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A daily rainfall erosivity model for Western Amazonia

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ABSTRACT: Rainfall erosivities as defined by the R factor from the universal soil loss equation were determined for all events during a two-year period at the station La Cuenca in western Amazonia. Three methods based on a power relationship between rainfall amount and erosivity were then applied to estimate event and daily rainfall erosivities from the respective rainfall amounts. A test of the resulting regression equations against an independent data set proved all three methods equally adequate in predicting rainfall erosivity from daily rainfall amount. We recommend the Richardson model for testing in the Amazon Basin, and its use with the coefficient from La Cuenca in western Amazonia.

SOIL erosion is promoted by raindrop impact and water runoff. Regardless of the relative importance of these erosive agents *(24)*, indices have been developed to express erosivity as a function of the kinetic energy of a rainfall event. The most widely used of these indices is the R-factor of the universal soil loss equation (USLE) *(26)*. For a given precipitation event, R is the product of the kinetic energy, E, and the maximum 30-minute rainfall intensity, I_{30}:

$$\mathbf{R=EI_{30}} \qquad [1]$$

where R is expressed in:

MJ•mm/ha•h

In practice E is computed for time intervals of equal intensity:

$$\mathbf{E_i=0.119+0.0873\log_{10}I_i,\ I_i\leq 76} \qquad [2]$$

or

$$\mathbf{E_i=0.283,\ I_i\leq 76} \qquad [3]$$

and summed over all time intervals of an event:

$$\mathbf{E=\Sigma E_iN_i} \qquad [4]$$

where E_i is the kinetic energy per mm of rain for time interval i, in MJ/ha, I_i is its rainfall intensity, in mm/h, and N_i is its rainfall amount in mm. This relationship is based on the drop size distributions of Laws and Parsons *(9)* for rainfall in Washington, D.C. The restriction expressed in equation [3] reflects the principle that the median drop size, and hence the kinetic energy, level off or even diminish slightly beyond a certain intensity *(3, 5, 6, 12, 18)*. The cutoff value of 76 mm/h (3 in/h) was proposed by Wischmeier and Smith *(26)*.

Other erosivity indices have been proposed, the better known ones being AI_m *(7)* and KE *(5)*. AI_m is defined as the product of the amount of rainfall per storm, A, in mm, and its maximum 7.5 minute intensity, I_m, in mm/h. The index then has the unit mm^2/h.

KE is computed according to:

$$\mathbf{KE=29.8\text{-}127.5/I,\ I > 25} \qquad [5]$$

where KE has the same units as E above, and I is the rainfall intensity in mm/h. In practice, KE is computed as outlined above for the R factor, except that time intervals with an intensity below 25 mm/h (1 in/h) are ignored.

Computing such indices requires a continuous record of rainfall intensity. In countries with a well-organized meteorological service, this information is available only from major weather stations; such records are not common in many parts of the world, including the Amazon Basin. At these locations the best temporal resolution of rainfall intensity available is the daily rainfall amount. To provide an input for erosion models, such as the widely-used USLE or its modified versions, a predictive model relating erosivity indices to daily rainfall amount would be desirable. A number of regression equations relating daily rainfall amount to an erosion index have been developed. Some are chiefly of regional interest, but have not been verified or tested in other areas, such as the equations de-

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rived by Roose *(16)*. A notable exception is the model developed by Richardson et al. *(15)*, which has been verified and tested extensively *(2, 4, 22)* on the North American continent.

This model is based on the general relationship between the erosivity index R and the daily or event rainfall N:

$$\mathbf{R=aN^b} \qquad [6]$$

where R and N are defined as above, and a and b are model parameters.

Richardson et al. *(15)* introduced the following restrictions: R is minimal for a given daily rainfall amount N, if this amount is distributed uniformly over the whole 24-hour period under consideration. Hence, $I_i=I_{30}=N/24$, and equations [1], [2], and [4] yield:

$$\mathbf{R_{min}=N^2(0.00364logN-0.000062)} \qquad [7]$$

R is maximal for a given daily rainfall amount N, if all the precipitation occurred in no more than 30 minutes. According to the restrictions expressed in equations [2] and [3], two cases may be distinguished:

$$\mathbf{R_{max}=N^2(0.259+0.1746logN)} \qquad [8]$$
$$\mathbf{N \le 38 \text{ or } I \le 76}$$

where N and I are in units as defined initially, and $I_i=I_{30}=N/0.5$, and

$$\mathbf{R_{max}=0.566N^2 \quad N > 38 \text{ or } I > 76} \qquad [9]$$

with terms and units as previously defined.

Based on R and N data from 11 locations in the United States east of the Rocky Mountains, the parameters a and b were estimated by linear regression of the ln-transformed variables. The b-value was found to be invariant in space and time, and hence assigned an average value of 1.81, while the a-value varied from station to station, and throughout the seasons. Hence, the a-value was determined twice for each station under consideration, once for the cool season, and once for the warm season, according to:

$$\mathbf{R=aN^{1.81}} \qquad [10]$$

This model was found to be operational on the North American continent *(2, 4, 22)*, which encompasses a wide range of types of rain, and hence lends itself to further testing in other areas of the world.

In our study, we provide a rationale for the selection of R as an erosivity index, establish a relationship between R and N based equation [6], apply the Richardson model (equation [10]) to the same data set, and compare the results.

Study methods

Precipitation data were collected at the research station La Cuenca situated in the Selva Central of Peru (75°5'W,10°13'S, 300m amsl). This region is in the western Amazon Basin (Figure 1). Based on a discontinuous record of 14 years from the nearby town of Puerto Bermudez, the mean annual precipitation is 3300 mm (130 in), and the mean annual temperature is 25.5°C (78°F).

A tipping bucket rain gauge was operated from September 1986 to April 1989, and charts were changed daily. All charts from the beginning of the operation through 1988 were analyzed to obtain short-term rainfall intensities, and according to the procedures outlined in *(26)*, the latter yielded 191 erosive precipitation events on which the following analysis is based.

To provide an idea of how representative the study period was, the monthly precipitation of the years 1987 and 1988 at La Cuenca is compared with the average monthly precipitation at Puerto Bermudez (Figure 2). In addition, the maximum daily precipitation in each month is compared to the respective average maximum daily precipitation and its range at Puerto Bermudez (Figure 3).

The parameters a and b from equation [6] first were estimated by a nonlinear regression between R and N, and second by a linear regression between ln R and ln N, following the procedures outlined in *(15)*. Third, the parameter a was estimated according to equation [10]. In all three cases, the sample consisted of all erosive events of the years 1987 and 1988.

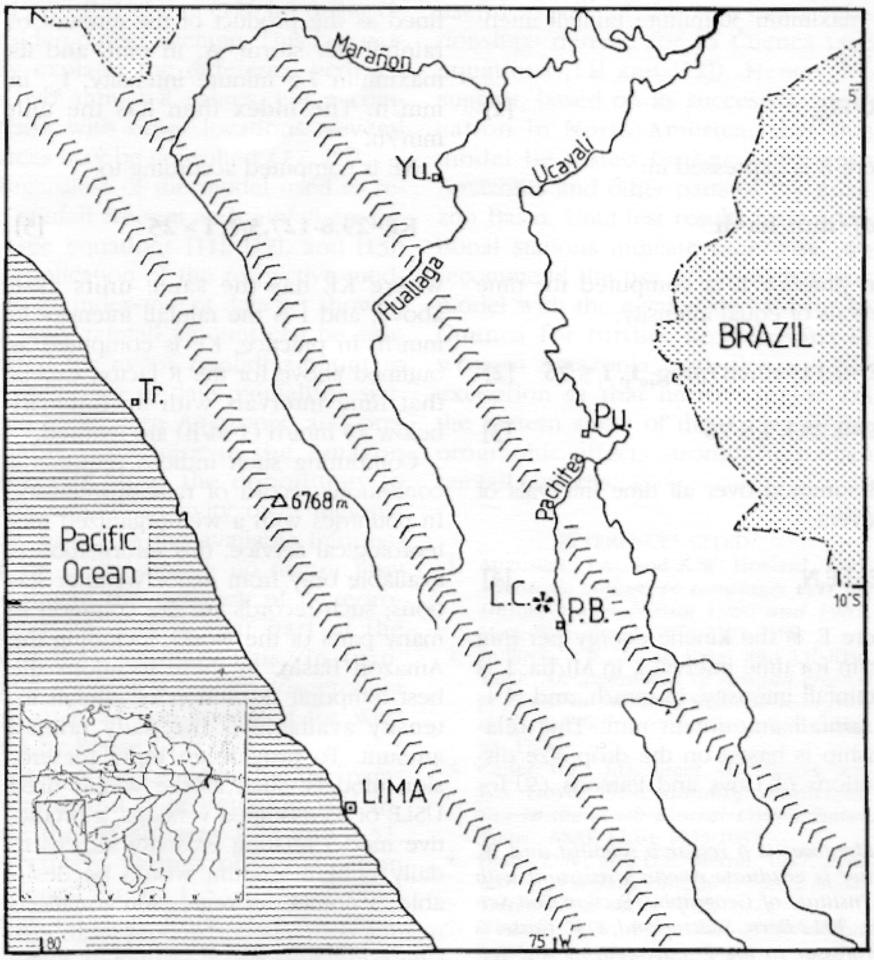

Figure 1. The location of the research catchment La Cuenca in Peru. L.C. = La Cuenca, P.B. = Puerto Bermudez, Pu. = Pucallpa, Yu. = Yurimaguas, Tr. = Trujillo.

The three resulting regression models were then tested against an independent data set. The data set comprised the rainfall amounts of all erosive events and days of the period September through December 1986. The event and daily R values computed in this manner were then compared with the R values for the same period obtained according to Wischmeier and Smith *(26)*. A significance level of 0.05 and 95 percent confidence limits were adopted for all statistical tests and parameter estimates, respectively.

SAS utilities were employed for all data analytical and statistical procedures *(19, 20, 21)*.

Results

Rainfall intensities at La Cuenca. Table 1 summarizes selected short-term rainfall intensities by means of descriptive statistics. The large number of events, on which Table 1 is based, compared to 191 erosive events in the sense of Wischmeier and Smith *(26)*, is due to the arbitrary criterion to separate individual events. We chose one hour to distinguish between two events, i.e. if there was a period of at least one hour during which less than 0.1 mm (.04 in) precipitation was recorded, the rain before and after that period was attributed to two different events. This arbitrary definition of an event does not imply any particular temporal or spatial scale with respect to the rainfall-generating process (convective cell, mesoscale convective complex, synoptic disturbance, etc.). We leave it at the reader's discretion to classify the intensities summarized in Table 1 as "high," "low," or "tropical." The threshold of 76 mm/h (3 in/h) (see equation [3]) was attained just once during the study period. We have no evidence from this station in the humid tropics that would support global statements about the alleged high intensity of "tropical" storms *(6, 8)*.

R values at La Cuenca. Table 2 shows some descriptive statistics of the 191 event R values used for the subsequent regression models. The total of all R values for 1987 is 18,378 MJ•mm/ha•h, and for 1988 is 16,613 MJ•mm/ha•h.

Nonlinear regression. The Gauss-Newton method *(19)* yielded the regression equation:

$$R=1.46N^{1.43} \qquad [11]$$

Table 1. Selected quantiles of rainfall intensity distributions.

Max. Intensity	Max	99%	95%	90%	75%	50%	25%	Min
	mm/h							
I_{60} (n=502)	50	45.6	29.5	24.2	14	6.2	2.3	0.2
I_{30} (n=655)	76	60	40	31	17.4	6.4	2.2	0.2
I_{10} (n=805)	111	69	54	42	24	9	2.4	0.6

Table 2. Selected quantiles of event R values

Max	99%	95%	90%	75%	50%	25%	Min
MJ • mm/ha •h							
1504	1167	821	522	287	120	53	8

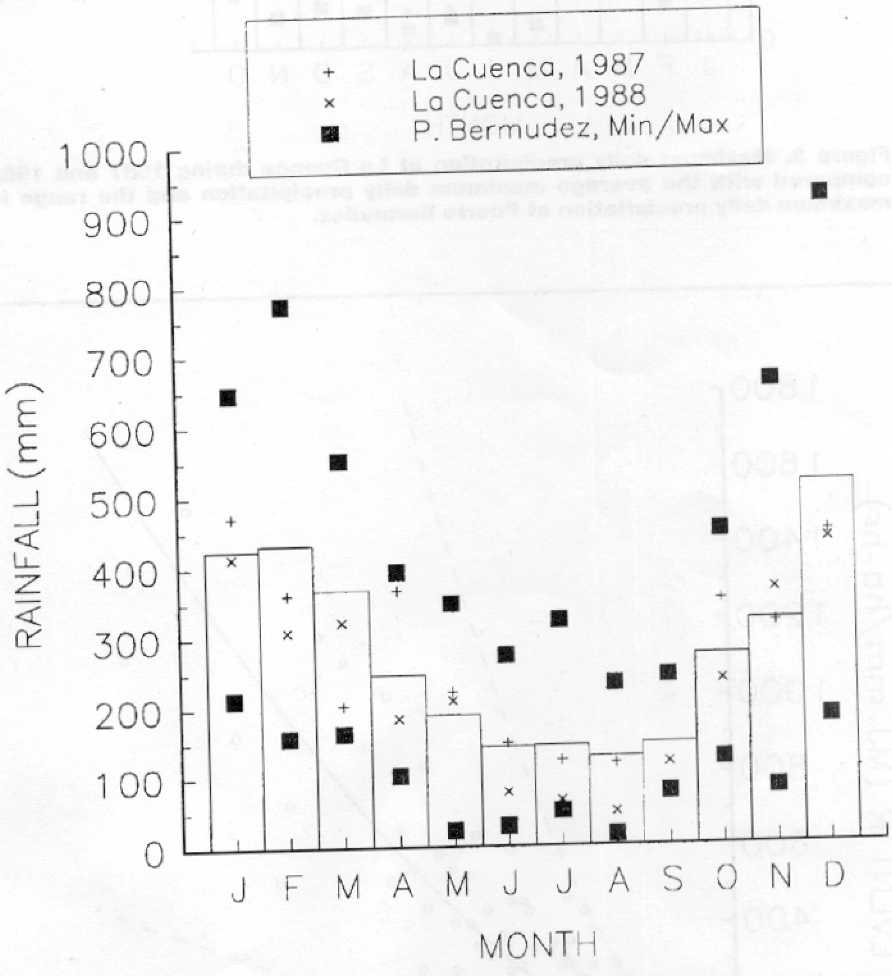

Figure 2. Monthly precipitation at La Cuenca during 1987 and 1988, compared with the average monthly precipitation and the range in monthly precipitation at Puerto Bermudez.

where R is the event erosivity, and N the event rainfall amount. The asymptotic upper and lower confidence limits for the parameters a and b are 0.76 and 2.16, and 1.32 and 1.55, respectively.

We used the two-sided Wilcoxon rank sum test to test the hypothesis that the event R values predicted according to equation [11] are not different from the event R values computed according to Wischmeier and Smith *(26)*, for all erosive events from Sep-

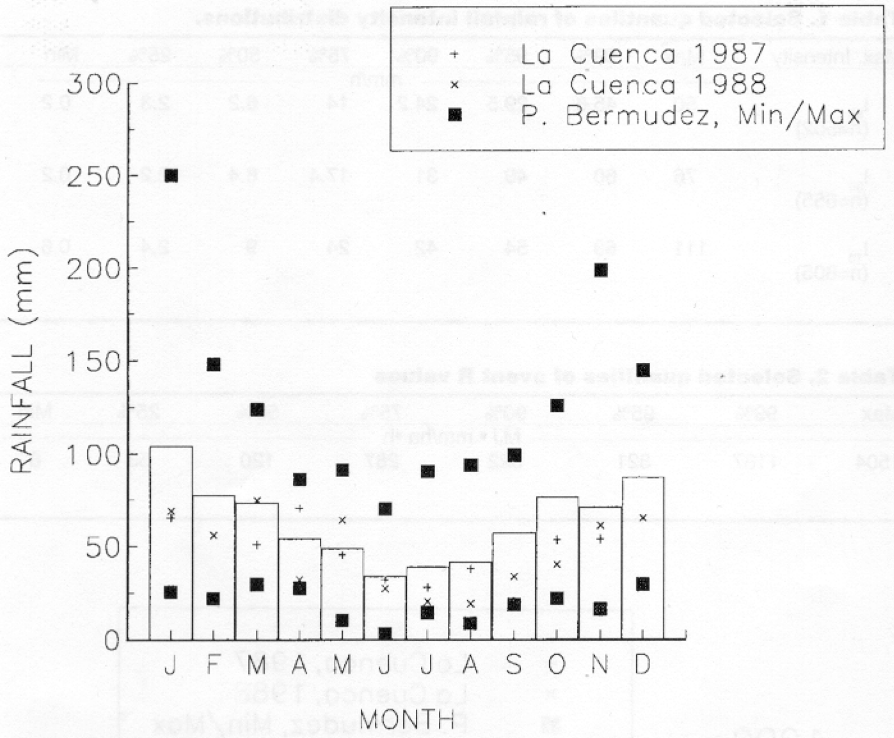

Figure 3. Maximum daily precipitation at La Cuenca during 1987 and 1988 compared with the average maximum daily precipitation and the range in maximum daily precipitation at Puerto Bermudez.

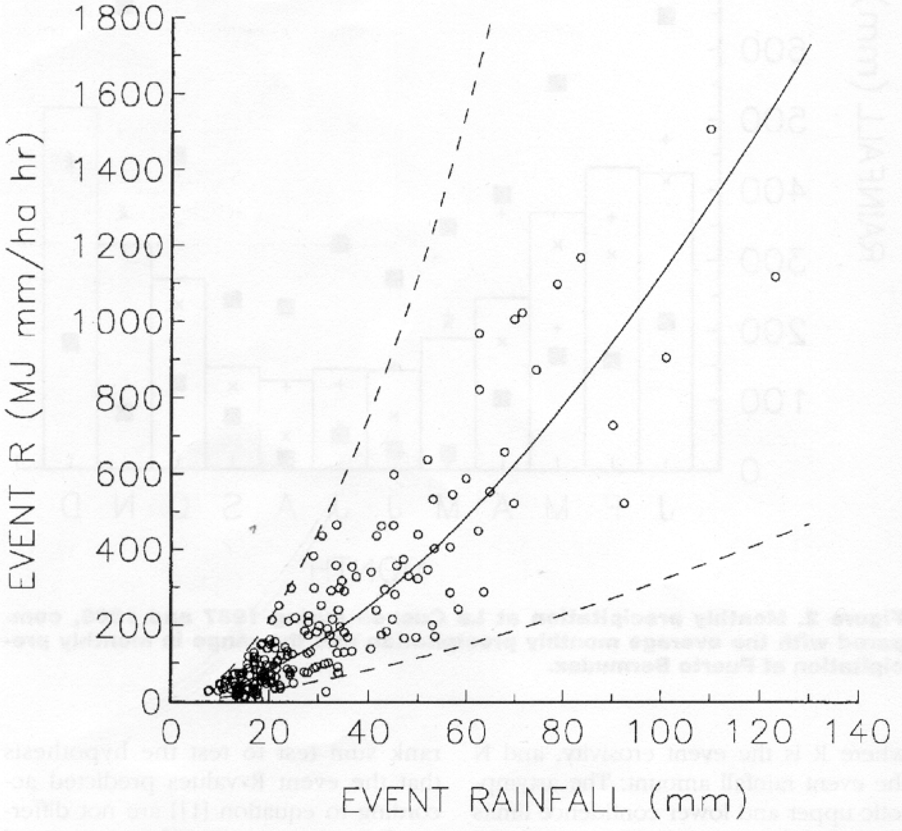

Figure 4. Scatterplot of R values against rainfall amount with the corresponding regression line, based on the regression of ln-transformed variables. The dashed lines represent the confidence band for individual predictions.

tember through December 1986. We selected this nonparametric test because the distribution of the R values is not Gaussian, and because the test involves a pair-wise comparison. The above hypothesis, i.e. the two samples are not different, was accepted (test statistic z=1.08, p=0.279).

To test the same hypothesis as above, but with the daily rainfall amount replacing the event rainfall amount as independent variable, we applied the two-sided Wilcoxon-Mann-Whitney test. This nonparametric test is valid for unpaired samples, as is the case in this comparison. Again, the hypothesis was accepted (test statistic z=-0.97, p=0.332). We concluded that the daily rainfall amount predicts the R value just as well as the event rainfall amount does if equation [11] is used.

Linear regression of ln-transformed variables. The ordinary least-square method yielded

$$\ln R = -0.54 + 1.64 \ln N \quad [12]$$

Upon re-transformation, the a-value of 0.58 is obtained; the b-value is 1.64. The respective confidence limits are 0.35 and 0.98, and 1.48 and 1.80, and the coefficient of determination is 0.71. Figure 4 displays the pertinent scatter plot, the regression line, and its confidence limits after re-transformation.

Strictly speaking, a test of this regression equation as above can be performed only with respect to ln-transformed R values computed according to Wischmeier and Smith *(26)*. For the sake of comparison with the other regression models presented in this paper, however, we performed the previously mentioned non-parametric tests. Based on the Wilcoxon rank sum test, the hypothesis was accepted that the event values predicted from equation [12] are not different from the respective R values computed according to Wischmeier and Smith *(26)* for all erosive events from September through December 1986 (test statistic z=0.38, p=0.703).

The Richardson model. Accepting the fixed b-value of 1.81 and following the procedures outlined in Richardson et al. *(15)*, we obtained the following relationship:

$$R = 0.335 N^{1.81} \quad [13]$$

Based on the two-sided Wilcoxon

rank sum test, the predicted and computed R values for the erosive events from September through December 1986 were not different (test statistic z=0.38, p=0.703). If daily rainfall amounts were used in equation [13], instead of event rainfall amounts, to predict the respective R values, the Wilcoxon-Mann-Whitney test supported the hypothesis that predicted and computed R values are not different for the test sample (test statistic z=0.41, p=0.682). As was the case with the non-linear regression model, the daily rainfall amount is as good a predictor of the R value as the event rainfall amount. Figure 5 shows a plot of predicted vs. observed R values for the test period September through December 1986.

Temporal variation of the a-value. Richardson et al. *(15)* observed a strong seasonal dependence of the a-value for stations east of the Rocky Mountains. To investigate the possibility of such a dependence for La Cuenca, we calculated monthly a-values by linear regression of the ln-transformed variables N and R. Since we could not detect a trend in the monthly b values, a constant b-value of 1.64 (see equation [12]) was adopted to compute monthly a-values. The result is shown in Figure 6, in comparison with one of the 11 stations used by Richardson et al. *(15)*. We conclude that the a-value at La Cuenca does not follow a seasonal trend.

Discussion

On the basis of rainfall intensities alone, we conclude that the R factor is a suitable erosivity index for this western Amazonian location, for these intensities certainly do not exceed what might be expected for the U.S. east of the Rocky Mountains, for which USLE was developed. This is not to say that the drop-size distribution of rain drops falling at La Cuenca equals that of Washington, D.C., on which the R factor is based. A recent investigation by McIsaac *(13)*, however, did not detect a conclusive latitudinal trend in median raindrop diameters, which might justify the assumption of a higher kinetic energy load of precipitation events at La Cuenca. We suggest that instead of emphasizing latitude as a factor in the assessment of rainfall erosivity, i.e. "tropical" versus "temperate", more attention should be paid to the influence of rainfall generating mechanisms on drop-size distributions

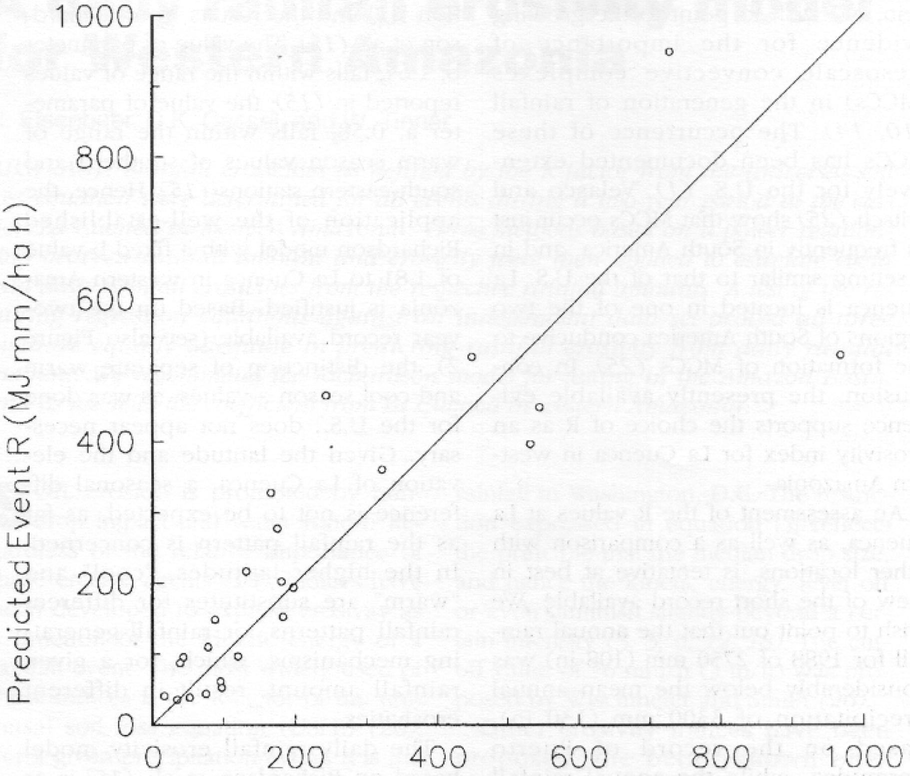

Figure 5. Scatterplot of predicted against observed R values. This independent test data set consists of all erosive events from September to December 1986. The predicted R values are based on equation [13].

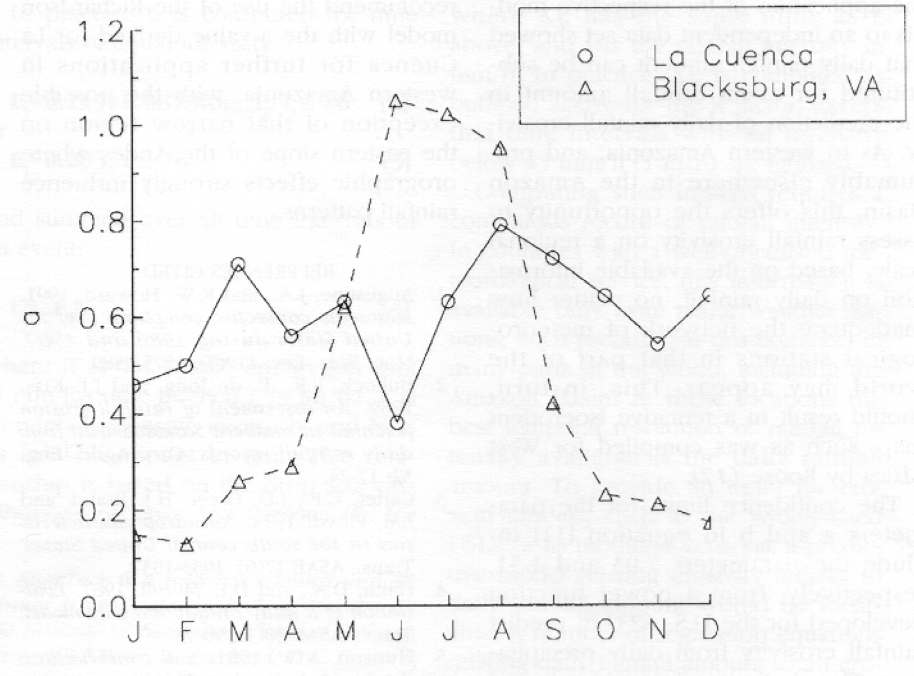

Figure 6. Monthly a-values for La Cuenca compared to a-values for Blacksburg, VA [15].

(11), and hence on erosivity. In this vein, we wish to point to the growing evidence for the importance of mesoscale convective complexes (MCCs) in the generation of rainfall *(10, 14)*. The occurrence of these MCCs has been documented extensively for the U.S. *(1)*. Velasco and Fritsch *(25)* show that MCCs occur just as frequently in South America, and in a setting similar to that of the U.S. La Cuenca is located in one of the two regions of South America conducive to the formation of MCCs *(25)*. In conclusion, the presently available evidence supports the choice of R as an erosivity index for La Cuenca in western Amazonia.

An assessment of the R values at La Cuenca, as well as a comparison with other locations, is tentative at best in view of the short record available. We wish to point out that the annual rainfall for 1988 of 2750 mm (108 in) was considerably below the mean annual precipitation of 3300 mm (130 in), based on the record of Puerto Bermudez, while the annual rainfall for 1987 of 3190 mm (126 in) might be considered near average. This fact partially explains the difference between the two annual R values. For a comparison with other locations, several sources may be consulted *(17, 26)*.

Regardless of the model used to relate rainfall amount and rainfall erosivity (see equations [11], [12], and [13]) the application of the respective models to an independent data set showed that daily rainfall amount can be substituted for event rainfall amount in the prediction of daily rainfall erosivity. As to western Amazonia, and presumably elsewhere in the Amazon Basin, this offers the opportunity to assess rainfall erosivity on a regional scale, based on the available information on daily rainfall, no matter how inadequate the network of meteorological stations in that part of the world may appear. This, in turn, should result in a tentative isoerodent map, such as was compiled for West Africa by Roose *(17)*.

The confidence limits for the parameters a and b in equation [11] include the parameters 1.03 and 1.51, respectively, from a power function developed for the U.S. *(23)* to predict rainfall erosivity from daily precipitation. The similarity, if not identity, of these parameters suggests that the relationship derived for the U.S. is also applicable at La Cuenca. A similar argument can be made based on equation [12] and the results from Richardson et al. *(15)*. The value of parameter b, 1.64, falls within the range of values reported in *(15)*; the value of parameter a, 0.58, falls within the range of warm season values of southern and south-eastern stations *(15)*. Hence, the application of the well-established Richardson model with a fixed b-value of 1.81 to La Cuenca in western Amazonia is justified. Based on the two-year record available (see also Figure 2), the distinction of separate warm and cool season a-values, as was done for the U.S., does not appear necessary. Given the latitude and the elevation of La Cuenca, a seasonal difference is not to be expected, as far as the rainfall pattern is concerned. In the higher latitudes, "cool" and "warm" are substitutes for different rainfall patterns, or rainfall-generating mechanisms, which, for a given rainfall amount, result in different erosivities.

The daily rainfall erosivity model based on Richardson et al. *(15)* is as useful as any of the two other relationships derived for La Cuenca (see equations [11] and [12]). Hence, we suggest, based on its successful application in North America, that this model be tested further in western Amazonia and other parts of the Amazon Basin. Until test results from additional stations indicate otherwise, we recommend the use of the Richardson model with the a-value derived for La Cuenca for further applications in western Amazonia, with the possible exception of that narrow region on the eastern slope of the Andes where orographic effects strongly influence rainfall patterns.

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