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NONLINEAR INTERACTION OF FARLEY-BUNEMAN WAVES

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Abstract. The nonlinear interaction of waves excited by the modified two-stream instability (Farley-Buneman instability) is considered. It is found that, during the linear stage of wave growth, the enhanced pressure of the high-frequency part of the waves locally generates a ponderomotive force. This force acts on the plasma particles and redistributes them. Thus an additional electrostatic polarization field occurs, which influences the low-frequency part of the waves. Then, the low-frequency waves also cause a redistribution of the high-frequency waves. In the paper, a self-consistent system of equations is obtained, which describes the nonlinear interaction of the waves. It is shown that the considered mechanism of wave interaction causes a nonlinear stabilization of the high-frequency waves' growth and a formation of local density structures of the charged particles. The density modifications of the charged particles during the non-linear stage of wave growth and the possible interval of aspect angles of the high-frequency waves are estimated.

1. Introduction

Recent high-resolution VHF radar and satellite measurements showed that in the auroral plasma a great variety of moving small-scale plasma structures occurs. The auroral radio scatter experiments EISCAT and STARE reveal that in the E-region of the auroral ionosphere rather intensive electrostatic structures form, which are connected with an essential modification of the density distribution of the charged background particles. According to a widespread point of view, these plasma structures can be a consequence of the excitation of the modified two-stream or Farley-Buneman (FB) plasma turbulence (Fejer and Kelley 1980, Schlegel 1983, Pfaff et al. 1984, Haldoupis and Schlegel 1990, Schlegel et al. 1990, Schlegel and Thiemann 1994, Sahr and Fejer 1996).

And indeed, the linear theory of the FB instability effectively explains many of the observed characteristics of the behaviour of radar echoes, e.g. the conditions necessary for the onset of these waves, the phase velocity of the waves, and their dominant wavelengths. Besides a lot of features of auroral backscattering data can be explained by the nonlinear theory of the FB instability (Volosevich et al. 1982, Sudan 1983, Hamza and St. Maurice 1993). But many problems remained unclear, in particular, the generation of the high-frequency small-scale (16 cm) irregularities, or how to explain the existence of auroral echoes with large aspect angles of 5^o, that means with angles of 5^o between the propagation direction and the plane perpendicular to the magnetic field. It is unclear why the waves propagate perpendicular to the electron drift velocity.

Within the classical linear theory, the following dispersion equation of the FB-waves in the auroral E-region is found (Volosevich et al. 1982, Dimant and Sudan 1995)

$$\omega = \frac{\vec{k}\vec{V}_{oe} + \hat{\psi}\vec{k}\vec{V}_{oi}}{1 + \psi(1 + k_{\|}^2\omega_{ce}^2/(k_{\perp}^2\nu_e^2) + \eta_i k^2)}, \quad \hat{\psi} = \psi\left(1 + \frac{k_{\|}^2}{k_{\perp}^2}\frac{\omega_{ce}^2}{\nu_e^2}\right) \tag{1}$$

where $\psi = \nu_e\nu_i/\omega_c^2$ is the altitude factor, $\omega_c^2 = \omega_{ce}\omega_{ci}$, $\eta = \alpha_\eta v_{ti}^2/\nu_i$ is the ion dynamical viscosity, α_η is a dimensionless constant depending on the type of collisions in the plasma. $k_{\|}$ and $k_{\perp}$ are the components of the wave vector $\vec{k}$ parallel and perpendicular to the magnetic field, $v_{ti} = (\gamma_i T_i/m_i)^{1/2}$, T_i represents the ion temperature, ν_e and ν_i are the collision frequencies of the electrons and ions with the neutrals, ω_{ce} and ω_{ci} designate the corresponding gyrofrequencies, and $\vec{V}_{oe}$ and $\vec{V}_{oi}$ represent the electron and ion drift velocities which are determined by the mean electrostatic field $\vec{E}_o$. The equation (1) is valid for plasma regions with magnetized electrons $\nu_e \ll \omega_{ce}$ and nonmagnetized ions $\nu_i \gg \omega_{ci}$, where the conditions $\omega < \nu_i$, $\omega < \nu_e$ are satisfied, and the aspect angle is small, $k_{\|}^2/k_{\perp}^2 \ll \nu_e^2/\omega_{ce}^2 \approx 10^{-5}$ for $\nu_e \approx 3\cdot 10^4$ s^{-1}, $\omega_{ce} \approx 6\cdot 10^6$ s^{-1}.

¿From the linear theory, one can conclude that the condition for the frequency of the FB waves $\omega < \nu_i$ is not satisfied if the irregularities have scales $L < 2\pi V_{oe}/\nu_i$ (At altitudes of $h \approx 100$ km, at which $\nu_i \approx 2\cdot 10^3$ and $V_{oe} \approx 6\cdot 10^2$ ms^{-1}, it follows $L < 2$ m). Besides, in the upper E-region also plasma conditions with $\omega \gtrsim \nu_e$ are possible. In the works (Lee and Kennel 1971, Schlegel 1994), it was found that the linearly growing wave mode excited by electron-neutral collisions, has a frequency $\omega < \omega_c$.

Further, considering the action of neutral winds in the E-region, it was shown that the dispersion equation has three solutions (Meister 1995, Liperovsky et al. 1996). Two wave modes are damped, and the third mode with frequencies, being about one order smaller than the frequencies of the damped waves, is linearly unstable. The unstable mode has wavelengths k of about 1/m $\lesssim k \lesssim$70/m, and the maximum growth rates amounting to about 400 Hz occur at $k \approx 27$/m$< 1/r_D \approx 100$/m (r_D is the Debye radius). The phase velocity at maximum wave growth was found to be about 500 m^{-1}. The unstable growing wave mode was excited if both electron-neutral collisions and an electron drift exist. In sporadic E-regions, the electron drift V_{oe} may be generated by neutral winds.

When kinetic effects are taken into account, such as Landau damping at the ions, the interval of possible values of the wave number k is limited. Within the frame of magnetohydrodynamics, Landau damping at the ions is equivalent to the consideration of dynamical viscosity in the dispersion equation of the waves (Gershman et al. 1984, Volosevich and Galperin 1997). Within the kinetic description (Volosevich 1978) follows, that the FB instability can be excited if the conditions $\omega > \nu_i$ and $\omega \lesssim \nu_e$ are satisfied. In the case $\omega > \nu_e$, instead of FB modes, lower hybrid waves with $\omega \approx \omega_c$ may occur.

The investigation of the nonlinear interaction of FB waves showed, that within a three-dimensional model and taking into account dispersive effects, the decay conditions $\vec{k} = \vec{k}_1 + \vec{k}_2$ and $\omega = \omega_1 + \omega_2$ are satisfied (Volosevich 1982). Then, for instance for large aspect angles $k_{\|}^2/k_{\perp}^2 > \nu_e^2/\omega_{ce}^2$ or $|\cos\varphi| \lesssim \pi/2$ (φ is the angle between the electron drift velocity and the wave propagation), it follows that a wave decay from the region of linear wave generation into the region of linear wave damping is possible. The frequency of the waves occurring as a result of the nonlinear interaction may be essentially smaller than the frequency of the linear waves.

Thus one may draw the following conclusions:

1) The FB instability can be excited in a wide frequency interval. The low frequency branch of the instability satisfies Eq. (1), but the high-frequency branch of the instability is not described by Eqs. (1).
2) Under real ionospheric conditions, for instance in the E-region auroral ionosphere, simultaneously with the low-frequency FB waves also high-frequency waves with $\omega \lesssim \omega_c$ may exist. Although their nature is different, these waves are similar to the electrostatic lower hybrid waves having a dispersion relation (see e.g. (Musher and Sturman 1975), (Shapiro et al. 1993))

$$\omega \approx \omega_{LH} \left(1 + \frac{k^2 R^2}{2} + \frac{m_i}{2m_e} \frac{k_{\|}^2}{k^2} \right), \quad R^2 = \frac{3T_i}{m_i \omega_{LH}^2} + \frac{2T_e}{m_e \omega_{ce}^2} \frac{\omega_{pe}^2}{(\omega_{pe}^2 + \omega_{ce}^2)}, \tag{2}$$

where $\omega_{LH}^2 = \omega_{pi}^2/(1 + \omega_{pe}^2/\omega_{ce}^2)$, and, for dense plasma with $\omega_{pe}^2 \gg \omega_{ce}^2$ and $R^2 \approx (3T_i + 2T_e)/(m_i \omega_c^2)$, $\omega_{LH} \approx \omega_c$. ω_{pe} and ω_{pi} are the plasma frequencies of the electrons and ions, respectively. Like the FB waves, the electrostatic lower hybrid waves also satisfy the condition $k_\perp \gg k_\|$. For example, for FB waves with $\lambda = 16$ cm, $k \approx 40$ m^{-1} and $V_o = 600$ ms^{-1}, one has $\omega \approx 2.4 \cdot 10^4 \approx \omega_c$.
3) Linearly generated high-frequency FB waves may nonlinearly interact with low-frequency FB waves which are also excited during the linear stage of wave generation.

2. Derivation of the wave equations

2.1. Equations for high-frequency waves

In this work the nonlinear interaction of high-frequency (HF) waves and low-frequency (LF) waves is considered. The frequencies of the HF waves satisfy the conditions $\omega > \nu_i$ and $\omega \lesssim \omega_c$, and the LF waves possess frequencies which are much lower than the frequencies of the HF waves. The nature of the interaction consists in the fact that sufficiently intensive high-frequency waves cause wave pressure and redistribute the density of the LF variations.

The kinetic description of the nonlinear interaction is rather difficult. Thus here magnetohydrodynamics is used to study the physical mechanism of the wave interaction.

The evolution of nonlinear FB waves may be described by a system of quasi-hydrodynamic equations of motion of the charged particles, together with the continuity equation and the Poisson equation for the electric field. It is supposed that the electrons are magnetized $\nu_e \ll \omega_{ce}$ while the ions are unmagnetized, $\nu_i > \omega_{ci}$.

$$\frac{d\vec{v}_e}{dt} = -\frac{e\vec{E}}{m_e} - \omega_{ce}[\vec{v}_e, \vec{e}_z] - \nu_e \vec{v}_e - v_{te}^2 \nabla \ln n_e, \tag{3}$$

$$\frac{d\vec{v}_i}{dt} = \frac{e\vec{E}}{m_i} - \nu_i \vec{v}_i - v_{ti}^2 \nabla \ln n_i + \eta_i \Delta \vec{v}_i, \tag{4}$$

$$\frac{\partial n_\alpha}{\partial t} + \nabla(n_\alpha \vec{v}_\alpha) = 0, \tag{5}$$

$$\nabla \vec{E} = \frac{e}{\varepsilon_o}(n_i - n_e), \tag{6}$$

where the used designations are standard, $\omega_{ce} = |e|B_o/m_e$, $\vec{B}_o = B_o\vec{e}_z$, $v_{t\alpha}^2 = k_B T_\alpha/m_\alpha$. T_α, m_α, n_α, v_α, ν_α are the temperature, mass, density, velocity and collision frequency of the particles of sort α, $\alpha = e$ - electron, $\alpha = i$ - ion. The electric field of a high-frequency wave is supposed to have the following form

$$\tilde{\vec{E}} = \vec{E}(\vec{r}, t)e^{-i\omega_o t} + c.c.$$

Here the fast time dependence is separated by the factor $e^{-i\omega_o t}$, and the slow time dependence is included in the complex amplitude $\vec{E}(\vec{r}, t)$. Then, from Eqs. (3, 4), one obtains

$$\vec{v}_{e\perp} = \frac{i\omega_e}{B\omega_{ce}}\vec{E}_\perp + \frac{1}{B}[\vec{E}, \vec{e}_z] + i\frac{\omega_e}{\omega_{ce}^2}v_{te}^2\nabla_\perp N_e - \frac{v_{te}^2}{\omega_{ce}^2}[\nabla N_e, \vec{e}_z] - \frac{1}{B\omega_{ce}}\frac{\partial \vec{E}_\perp}{\partial t}, \tag{7}$$

$$\vec{v}_{e\parallel} = \frac{e\vec{E}_\parallel}{i\omega_e m_e} + v_{te}^2\frac{\nabla N_e}{i\omega_e}, \tag{8}$$

$$\vec{v}_i = \frac{ie\vec{E}}{m_i\omega_i} - i\frac{v_{te}^2\nabla N_i}{\omega_o} - \eta_i\frac{e}{m_i\omega_i^2}\Delta\vec{E} + \frac{e}{m_i\omega_i^2}\frac{\partial \vec{E}}{\partial t}. \tag{9}$$

where $\omega_e = \omega_o + i\nu_e$, $\omega_i = \omega_o + i\nu_i$, $N_\alpha = (n_\alpha - n_o)/n_o$

¿From Eq. (3), one finds the following equation for the high-frequency density perturbations

$$N_i = \frac{1}{i\omega_o}\frac{\partial N_i}{\partial t} + \frac{e}{im_i\omega_o\omega_i^2}\frac{\partial \nabla\vec{E}}{\partial t} + \frac{e}{m_i\omega_i\omega_o}\nabla\vec{E} - \frac{v_{ti}^2}{\omega_o\omega_i}\Delta N_i - \eta_i\frac{e}{im_i\omega_o\omega_i^2}\Delta\nabla_\perp\vec{E}, \tag{10}$$

$$N_e = \frac{1}{i\omega_o}\frac{\partial N_e}{\partial t} - \frac{1}{iB\omega_o\omega_e}\frac{\partial \nabla\vec{E}_\perp}{\partial t} + \frac{\vec{v}_o\nabla N_e}{i\omega_o} + \frac{e\omega_e}{\omega_o m_e\omega_{ce}^2}\Delta_\perp\vec{E} - \frac{e}{m_e\omega_o\omega_e}\nabla_\parallel\vec{E}_\parallel \tag{11}$$
$$+\frac{\omega_e v_{te}^2}{\omega_o\omega_{ce}^2}\Delta_\perp N_e - \frac{v_{te}^2}{\omega_o\omega_e}\Delta_\parallel N_e + \frac{1}{i\omega_o B}[\nabla N_s, \vec{E}]\cdot\vec{e}_z.$$

Here

$$\Delta_\perp = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \quad \Delta_\parallel = \frac{\partial^2}{\partial z^2}, \quad \nabla_\perp = \frac{\partial}{\partial x}\vec{e}_x + \frac{\partial}{\partial y}\vec{e}_y.$$

N_i, N_e, $\vec{E}$ designate complex amplitudes. In the Eqs. (10, 11) the main nonlinear contributions caused by the density disturbances because of low-frequency waves N_s is considered.

Besides the constant electron drift velocity $\vec{v}_o$ caused by the constant mean electric field is taken into account. The last term in Eq. (11) describes a vector nonlinearity which occurs because of the electron drift motion. Only in the case of two-dimensional perturbations, this term is a non-vanishing one. In a one-dimensional model it vanishes.

Further, it will be taken into account that during the generation of high-frequency waves with $\omega > \nu_i$ and $\omega < \omega_c$ a local disturbance of the quasi-neutrality condition of the plasma is possible. The investigation of the high-frequency part of the FB waves showed, that their wave frequencies and the growth rates depend on the density of the charged particles of the background plasma. This means, that the high-frequency waves have a dispersion which occurs because of the disturbance of the quasi-neutrality of the plasma.

For convenience, the Eqs. (10, 11) can be transformed into the relations

$$N_i = \frac{1}{i\omega_o}\frac{\partial N_i}{\partial t} + a_i\nabla\vec{E} + b_i\Delta N_i + \tilde{\eta}_i\Delta\nabla_\perp\vec{E} + \frac{c_i}{i\omega_o}\frac{\partial}{\partial t}\nabla\vec{E}, \tag{12}$$

$$N_e = \frac{1}{i\omega_o}\frac{\partial N_e}{\partial t} + a_{e\perp}\nabla_\perp\vec{E} + b_{e\perp}\Delta N_e + \vec{\sigma}\nabla N_e + a_{e\|}\nabla\vec{E}_\| \tag{13}$$

$$+\frac{c_e}{i\omega_o}\frac{\partial}{\partial t}\nabla_\perp E + b_{e\|}\Delta_\| N_e + \frac{1}{i\omega_o B}[\nabla N_s, \vec{E}]\cdot\vec{e}_z,$$

where

$$a_i = \frac{e}{m_i\omega_o\omega_i}, \quad b_i = -\frac{v_{ti}^2}{\omega_i\omega_o}, \quad \tilde{\eta}_i = \frac{i\eta_i e}{m_i\omega_o\omega_i^2}, \tag{14}$$

$$a_{e\perp} = \frac{e\omega_e}{m_e\omega_o\omega_{ce}^2}, \quad b_{e\perp} = \frac{\omega_e v_{te}^2}{\omega_o\omega_{ce}^2}, \quad a_{e\|} = -\frac{e}{m_e\omega_o\omega_e}, \quad b_{e\|} = -\frac{v_{te}^2}{\omega_o\omega_e},$$

$$\vec{\sigma} = \frac{\vec{v}_o}{i\omega_o}, \quad c_i = \frac{e}{m_i\omega_i^2}, \quad c_e = \frac{e}{m_i\omega_c^2}.$$

Subtracting Eq. (13) from Eq. (12), and using the Poisson equation (6), one gets

$$-\frac{ic}{\omega_o}\frac{\partial}{\partial t}\nabla_\perp\vec{E} + (a_\perp - \lambda)\nabla_\perp\vec{E} + b_{e\perp}\Delta N_e + (b_i\lambda + \tilde{\eta}_i)\Delta\nabla_\perp\vec{E} - \vec{\sigma}\nabla N_e \tag{15}$$

$$-a_{e\|}\nabla_\|\vec{E}_\| - b_{e\|}\Delta_\| N_e + \frac{1}{i\omega_o B}[\nabla N_s, \vec{E}]\cdot\vec{e}_z = 0,$$

where $c = c_i - c_e + \lambda$, $a_\perp = a_i - a_{e\perp}$, $b_\perp = b_i - b_{e\perp}$, and $\lambda = e/(m_i\omega_{oi}^2)$.

The relation between the electrostatic field and the fluctuations of the electron density can be found from Eqs. (12, 13). In linear approximation, neglecting small dispersive contributions and assuming $k_\|^2/k_\perp^2 \ll 1$, one has

$$bN_e = (\beta + \lambda b_e)\nabla\vec{E} - \lambda b_i b_e\Delta\nabla_\perp\vec{E} + b_i a_i(\vec{\sigma}\nabla)\nabla\vec{E}. \tag{16}$$

Further, using Eq. (16), Eq. (15) can be transformed into

$$-\frac{i\tilde{c}}{\omega_o}\frac{\partial}{\partial t}\nabla_\perp\vec{E}+\tilde{a}\nabla_\perp\vec{E}+\tilde{b}\Delta\nabla_\perp\vec{E}-\tilde{\sigma}\nabla_\perp\vec{E}+\beta_\|\nabla_\|\vec{E}_\|=\frac{\omega_o}{i\omega_{ci}}[\nabla N_s,\vec{E}]\cdot\vec{e}_z, \qquad (17)$$

$$\tilde{a}=\frac{a_\perp-\lambda}{\alpha},\quad \tilde{\sigma}=\frac{a_i-\lambda}{\alpha},\quad \tilde{c}=\omega_o^2\left(\frac{1}{\omega_i^2}+\frac{1}{\omega_{oi}^2}+\frac{1}{\omega_c^2}\right),\quad \omega_{oi}^2=\frac{e^2n_o}{\varepsilon_o m_i},$$

$$\beta=a_ib_e-a_eb_i,\quad \alpha\beta_\|=a_i-a_{e\|},\quad \alpha\tilde{b}=\tilde{\eta}_i+(b_i+b_e)\lambda+\beta-\tilde{\sigma}^2,\quad \alpha=\frac{e}{m_i\omega_o^2}. \qquad (17a)$$

The frequency ω_o will be determined by the condition

$$\tilde{a}\nabla_\perp E-(\vec{\sigma}\nabla)\nabla\vec{E}=0 \qquad (18)$$

which corresponds to the dispersion equation. Substituting the expressions Eq. (14) for the coefficients into Eq. (18), follows

$$\frac{\omega_{oi}^2}{\omega_o^2}\left(\frac{\omega}{\omega_i}-\frac{\omega_e\omega_o}{\omega_c^2}-\frac{kv_o}{\omega_i}\right)-\left(1+\frac{kv_o}{\omega_o}\right)=0. \qquad (19)$$

Then, for the low-frequency waves follows

$$\frac{\omega_o-kv_o}{\omega_i}=\frac{\omega_e\omega_o}{\omega_c^2}.$$

And in the case Re $(\omega_i\omega_e)=\omega_o^2-\nu_e\nu_i\approx-\nu_e\nu_i$, the usually considered dispersion relation of the FB waves will be found,

$$\omega_o=\frac{kv_o}{1+\psi},\quad \psi=\frac{\nu_e\nu_i}{\omega_c^2},\quad \gamma_L=\frac{\omega_o^2\nu_e}{\omega_c^2}. \qquad (20)$$

γ_L is the linear growth rate of the waves. In the case of high frequencies, in which the dispersive contributions have to be taken into account, the dispersion relation can be found from Eq. (19) ($\omega_o>\nu_i$, $\omega_i\approx\omega_o$)

$$\frac{\omega_c^2}{\omega_o^2}-\left(1+\frac{\omega_c^2}{\omega_{oi}^2}\right)-\frac{kv_o}{\omega_o}\left(\frac{\omega_c^2}{\omega_o^2}+\frac{\omega_c^2}{\omega_{oi}^2}\right)=0. \qquad (21)$$

Omitting the second contribution in Eq. (21), for instance for $kv_o\approx 0$, from Eq. (21) follows

$$\omega_o^2=\omega_c^2\left(1+\frac{\omega_c^2}{\omega_{oi}^2}\right)^{-1}=\omega_{oi}^2\left(1+\frac{\omega_{oi}^2}{\omega_c^2}\right)^{-1}, \qquad (22)$$

thus ω_o equals the usual lower-hybrid frequency ω_{LH} which depends on the background density n_o. In the more general case with $v_o\neq 0$ and $\omega\lesssim\nu_e$, the dispersion relation $\omega_o\approx$

$\omega_c + \Delta = \omega_c(1 - \vec{k}\vec{v}_o/\omega_c) \lesssim \omega_c$ is obtained. The high-frequency FB instability was numerically investigated in [Lee et al. 1971, Schlegel 1983, Volosevich 1978). This oscillation branch corresponds to frequencies $\omega < \omega_{oi}$, $\omega_c > \omega > \nu_i$, and it is excited by collisions of the electrons with the neutral particles. The collisions of the ions with the neutrals cause a wave damping.

Equation (17) describes the evolution of high-frequency FB waves under arbitrary plasma conditions. Taking Eq. (18) into account, Eq. (17) may be presented in the form

$$-\frac{i\tilde{c}}{\omega_o}\frac{\partial}{\partial t}\nabla_\perp \vec{E} + \tilde{b}\Delta\nabla_\perp \vec{E} + \beta_\parallel \nabla_\parallel \vec{E}_\parallel = \frac{\omega_o}{i\omega_{ci}}[\nabla N_s, \vec{E}] \cdot \vec{e}_z. \tag{23}$$

Equation (23) describes the evolution of high-frequency waves in collisional and collisionless plasmas. In the case $\omega_o = \omega_{LH}$ the equation corresponds to the well-known relation for the lower-hybrid instability. As Eq. (23) was derived within the frame of magnetohydrodynamics, for frequencies $\omega \gtrsim \nu_i$ kinetic effects have to be added. As an analogon of a kinetic effect, here the ion viscosity is introduced guaranteeing collisional dispersion. The second term on the left side of Eq. (23) takes dispersive effects into account. These effects may be caused by viscosity (coefficient η_i, with different sign in dependence on the proposed model), or by the finite values of the Debye radius $r_D = v_{te}/\omega_{pe}$ in the case of strongly magnetized plasma $\omega_c^2 \gg \omega_{oi}^2$, and of the electron Larmor radius $r_{ce} = v_{te}/\omega_{ce}$ in a dense plasmas with $\omega_{pe} \gg \omega_{ce}$.

The third contribution on the left side of Eq. (23) is caused by the motion of the charged particles along the magnetic field lines. Here it should be mentioned, that taking into account the relation $k_\parallel^2/k_\perp^2 \ll 1$, in this work electromagnetic effects were not considered. But, if the condition $k_\parallel^2/k_\perp^2 \gtrsim \nu_e^2/\omega_{ce}^2$ is satisfied, then the waves propagate under a small angle with respect to a plane which is normal to the magnetic field. This small angle plays an important role in the nonlinear dynamics of the waves, as the region of small angles corresponds to an effective energy absorption during the nonlinear wave interaction. During the nonlinear wave interaction, the angle $\psi = \mathrm{arctg}\ k_\parallel^2/k_\perp^2 \approx 0$ formed during the stage of linear wave growth changes to finite ψ-values. It should be mentioned, that in the case of lower hybrid waves with $\omega \approx \omega_{LH}$ and $\beta_\parallel = m_i/m_e$, and in the case of FB waves under the condition $\omega < \nu_e$, $\beta_\parallel \approx \omega_{ce}^2/\nu_e^2$, the value of the angle may change by even one order.

¿From the definition of the parameter $\tilde{b}$, which contains dispersive contributions, it can be seen that the dispersion caused by the collisional viscosity may increase or compensate the dispersion caused by the disturbance of the quasi-neutrality.

2.2. Equations for low-frequency waves

Further, the evolution of the low-frequency oscillations of the quasi-neutral plasma will be considered, in which HF waves are excited and act on the charged particles by the pressure force and the ponderomotive force (PMF, Miller force).

The system of magnetohydrodynamic equations Eq. (1-4) will be studied assuming that the frequency of the disturbances is small. Under such conditions, the collisions play an essential role in the dynamics of the charged particles. It is assumed that the electrons are magnetized, $\nu_e \ll \omega_{ce}$, and the ions are non-magnetized, $\nu_i > \omega_{ci}$. Further it is suggested that in a certain local region of the plasma, a HF distribution of the electromagnetic field exists, and its mean action will be taken into account by the ponderomotive force $\vec{F}_\alpha = m_\alpha\langle(\vec{v}_\alpha\nabla)\vec{v}_\alpha^*\rangle$. Neglecting in Eqs. (4, 5) the ion viscosity for the LF waves and omitting all nonlinear terms, one gets the relations

$$\frac{\partial\vec{v}_i}{\partial t} = -\nabla\frac{e\Phi_s}{m_i} + v_{ti}^2\nabla\ln N_i - \nu_i\vec{v}_i - \frac{\vec{F}_i}{m_i}, \tag{24}$$

$$\frac{\partial N_i}{\partial t} + \nabla\vec{v}_i = 0, \quad N_i = \frac{n_i}{n_o}. \tag{25}$$

Combining Eqs. (24, 25) and excluding $\vec{v}_i$, an equation for the ion density is obtained

$$\frac{\partial^2 N_i}{\partial t^2} - \nabla\frac{e\Phi_s}{m_i} - v_{ti}^2\nabla\ln N_i + \nu_i\frac{\partial N_i}{\partial t} - \frac{\nabla\vec{F}_i}{m_i} = 0. \tag{26}$$

In Eq. (26), all nonlinear terms are neglected, and only actions on the particles by the ponderomotive force are taken into account.

Considering the action of the PMF on the electrons analogously, and determining the velocity components of the electrons using the equation of motion in drift approximation, one gets

$$\vec{v}_\perp^e = -\frac{1}{B}[\nabla_\perp\Phi_s,\vec{e}_z] + \frac{\nu_e}{B\omega_{ce}}\nabla_\perp\Phi_s - \frac{v_{te}^2}{\omega_{ce}}[\nabla N_e,\vec{e}_z] - \frac{\nu_e v_{te}^2}{\omega_{ce}^2}\nabla_\perp N_e + \frac{1}{m_e\omega_{ce}}[\vec{F},\vec{e}_z] - \frac{\nu_e}{m_e\omega_{ce}^2}\vec{F}_{e\perp}, \tag{27}$$

$$\vec{v}_\parallel^e = \frac{e}{m_e\nu_e}\nabla_\parallel\Phi_s - \frac{v_{te}^2}{\nu_e}\nabla_\parallel N_e - \frac{\vec{F}_{e\parallel}}{m_e\nu_e}. \tag{28}$$

Here, $\vec{F}_e = m_e\langle(\vec{v}\nabla)\vec{v}^*\rangle$ is the mean force acting on the electrons. Further, substituting Eqs. (27, 28) into the equation of motion, the following equation for the electron density is obtained

$$\frac{\partial N_e}{\partial t} + \vec{v}_{oe}\nabla N_e + \frac{\nu_e}{B\omega_{ce}}\Delta_\perp\Phi_s - \frac{\nu_e v_{te}^2}{\omega_{ce}^2}\Delta_\perp N_e + \frac{e}{m_e\nu_e}\Delta_\parallel\Phi_s - \frac{v_{te}^2}{\nu_e}\Delta_\parallel N_e \tag{29}$$
$$-\frac{\nu_e}{m_e\omega_{ce}^2}\nabla_\perp\vec{F}_{e\perp} + \frac{1}{m_e\omega_{ce}}[\nabla_\perp,\vec{F}_e]\cdot\vec{e}_z - \frac{1}{m_e\nu_e}\nabla_\parallel\vec{F}_{e\parallel} = 0.$$

Using the relations for the electron and ion velocities under the action of the HF field (given by Eqs. (7-9)), the ponderomotive force on the electrons and ions can be found,

$$\vec{F}_i = \frac{m_i}{2B^2}\left(\frac{\omega_{ci}}{\omega_o}\right)^2\nabla|E^2|. \tag{30}$$

Introducing the electron mobilities

$$\mu_\perp = \frac{1}{B}, \quad \mu_e = \frac{i\omega_e}{B\omega_{ce}}, \quad \mu_\parallel = \frac{i\omega_{ce}}{B\omega_e},$$

it follows

$$F_x^e = \frac{m_e}{2}\left(\mu_\perp^2 \frac{\partial f}{\partial y} + \frac{\mu_\perp \mu_\parallel}{2}\frac{\partial}{\partial y}|\frac{\partial \varphi}{\partial z}|^2\right) + c.c., \tag{31}$$

$$F_y^e = \frac{m_e}{2}\left(-\mu_\perp^2 \frac{\partial f}{\partial x} - \frac{\mu_\perp \mu_\parallel}{2}\frac{\partial}{\partial x}|\frac{\partial \varphi}{\partial z}|^2\right) + c.c.,$$

$$F_z^e = \frac{m_e}{2}\left(\mu_\perp \mu_\parallel \frac{\partial f}{\partial z} - \frac{\mu_\parallel^2}{2}\frac{\partial}{\partial z}|\frac{\partial \varphi}{\partial z}|^2\right) + c.c.,$$

where φ is the HF potential, and $f_e = [\nabla\varphi^*, \nabla\varphi] \cdot \vec{e}_z$ is defined by the vector nonlinearity, that means by the Poisson brackets

$$f_e = \frac{\partial \varphi^*}{\partial x}\frac{\partial \varphi}{\partial y} - \frac{\partial \varphi^*}{\partial y}\frac{\partial \varphi}{\partial x}.$$

Assuming that the HF potential is given by

$$\varphi(\vec{r}, t) = A(\vec{r}, t) \cdot e^{i\psi(\vec{r}, t)},$$

where A and φ are the amplitude and the phase, then

$$f_e = i[\nabla\psi, \nabla A^2],$$

and the sign of the PMF is determined by the phase shift of two components of the HF field $-\partial\varphi/\partial x = E_x$, $-\partial\varphi/\partial y = E_y$. It can be easily shown that $\nabla_\perp \vec{F}_e = 0$ and $[\nabla_\perp, \vec{F}_e] = 0$, if for the HF field no influence of collisions is taken into account. Separating the real part of $\vec{F}_e$, and estimating it contributions, Eq. (29) may be transformed into

$$\frac{\partial N_e}{\partial t} + \vec{v}_{oe}\nabla N_e + \frac{\nu_e}{B\omega_{ce}}\Delta_\perp \Phi_s - \frac{\nu_e v_{te}^2}{\omega_{ce}^2}\Delta_\perp N_e + \frac{e}{m_e \nu_e}\Delta_\parallel \left(\Phi_s - T_e \ln N_e - \tilde{f}_e\right) = 0, \tag{30}$$

$$\vec{F}_{e\parallel} = m_e \frac{\omega_{ce}}{\omega_o B^2}\nabla_\parallel f_e = \nabla_\parallel \tilde{f}_e.$$

Thus, the Eqs. (23, 26, 30) form a closed system of equations for the determination of the parameters N_i, N_e and Φ_s. Using the quasi-neutrality condition $N_e = N_i$, from Eqs. (26, 30) a relation between $N = N_e = N_i$ and Φ_s can be found. Neglecting all nonlinear effects besides the PMF, it follows

$$v_d \nabla N + e\frac{1+\psi}{m_i \nu_i}\Delta_\perp \Phi_s + \frac{v_{ti}^2}{\nu_i}(1-\psi\tau)\Delta_\perp \ln N \tag{31}$$

$$+\frac{1}{m_e \nu_e}\Delta_\parallel \left(e\Phi_s(1+\psi_1) - T_i(\tau-\psi_1)\ln N - f_e(1+R_1 f_i)\right) = 0,$$

$\vec{v}_d = \vec{v}_{oe} - \vec{v}_{oi}$, $\psi = \nu_i\nu_e/\omega_c^2$, $\psi_1 = m_e\nu_e/(m_i\nu_i)$, $\tau = T_e/T_i$. The formula Eq. (31) determines the character of the disturbances in the plasma and the dynamics of the process, the character of the density structure of the charged particles and the electrostatic potential. Equation (31) is equivalent to the equation

$$\mathrm{div}\vec{j}_\perp = -\nabla_\parallel j_\parallel, \tag{32}$$

and it determines the dynamics of the particle motion in a three-dimensional model. It is very difficult to find a general solution of Eq. (32). In the case $v_d \neq 0$, and if the field-aligned motion is not essential because of the condition $k_\parallel^2/k_\perp^2 \ll \nu_e^2/\omega_{ce}^2$, then $\Delta_\parallel \ll \Delta_\perp$ and the first three terms in Eq. (31) give the main contributions. Under such conditions, the potential may be approximately found

$$e\Delta_\perp \Phi_s = -\frac{m_i\nu_i}{1+\psi} v_d \nabla N - T_i(1-\psi\tau)\Delta_\perp \ln N. \tag{33}$$

This is the main condition of the linear theory of FB waves. Under this condition, the linear dispersion equation Eq. (1) is satisfied. The additional contributions connected with the action of $\Delta_\parallel$ are of the order of $k_\parallel^2/k_\perp^2$.

But, in the contrary case, if one suggests that the field-aligned motion of the particles determines the physical process, Eq. (31) may be solved in general. Suggesting approximately that the potential Φ_s determines the polarization electric field which is directed along the magnetic field lines because of the particle motion along $\vec{B}_o$, one obtains

$$\Delta_\parallel \left(e\Phi_s(1+\psi_1) - T_i(\tau-\psi_1)\ln N_e - f_e\right) = 0$$

as $\psi_1 f_i \ll 1$, and one has approximately

$$e\Phi_s = T_e \ln N_e + \tilde{f}_e. \tag{34}$$

The relation Eq. (34) corresponds to the Boltzmann distribution of the electrostatic potential at $\tilde{f}_e = 0$. Further, substituting Eq. (34) into the equation for the electron and ion motion, follows

$$\frac{\partial N_e}{\partial t} + \vec{v}_o \nabla N_e + \frac{\nu_e}{m_i \omega_o^2}\Delta_\perp \tilde{f}_e = 0, \tag{35}$$

$$\frac{\partial^2 N_i}{\partial t^2} - \frac{T_e}{m_i}\Delta \ln N_e - \frac{T_i}{m_i}\Delta \ln N_i + \nu_i \frac{\partial N_i}{\partial t} - \frac{\Delta \tilde{f}_e}{m_i} = 0. \tag{36}$$

If the quasi-neutrality condition $N = N_e = N_i$ is satisfied, instead of Eqs. (35, 36), one can write only one equation

$$\frac{\partial^2 N}{\partial t^2} - c_s^2 \Delta \ln N - \nu_i \vec{v}_o \nabla_\perp N - (1+\psi)\frac{\Delta \tilde{f}_e}{m_i} = 0. \tag{37}$$

In the case that the waves propagate perpendicular to the drift velocity outside the cone of linear wave excitation, the third term in Eq. (37) is small, and a final relation

$$\frac{\partial^2 N}{\partial t^2} - c_s^2 \Delta \ln N = (1+\psi)\frac{\Delta \tilde{f}_e}{m_i} \tag{38}$$

is obtained. The equations (15) and (38) form a self-consistent system of equations for low-frequency density waves and high-frequency FB waves.

In the quasi-stationary state with $\partial/\partial t = -v\partial/\partial z$ follows

$$N \approx \frac{1}{(M^2-1)}\frac{\tilde{f}_e}{m_i c_s^2}(1+\psi), \tag{39}$$

with $M = v/c_s$, v being the velocity of the moving density structure, $\tilde{f}_e$ is given by Eq. (27), and

$$N \approx \frac{1}{M^2-1}\sqrt{\frac{m_e}{m_i}}\frac{\tilde{f}_e}{B^2 c_s^2}. \tag{40}$$

Substituting Eq. (40) into Eq. (23), one finally obtains

$$-i\frac{\tilde{c}}{\omega_o}\frac{\partial}{\partial t}\nabla_\perp \vec{E} + \tilde{b}\Delta\nabla_\perp \vec{E} + \beta_\parallel \nabla_\parallel \vec{E}_\parallel = -iS[\nabla \hat{f}_e, \vec{E}]\cdot \vec{e}_z, \tag{41}$$

$$S = \frac{\omega_o}{\omega_{ci}}\sqrt{\frac{m_e}{m_i}}\frac{1}{M^2-1}, \quad \hat{f}_e = \frac{\tilde{f}_e}{B^2 c_s^2} = \frac{\tilde{f}_e}{E_o^2},$$

where S represents the coefficient of nonlinear wave interaction of FB waves.

3. Discussion

In the paper, the nonlinear interaction between the high-frequency and low-frequency parts of waves, excited by the modified two-stream instability during the linear stage of wave excitation, is considered.

The derived system of equations (17) and (37) allows the investigation of the nonlinear interaction of waves satisfying the decay conditions. The relations are a generalization of former equations obtained by Musher and Sturman (1975) for the case of the collisional plasma.

It is shown that in the two limiting cases of the collisionless plasma ($\omega > \nu_e$) and the plasma with intensive collisions ($\omega < \nu_e, \nu_i$), the relations decribe the evolution of lower-hybrid and Farley-Buneman waves, respectively. In the intermediate region with frequencies $\nu_i < \omega < \nu_e$, a special high-frequency branch of Farley-Buneman waves exist.

As follows from Eq. (23), the dispersion of the high-frequency Farley-Buneman modes depends on the viscosity caused by the ion collisions (coefficient $\tilde{\eta}_i$), on the Debye wavelength $(b_i + b_e)\lambda/\alpha$ (via the small contribution proportional to $\tilde{b}$), and on the electron Larmor radius (via the small contribution proportional to the coefficient β). Besides the dispersion is influenced by the value of the electron drift velocity $\vec{v}_o$. It should be mentioned, that in a collisional plasma the coefficients determining the nonlinear interaction have complex values, that means, the wave interactions cause both, a nonlinear frequency shift and a modification of the linear wave growth rate. For instance, in the case that the dispersive and collisional contributions compensate, a stationary regime may occur.

Further it should be underlined, that the disturbances of the background plasma described by Eq. (40) may compensate the damping of the waves because of the wave propagation non-perpendicular to the magnetic induction $\vec{B}_o$. Under such conditions, the interval of possible aspect angles can be essentially widened. Thus, e.g. from Eq. (41) follows

$$\frac{k_\|^2}{k_\perp^2} \approx \frac{\nu_e}{\omega_{ce}} \sqrt{\frac{m_e}{m_i}} \frac{|\nabla\Phi|^2}{E_o^2 |M-1|},$$

and, taking into account the presented additional small contributions for the ionospheric plasma with $\nu_e/\omega_{ce} \approx 1.2 \cdot 10^{-2}\sqrt{m_e/m_i} \approx 5 \cdot 10^{-3}$, $M \approx 0.8$, one finds $k_\|^2/k_\perp^2 \approx 3 \cdot 10^{-3}|\nabla\Phi|^2/E_o^2$. In the case that locally $|\nabla\Phi|^2/E_o^2 \approx 10$, one has $|k_\|/k_\perp| \approx 0.17$. Thus it can be seen, that the aspect angle (that means $k_\|^2/k_\perp^2$) depends on the mass ratio of the electrons to the ions, on the relation between the frequency ω_o and the collision frequency, and on the structure of the electrostatic potential. That means, the aspect angle is determined by the value and the form of the gradient $\nabla\Phi$. Locally, the value of $|\nabla\Phi|^2/E_o^2$ may be very large in the auroral ionosphere, and correspondingly, one may observe by radio scatter experiments waves propagating under very high aspect angles.

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