

Global existence of a shock for the supersonic flow past a curved wedge

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This note is devoted to the study on the global existence of a shock wave for the supersonic flow past a curved wedge. When the curved wedge is a small perturbation of a straight wedge and the angle of the wedge is less than some critical value, we show that a shock attached at the wedge will exist globally.

§1. Introduction

This note is concerned with the global existence problem of a shock wave for the stationary supersonic flow past an infinite curved wedge. When the wedge is straight (that is, it is formed by two planes), such a problem has been solved well in the book [1] by the shock polar. For the curved wedge, if the direction of the supersonic coming flow is perpendicular to the edge of the wedge and the angle of the wedge is less than a critical value, which is determined by the velocity, density and adiabatic exponent of coming flow, then the local existence of a shock attached at the edge is proved in [2-4]. Besides, the author in [5] established the global existence of a weak solution by use of the Glimm's scheme for the isentropic steady supersonic flow past a 2-D infinite curved wedge with a sharp angle. Our main interest is on the structure of the global weak solution of such a problem. The aim of this paper is to show the global existence of a shock as observed in physical experiments, moreover we will remove the restriction on the sharp angle in [5].

Now let's give a detailed description of our main result. Suppose that the stationary inviscid supersonic flow past a 2-D wedge is governed by the steady Euler system. Under the assumptions that the flow is isentropic and irrotational, the system can be written as

$$\begin{cases} \partial_x(\rho u) + \partial_y(\rho v) = 0 \\ \partial_x(\rho u^2) + \partial_y(\rho uv) + \partial_x P(\rho) = 0 \\ \partial_x(\rho uv) + \partial_y(\rho v^2) + \partial_y P(\rho) = 0 \end{cases} \quad (1.1)$$

where ρ , (u, v) and $P(\rho)$ represent the density, the velocity and the pressure respectively.

Suppose that there is a uniform supersonic flow $(u, v) = (q_0, 0)$ with constant density $\rho_0 > 0$, which comes from minus infinity. The flow hits a 2-D wedge along the direction perpendicular to the edge of the wedge. When the angle of the wedge is less than a critical value b_* , then the flow on the upper part and on the lower part can be determined independently. Without loss of generality, in this paper we only take into account the upper part of the wedge, whose equation in $x \geq 0$ is described by a smooth function $y = b(x)$ with $b(0) = 0$ and $b'(0) = b_0 > 0$.

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If $b_0 < b_*$, then there will be a shock attached at the point $(0, 0)$. Denote by $y = \chi(x)$ the equation of the shock front for $x \geq 0$, here $\chi(0) = 0$. On the wedge and the shock front, the following boundary conditions should be satisfied

$$ub'(x) - v = 0, \quad \text{on} \quad y = b(x) \quad (1.2)$$

and

$$\begin{cases} [\rho u]\chi'(x) - [\rho v] = 0, \\ [\rho u^2 + P(\rho)]\chi'(x) - [\rho uv] = 0, \\ [\rho uv]\chi'(x) - [\rho v^2 + P(\rho)] = 0, \end{cases} \quad \text{on} \quad y = \chi(x) \quad (1.3)$$

where the system (1.3) comes from the Rankine-Hugoniot conditions.

Since the flow is irrotational, then one can deduce from (1.1) that

$$\nabla_{x,y} \left(\frac{1}{2}(|u|^2 + |v|^2) + h(\rho) \right) = 0 \quad (1.4)$$

where $h(\rho)$ is the specific enthalpy, and $h'(\rho) = \frac{c^2(\rho)}{\rho}$, $c^2(\rho) = P'(\rho) > 0$ for $\rho > 0$.

The integration of (1.4) leads to the Bernoulli's law

$$\frac{1}{2}(u^2 + v^2) + h(\rho) = \frac{1}{2}q_0^2 + h(\rho_0) \equiv C_0 \quad (1.5)$$

Then by the implicit function theorem we have

$$\rho = h^{-1}\left(C_0 - \frac{1}{2}(u^2 + v^2)\right) \equiv H(u^2 + v^2) \quad (1.6)$$

Consequently, substituting (1.6) into the first equation in (1.1) we can reduce the system (1.1) into the following 2×2 system on (u, v)

$$\begin{cases} \partial_x(Hu) + \partial_y(Hv) = 0 \\ \partial_x v - \partial_y u = 0 \end{cases} \quad (1.7)$$

It is easy to verify that (1.7) is strictly hyperbolic with respect to x -direction if $u > c(\rho)$.

Indeed, for $u > c(\rho)$, (1.7) has two distinct real eigenvalues $\lambda_1(u, v) = \frac{uv - c(\rho)\sqrt{u^2 + v^2 - c^2(\rho)}}{u^2 - c^2(\rho)}$ and $\lambda_2(u, v) = \frac{uv + c(\rho)\sqrt{u^2 + v^2 - c^2(\rho)}}{u^2 - c^2(\rho)}$.

Furthermore, by (1.3) we obtain the following free boundary conditions on $y = \chi(x)$

$$\begin{cases} [Hu]\chi'(x) - [Hv] = 0 \\ [v]\chi'(x) + [u] = 0 \end{cases} \quad (1.8)$$

The main conclusion in this paper can be stated as:

Theorem 1.1. Suppose that the upper part of a curved wedge is given by $y = b(x)$, which satisfies

$$b(0) = 0, \quad b'(0) = b_0 \quad (1.9)$$

$$|(1+x)^k \frac{d^k}{dx^k}(b(x) - b_0 x)| \leq \varepsilon_0 \quad \text{for} \quad 0 \leq k \leq 2, \quad x \geq 0, \quad (1.10)$$

Assume that a uniform supersonic flow parallel to the x -axis comes from minus infinity with velocity $(q_0, 0)$, density $\rho_0 > 0$ satisfying $q_0 > c(\rho_0)$. Additionally, $b_0 > 0$ is assumed to be less than b_* . Then (1.7) with the boundary conditions (1.2) and (1.8) admits a global weak entropy solution with a shock attached at the origin. Moreover, the location of the shock front and the flow field between the shock and the wedge tend to the corresponding ones for the flow past the unperturbed straight wedge $y = b_0 x$ with the decay rate $(1 + x)^{-\delta_0}$, here δ_0 is some constant independent of ε_0 .

Remark 1.1. We emphasize that there are no other discontinuities in our solution besides the main curved shock. This is contrast with the results in [5]. The condition (1.9) comes from the local existence theorem in [2-4] and the condition (1.10) especially gives a restriction on the perturbation of the wedge for large x . Our result demonstrates that the shock which produced by the supersonic flow past a wedge is structurally stable in a global sense.

Remark 1.2. Since the shock is weak, then the isentropic equation (1.7) actually gives a good approximation for the general 2-D stationary compressible Euler equations (see [6-7]).

In order to prove Theorem 1.1, we need to establish some global uniform estimates for the solution and its derivatives by the characteristics method. This is also the spirit of the results in [8]. To obtain the needed uniform estimates, we have to give some detailed computations in terms of the physical properties of shock wave as well as the boundary conditions (1.2) and (1.8).

This note is organized as follows: In section 2, we give a reformulation of problem (1.7). In section 3, we complete the proof of Theorem 1.1.

§2. The reformulation of main problem

In this section, we firstly recall some basic properties for the supersonic flow past the straight wedge $y = b_0 x$ in $x \geq 0$. On the left of shock wave, the flow has the density ρ_0 and the velocity $(q_0, 0)$, between the shock and the fixed boundary, the constant solution (ρ_+, u_+, v_+) can be obtained by use of the shock polar curve (see [1]). Here and below, $u_+ > c_+ = c(\rho_+)$ will be assumed.

Now we list some properties on the solution (ρ_+, u_+, v_+) . Denote by $y = s_0 x$ the equation of shock wave, then the Rankine-Hugoniot condition (1.8) implies

$$\begin{cases} \rho_+ v_+ - s_0(\rho_+ u_+ - \rho_0 q_0) = 0 \\ u_+ + s_0 v_+ - q_0 = 0 \end{cases} \quad (2.1)$$

by use of the fixed boundary condition $v_+ = b_0 u_+$ in (1.2), one can get

$$\begin{cases} u_+ = \frac{q_0}{1+s_0 b_0} \\ v_+ = \frac{b_0 q_0}{1+s_0 b_0} \\ \rho_+ = \frac{s_0 \rho_0 (1+s_0 b_0)}{s_0 - b_0} \end{cases} \quad (2.2)$$

Meanwhile, s_0 is determined from the Bernoulli's law (1.5), namely, s_0 satisfies

$$\frac{(1 + b_0^2) q_0^2}{2(1 + s_0 b_0)^2} + h\left(\frac{s_0 \rho_0 (1 + s_0 b_0)}{s_0 - b_0}\right) \equiv C_0 \quad (2.3)$$

From the book [1], for $u_+ > c_+$, we know that the algebraic equation (2.3) on s_0 has three distinct real roots when b_0 is less than the critical angle b_* . Physically, s_0 takes the intermediate value, it corresponds to the formation of weak oblique shock. Since the shock is weak, without loss of generality we will assume

$$v_+ \leq 2c(\rho_0) \quad (2.4)$$

Additionally, the P.Lax's entropy condition on 2-shock holds:

$$\begin{cases} \lambda_1(u_+, v_+) < s_0 < \lambda_2(u_+, v_+) \\ \frac{c(\rho_0)}{\sqrt{q_0^2 - c^2(\rho_0)}} < s_0 \end{cases} \quad (2.5)$$

From (2.5), one easily gets

$$\frac{s_0 u_+ - v_+}{\sqrt{1 + s_0^2}} < c(\rho_+) \quad (2.6)$$

The physical sense of (2.6) is: across the shock, the normal velocity of supersonic flow becomes subsonic.

Based on the above preparations, now we can reformulate the nonlinear problem (1.7) since the solution of (1.7) is actually a small perturbation of (ρ_+, u_+, v_+) .

Introducing two Riemann invariants as follows

$$\begin{aligned} w_1 &= \arctg \frac{v}{u} + F(\sqrt{u^2 + v^2}) - \arctg b_0 \\ w_2 &= \arctg \frac{v}{u} - F(\sqrt{u^2 + v^2}) - \arctg b_0 \end{aligned} \quad (2.7)$$

where $F'(q) = \frac{\sqrt{q^2 - c^2(H(q^2))}}{qc(H(q^2))}$ and $F(\frac{\sqrt{1+b_0^2}}{1+s_0 b_0} q_0) = 0$, then we can change (1.7) into a 2×2 system on (w_1, w_2) :

$$\begin{cases} \partial_x w_1 + \lambda_1(w) \partial_y w_1 = 0 \\ \partial_x w_2 + \lambda_2(w) \partial_y w_2 = 0 \end{cases} \quad (2.8)$$

where $\lambda_i(w) = \lambda_i(u(w_1, w_2), v(w_1, w_2))$, $i = 1, 2$, $u(w_1, w_2)$ and $v(w_1, w_2)$ are the inverse functions of (2.7), moreover $u(0, 0) = u_+$ and $v(0, 0) = v_+$.

Correspondingly, the boundary condition (1.8) on $y = \chi(x)$ becomes

$$\begin{cases} H(w_1, w_2) v^2(w_1, w_2) + (H(w_1, w_2) u(w_1, w_2) - \rho_0 q_0)(u(w_1, w_2) - q_0) = 0, & (2.9) \\ \chi'(x) = \frac{q_0 - u(w_1, w_2)}{v(w_1, w_2)}, & (2.10) \end{cases}$$

and the fixed boundary condition (1.2) is

$$u(w_1, w_2) b'(x) - v(w_1, w_2) = 0 \quad \text{on} \quad y = b(x) \quad (2.11)$$

By Taylor's formula and the implicit function theorem, for the small $|w_1|$ and $|w_2|$, (2.9) can be rewritten as

$$w_1 = A w_2 + f_1(w_2) \quad \text{on} \quad y = \chi(x) \quad (2.12)$$

where

$$A = \frac{c_+ u_+ + v_+ \sqrt{u_+^2 + v_+^2 - c_+^2}}{c_+ v_+ + u_+ \sqrt{u_+^2 + v_+^2 - c_+^2}} \frac{m_1 - m_2 \lambda_2(u_+, v_+)}{m_1 - m_2 \lambda_1(u_+, v_+)}$$

$$m_1 = -\frac{\rho_+}{c_+^2} v_+^3 + 2\rho_+ v_+ + \frac{\rho_+}{c_+^2} u_+ v_+ (q_0 - u_+)$$

$$m_2 = -\frac{\rho_+}{c_+^2} u_+ v_+^2 + \left(\frac{\rho_+}{c_+^2} u_+^2 - \rho_+\right) (q_0 - u_+) + (\rho_+ u_+ - \rho_0 q_0)$$

and $f_1(0) = f_1'(0) = 0$, $f_1(w)$ is smooth.

Similarly, from the boundary condition (2.11), one can derive

$$w_2 = B w_1 + f_2(b'(x) - b_0, w_1) \quad \text{on} \quad y = b(x) \quad (2.13)$$

where

$$B = \frac{c_+ v_+ + u_+ \sqrt{u_+^2 + v_+^2 - c_+^2}}{c_+ u_+ + v_+ \sqrt{u_+^2 + v_+^2 - c_+^2}} \frac{1 + b_0 \lambda_1(u_+, v_+)}{1 + b_0 \lambda_2(u_+, v_+)}$$

and $f_2(0, 0) = (\partial_2 f_2)(0, 0) = 0$, f_2 is smooth.

Hence in order to prove Theorem 1.1, by the local existence of solution in [2-4], we only need to solve the problem (2.8), (2.10), (2.12), (2.13) with the small initial data $w_i(x, y)|_{x=1}$ ($i = 1, 2$) and $(\chi(x) - s_0 x)|_{x=1}$ in the domain $\{(x, y) : x \geq 1, b(x) \leq y \leq \chi(x)\}$. The smallness means

$$\sum_{|\alpha| \leq 1} \sup_{b(1) \leq y \leq \chi(1)} |\nabla_{x,y}^\alpha w_i(1, y)| \leq C\varepsilon_0, \quad |\chi(1) - s_0| \leq C\varepsilon_0, \quad |\chi'(1) - s_0| \leq C\varepsilon_0 \quad (2.14)$$

where ε_0 is given in Theorem 1.1. We note that (2.14) can be derived from the result on the local existence and stability in [2-4].

§3. The proof of Theorem 1.1

In order to prove Theorem 1.1, as the first step, we need to verify that the condition $|AB| < 1$ holds, where the constants A and B appear in (2.12) and (2.13) respectively. The condition $|AB| < 1$ will lead to the uniform bound of solution to (2.8) in the domain $\{(x, y) : x \geq 1, b(x) \leq y \leq \chi(x)\}$.

Lemma 3.1. In (2.12), $m_1 > 0$ and $m_2 > 0$.

Proof. In terms of the expressions of u_+, v_+ in (2.2), we have

$$m_1 = \frac{\rho_+ v_+}{c_+^2} (2c_+^2 - v_+^2 + u_+ (q_0 - u_+)) > \frac{\rho_+ v_+}{c_+^2} (-v_+^2 + u_+ (q_0 - u_+)) = \frac{\rho_+ v_+}{c_+^2} \frac{b_0 q_0^2 (s_0 - b_0)}{(1 + s_0 b_0)^2} > 0$$

Similarly, we can get

$$m_2 = \frac{\rho_+}{c_+^2} \left((u_+^2 - c_+^2) (q_0 - u_+) - u_+ v_+^2 \right) + (\rho_+ u_+ - \rho_0 q_0)$$

$$= \frac{b_0 \rho_0 q_0}{(s_0 - b_0) c_+^2} \left(s_0 (s_0 - b_0) u_+^2 + (1 - s_0^2) c_+^2 \right)$$

If $s_0 \leq 1$, obviously one has $m_2 > 0$.

If $s_0 \geq 1$, then the entropy condition (2.6) leads to

$$m_2 > \frac{b_0 \rho_0 q_0}{(s_0 - b_0) c_+^2} \left(s_0 (s_0 - b_0) u_+^2 + (1 - s_0^2) \frac{(s_0 - b_0)^2 u_+^2}{1 + s_0^2} \right) > \frac{b_0^2 \rho_0 q_0 u_+^2}{c_+^2} > 0$$

Hence Lemma 3.1 holds.

Lemma 3.2. 1). $1 + b_0 \lambda_1(u_+, v_+) > 0$.

2). Under the condition (2.4), $|AB| < 1$ holds.

Proof. 1). By a direct computation, one has

$$1 + b_0 \lambda_1(u_+, v_+) = \frac{u_+^2 - c_+^2 + b_0 u_+ v_+ - b_0 c_+ \sqrt{u_+^2 + v_+^2 - c_+^2}}{u_+^2 - c_+^2} = \frac{(1 + b_0^2) \sqrt{u_+^2 + v_+^2 - c_+^2}}{\sqrt{u_+^2 + v_+^2 - c_+^2} + b_0 c_+} > 0$$

2). Firstly, under the condition (2.4), we assert

$$m_1 - m_2 \lambda_1(u_+, v_+) > 0 \tag{3.1}$$

Indeed, if $v_+ \leq c_+$, then $\lambda_1(u_+, v_+) \leq 0$, so by lemma 3.1, (3.2) obviously holds. If $v_+ \geq c_+$, then $\lambda_1(u_+, v_+) \geq 0$, moreover by (2.4) we have $v_+ < 2c_+$. A direct computation yields

$$\begin{aligned} m_1 - m_2 \lambda_1(u_+, v_+) &= \frac{\rho_+}{c_+^2} (2c_+^2 v_+ - v_+^3 + c_+ (q_0 - u_+) \sqrt{u_+^2 + v_+^2 - c_+^2} + u_+ v_+^2 \lambda_1(u_+, v_+)) \\ &\quad - (\rho_+ u_+ - \rho_0 q_0) \lambda_1(u_+, v_+) \\ &> \frac{\rho_+ v_+}{c_+^2} (2c_+^2 - v_+^2 + c_+ v_+) + \rho_+ v_+ \left(\frac{u_+ v_+}{c_+^2} - \frac{1}{s_0} \right) \lambda_1(u_+, v_+) \\ &> \frac{\rho_+ v_+}{c_+^2} (c_+ + v_+) (2c_+ - v_+) + \rho_+ v_+ \left(\frac{1}{b_0} - \frac{1}{s_0} \right) \lambda_1(u_+, v_+) > 0 \end{aligned}$$

Now we show $|AB| < 1$.

If $m_1 - m_2 \lambda_2(u_+, v_+) \geq 0$, by (3.1) and Lemma 3.1 then

$$1 - |AB| = \frac{(m_2 + b_0 m_1)(\lambda_2(u_+, v_+) - \lambda_1(u_+, v_+))}{(m_1 - m_2 \lambda_1(u_+, v_+))(1 + b_0 \lambda_2(u_+, v_+))} > 0$$

If $m_1 - m_2\lambda_2(u_+, v_+) < 0$, by (3.1) and Lemma 3.1 then

$$\begin{aligned}
1 - |AB| &= \frac{2m_1 - 2b_0m_2\lambda_1(u_+, v_+)\lambda_2(u_+, v_+) + (b_0m_1 - m_2)(\lambda_1(u_+, v_+) + \lambda_2(u_+, v_+))}{(m_1 - m_2\lambda_1(u_+, v_+))(1 + b_0\lambda_2(u_+, v_+))} \\
&= \frac{2\rho_+v_+}{c_+^2(u_+^2 - c_+^2)(m_1 - m_2\lambda_1(u_+, v_+))(1 + b_0\lambda_2(u_+, v_+))} \left\{ [2c_+^2 + (s_0b_0 - b_0^2)u_+^2](u_+^2 - c_+^2) \right. \\
&\quad - [(1 - b_0^2)(s_0 - b_0)u_+^2 - (s_0 + 2b_0 - \frac{1}{s_0})c_+^2]b_0u_+^2 - [b_0(s_0 - b_0)u_+^2 \\
&\quad \left. - b_0(s_0 - \frac{1}{s_0})c_+^2](b_0^2u_+^2 - c_+^2) \right\} \\
&= \frac{2\rho_+v_+}{(u_+^2 - c_+^2)(m_1 - m_2\lambda_1(u_+, v_+))(1 + b_0\lambda_2(u_+, v_+))} \left\{ [2 + b_0(s_0 + 2b_0 - \frac{1}{s_0}) \right. \\
&\quad \left. + b_0^3(s_0 - \frac{1}{s_0})]u_+^2 - [2 + b_0(s_0 - \frac{1}{s_0})]c_+^2 \right\} \\
&> \frac{2b_0^2\rho_+v_+u_+^2}{(u_+^2 - c_+^2)(m_1 - m_2\lambda_1(u_+, v_+))(1 + b_0\lambda_2(u_+, v_+))} (1 + s_0b_0) > 0
\end{aligned}$$

Therefore $|AB| < 1$ holds.

To prove Theorem 1.1, now we give a uniform estimate on w and its derivatives.

Lemma 3.3. Set $D_T = \{1 \leq x \leq T, b(x) \leq y \leq \chi(x)\}$ for any $T > 1$. If $w \in C^2(D_T)$ satisfies (2.8)-(2.11) and (2.14), then there exist two positive constants C_0 and \tilde{C}_0 independent of ε_0 and T , such that $|\partial_{x,y}^\alpha w_i| \leq \frac{C_0\varepsilon_0}{(1+x)^{\delta_0+|\alpha|}}$ in D_T for $|\alpha| \leq 1$, $i = 1, 2$, and $|\partial_x^j(\chi'(x) - s_0)| \leq \frac{\tilde{C}_0\varepsilon_0}{(1+x)^{\delta_0+j}}$ in $[1, T]$ when $0 \leq j \leq 1$, here $\delta_0 > 0$ is an appropriate constant.

Proof. We shall use the reflected characteristics method to obtain the needed estimates (such a method has been used in Lemma 2.1 of Chapter 5 of [8]). By the local existence result of solution in [2-4] and the continuity induction, we only need to prove

For some constants $C_0 > 0, 0 < \delta_0 < 1$, if $|\partial_{x,y}^\alpha w_i| \leq \frac{C_0\varepsilon_0}{(1+x)^{\delta_0+|\alpha|}}$ in D_T for $|\alpha| \leq 1, i = 1, 2$, then there exist two positive constants $C'_0 < C_0$ and \tilde{C}_0 such that $|\partial_{x,y}^\alpha w_i| \leq \frac{C'_0\varepsilon_0}{(1+x)^{\delta_0+|\alpha|}}$ in D_T for $|\alpha| \leq 1$, and $|\partial_x^j(\chi'(x) - s_0)| \leq \frac{\tilde{C}_0\varepsilon_0}{(1+x)^{\delta_0+j}}$ in $[1, T]$ when $0 \leq j \leq 1$. (3.2)

Below we denote by C various strictly positive constants independent of ε_0 and T . If $(x, y) \in D_T, x < T$, we shall denote by $\gamma_j^-(t, x, y) (j = 1, 2)$ the backward j -th characteristic curve pass the point (x, y) , namely

$$\begin{cases} \frac{d\gamma_j^-(t, x, y)}{dt} = \lambda_j(w(t, \gamma_j^-(t, x, y))), & t \leq x \\ \gamma_j^-(t, x, y)|_{t=x} = y \end{cases} \quad (3.3)$$

By the assumptions in (3.2), one has

$$\left| \frac{d\gamma_j^-(t, x, y)}{dt} - \lambda_j(u_+, v_+) \right| \leq \frac{C\varepsilon_0}{(1+x)^{\delta_0}} \quad \text{in } D_T \quad (3.4)$$

If $\{(t, \gamma_1^-(t, x, y))\} \cap \{(t, z) : z = \chi(t)\} = (\Gamma_1(x, y), \xi_1(x, y))$, $\{(t, \gamma_2^-(t, x, y))\} \cap \{(t, z) : z = b(t)\} = (\Gamma_2(x, y), \xi_2(x, y))$, then from the system (2.8) we have

$$w_i(x, y) = w_i(\Gamma_i(x, y), \xi_i(x, y)) \quad (3.5)$$

If $\{(t, \gamma_1^-(t, \Gamma_2(x, y), \xi_2(x, y)))\} \cap \{(t, z) : z = \chi(t)\} = (\pi_1(x, y), \eta_1(x, y))$ and $\{(t, \gamma_2^-(t, \Gamma_1(x, y), \xi_1(x, y)))\} \cap \{(t, z) : z = b(t)\} = (\pi_2(x, y), \eta_2(x, y))$, then by use of the characteristics method and the boundary conditions (2.12) and (2.13) we get

$$\begin{aligned} |w_2(x, y)| &= |Bw_1(\Gamma_2(x, y), \xi_2(x, y)) + f_2(b'(\Gamma_2(x, y)) - b_0, w_1(\Gamma_2(x, y), \xi_2(x, y)))| \\ &= |Bw_1(\pi_1(x, y), \eta_1(x, y)) + f_2(b'(\Gamma_2(x, y)) - b_0, w_1(\pi_1(x, y), \eta_1(x, y)))| \\ &\leq |ABw_2(\pi_1(x, y), \eta_1(x, y))| + |Bf_1(w_2(\pi_1(x, y), \eta_1(x, y)))| \\ &\quad + |f_2(b'(\Gamma_2(x, y)) - b_0, Aw_2((\pi_1(x, y), \eta_1(x, y)) + f_1(w_2(\pi_1(x, y), \eta_1(x, y)))))| \end{aligned}$$

Noting that for small ε_0 , one has

$$\begin{aligned} d_i - C\varepsilon_0 &\leq \frac{\Gamma_i(x, y)}{x} \leq 1 + c\varepsilon_0 \\ d_1 d_2 - C\varepsilon_0 &\leq \frac{\pi_i(x, y)}{x} \leq d_i + C\varepsilon_0 \end{aligned} \quad (3.6)$$

where $d_1 = \frac{b_0 - \lambda_1(u_+, v_+)}{s_0 - \lambda_1(u_+, v_+)} > 0$ and $d_2 = \frac{\lambda_2(u_+, v_+) - s_0}{\lambda_2(u_+, v_+) - b_0} > 0$.

Assume $|f_2(b'(x) - b_0, w_2)| \leq C_1|b'(x) - b_0|$ for small $|w_2|$, then by assumption in (3.2) and by use of (3.6) and $d_1 d_2 < 1$, we obtain

$$|w_2(x, y)| \leq \left(\frac{|AB|}{(d_1 d_2)^{\delta_0}} C_0 + C_1 + C\varepsilon_0 \right) \frac{\varepsilon_0}{(1+x)^{\delta_0}}$$

Similarly, one can obtain

$$|w_1(x, y)| \leq \left(\frac{|AB|}{(d_1 d_2)^{\delta_0}} C_0 + C\varepsilon_0 \right) \frac{\varepsilon_0}{(1+x)^{\delta_0}}$$

If $\{(t, \gamma_1^-(t, x, y))\} \cap \{(t, z) : z = \chi(t)\} = \emptyset$, or $\{(t, \gamma_2^-(t, x, y))\} \cap \{(t, z) : z = b(t)\} = \emptyset$, or $\{(t, \gamma_1^-(t, \Gamma_2(x, y), \xi_2(x, y)))\} \cap \{(t, z) : z = \chi(t)\} = \emptyset$, or $\{(t, \gamma_2^-(t, \Gamma_1(x, y), \xi_1(x, y)))\} \cap \{(t, z) : z = b(t)\} = \emptyset$, then by (3.4), (2.5) and the initial data (2.14) we can conclude

$$x \leq \bar{C}, |w_i(x, y)| \leq \bar{C}\varepsilon_0$$

where \bar{C} is independent of ε_0 .

Therefore, if we choose $0 < \delta_0 < 1$ such that $\frac{|AB|}{(d_1 d_2)^{\delta_0}} < 1$, and set $C_0 = \max\left\{\frac{2C_1}{1 - \frac{|AB|}{(d_1 d_2)^{\delta_0}}}, 2\bar{C}(1 + \bar{C})^{\delta_0}\right\}$ and $C'_0 = \max\left\{\frac{C_1(1 + \frac{|AB|}{(d_1 d_2)^{\delta_0}})}{1 - \frac{|AB|}{(d_1 d_2)^{\delta_0}}}, \bar{C}(1 + \bar{C})^{\delta_0}\right\}$, then we know (3.2) holds for $|\alpha| = 0$.

At this time, one has

$$|\chi'(x) - s_0| \leq \left| \frac{u(w_1, w_2) - u_+}{v(w_1, w_2)} \right| + \left| \frac{(u_+ - q_0)(v(w_1, w_2) - v_+)}{v_+ v(w_1, w_2)} \right| \leq \frac{\tilde{C}_0 \varepsilon_0}{(1+x)^{\delta_0}}$$

Next we prove (3.2) for $|\alpha| = 1$.

Denote by $Z = \partial_x + b'(x)\partial_y$ the tangent vector field of boundary $y = b(x)$. Set $\bar{w}_i = Zw_i$, $i = 1, 2$, then from (2.8) we get

$$\begin{cases} \partial_x \bar{w}_1 + \lambda_1(w) \partial_y \bar{w}_1 = f_1 \\ \partial_x \bar{w}_2 + \lambda_2(w) \partial_y \bar{w}_2 = f_2 \end{cases} \quad (3.7)$$

where $f_i = b''(x)\partial_y w_i - (\partial_{w_1} \lambda_i \bar{w}_1 + \partial_{w_2} \lambda_i \bar{w}_2)\partial_y w_i$.

From (2.13), one has

$$\bar{w}_2 = (B + (\partial_2 f_2)(b'(x) - b_0, w_1))\bar{w}_1 + (\partial_1 f_2)(b'(x) - b_0, w_1)b''(x) \quad (3.8)$$

To get the boundary condition of \bar{w} on $y = \chi(x)$, one should notice that the vector field $V = \partial_x + \chi'(x)\partial_y$ tangent to $y = \chi(x)$ can be expressed as follows

$$V = \frac{1}{\lambda_i(w) - b'(x)} \{ (\lambda_i(w) - \chi'(x))Z + (\chi'(x) - b'(x))(\partial_x + \lambda_i(w)\partial_y) \}$$

So on the shock $y = \chi(x)$, from (2.12) and (2.8) we have

$$\bar{w}_1 = \frac{(\lambda_2(w) - \chi'(x)(b'(x) - \lambda_1(w)))}{(\lambda_2(w) - b'(x))(\chi'(x) - \lambda_1(w))} (A + f'_1(w_2))\bar{w}_2 \quad (3.9)$$

By the assumptions in Theorem 1.1 and (3.2), we have

$$|f_i(x, y)| \leq C\varepsilon_0^2 \left(\frac{1}{(1+x)^{3+\delta_0}} + \frac{1}{(1+x)^{2+2\delta_0}} \right) \leq \frac{C\varepsilon_0^2}{(1+x)^{2+2\delta_0}} \quad (3.10)$$

Using the notations above, if $\gamma_1^-(t, x, y)$ and $\gamma_2^-(t, x, y)$ both intersect with fixed boundary and shock front, then by the characteristics method, (3.10) and the boundary conditions (3.8), (3.9), one has

$$\begin{aligned} |\bar{w}_2(x, y)| &\leq |\bar{w}_2(\Gamma_2(x, y), \xi_2(x, y))| + \int_x^{\Gamma_2(x, y)} \frac{C\varepsilon_0^2}{(1+s)^{2+2\delta_0}} ds \\ &\leq |B\bar{w}_1(\Gamma_2(x, y), \xi_2(x, y))| + \frac{C\varepsilon_0^2}{(1+x)^{1+2\delta_0}} \\ &\leq |B\bar{w}_1(\pi_1(x, y), \eta_1(x, y))| + \frac{C\varepsilon_0^2}{(1+x)^{1+2\delta_0}} \\ &\leq |AB|d_1d_2|\bar{w}_2(\pi_1(x, y), \eta_1(x, y))| + \frac{C\varepsilon_0^2}{(1+x)^{1+2\delta_0}} \\ &\leq \frac{|AB|}{(d_1d_2)^{\delta_0}} \frac{C_0\varepsilon_0}{(1+x)^{1+\delta_0}} + \frac{C\varepsilon_0^2}{(1+x)^{1+2\delta_0}} \end{aligned}$$

Similarly, we have

$$|\bar{w}_1(x, y)| \leq \frac{|AB|}{(d_1 d_2)^{\delta_0}} \frac{C_0 \varepsilon_0}{(1+x)^{1+\delta_0}} + \frac{C \varepsilon_0^2}{(1+x)^{1+2\delta_0}}$$

Noting $\partial_x w_i = \frac{\lambda_i(w) \bar{w}_i}{\lambda_i(w) - b'(x)}$, $\partial_y w_i = -\frac{\bar{w}_i}{\lambda_i(w) - b'(x)}$ and $\frac{|AB|}{(d_1 d_2)^{\delta_0}} < 1$, then similar to the proof on w in the above we know that (3.2) also holds for $|\alpha| = 1$.

Since $\chi''(x) = \sum_{i=1}^2 \partial_{w_i} \left(\frac{q_0 - u(w_1, w_2)}{v(w_1, w_2)} \right) (\partial_x w_i + \partial_y w_i \chi'(x))$ on $y = \chi(x)$, then by the estimates on w_i and $\partial_x w_i$, $\partial_y w_i$, it is easy to conclude

$$|\chi''(x)| \leq \frac{\tilde{C}_0 \varepsilon_0}{(1+x)^{1+\delta_0}}$$

Therefore (3.2) and Lemma 3.3 are proved.

Proof of Theorem 1.1. Since the local existence of the solution of (2.8)-(2.11) near the origin $(0, 0)$ is achieved in [2-3], while for any given x_0 , the solution of (2.8) with the initial data given on $x = x_0$ as in (2.14) and the boundary conditions (2.9) -(2.11) in $[x_0, x_0 + \frac{C}{\varepsilon_0}]$ can be obtained by use of the characteristics method. Therefore, by Lemma 3.3, we can get the smaller initial data of w on $\frac{C}{\varepsilon_0}$, then the solution can be extended continuously to the whole domain.

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