delivered the proof that – as long as a Bernoulli utility function is not under consideration – even in case of an agreement on these two problems a conclusion on the type of tax rate structure is not possible as long as the initial value problem remains unsolved. Therefore, we are still – to quote Cohen Stuart, who has attributed this citation to *McCulloch* (1845) – "... at sea without rudder or compass".[22]

(2) In this analysis the problem has been widened and all relevant utility functions have been taken into consideration. The used concept enables us to determine the exact type of tax schedule.

(3) The results are of specific interest in the lower income brackets. The need for a negative income tax does not only exist in case of the MS principle but also for the ERS principle if it is combined with a Bernoulli utility function. In the latter case a continuous delayed progression results which is in accordance with Pigou's tax scale criteria.[23]

(4) Beyond that it is interesting to note that justice hypotheses based on marginal utility criteria incorporate elements of lump sum taxation, which is

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[22] *Cohen Stuart* (1958, p. 67).

[23] See e.g., *Petersen* (1976).

especially true for Gossen utility functions. Additionally with the conventional approaches it could be demonstrated that the tax scale types are not constant for the whole income range but changing with increasing income.

Table 3
Tax Scales for the ERS Principle and Alternative Initial Values

Utility Function	$d,e=0$	$d,e=+1$	$d,e=-1$
general COHEN STUART utility function $U=2\cdot\sqrt{x}+d$	proportional scale	(1) accelerated regression (2) delayed regression	delayed progression
general GOSSEN utility function $U=e+0{,}7\cdot x-0{,}5\cdot 0{,}07\cdot x^2$ (in the relevant range)	(1) delayed progression (2) accelerated progression	(1) delayed regression (2) accelerated progression	delayed progression

Source: Own calculations.

(5) The combination of the ERS principle with the Bernoulli utility function generates types of tax scales which are quite similar to those in optimal taxation approaches, without considering the “disincentives” that are included in that analysis.

(6) The assumption that all individuals do face the same utility function is extremely restrictive. If this assumption is skipped – e.g., all individual utility functions (being of the types Bernoulli, Cohen Stuart and/or Gossen) are additively (Bentham) or multiplicatively (Nash) aggregated in form of a social welfare function[24] –, the approach is not any longer mathematically resolvable and the results for the tax scales are totally undetermined.

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[24] For more details see *Petersen* (2004).

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