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Allgemeinverständliche Zusammenfassung zur Dissertation

Seit 1971 wurde an dem renommierten Freudenthal Institut in Utrecht ein als Realistic Mathematics Education (RME) bezeichneter mathematikdidaktischer Ansatz entwickelt. Die Philosophie von RME beruht auf Hans Freudenthals Auffassung von Mathematik als menschlicher Aktivität. Der Mathematiker und Didaktiker Prof. Hans Freudenthal (1905 – 1990) plädierte dafür, dass Mathematik an den Schulen nicht als Fertigprodukt unterrichtet werden sollte. Im Gegensatz dazu forderte er, den Schülern an ‚realistischen' Situationen nicht-formale und formale Mathematik wieder entdecken zu lassen.

Obwohl die mathematische Schulbildung in Vietnam in den letzten Jahrzehnten schon einige Fortschritte gemacht hat, steht sie noch vor großen Herausforderungen. Derzeit ist die Reform der Unterrichtsmethoden eine dringliche Aufgabe in Vietnam. Augenscheinlich ermangelt es der Mathematikdidaktik in Vietnam an dem dazu notwendigen theoretischen Rahmen. Die Philosophie von RME eignet sich grundsätzlich als Orientierung für die Reform der Unterrichtsmethoden in Vietnam. Allerdings ist die Potenz von RME für die mathematische Schulbildung in Vietnam und die Möglichkeiten, RME im Mathematikunterricht anzuwenden, noch zu klären.

Das Hauptziel dieser Arbeit war zu erforschen, wie RME beim Mathematik-Lernen und -Lehren in Vietnam eingesetzt werden kann und die Frage zu beantworten: Wie kann RME den Mathematikunterricht in Vietnam bereichern? Dazu wurde insbesondere der Geometrieunterricht in der Sekundarstufe I betrachtet.

Im Einzelnen beinhaltet die Untersuchung:

- eine Analyse der vietnamesischen Mathematikdidaktik in der 'Reformperiode' (etwa von 1980 bis 2000)
- die Konzeption, Durchführung und Auswertung einer Befragung von 152 Mittelschullehrern aus verschiedenen vietnamesischen Provinzen und Städten zum Mathematikunterricht in Vietnam
- eine Analyse von RME einschließlich der Freudenthalschen Sicht von RME und der Charakteristika von RME
- die Diskussion, wie man RME-basierten Unterrichtseinheiten gestalten und diese in den Mathematikunterricht in Vietnam integrieren kann
- Test solcher Einheiten in vietnamesischen Mittelschulen
- Analyse der Rückmeldungen anhand der Schülerarbeitsblätter und der Lehrerberichte
- Diskussion der Chancen und Probleme von RME-basierten Unterrichtseinheiten im Geometrieunterricht vietnamesischer Mittelschulen
- Diskussion von Vorschläge zur Entwicklung und zum Einsatz RME- basierter Unterrichtseinheiten in Vietnam, einschließlich von Hinweisen für Lehrende und der Konzeption von Ausbildungs- und Fortbildungskursen zu RME

Die Untersuchung zeigt, dass – obwohl Lehrer wie Schüler zunächst einige Hindernisse beim Lehren und Lernen mit RME- basierten Unterrichtseinheiten zu bewältigen haben werden – RME ein mächtiger mathematikdidaktischer Ansatz ist, der wirkungsvoll im Lehren und Lernen von Mathematik in vietnamesischen Schulen angewandt werden kann.

Abstract

Since 1971, the Freudenthal Institute has developed an approach to mathematics education named Realistic Mathematics Education (RME). The philosophy of RME is based on Hans Freudenthal's concept of 'mathematics as a human activity'. Prof. Hans Freudenthal (1905-1990), a mathematician and educator, believes that 'ready-made mathematics' should not be taught in school. By contrast, he urges that students should be offered 'realistic situations' so that they can rediscover from informal to formal mathematics.

Although mathematics education in Vietnam has some achievements, it still encounters several challenges. Recently, the reform of teaching methods has become an urgent task in Vietnam. It appears that Vietnamese mathematics education lacks necessary theoretical frameworks. At first sight, the philosophy of RME is suitable for the orientation of the teaching method reform in Vietnam. However, the potential of RME for mathematics education as well as the ability of applying RME to teaching mathematics is still questionable in Vietnam. The primary aim of this dissertation is to research into abilities of applying RME to teaching and learning mathematics in Vietnam and to answer the question "how could RME enrich Vietnamese mathematics education?". This research will emphasize teaching geometry in Vietnamese middle school.

More specifically, the dissertation will implement the following research tasks:

- Analyzing the characteristics of Vietnamese mathematics education in the 'reformed' period (from the early 1980s to the early 2000s) and at present;
- Implementing a survey of 152 middle school teachers' ideas from several Vietnamese provinces and cities about Vietnamese mathematics education;
- Analyzing RME, including Freudenthal's viewpoints for RME and the characteristics of RME;
- Discussing how to design RME-based lessons and how to apply these lessons to teaching and learning in Vietnam;
- Experimenting RME-based lessons in a Vietnamese middle school;
- Analyzing the feedback from the students' worksheets and the teachers' reports, including the potentials of RME-based lessons for Vietnamese middle school and the difficulties the teachers and their students encountered with RME-based lessons;
- Discussing proposals for applying RME-based lessons to teaching and learning mathematics in Vietnam, including making suggestions for teachers who will apply these lessons to their teaching and designing courses for in-service teachers and teachers-in training.

This research reveals that although teachers and students may encounter some obstacles while teaching and learning with RME-based lesson, RME could become a potential approach for mathematics education and could be effectively applied to teaching and learning mathematics in Vietnamese school.

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Potsdam, December 2006

Le Tuan Anh

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Some mathematical signs used in the dissertation

Sign	*Meaning*	*Vietnamese style*
$\overline{AB}$	Segment	Segment AB
AB	The measure (length) of $\overline{AB}$	AB
$AB = CD$	The length of $\overline{AB}$ is equal to the length of $\overline{CD}$	AB = CD
$AB > CD$	The length of $\overline{AB}$ is greater than the length of $\overline{CD}$	AB > CD
$\overline{AB} \cong \overline{CD}$	$\overline{AB}$ is congruent to $\overline{CD}$	AB = CD (two segments AB and CD are equal)
$\overline{AB} \parallel \overline{CD}$	$\overline{AB}$ is parallel to $\overline{CD}$	AB // CD (segment AB is parallel to segment CD)
$\overline{AB} \perp \overline{CD}$	$\overline{AB}$ is perpendicular to $\overline{CD}$	AB $\perp$ CD (segment AB is parallel to segment CD)
$\overrightarrow{Ox}$	Ray	Ray Ox
$\overrightarrow{AB}$	Ray	Ray AB
$\vec{a}$	Straight line	Straight line a
$\overleftrightarrow{AB}$	Straight line	Straight line AB
$\overleftrightarrow{h} \perp \overleftrightarrow{k}$	$\overleftrightarrow{h}$ is perpendicular to $\overleftrightarrow{k}$	h $\perp$ k (straight line h is perpendicular to straight line k)
$\overleftrightarrow{h} \parallel \overleftrightarrow{k}$	$\overleftrightarrow{h}$ is parallel to $\overleftrightarrow{k}$	h // k (straight line h is parallel to straight line k)
$\angle A$, $\angle xOy$ and $\angle ABC$	Angles	$\angle$A, $\angle$xOy and $\angle$ABC $\hat{A}$, $x\hat{O}y$ and $A\hat{B}C$ Angles A, xOy and ABC
m$\angle A$, m$\angle xOy$, m$\angle ABC$	The measure of $\angle A$, $\angle xOy$ and $\angle ABC$ respectively	The measure of angles A, xOy and ABC
m$\angle A = 30^0$	The measure of $\angle A$ is 30^0	$\hat{A} = 30^0$
m$\angle A$ = m$\angle B$	The measure of $\angle A$ is equal to the measure of $\angle B$	$\hat{A} = \hat{B}$
$\angle A \cong \angle B$	$\angle A$ is congruent to $\angle B$	$\hat{A} = \hat{B}$ (two angles are

		equal)
$m\angle A > m\angle B$	The measure of $\angle A$ is greater than the measure of $\angle B$	$\hat{A} > \hat{B}$
ΔABC	Triangle	$\Delta \mathrm{ABC}$
$\Delta ABC \cong \Delta DEF$	ΔABC is congruent to ΔDEF	$\Delta \mathrm{ABC} = \Delta \mathrm{DEF}$ (two triangles ABC and DEF are equal)
$(O; R)$	A circle with center O and radius R	(O; R)

Abbreviations in the dissertation

Abbreviation	Meaning
2-D	Two-dimensional (geometry)
3-D	Three-dimensional (geometry)
DG	Dynamic Geometry
ICT	Information and Communication Technology
IT	Information technology
ICT	Information and communication technology
MoET	The Ministry of Education and Training (Vietnam)
n.d.	No date
NIES	The National Institute for Educational Sciences (Vietnam)
NIESAC	The National Institute for Educational Strategy and Curriculum Development (Vietnam)
OW & OC	The Research Group on Mathematics Education and Educational Computer Centre (the Netherlands)
RME	Realistic Mathematics Education

Introduction

Recently, education in Vietnam has been transitioned from the third educational reform to the next movement. Because of the fact of primitive and conventional teaching methods, a reform for teaching methods has become an essential task in Vietnam. The Vietnamese Ministry of Education and Training (MoET) has set up the orientation for this reform with the motto “activate students’ activities”. At first sight, RME approach which has been developed at the Freudenthal Institute, the Netherlands since the early 1970s seems to be suitable for the MoET’s orientation of teaching methods and can become a promising approach for mathematics education in Vietnam. However, the potential, ability and efficacy of the application of RME in Vietnamese school are still questionable. For these reasons, it is worth conducting the application of RME to mathematics education in Vietnam.

This section, firstly, establishes the objective and includes some specific tasks and research methods of the dissertation.[1] Later, this section provides a brief overview to the chapters in this dissertation.

The primary aim of this dissertation is to explore and consider the efficacy of Realistic Mathematics Education (RME) when applied in mathematics education in Vietnamese school.[2] This purpose can be divided into the following research tasks:

• researching the characteristics of mathematics education (mathematics curricula and textbooks, teaching methods, teacher staff, examinations and assessment, etc.) in Vietnam;

• studying Realistic Mathematics Education (a brief overview of history, basic ideas and characteristics);

• applying RME in teaching and learning mathematics in Vietnam (differences and similarities between Vietnamese and (Dutch) RME curricula and textbooks; possible potentials of RME for enriching Vietnamese mathematics education; a method, advantages and disadvantages of applying RME in Vietnamese school; and proposals for using RME in Vietnam).

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[1] The research questions are presented at the end of chapter one.

[2] Some researchers consider RME as a theory, while the others regard it as a philosophy. In this dissertation, RME is considered as a theory.

Here are some of the research methods and strategies applied in this study:

• Literatures related to Vietnamese mathematics education as well as RME (mathematics curricula and textbooks, research articles, books and so forth) were reviewed and analyzed.

• A questionnaire about mathematics curricula and textbooks, teaching methods, teachers' difficulties in reform of teaching methods, students' difficulties in learning mathematics, teaching mathematics application in school, working in groups, etc. was distributed in 2005 and at the beginning of 2006 to 152 middle school mathematics teachers from several Vietnamese provinces and cities, including Kien Giang, Thai Binh, Ha Nam, Hai Duong, Ha Tay and Hanoi. In addition, the teachers were also asked to give their ideas about a problem called 'Polar Bear' (see Van den Heuvel-Panhuizen, 1996, p. 95) and solutions of a problem named 'T-shirts and Sodas' (see Van Reeuwijk, 1995, pp. 2-4; De Lange, 1996, p. 63-64[3]). The questionnaire form is presented in appendix B.

• RME-based lessons were introduced and tested in mathematics lessons in Vietnam, and the participating teachers and students' feedback was carefully analyzed.

The dissertation is divided into six chapters. The following paragraphs briefly introduce these chapters.

Chapter one discusses the characteristics of mathematics education in Vietnam. Vietnamese education has been influenced by countries such as China, France, the U.S.A and some countries in the Social System[4], especially the former Union of Soviet Socialist Republics (U.S.S.R.) (see, for example, Fraser, 1984, pp. 78-82; Nguyen Thanh Thuy, 2005, p. 3). In the history of education, Vietnam implemented three reforms which started in 1950, 1956 and 1979 to overcome educational weaknesses because of consequences of the French domination (1858-1945) and the Vietnam War (1954-1975) (see, for instance, Pham Minh Hac[5], 2002; Bui Minh Hien, 2005). After the national unity in 1975, there were two different educational systems in two parts of the country (the Northern and Southern Vietnam) because of the Vietnam War. Due to the third reform, a national-wide series of curricula and 'reformed' textbooks for all grades has been used since 1992. Recently, Vietnamese education has been transitioned

[3] Prof. Dr. De Lange is the former director of the Freudenthal Institute, Utrecht University in the Netherlands.

[4] To some extent, Vietnamese mathematics education is a little (structurally) similar to the former German Democratic Republic's mathematics education which is described by Henning (n.d).

[5] Prof. Dr. Pham Minh Hac is a former Minister of Education and Training in Vietnam.

from the third reform to the next educational movement. A new series of curricula and textbooks for grades 1 until 10 has been used in school instead of the old one since the school-year 2006-2007. In the school-year 2008-2009, this series will be used for the last grade (12) in the system of general education. Because of this situation, chapter one discusses some characteristics of mathematics education in the 'reformed' period (from 1979 to the early 2000s), some changes and efforts of the Ministry of Education and Training to overcome disadvantages of this period. Finally, some research questions are posed at the end of this chapter.

In general, mathematics education is quite rigid in the 'reformed' period. Firstly, the mathematics curriculum and textbooks emphasize formal mathematics phase rather than pedagogic phase. Moreover, there is an imbalance between amount of content in the textbooks and amount of time teachers and students have for their teaching and learning. Secondly, the teaching style of most classrooms is distinctly teacher-centered. Thirdly, there is a general lack of research on examinations and assessment, and written examination is the primary mode of evaluation in this period. Fourthly, in some parts, teachers are not well-trained. More specifically, these teachers do not have qualifications which are requisite for teaching in school because of a confluence of historical factors, including the consequences of wars. Finally, teaching and learning resources are insufficient in Vietnamese school.

Recently, the Vietnamese Ministry of Education and Training (MoET) has implemented many important national projects to combat, and, remedy ultimately the weaknesses of education. There are significant changes in the new curricula and textbooks. Students from grades 1 through 10 have been using new textbooks. Generally, these series of curricula and textbooks have many advantages compared to the 'reformed' ones. A movement of method teaching reforms has become an urgent task with the motto, "activate students' activeness" (MoET, 2002 a) or "learning through activities" (Nguyen Ba Kim[6], 2002, p. 112). Solutions to problems of sub-standard teachers and insufficiency of teaching and learning facilities have been discussed and gradually executed. Overall, chapter one provides some examples of efforts and changes related to mathematics education.

[6] Prof. Dr. Sc. Nguyen Ba Kim is the Project Manager of the Middle School Teacher Training Project of the MoET.

A reform of teaching and learning mathematics has become an essential task in Vietnam because of the weaknesses of teaching methods (Nguyen Ba Kim, 2002, pp. 111-112). Some non-traditional (modern) approaches were introduced in methodology courses for pre-service and in-service mathematics teachers (see, for instance, Nguyen Ba Kim, 2002, pp. 178-286; Pham Gia Duc, Nguyen Manh Cang, Bui Huy Ngoc & Vu Duong Thuy, 1998 a, pp. 91-120). In general, each non-traditional approach has its own advantages and disadvantages when it is applied in teaching and learning in Vietnam. Hence, beyond these approaches, finding and applying other approaches which are suitable for the orientation of the teaching method reform are necessary. This dissertation places special emphasis on the researching potentials of Realistic Mathematics Education (RME) which has been developing in the Netherlands since 1970s for mathematics education in Vietnam. However, it restricts itself from teaching and learning grade-seven geometry because it appears that Vietnamese mathematics curricula and textbooks focuses on formal (deductive Euclidean) geometry, an area with which students usually struggle. At the end of chapter one, some research questions are posed.

Chapter two presents RME theory, including a brief overview of history, basic ideas of Freudenthal, characteristics of RME and some selected examples. As RME is new for nearly all mathematics educators, curriculum developers, textbooks authors and teachers in Vietnam, this chapter tries to present the basics of this theory clearly.

Chapter three discusses foundations for designing RME-based geometry lessons for grade 7 in Vietnam. First, the chapter discusses some controversial phases in curricula and textbooks and insights into the Vietnamese ones in the 'reformed' period and at present. In general, there are different viewpoints on creating school geometrical curricula and textbooks. Second, this chapter explains why it is not reasonable for creating and applying a RME curriculum for teaching and learning mathematics in Vietnam at least in the foreseeable future. However, creating RME-based lessons for Vietnamese school is possible. Next, the chapter discusses the foundations for designing Vietnamese RME-based geometry lessons for grade 7. Finally, at the end of this chapter, RME-based lesson for the Triangle-Angle Sum Theorem is introduced as an example. Details of some other RME-based geometry lessons are presented in appendix C.

The aim of the fourth chapter is to examine how RME-based lessons worked in Vietnamese school. The lessons were offered to several middle school mathematics teachers; however, only two young teachers among them were willing to test out these lessons. Their reservations stemmed from a number of reasons, such as teaching time pressure, refusal to change teaching habits and disapproval of their head teachers. Because these two teachers could not read English, these RME-based lessons, as well as some related materials, were translated into Vietnamese. They used these lessons in the school-year 2005-2006. They were not obliged to follow these lessons completely. In contrast, they could alter certain aspects to make these lessons better suited to the circumstances of their teaching. For several reasons, especially the teaching of time pressure, the teachers used four RME-based lessons. The author of this dissertation could not attend the lessons. This chapter analyzes the feedback, including the teachers' comments and their students' worksheets. The advantages and disadvantages of RME-based lessons are discussed in the chapter. At the end, some adjustment of RME-based lessons is discussed.

Chapter five discusses proposals for applying RME-based geometry lessons in teaching and learning in Vietnam. Firstly, the chapter proposes some necessary changes of viewpoints on mathematics education related to *mathematics as a ready-made product* and *mathematics as a human activity*, *guided reinvention*, *informal knowledge* in teaching and learning school mathematics, teaching mathematics application and emergent modeling (see, for example, Freudenthal, 1973, 1983, 1991; Gravemeijer, 2002, 2004). It is argued that these are prerequisites for applying RME-based lessons. Secondly, it refers to conditions for applying RME-based lessons in teaching and learning mathematics in Vietnam. Thirdly, training pre-service and in-service mathematics teachers is discussed.

Conclusions and suggestions are presented in the last chapter of the dissertation. First, this chapter makes some conclusions related to Vietnamese mathematics education, potentials and difficulties of applying RME in teaching and learning and proposals for utilizing RME in Vietnam. This chapter also gives answers to the research questions posed at the end of chapter one. Second, this chapter provides some suggestions for further possible research in this area.

Chapter 1 Characteristics of mathematics education in Vietnam

Education in Vietnam is evolving from the third education reform (from 1979 until the early 2000s) to the next educational movement.[7] This chapter, firstly, discusses some characteristics of mathematics education, including school mathematics curriculum and textbooks, teaching methods, assessment and examinations and some other factors during the third education reform in Vietnam. Secondly, it discusses some efforts and changes of the Vietnamese Ministry of Education and Training (MoET) to overcome weakness of education as well as mathematics education. Finally, some research questions are posed.

On September 2, 1945, President Ho Chi Minh declared independence for the Democratic Republic of Socialist Vietnam. After the declaration of the independence, young Vietnamese education faced many problems as a result of a French domination. In addition, Vietnamese education was also affected seriously by the Vietnam War (1954-1975). To solve these problems, the government of Vietnam implemented several education reforms. The third education reform was established in Vietnam in 1979. This reform plays its role in the process of education development; however, it reveals some problems. For this reason, recently, the third education reform's school curricula and textbooks, known as 'reformed' ones, have been gradually replaced by new ones since the early 2000s.

Teaching and learning mathematics have played an important role in the process of education development in Vietnam.[8] Although Vietnamese school mathematics education has many achievements, such as students' high performances in the International Mathematics Olympiads (IMO) and Asian Pacific Mathematics Olympiad (APMO) for high school students, the International Junior Science Olympiad (IJSO) and the Mathematics and Science Olympiad for primary students from ASEAN (see, for example, International Mathematical Olympiad, n.d.; Canadian Mathematical Society, n.d.; Vu Kim Thuy, 2006), it has still faced some challenges.[9]

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[7] Recently, new curricula and textbooks have been being written and utilized in Vietnamese schools.

[8] In Vietnam, mathematics is often considered as one of the main school subjects. Mathematics is taught from grade 1 to grade 12 in Vietnamese school. Moreover, mathematics is always appears in middle and high school diploma examination. In addition, most blocks of university entrance examinations contain mathematics examination.

[9] There are few studies on comparison students' achievement in Vietnam and other countries. Nevertheless, one study on comparison students' achievement and motivation in Hanoi and Munich shows that students from Hanoi get higher achievement in mathematics than those from Munich;

In this chapter, some shortcomings relating to Vietnam's mathematics curriculum and textbooks, teaching methods, examinations and assessment and classroom organization are carefully discussed. The weak points mainly appeared from the early 1980s until the early 2000s. The analyses of these shortcomings do not aim at blaming students, mathematics teachers, mathematics educators, mathematics textbook authors, curriculum developers, policy makers and education managers. Their primary purposes are to give a factual overview of teaching and learning mathematics so that possible ways of enriching mathematics education are hopefully found. Several characteristics discussed below might appear in teaching and learning not only mathematics but also some other school subjects.

1.1 Brief introduction to educational reforms in Vietnam after 1945

This section introduces a brief history of three education reforms in Vietnam after it became the independent state in September 1945. The brief description of the first three education reforms, including the starting time, main reason and range of each reform is presented in table 1.1 (Pham Minh Hac, 2002, pp. 65-70; Bui Minh Hien, 2005, pp. 136-137, 153-155 & 181-183).

Table 1.1: The first three Educational Reforms in Vietnam

Education reform	Starting time	Main reason	Range
1st	1950	Extremely primitive education	Liberated areas
2nd	1956	Necessary unification of general education after the restoration of peace in the Northern Vietnam	The Northern Vietnam
3rd	1979	Necessary unification of general education after the war	The whole country

After becoming independent in September 1945, Vietnamese education faced many serious problems. For example, at that time more than 95% of the population were illiterate, and only around 3% of the population were attending school (Pham Minh Hac, 2002, pp. 45-46). Because of the war, until July 1950, a project of

furthermore, Hanoi students have more desire to learn mathematics than Munich counterparts (Helmke et al., 2003, pp. 195-198).

educational reform was passed, and the *first educational reform* was implemented in liberated areas; and as a result of this reform, a nine-year educational system, including level I (from grade 1 to grade 4), level II (from grade 5 to grade 7) and level III (grades 8 and 9) was implemented, whereas in French-occupied territories, curricula which were similar to pre-1945 curricula were being used (Pham Minh Hac, 2002, pp. 65-67; Bui Minh Hien, 2005, pp. 137-138). In 1954, peace was restored in Northern Vietnam. Since 1954, there were two different general educational systems in the Northern Vietnam: the first educational reform-based one with nine-year duration and the other one with twelve-year duration in pre-French occupied territories (Pham Minh Hac, 2002, p. 67). Therefore, one urgent task at that time was to unify the education system (Pham Minh Hac, 2002, p. 67; Bui Minh Hien, 2005, pp. 153-154).

In May 1956, a project of the *second educational reform* was passed and implemented by the Ministry of Education, and according to this project, the general educational system included 10 grades (level I: from grade 1 to grade 4, level II: from grade 5 to grade 7 and level III: from grade 8 to grade 10) (Pham Minh Hac, 2002, pp. 67-68; Bui Minh Hien, 2005, p. 154).

Although Vietnam was unified, after securing peace in April 1975, it still had two different systems of general education: the Northern Vietnam's system with duration of 11 years and the Southern Vietnam's one with 12 years as a result of the wars (Pham Minh Hac, 2002, p. 68; Bui Minh Hien, 2005, pp. 181-182). Eventually, Vietnam needed to have one united system for education. Moreover, despite the fact that Vietnamese education had many important achievements from the second educational reform begun in 1956, it "did not keep up with the development of society, science, and technique and satisfy requirements of the country's postwar rebuilding" (Pham Minh Hac, 2002, p. 68). This is why the Vietnamese government implemented the *third educational reform* in 1979. As a result of this reform, in the school-year 1981-1982, textbooks for grade 1 called 'reformed' textbooks were originally used (Pham Minh Hac, 2002, pp. 69-70; Bui Minh Hien, 2005, p. 183). Subsequently, the 'reformed' textbooks were applied for grade 12 in the school-year 1992-1993. Vietnam succeeded in unifying its general educational system with a duration of 12 years (grades 1 to 12), and all schools in the whole country used a common set of curricula and textbooks (Pham Minh Hac, 2002, pp. 69-70; Bui Minh Hien, 2005, p. 183).

The aforementioned set of curricula and textbooks assumed an important role in the process of educational development in Vietnam. However, it also revealed some shortcomings. This is why there is a great need of a new series of curricula and textbooks for Vietnamese school. Since the early 2000s, this new series has been tested and gradually applied in school.

1.2 General education in Vietnam

This section briefly introduces general education (primary, middle and high school education) in the Vietnamese education system.

According to Vietnam National Assembly (1998, chapter 1, article 6; 2005, chapter 1, article 4), the national education system includes pre-school education, general education, vocational education and graduate and postgraduate education. The general education including primary, middle and high school education can be illustrated in figure 1.1 (Vietnam National Assembly, 1998, chapter 2, article 22; 2005, chapter 2, article 26). The full of the Vietnamese national education system can be found in MoET (n.d. a).

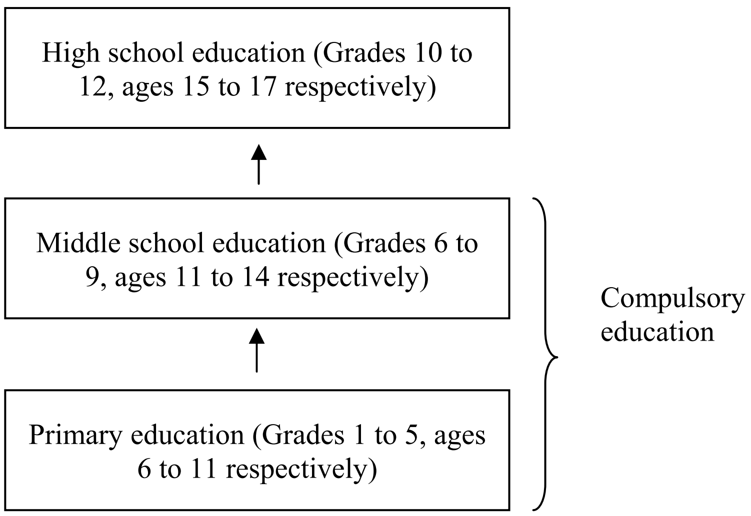

Figure 1.1: The general education system in Vietnam

Typically, there is a unique nationwide series of curricula and textbooks for Vietnamese school. Exceptionally, during the early 1990s until the late 1990s, there were three series of high school textbooks (for some subjects) written by three different groups of authors from Hanoi University of Education, Ho Chi Minh City's University of Pedagogy and the National Institute for Educational Sciences (NIES). However, in 2000, these series were corrected, edited and unified to make a unique series called ‘corrected and unified the year 2000 textbook’ one. Furthermore, the MoET implemented an experiment according to which high school students were divided into three streams: Natural Sciences (stream A), Technique (Stream B) and Social Sciences (Stream C) from 1992 until 1997.[10] However, this experiment did not succeed for many reasons. One reason for this experiment is ultimate failure is that very few students chose the Technique Stream. Recently, there has been a unique series of curricula and textbooks for primary as well as middle school. In high school level, grade-ten students have been divided (since the school-year 2006-2007) and grade-eleven and grade-twelve students are going to be divided (from the school-year 2007-2008 and 2008-2009, respectively) into three streams named a Basic Stream, a Natural Stream and a Social Stream. In the school-year 2006-2007, most grade-ten students (about 73%) choose the Basic Stream; in contrast, very few students (around 6%) choose the Social Stream (see, for instance, Kim Dung, 2006 and related information in Vietnam News, 2006).

1.3 Characteristics of mathematics education in the ‘reformed’ period

1.3.1 School mathematics curriculum and textbooks

This section presents some characteristics of the school mathematics curriculum and textbooks in the ‘reformed’ period (from 1979 until the early 2000s). First, it briefly introduces the mathematics curriculum and textbooks. Second, it discusses specific characteristics of these curriculum and textbooks. Finally, additional characteristics are presented and discussed.

1.3.1.1 Overview of mathematical curriculum and textbooks

Vietnamese national mathematical curriculum is designed and determined by the Ministry of Education and Training (MoET). Based on the national mathematical curriculum, the authors write experimental mathematics textbooks. These textbooks are tested in some schools in different areas during a span of several years and upgraded before they are official used in all schools. All Vietnamese school textbooks are published by a special publisher named the Educational Publishing House. Mathematics textbooks are divided into several chapters containing lessons. A mathematics lesson often has some formal mathematics definitions, theorems,

[10] Recently, the MoET has been experimented and applied a new policy. According to this policy, high school students are divided into three streams: Natural Science, Social Science and Basis streams.

regulations, or formulae. Some contents of probability, analysis and analytic geometry have been presented in high school textbooks since the early 1990s.

Typically, there is one series of mathematical textbooks in school despite that students live in different regions such as urban, rural and remote (mountainous and islandish) areas.[11] In general, Vietnamese students' achievement differs from one region to another region. From 1998 to 2000, the National Institute for Educational Sciences (NIES) conducted a study on third and fifth grade students' achievement of fourteen primary schools in five provinces. The research demonstrates that there are "big gaps in pupils' achievement not only among provinces but also among schools in one province", and it is surprising that "grade 3 pupils in the capital, Hanoi had levels of achievement in mathematics and reading comprehension in Vietnamese that were higher than grade 5 pupils in the other four, more rural, provinces" (The World Bank, 2004). The Primary Education for Disadvantaged Children Project also confirms an inhomogeneity of education opportunity [for students] and education quality in the whole country although the MoET has tried to reduce the gap (MoET, n.d. b).

Using the common set of textbooks in Vietnam has advantages (see section 3.1.2.3 of chapter 3). However, it also has some disadvantages (see section 3.1.2.3 of chapter 3). For instance, students and teachers have no choice but to use this specific series of textbooks. Moreover, a common textbook should be conformable to different levels of students' capacity. As discussed above, it should be noted that there are significant gaps of Vietnamese students' achievement from urban, rural and remote (mountainous and islandish) areas; moreover, the teaching and learning conditions in urban areas are much better than the teaching and learning conditions in remote areas (see also Nhan Dan, 2006).

On the contrary, there are different sets of mathematical textbooks in many countries. As an example, in Korea, about ten sets of mathematics textbooks are available for secondary school, although there is one series of textbooks for primary school (KSICMI, 2004, pp. 40 & 69). In Japan, there are six series of mathematics textbooks for middle school (Kunimune & Nagasaki, 1996). In China, in the past, there were unique series of curricula and textbooks; however, several series of textbooks

[11] Recently, there have been different points of view about the unique school textbook set in Vietnam. See a related debate in section 3 of chapter 3.

have been used since 1998 (Zhongru, 2004, p. 3). According to Hoyles[12], Küchenmann and Foxman (2003 a, p. 4), there are about six series of mathematics textbooks available in the Netherlands. Furthermore, Dutch school teachers can choose a mathematics textbook for their teaching, or they themselves can design a curriculum (Van den Heuvel-Panhuizen, 2000, p. 10). There are 16 states (Bundesländer) in Germany, and each state has its own education system (Weidig, n.d.). There are different series of mathematics textbooks used in Germany; moreover, the teachers of a school can choose the most suitable one from a list of textbooks accepted by the state, or they can personally develop teaching materials (Keitel-Kreidt, n.d.; Haggarty & Pepin, 2001, p. 119-120). There are several main sets of middle school mathematics textbooks in the United States such as *Mathematics in Context*[13], *Math Thematics*, *Connected Mathematics*, *MathScape* and *Pathways to Algebra and Geometry* (Meyer, Dekker & Querelle, 2001, p. 522). A project called Project 2061 analyzed thirteen sets of the middle school mathematics textbooks in the United States; however, only four of them were found "to be satisfactory" (Kulm, Roseman & Treistman, 1999).

1.3.1.2 Rigorous characteristic of mathematical curriculum and textbooks

One can compare the mathematical curriculum and textbooks in Vietnam with those in some other countries. However, it is a quite hard task because it is not easy to select a suitable country in order to consider this country's mathematical curriculum and textbooks and Vietnam's ones. Moreover, finding criteria for the comparison can be quite difficult. In the following, we mainly consider the relationship between amount of content in mathematics textbooks and amount of time a teacher has in a school.

There is a tension between the amount of content in mathematical textbooks and the amount of time teachers can spend in school. Some mathematics teachers complain that they do not have enough time for their teaching, and they have to transfer knowledge in textbooks to their students by anyway they can without paying attention to teaching methods they use (Tran Viet Luu, 2001, p. 19; Nguyen Thi Quy[14], 2004).

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[12] Prof. Dr. Celia Hoyles was awarded the first (2003) Hans Freudenthal Medal of International Commission on Mathematical Instruction (ICME) for her outstanding contribution in the technology and mathematics education domain.

[13] This set of textbook which is based on RME is created by researchers at the Freudeuthal Institute in the Netherlands.

[14] Dr. Nguyen Thi Quy is the Vice-Director of the Institute for Educational Research, Ho Chi Minh City University of Pedagogy.

One study of Do Dinh Hoan[15] shows that Vietnamese primary students spend less time than students from some other countries learning in school.[16] Because of this phenomenon, he suggests that primary students need to learn both in the morning and in the afternoon (two shifts) in school (Do Dinh Hoan, 2003, p. 14). Some studies based on the contradiction between knowledge quantity and total time (teachers can use for their teaching in school) to propose that all elementary students should participate in school both in the morning and in the afternoon (see, for instance, Nguyen Thi Quy, 2004).

Furthermore, some researchers claim that some parts of middle and high school students also need to learn two shifts in school (some other parts of middle school students in rural areas usually have to help their parents do housework and fieldwork. For this reason, these students cannot attend school in the morning and in the afternoon) (Do Dinh Hoan, 2003, p. 14; Nguyen Thi Quy, 2004; Huynh Cong Minh[17], 2004).

Apart from learning in official classes in school, students usually have to take part in some extra-lessons to keep up with curricula or pass examinations, and their parents have to pay these lessons. In some big cities or towns, even primary students have to attend these types of lessons. Students must attend extra-lessons so often that they have little time for their self-study or leisure activities. Among subjects in school, the number of students taking the extra-lessons of mathematics is highest. The abuse of extra-lessons has become one serious social problem and attracts attention of educators and society (see, for example, Nguyen Thi Quy, 2004; Huynh Cong Minh, 2004; Ho Thieu Hung, 2004; Binh Thanh Department of Education and Training, 2004, Tran Thu Ha et al., n.d.). The National Assembly has discussed the possibility of reducing school-induced stress (Nguyen Van An[18], 2004). The MoET has implemented certain strategies designed to cope with this problem; however, it appears that it is an extremely complicated social problem, and dealing with this problem is an especially

[15] Associate Prof. Dr. Do Dinh Hoan is the Director of the Center of Curriculum and Method for General Education of the MoET.

[16] Chapman and Adams (1998, p. 646) gave detailed data:

> Primary school students in Vietnam receive about 660 h of instruction, about ¾ of the worldwide average of 880 h for primary education. This is the result of both a short school year (165 d, below the international norm of 180 d) and short school days (rarely exceeding four hours a day).

[17] Huynh Cong Minh is the Director of Ho Chi Minh City's Education and Training Department.

[18] Nguyen Van An is the former President of Vietnam National Assembly.

difficult task (Nguyen Minh Hien[19], 2000 & 2005; Ho Thieu Hung, 2004; Huynh Cong Minh, 2004).

The same situation is also found in some other Asian countries. According to Heinze, Cheng and Yang (2004, p. 166), a number of Taiwanese students taking part in out-school-studying are quite high (more than 60%).[20]

The Institute for Educational Research at Ho Chi Minh City University of Pedagogy surveys of 35 schools, including primary, middle and high schools in 14 districts of Ho Chi Minh City, including 2,384 students and concludes that most of the students in these schools have to take extra-lessons (table 1.2), and there is no great difference among high-, middle-, low- and very low-achieving students (figure 1.2).

Table 1.2: The percentage of students who take extra-lessons according to grades (Nguyen Thi Quy, 2004)

	Grade 6	Grade 7	Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
The percentage of students who take extra-lessons (%)	60.87	85.23	88.80	92.70	81.80	88.56	85.86

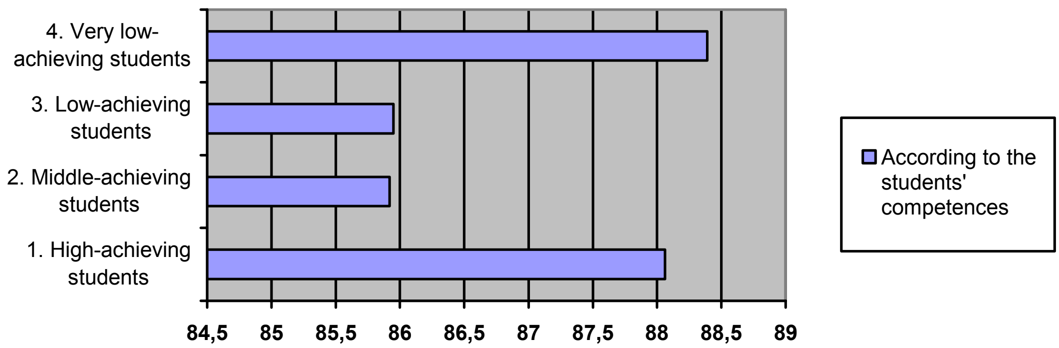

Figure 1.2: The percentage of student who take extra-lessons according to their competence (Nguyen Thi Quy, 2004)

In the research of Nguyen Thi Quy (2004), 2,151 students are asked about the contents in the extra-classes. The number of students who answer “learn more carefully content in the official classes” is highest (954 students, figure 1.3).

[19] Associate Prof. Dr. Nguyen Minh Hien is the former Minister of Education and Training.
[20] Taiwanese students also attend cram schools. However, there is no detailed information about a number of Taiwanese students taking part in this type of schools (Heinze, Cheng & Yang, 2004, p. 166).

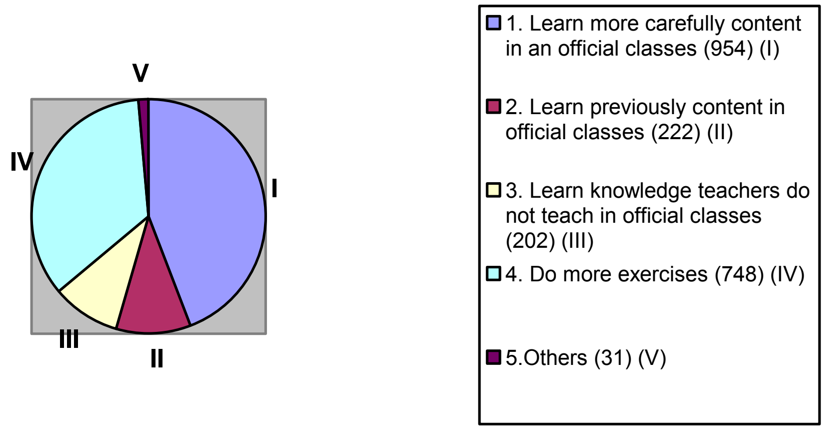

Figure 1.3: Contents are taught in the extra-lessons (Nguyen Thi Quy, 2004)

According to an interview of 416 parents, children usually spend anywhere from 5 to 25 hours a week for the extra-lessons, and 75.7 % parents said that their children do not have enough time for self-study (Nguyen Thi Quy, 2004). More seriously, 49.3 % of parents report that health and spirit of their children become weaker because of taking too many extra-lessons (Nguyen Thi Quy, 2004).

Most of the related studies confirm that among subjects in school, a number of students taking the extra-lessons of mathematics are highest from primary through high school (Ho Thieu Hung, 2004).

One study of an innovative long-term international research project, "Young Lives", is conducted in five Vietnamese provinces: Lao Cai, Hung Yen, Da Nang, Phu Yen and Ben Tre (Tran Thu Ha et al., n.d.). About 1,000 eight-year-old primary students are involved in the project. This research shows that the percentage of eight-year-old children take extra-lessons, in which mathematics and Vietnamese are the main subjects, in urban, rural and mountainous areas are 58 %, 56% and 7% respectively, and on average, children must spend about 9 hours per week for these lessons (7.8 hours, 8.9 hours and 9.5 hours for children in urban, rural and mountainous areas, respectively) (Tran Thu Ha et al., n.d.).

In comparison with curricula and textbooks from some other countries, some contents are presented earlier in Vietnamese ones.[21] In the following section, some examples are discussed.

[21] There are a few studies which compare mathematics education in Vietnam and in other countries.

Children in Vietnam are taught two-digit multiplication as early as second grade, whereas children in the United State learn multiplication more than a year later, around third grade (Orey & Nguyen, n.d.).

1.3.1.3 Theoretical knowledge in mathematics curriculum and textbooks

Generally, the Vietnamese mathematical curriculum and textbooks which emphasize mathematics structures and formal mathematics include some theoretical knowledge which is not suitable for school students:

• There are many complicated theoretical proofs in the middle and high school mathematics textbooks (see, for instance, some lessons in the 'reform' mathematics textbooks in appendix A). Furthermore, some contradiction proofs are introduced in the middle school mathematical textbooks. The fact of teaching mathematics shows that it is quite difficult for middle school students to understand such proofs (MoET, 2002 a, p. 9).

• After comparing geometrical content in mathematics textbooks of some countries, Hoang Chung (1999, p. 15) concludes that:

> The Vietnamese middle school geometrical textbooks mainly focus on practicing logical thought and reasoning deductively for students. In the whole geometrical textbooks from 7 to 9 (and geometrical parts in textbooks 6), there are mainly abstract geometry, definitions and proofs.

Hoang Chung also confirms that students could not acquire such abstract logical thought, or if they could acquire something, this is only the form, not the insight and meaning of geometry (1999, p. 16).

• As discussed in section 1.3.1.1 (this chapter), some content of probability and statistics, analysis and analytic geometry was first introduced in high school in Vietnam in the early 1990s. However, some knowledge is presented in the mathematical textbooks without taking notice of levels of students' awareness such as limit (ε-δ notation), tangent, derivative and integral concepts. It is rather difficult for students in high school to grasp fully concepts of a sequence and a function used ε-δ notion. Indeed, many mathematical teachers cannot teach these concepts, and they only teach students how to apply formulae in calculating a limit (Le Van Tien, 2000, p. 25; Nguyen Manh Chung, 2001, pp. 8-9; Nguyen Phu Loc, 2005, p. 30).

1.3.1.4 Lack of practical knowledge in mathematics curriculum and textbooks

Mathematics in the textbooks as well as examinations in school offer few examples or applications relating to real life or real world (see, for example, Do Dat, 2000). The relations between mathematics and other activities such as sports, politics, art and environment are not considered in the mathematics curriculum and textbooks, especially in secondary level. In general, application role of mathematics is not sufficiently appreciated in teaching-learning process.[22]

Vietnamese students often struggle to apply mathematical knowledge in reality. Despite the fact that they can find solutions to problems with pure mathematics content, it is difficult for them to use this knowledge to solve problems with practical contents or apply it in other disciplines. One reason for this situation is that the mathematics curriculum and textbooks do not pay enough attention to applying mathematics in reality. For instance, both the textbook and exercise-book of algebra for grade 10 have *only 5 exercises* with content related to reality (Dinh Quang Minh, 2003, p. 17). The same situation also occurs in other textbooks for other grades (see, for instance, some mathematics lessons in appendix A). Moreover, these exercises rarely appear in mathematical examinations (Dinh Quang Minh, 2003, p. 17).

Nguyen Ngoc Anh (1999, p. 18) emphasizes a role of mathematical application in teaching and learning mathematics in school; however, he claims that mathematics teachers do not train their students in applying mathematics in real life or in other subjects. He shows some main causes of this situation. Firstly, application of mathematics in other subjects in school or in real life is not focused in mathematical textbooks and referent books; secondly, it is not considered as an essential task in assessment; furthermore, such contents do not appear in mathematical examinations, and teachers often notice other contents which their students need for passing examinations (Nguyen Ngoc Anh, 1999, p. 18). For this reason, students are generally incapable of applying mathematics. One of Nguyen Ngoc Anh's surveys in the school-year 1994-1995 shows that although 48% of students know how to find extreme values of a function, only 28% of them can solve a simple problem of extreme values with practical contents (Nguyen Ngoc Anh, 1999, p. 18).

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[22] Recently, the present mathematics curriculum and textbooks significantly emphasize mathematics application. However, the way in which mathematics applications are used is still primitied and limited (see section 3.2.2.3).

Tran Kieu [23](2001, p. 9) stresses that "practical roots and various applications of mathematics are acknowledged most in teaching in school". According to Tran Kieu (2001, p. 9), mathematics is often regarded as a necessary tool for understanding the sciences. Despite mathematics' noted centrality, Vietnamese schools neglect to present mathematical applications to real life situations.

In comparison with trends of mathematics education in the world, the MoET confirms that the Vietnamese mathematics curriculum and textbooks in middle school immoderately focuses on pure theory and did not take enough notice of practical knowledge (MoET, 2002 a, p. 2).

Mathematical exercises related to reality do not appear in examinations (see some mathematics examinations in Do Dat, 2000).

1.3.1.5 Other characteristics

In general, most of the 'reformed' mathematics textbooks are well-structured. However, the form and content of mathematics textbooks used from the early 1980s through the early 2000s appeared more advanced and theoretical and less pedagogically- suitable. More specifically, although these textbooks ensured scientific and precise characteristics, pedagogical phases were not sufficiently considered.

Typically, a formula or a theorem is presented in the mathematics textbooks as follows:

- Step1: Content of a formula or a theorem
- Step 2: A proof of the formula or theorem
- Step 3: Application of the formula or theorem in some pure mathematics examples

Similarly, a concept is often performed as follows:

- Step 1: Definition
- Step 2: Some examples of the concept
- Step 3: Characteristics

However, some parts of some mathematical textbooks are not well-structured, for example:

> There is an imbalance between teaching time for natural numbers and for decimal numbers. Time for teaching natural numbers starts from the beginning of the first year

[23] Asscociate Prof. Tran Kieu is an Ex-director of the The National Institute for Educational Sciences in Vietnam.

> to half of the first term of year 5, but time for teaching decimal numbers only starts at the end of the first term of year 5. Therefore pupils' calculating skills with decimal numbers are weak
>
> (Do Dat, 2000, p. 4)

1.3.2 Methods of teaching mathematics

1.3.2.1 Typical methods of teaching mathematics in school

In general, methods of teaching school subjects, in general and mathematics, in particular are poorly developed in Vietnam (Ton Than, 2003 a, p. 31; Hoang Huy Lap, 2004, p. 14; Nguyen Minh Thuyet, 2005, p. 3). Most school mathematics teachers in Vietnam use and prefer two teaching styles called 'thày đọc, trò chép' and 'luyện thi', meaning 'a teacher reads, students write' and 'training for exams', respectively. It is better if the first one is called 'a teacher explains, students listen and write down'. The latter can be described as follows: a teacher introduces different forms of problems which are related to specific knowledge and gives illustrated examples for each form, then student imitate the teacher' solutions to deal with other similar problems. The 'teacher reads, students write' method is quite popular in teaching mathematics in Vietnamese school (see, for instance, Ton Than[24], 2003 a, p. 31).

Dinh Quang Bao, a former Rector of Hanoi University of Education (from 1998 to 2006), states that:

> Teaching methods in both school and university are underdeveloped. Main method is a lecture method. Teachers slowly and monotonously present and even slowly read their teaching plan. School and university students write down carefully. At home, they only learn what are written down in their notebooks.
>
> (cited in Hoang Thanh Hai, n.d.)

Hoang Xuan Sinh, a professor of mathematics, also confirms that: "until the master level, we have only one way of teaching: teachers lecture, and students take notes" (cited in Hoang Thanh Hai, n.d.).

Bui Huy Hien (2005, p. 221) writes in his book about the history of Vietnamese education about the fact of the current teaching methods in Vietnam:

[24] Associate Prof. Dr. Ton Than is the Chief Editor of the current middle school mathematics textbooks in Vietnam.

> Methods of teaching are not improved: normally, a teacher lectures, and students inactively acquire. This occurs not only in school education but also in graduate and post-graduate education.

Similarly, Do Dat describes typical methods of teaching mathematics in primary school as follows:

> In most primary schools, teaching methods are explained and illustrated. The teacher mainly imparts content and knowledge, pupils learning by the examples. The majority of teachers generally prefer explanations, lectures and samples with frequent incidental questioning. The teachers are not monitored, they do not help pupils to create problems and 'occupy' new knowledge.

(Do Dat, 2000, p. 4)

1.3.2.2 Impact of examinations on methods of teaching mathematics

Typically, schools, local Departments of Education and Training in a province or city and the Ministry of Education and Training often regard results of examinations as important indicators and goals of teaching and learning.

Mathematics teachers usually use 'training for exams' method to help their students carefully practice forms of problems which usually appear in examinations. One 'real goal' of teaching mathematics in school is to help students score higher on examinations.

1.3.3 Assessment and examinations

This section, firstly, introduces examinations which school students must take from primary through high school. Secondly, it discusses some characteristics of school assessment and examinations. Finally, lack of scientific foundations for school assessment and examination is considered.

1.3.3.1 Examinations

Apart from examinations at the beginning of the first semester and at the end of every semester of school-years, there are five major and hard examinations (MoET, 2002 c; 2006, p. 4):

- Examination for diploma of primary school (two subjects: mathematics and Vietnamese)[25];

[25] Recently, the National Assembly of Vietnam has made a decision of deletion of this examination (see, for instance, Hoang Van Tu, Le Van Binh, Vu Anh Tuan, Vu Lan Anh, Phan Thanh Ha, 2005, p. 29).

• Examination for diploma of middle school (four subjects: mathematics, literature and two of the other subjects);

• Examination prior to entering high school (two subjects: mathematics and literature; in some provinces or cities, students' enter high school are considered through their learning results in middle school);

• Examination for diploma of high school (six subjects: mathematics, literature, foreign language and three of the other subjects);

• College or university entrance examinations (there are four main blocks: A (mathematics, physics and chemistry), B (mathematics, chemistry and biology), C (literature, history and geography) and D (mathematics, literature and foreign language). This is an exceptionally competitive examination.

Nearly all of the mentioned examinations include mathematics. Written examinations are mainly used. All mentioned mathematics examinations in this section are in the form of written examinations. Each of the examinations for diploma and college or university entrance (national examinations) is organized on the same days throughout the country (MoET, 2002 c, p. 1) The examination for diploma of middle school is composed by a committee of the local department of education and training in a city or province, while the examination for diploma of high school and the university entrance examination are composed by the MoET (MoET, 2002 c, pp. 8-9).

1.3.3.2 Memorization and creativity in examinations

In general, content of examination in school forces students to learn by heart and does not require creativity (Tran Viet Luu, 2001, p. 19 & 22; Nguyen Thi Quy, 2004).

De Lange uses a pyramid model to present the levels of understanding (cited in Verhage & De Lange, 1996, pp. 1-5; Van den Heuvel-Panhuizen, 1996, pp. 137-139). He distinguishes three levels: a lower, middle and high level (Verhage & De Lange, 1996, p. 2):

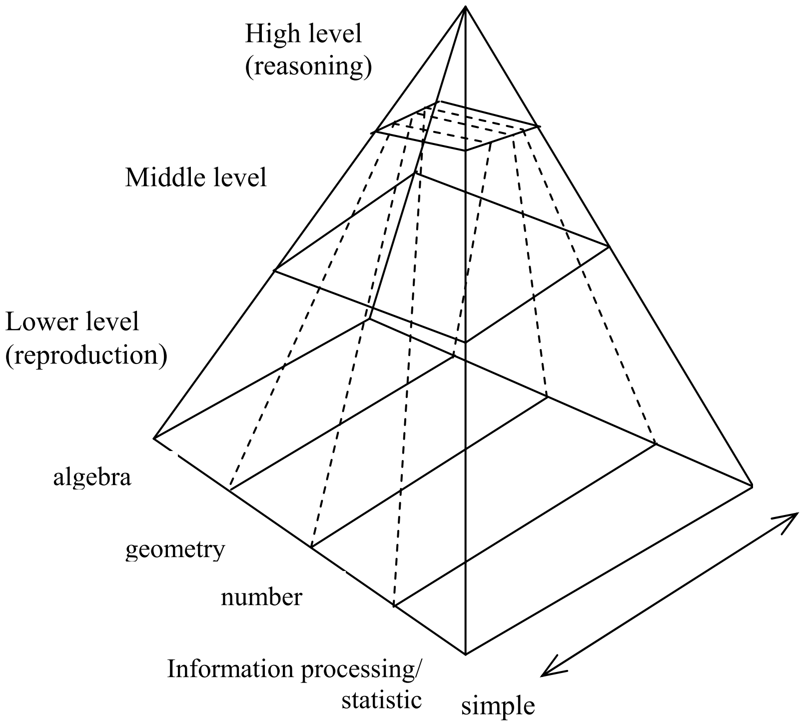

Figure 1.4: De Lange' s Pyramid model

Generally, assessment in Vietnamese school often focuses on the lower level. Most of problems appearing in mathematics examinations are algorithm-based problems which usually assess students' ability of rote learning. In other words, students have to memorize mathematical regulars and formulae and then apply them to solve problems. Or students are introduced to one form of mathematical problems and then try to understand the solution to these problems and how to apply this solution in similar situations. Sometimes "problems that offer different strategies for solving, or offer more than one approach to solve" (Verhage & De Lange, 1996, p. 3) are used in assessment. However, as a whole, the second level (middle level) of De Lange (Verhage & De Lange, 1996, pp. 3-4) is rarely used in assessment in Vietnam.

There is a familiar problem of 'Polar Bear' in the Netherlands: "A polar bear weights 500 kilograms. How many children together weigh as much as one polar bear?" (Van den Heuvel- Panhuizen, 1996, p. 95). Verhage and De Lange (1996, pp. 4-5) offer this problem as an example for the third level. In order to solve this problem, students, first of all, have to estimate the average weight of children. The solution to the problem is not fixed, and it depends on estimations of students. The aim of assessment is not only to test students' ability of applying division regulations (with remainder), but also to test their estimation and use of different (informal) strategies to solve the problem. The author of this dissertation used a questionnaire, including this problem to interview 152 Vietnamese mathematics teachers from different middle schools in Hai Duong, Ha Nam, Ha Tay, Kien Giang, Thai Binh and Hanoi (see the questionnaire form in appendix B). Most of them believed that not enough 'given' information was provided because the weight of a child or the average of children' weight is unknown (see details in section 5.2.5 of chapter 5). Moreover, they thought that this problem is not worth using in school.

Nguyen Quang Trung (2004) describes examinations in Vietnamese school as follows:

> Examinations only aim at assessing students' abilities of memorization. As a result, students often focus on learning their teachers' lectures by heart. They do not know how to learn creatively […]

Similarly, Le Thi Thanh Thao states that most of the examinations only require abilities of "rote memorization, repetition and understanding through mechanical impractical applications" (Le Thi Thanh Thao, 2004). She explains that the applications "often require tricks and forms", and if students do not have chances to pre-train, they cannot solve problems of the applications (Le Thi Thanh Thao, 2004).

1.3.3.3 Lack of scientific foundations for assessment and examinations

Some research shows that there is a derth of scientists who specialize in assessment and examinations (see, for example, Le Thi Thanh Thao, 2004). According to Le Thi Thanh Thao (2004), one reason for this situation is that the role of assessment and examinations is not adequately considered; consequently, there is no strategy for developing staff in this field. Le Thi Thanh Thao explains why teachers often meet difficulties with assessment innovation: "[…] all teachers who graduated within several tens years ago were not equipped with knowledge about assessment and examination; therefore, they are faced many difficulties when they would like to reform methods of assessment and examination." (Le Thi Thanh Thao, 2004).

1.3.4 Classroom organization

1.3.4.1 Curriculum distribution and teaching plan

The local department of education and training in a province or city determined a year curriculum distribution, which indicates how much time should be devoted to each lesson. A teacher must prepare a written teaching plan. The headmaster of a school or the head of the school mathematics department can check whether or not a mathematics teacher has her or his own teaching plan.

The typical classroom in Vietnam rigidly adheres to set curriculum. Because teachers are so dependent on time regulations of the curriculum distribution, it is difficult for them to adjust their teaching method. These time allowances specified in the curriculum distribution will stifle certain classrooms by imposing time restraints on the lesson plan. Moreover, as discussed in section 1.3.1.1 of this chapter, students' achievement varies between regions and ethnicities. One standardized curriculum distribution established by the province or city's department of education cannot accommodate students' many different needs and capabilities.

1.3.4.2 Tools and activities of teachers in lessons

In classroom, mathematics teachers usually use pieces of chalk, a blackboard, a compass and a ruler. Teachers rarely use overhead projectors, computers, beamers, videos and other tools for their teaching. In the past, teachers were not sufficiently equipped these teaching tools. Recently, although Vietnamese schools have been gradually equipped necessary teaching facilities, teachers are not familiar with these tools.

Nguyen Quang Trung observes that most of school teachers, especially teachers of mathematics and English, suffer from overload of lecturing; and consequently, the teachers do not have sufficient time for self-studying, upgrading knowledge and reforming their teaching methods (Nguyen Quang Trung, 2004).

1.3.4.3 Tools and activities of students in lessons

The teacher typically assigns each student to a specific seat in the classroom for the semester or school-year. Normally, students are not allowed to change their seats.

Students rarely work in groups or in pairs. In primary school, for example, only 2% of classtime is allocated for group work (Do Dat, 2000, p. 4).

Students typically use pens, pencils, rulers, compasses and papers.

Students do not have the chance to play games, which can help them to learn mathematics.

1.3.4.4 High number of students in a class

According to a regulation of the Vietnamese Ministry of Education and Training, a class in a high school must have fewer than 50 students (table 1.3).

Table 1.3: A regulation about a number of students in a class (The Institute for Educational Development, 1998)

	A number of students per a class (according to regulation)
Primary school	From 35 to 40
Middle school	From 40 to 45
High school	From 45 to 50

However, the number of students per class is actually much higher than this regulation mandates. Different researches give different numbers. For example, the number of students in middle and high school in the city center are quite high (more than 40 students per class) (Huynh Cong Minh, 2004), and because of this high number of the students, the classroom is too crowded (on average from 40 to 55 students per a class) (Binh Thanh Department of Education and Training, 2004). Trinh Quoc Thai (2002, p. 23) confirms that the average number of students in a primary class is 60-70.[26]

According to the General Statistics Office of Vietnam (2005), an average number of students are presented in table 1.4 according to school-years[27] and a level (primary, middle and high school). In general, the average number of students in a high school class ranges from 46 to 50 students, while the average number of students in a primary class ranges 28 to 31 students. It should be noted that the number of students in a class in an urban area is significantly higher than the number of students in a class in a rural or mountainous area.

[26] It appears that he mentions primary classes in densely-populated cities.
[27] A Vietnamese school-year often lasts from September of a year to Mai of next one.

Table 1.4: An average number of students per class

School-year / Level	1998 -1999	1999 -2000	2000 -2001	2001 -2002	2002 -2003	2003 -2004
Primary school	31	31	30	30	29	28
Middle school	41	41	41	41	40	40
High school	49	50	48	46	47	46

According to Stigler and Hiebert (1999, p. 28), the national average for eighth-grade class size is twenty-five students per class in Germany and the United States and thirty-seven students per class in Japan. There are roughly thirty-five students per middle school class in Japan (Kunimune & Nagasaki, 1996). In comparison with these countries, the number of students in a class in Vietnam is considerably higher.

1.3.5 Teacher staff

Due to historical factors, a percentage of teachers in general studies, in particular teachers of mathematics, were not properly trained. As a consequence, they lack not only mathematical, but also pedagogical, knowledge. Hence, these teachers have, and continue to, struggle to modify or reform their teaching approaches (Nguyen Quang Trung, 2004). Furthermore, teachers' individual competence and capabilities differ widely among urban, rural and remote areas. The National Institute for Educational Sciences (NIES) conducted a study on three and five grade students' achievement in reading and mathematics of fourteen primary schools in five provinces. This study reveals that, "some Grade 5 pupils performed better than some Grade 5 teachers" (see details in The World Bank, 2004).

According to the MoET (cited in Bui Huy Hien, 2005, p. 222), the percentage of teachers who do not satisfy *training standards* is quite high (12,28%, 8,47% and 4,75% teachers in primary, middle and high school respectively in the school-year 2001-2002) (see details in figures 1.4, 1.5 and 1.6 below). In order to receive the teaching certification, primary teachers-in-training spend two years studying in professional schools to obtain diplomas; middle school teachers-in-training spend three years studying in colleges to obtain their degrees; and high school teachers-in-training spend four years studying in universities for qualification (Vietnam National

Assembly, 2005, chapter 4, item 2, article 77). In case the students who graduated from either non-pedagogic professional schools or non-pedagogic colleges and universities, they need to participate in some extra-lessons and receive certification for educational profession (Vietnam National Assembly, 2005, chapter 4, item 2, article 77). Recently, school standards for primary teachers have been discussed and created in Vietnam (see draft of standards for primary teachers in Educational Review, 2003, pp. 3-6).

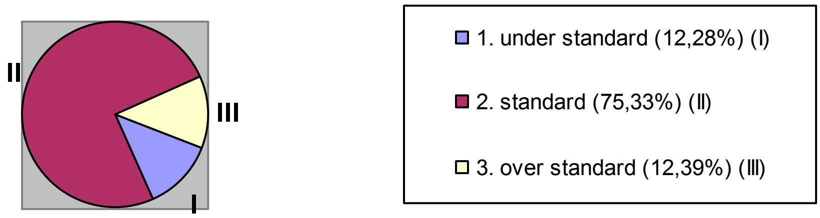

Figure 1.5: The percentage of primary teacher according to training standards

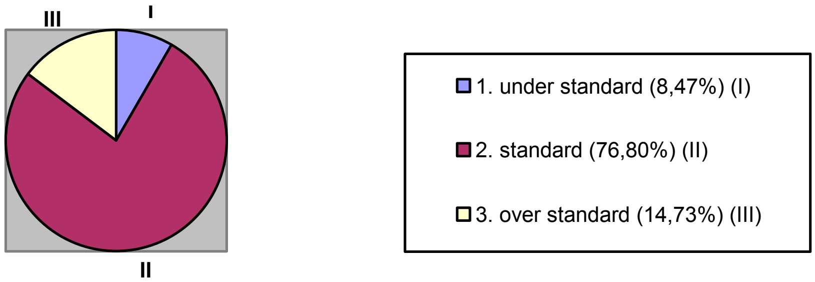

Figure 1.6: The percentage of middle school teachers according to training standards

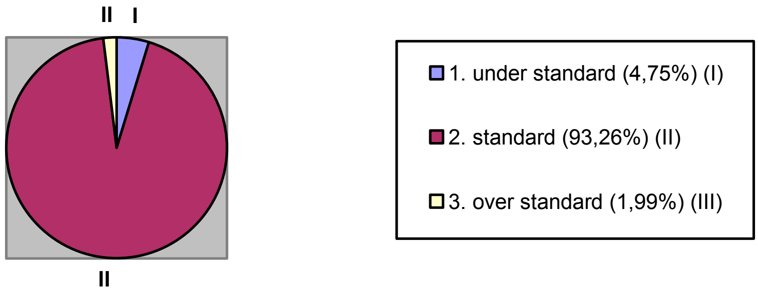

Figure 1.7: The percentage of high school teachers according to training standards

One study shows that about 10% of primary teachers would change jobs in order to ensure teaching and learning quality following the new curricula in Vietnam (Do Dinh Hoan, 2003, p. 14).

1.4 General ideas to improve mathematics education

As discussed above (section 1.3), Vietnamese mathematics education has encountered some challenges. To solve these problems is an extremely difficult and complicated long-term task. The Vietnamese Ministry of Education and Training (MoET) has been carrying out some important projects, including the Renovation of the Curriculum and Teaching Methods, Training of Information Technology (IT) officers and introducing of IT into school education, Primary Education Project, Primary Teacher Development Project, Lower Education Project and Lower Education Teacher Development Project, in order to reform and improve school education (see, for instance, MoET, n.d. c). In the following sections, some general ideas for improving mathematics education are considered and expounded.

1.4.1 Mathematics curricula and textbooks

As discussed in section 1.1 (this chapter), the ‘reformed’ series of curricula and textbooks has been gradually replaced by new curricula and textbooks. The new curricula and textbooks are noticeably different than the former ‘reformed’ materials. This section will explore some of these major modifications of the primary, middle and high school mathematics curricula and textbooks.

1.4.1.1 Primary school

According to Do Dat (2000, p. 6), the Primary Mathematics Curriculum for 2000 includes the following modifications:

- Minimizing unnecessary difficult knowledge;[28]
- Increasing mathematical application in reality;
- Adjusting time for teaching natural numbers, fractions, decimal and percentage;
- Introducing more geometrical shapes such as cylinder and sphere.

1.4.1.2 Middle school

Recently, a new series of textbooks based on a Middle School Mathematics Curriculum promulgated by the MoET on January 24, 2002 has been written (MoET, 2002 b). Most mathematics educators in Vietnam believe the new curriculum and textbooks are generally superior to the ‘reformed’ curriculum and textbooks.

Some of the principles for building the middle school mathematics curriculum are:

[28] It appears that the new series of primary curricula is still quite rigorous. According to the MoET, the new primary curricula need to be reduced about 15% although these curricula have just been used for a few years.

• do not pay excessive attention to structural and precise characteristics of the system of mathematical contents in the curriculum;

• do not present results with pure theoretical meaning and long complicated proofs which are not suitable for majority of students to the curriculum;

• allow students to practice and exercise calculation skills and applying mathematical knowledge in life and other school disciplines.

(MoET, 2002 a, p. 2)

According to Nguyen Minh Phuong (2001, pp. 9 & 11), Ton Than (2000, 2003 a, b & c), Tran Phuong Dung (2003), Pham Gia Duc (2003) and Vu Huu Binh (2004), the new series of middle school mathematics textbooks has the following advantages:

• *Inappropriate mathematical content is reduced* in the textbooks. Mathematical concepts, theorems and exercises which are not suitable for students at one grade are moved to the next grades or removed from the curriculum and textbooks. Additionally, some theorems are introduced without proofs if these proofs are too long or complicated, and some abstract mathematical definitions are replaced by simple descriptions.

• *Various types of exercises* are presented in the textbooks. There are exercises which seek to help students practice calculating, draw figures, predict characteristics, reason, create proofs, etc. In addition, there are *various forms for exercises* such as word problems, gap fillings, 'true-false' and multiple choice questions, crossword puzzles and mistake findings in given problems' solutions, whereas most of the exercises in the former textbooks were conventional word problems.

• The new series emphasizes *problem situations* and *self-study*.

• While the old set of textbooks did not strongly incorporate mathematics application, the *new one places special importance on mathematics application* in real life and other school disciplines.[29]

• The new series also *hopefully attracts students' interest by providing them interesting stories about history of mathematics*.

• The new curriculum covers *calculator usage* in mathematical calculation and reality.

[29] Although new mathematics curriculum and textbooks pay special attention to mathematics application, the way in which application is presented is quite conventional. That is, formal mathematics is taught first, and then formal mathematics is applied on solving application problems. See detailed discussion in section 2 of chapter 3.

In geometry:

• *Absolute axiom-based viewpoint* in geometry is replaced by *using alternative visual and reasoning*. Geometry in middle school is not constructed as *a pure deduction science.* This means that geometry does not introduce a system of axiomatic and then builds mathematical theorems and characteristics by using mathematical proof. Instead, in some cases, students can measure, observe, experiment and then draw conclusions (without proofs). Geometrical proofs are reduced significantly in grades 6 and 7.

• By *drawing* and *observing* figures, *measuring* segments and angles, *folding* and *cutting* paper, students have the opportunity to *comment on events* and *predict characteristics* before geometrical theorems and their proofs are officially introduced.

1.4.1.3 High school

Three sets of high school mathematics ‘reformed’ textbooks were corrected and edited to build new one, which has been used since 2000 (see section 1.2 of this chapter).[30] This series is called a set of the ‘2000 corrected unified high school’ mathematics textbooks. Recently, a new high school mathematics curriculum was approved, and new set of textbooks has been tested and gradually replaced the ‘2000 corrected unified high school’ series.

In comparison with the ‘reformed’ textbooks (see section 1.3.1, this chapter), the ‘2000 corrected unified high school’ ones have the following modifications (Nguyen Huy Doan, 2000):

• Reducing theoretical knowledge and increasing practical knowledge (while more abstract, complicated, theoretical knowledge is significantly reduced in this new curriculum, more applicable practical knowledge includes);

• Unifying mathematical signs and terms in textbooks;

• Removing extremely difficult exercises and using moderated number of exercises.

Recently, a new series of high school mathematics textbooks has been tested and gradually used in high school (see section 1.2, this chapter). In comparison with the ‘reformed’ curriculum and textbooks, new ones have following changes (MoET, 2002 b; Tran Van Hao, 2003, p. 20):

• Adjusting structure of mathematics contents;

[30] Typically, there is a unique series of mathematics textbooks in Vietnamese school. There was an exception for high school because there were three series of mathematics textbooks available from the early 1990s untill the late 1990s. However, these three sets of textbooks were unified in 2000.

• Reducing theoretical knowledge and paying attention to applications of mathematics (some complicated theoretical contents are omitted; in some cases, formal mathematical definitions are replaced by intuitive descriptions; some complicated formal mathematical proofs are not presented);
• Changing ways of presenting mathematical knowledge in textbooks (unlike the ‘reformed’ mathematics textbooks (see section 1.3.1.5 of this chapter), the new mathematics textbooks often offer students some tasks before mathematics definitions, theorems, rules and formulae are formally presented);
• Introduction to the use of calculators in teaching and learning mathematics;
• Introduction to the use of tests in examination;
• Emphasizing and encouraging students’ self-study.

1.4.2 Methods of teaching

Because of the fact of underdeveloped teaching methods (section 1.3.2, this chapter), a reform of teaching methods is urgently required to remedy mathematics education in Vietnam (Nguyen Thi Quy, 2004).

General orientation for methods of teaching mathematics in middle school, determined by the MoET, is “activate students’ activities; focus on students’ abilities of self-studying, discovering and solving problems in order to form and to develop active, independent and creative characteristics for students” (MoET, 2002 a). According to this orientation, teachers should play the role of designers, organizers, guiders and controllers, and students should learn how to self-study and self-practice, in order to develop their personality and prepare themselves to fulfill the requirements of new employees eventually (MoET, 2002 a).

According to MoET (2002 b), general orientation for teaching high school mathematics is:

• Paying attention to activeness, initiative and ability of self-study for students;

• Using advantages of each teaching methods and paying attention to using problem solving approach;

• Providing students with the necessary knowledge and skills for real life.

1.4.3 Assessment and examinations

Recently, an assessment reform in Vietnamese school has become a necessary task. Here are some orientations for this reform:

• The MoET considered assessment as a part of the current mathematics curriculum. In this curriculum, the MoET suggests some changes of assessment (MoET, 2002 a, b & c).

• More research of assessment in mathematics is required (Le Thi Thanh Thao, 2004).

• Assessments and examinations are necessary to research and reform: develop students' creativeness and reduce memorization (Hoang Thi Tuyet, 2004).

• Conduct other types of assessment and experiment use of tests for examinations (Hoang Thi Tuyet, 2004; Nguyen Quang Trung, 2004).

1.5 Research question

The reform of the teaching methods has become urgent for mathematics education in Vietnam. For this reason, seeking a suitable teaching and learning approach is one of several possible necessary tasks for mathematics education in Vietnam. Considering the problem about the fact of the underdeveloped methods in teaching and learning mathematics (section 1.3.2 of this chapter), Realistic Mathematics Education (RME) may become a potential and suitable approach which will help mathematics education in Vietnam overcome its obstacles. However, RME must be expanded, and its application to Vietnamese mathematics education must be considered carefully.

1.5.1 Research question

This dissertation focuses on the general question: *how can RME enrich teaching and learning mathematics in Vietnamese school?*

This question, however, must be narrowed down so that it fits within the range of a dissertation. Because the author of this dissertation specializes in training (pre-service and in-service) middle and high mathematics teachers, special emphasis is placed on the middle and high school level. Furthermore, since RME is quite new for mathematics curriculum developers, textbook authors, educators and teachers in Vietnam, the middle school level should be carefully considered. Finally, as discussed in chapter one, although teaching and learning mathematics have been noticeably improved, they still have their own weaknesses. The potentials of RME for overcoming shortcomings of mathematics education in Vietnam hence must be considered. In sum, the research question of this dissertation is:

How can RME be used as a potential teaching and learning approach which can help mathematics education in middle school overcome its disadvantages?

1.5.2 Sub-questions

1.5.2.1 Sub-question 1 (concerning a grade and a mathematics strand)

From primary school through high school in Vietnam, the abstraction of mathematics significantly increases. In addition, typically, Vietnamese mathematics curricula and textbooks emphasize formal mathematics. It appears that (formal deductive) geometry is one of the most difficult topics for Vietnamese middle school students. Accordingly, this dissertation focuses on middle school geometry. Furthermore, in the present middle school mathematics curriculum and textbooks, geometrical proofs are first introduced in grade 7, and a number of proofs are gradually increased in grade 8 and 9. It is worth considering how RME can be applied in teaching and learning grade-seven geometry. The first sub-question is:

How can RME be applied in teaching and learning grade-seven geometry in Vietnamese school?

1.5.2.2 Sub-question 2 (considering difficulties teachers and students may meet)

Teaching and learning styles in Vietnam are different from those in western countries, particularly in the Netherlands. Studies about applying RME to teaching and learning mathematics in Vietnam should bear this difference in mind.

What difficulties do teachers and students meet while RME is applied in teaching and learning middle school geometry?

1.5.2.3 Sub-question 3 (concerning potentials of RME)

In general, RME seems to be especially suitable for the orientation of the reform of mathematics teaching methods in Vietnam (see section 1.4.2 of this chapter). However, it is necessary to investigate possible effects of RME in Vietnamese mathematics education. Sub-question 3 should be:

What is the potential of RME, and how can this potential help mathematics education in Vietnam overcome its shortcomings?

1.5.2.4 Sub-question 4 (considering possible proposals for applying RME)

Generally, teaching styles are quite rigid and conventional in Vietnam. Teachers and students may face difficulties while working with RME. For this reason, possible solutions or suggestions should be found so that teachers can deal with these difficulties. Sub-question 4 is:

What and how proposals should be made so that RME can be applied in teaching and learning in Vietnam?

1.5.2.5 Sub-question 5 (concerning frequency use of RME in teaching and learning in Vietnam)

In general, there is a strict regulation about a length of time and timetable for each lesson in Vietnam. Moreover, there is the contradiction between the amount of content in mathematical textbooks and the amount of time which teachers and students have in school (see section 1.3.1.2 of this chapter). In addition, in Vietnam the unique set of textbooks is considered as an official material for teaching and learning in school (see section 1.3.1.1 of this chapter).

One question is posed: *How often should RME be implemented in teaching and learning in middle school?*

Chapter 2 Realistic Mathematics Education

> Realistic Mathematics Education (RME) is a theory of mathematics education that offers a pedagogical and didactical philosophy on mathematical learning and teaching as well as on designing instructional materials for mathematics education.
>
> Bakker, A. (2004, p. 5)

Because *Realistic Mathematics Education* (RME) is new to most mathematics teachers, textbooks authors and educators in Vietnam, this chapter presents the basic ideas of RME theory. An overview of RME history is given, followed by the essential ideas of Freudenthal for RME (mathematics as a human activity, guided reinvention and didactical phenomenology) and discussions of the principles of RME which combine Van Hiele's levels of learning mathematics, Freudenthal's didactical phenomenology and Treffers's progressive mathematization. Finally, some selected examples about RME, including developing long division and the empty number line for addition and subtraction up to 100, are presented.

2.1 Overview of RME history

In this section, the *Wiskobas* project is discussed. In the Netherlands, the innovative Wiskobas project inaugurated the period of elementary mathematics education in which RME was formulated and developed.

Beginning in the Netherlands in 1968, the Wiskobas (mathematics in the elementary school) project sought to create innovations in national mathematics education by reforming teacher training. An attempt to devise a new elementary mathematics curriculum in the Netherlands, the researchers of the project analyzed different trends of mathematics education, not only inside, but also outside, the Netherlands such as: the *Arithmetical (Mechanistical)*, *Structural*, *Empirical Trend* and *Dutch Arithmetic Education* (Treffers, 1987, pp. 14-17). Apart from the necessary "pre-institutional stage" (1968-1971), the Wiskobas project has three important periods: "an exploratory phase" (1971-1973), "an integration phase" (1973-1975) and "spin-off, further development and research" (1975-1977) (Treffers, 1987, pp. 11-13). As a result of this project, Dutch (elementary) mathematics education was not influenced by the "New Math" approach (Van den Heuvel-Panhuizen, 2000, p. 3). The founding principles of what later became RME approach appeared in the Netherlands in 1970s. The essential ideas of present RME form are mainly based on Freudenthal's philosophy on mathematics and mathematics education (Van den Heuvel-Panhuizen, 2000, p. 3). RME also incorporates elements from the aforementioned educational approaches. Moreover, it takes advantages from these approaches (Treffers, 1987, pp. 14-18).

For over thirty years, RME has been primarily developed by mathematics educators in the Freudenthal Institute of Utrecht University and other research institutions in the Netherlands. Currently, about 75% of the elementary Dutch schools use RME-based textbooks (Treffers, 1991, p. 11). Although RME is already 30 years old, it is still in development (Van den Heuvel-Panhuizen, 1998 & 2000, p. 3). Many dissertations and research projects conducted at the Freudenthal Institute and other institutions in the Netherlands have been involved in developing RME. The following paragraphs provide a brief introduction to some of these projects.

The Hewet project (1981-1985) developed a mathematics curriculum (*Mathematics A*), a high school RME curriculum, specially designed for students whose field of study in university will most likely be in either the humanities or the social sciences (De Lange, 1987, pp. 1-2 & 8-9). De Lange's dissertation (1987) includes an intense examination of historical context, development, content structure, theoretical framework, methods and assessment (tests) of *Mathematics A*.

Gravemeijer's dissertation (1994) entitled, "Developing Realistic Mathematics Education" thoroughly analyzes and discusses "instructional design as a learning process", "an instruction-theoretical reflection on the use of manipulatives", "mediating between concrete and abstract", "educational development and developmental research in mathematics education" and "implementation and effect of realistic curricula".

The development of RME assessment strategies (the starting period, the present period and written assessment) and the MORE project are sufficiently discussed, analyzed and expanded in Van den Heuvel's dissertation (1996).

The ideas of RME are applied to create a series of U.S. middle school mathematics textbooks called *Mathematics in Context* - one of the major middle school mathematics textbook series in the U.S.A (see, for instance, Romberg, 2001; Meyer et al., 2001). Although most research on RME focuses on school mathematics education, some even apply RME in undergraduate teaching and learning mathematics (Rasmussen & King, 2000; Kwon, 2002; Ju & Kwon, 2004).

2.2 Some basic ideas of Freudenthal for RME

In the following section, some basic ideas of Freudenthal for RME theory, including mathematics as a human activity, guided reinvention and didactical phenomenology are discussed. It should be noted that these Freudenthal' ideas are often interactive, and they relate to his question "Why […] teach mathematics so as to be useful", a lecture name of Freudenthal (cited in Van den Heuvel-Panhuizen, 1996, p. 10) and a name of his first article in the Educational Studies in Mathematics (Freudenthal, 1968).

2.2.1 Mathematics as a human activity

2.2.1.1 Mathematics and common sense

After offering some examples about common sense and analyzing how it is often rejected by the natural sciences (e.g. physics, chemistry and astronomy) and their didactical principles, Freudenthal (1991, p. 6) stresses the importance of common sense in instruction, especially in mathematics instruction:

> […] I believe that in instruction it would be more recommendable to start with common sense ideas rather than to reject them as outdated and better being suppressed. This belief is supported in any case by the fact of the more or less spontaneous development of mathematics.

Freudenthal discusses the poor relationship between classroom and school experience and life experience; however, education should emphasize real life experience, argues Freudenthal (1991, pp. 4-6).

Below, Freudenthal (1991, p. 9) discusses strategies to repair this relationship:

> Common sense, in order to become genuine mathematics and in order to progress, had to be systematised and organised. Common sense experiences, as it were, coalesced into rules (such as the commutativity of addition), and these rules again became common sense, say of a higher order, as a basis of even higher order mathematics-a tremendous hierarchy, built thanks to a remarkable interplay of forces.

According to Freudenthal (1991, p. 9), mathematics is the oldest science and "was more easily invented" than other sciences. After analyzing the differences between mathematics and other sciences, Freudenthal (1991, p. 11) suggests that mathematics should be learned and taught differently, "that is, neither as form nor as content but while maintaining respect for the interplay between them, acted out in the teaching/ learning process!".

2.2.1.2 Mathematics as a ready-made product and mathematics as a human activity

Freudenthal discusses two different approaches to mathematics. The first approach considers mathematics as *a ready-made product*, and the second one regards mathematics as *an activity*.

Freudenthal emphasizes idea of *mathematics as a human activity.* He explains that: "Mathematics as an activity is a point of view quite distinct from mathematics as printed in books and imprinted in minds." (Freudenthal, 1991, p. 14) Products of mathematical activity which are understood as a broad meaning include not only propositions and theorems, but also, "proofs, even definitions and notations, as well as the layout, in print and thought" (Freudenthal, 1991, pp. 14-15).

Freudenthal (1991) considers mathematizing to be one of main characteristics of mathematical activity. Details of mathematizing are discussed in section 2.3.1 of this chapter.

In mathematics education, Freudenthal strongly objects to what he called *anti-didactical inversion*: teaching mathematics by beginning with *ready-made mathematics* (Freudenthal, 1973, p. 106, 1983, p. ix; Gravemeijer & Terwel 2000, p. 780). Instead, Freudenthal believes that mathematics should be taught as an activity.

2.2.2 Guided reinvention

Freudenthal underscores the importance of *guided reinvention*.[31] He explains the term inventions "[…] are steps in learning processes, which is accounted for by the "re" in reinvention, while the instructional environment of the learning process is pointed to by the adjective "guided"." (Freudenthal, 1991, p. 46) In Freudenthal's own words (1973, p. 120):

> What I have called re-invention, is often known as discovery or re-discovery. I have also used these terms a few times, and it would not really matter which are used […]

Perhaps the term "invention" was chosen because students are expected to find something which is new and un-known to them but well-known to the instructor. He also explains why he prefers the notion of 'guided reinvention' to other notions such as *problem solving*, *discovery learning*, *heuristics* and *genetic method* (Freudenthal, 1991, pp. 45-48).

[31] The adjective 'guided' here might refer to guide from not only teachers and learning materials but also other peers (students) (Freudenthal, 1991, p. 47).

Students are not expected to "repeat the learning process of mankind" (Freudenthal, 1991, p. 48); however, they should be given the chance of reinventing mathematics under guidance of their teacher and learning materials. Freudenthal (1991, pp. 49-66) poses some basic questions which he later answers related to "guided reinvention". He suggests that students should be guided to reinvent mathematizing reality which means abstracting, schematizing and formalizing reality.[32] To answer the question of how students should be guided, he mentions five tenets of Treffers, including choosing learning situations within the learner's current reality, offering means and tools for vertical mathematizing, interactive instruction, the learner' own production and intertwining learning strands (Freudenthal, 1991, pp. 56-57).

2.2.3 Didactical phenomenology

Freudenthal explains that the way in which mathematics is published and presented is different from the way in which it is invented (Freudenthal, 1983). From this he suggests that:

> […] one should recognise that the young learner is entitled to recapitulate in a fashion the learning process of mankind. Not in the trivial manner of an abridged version, but equally we cannot require the new generation to start just at the point where their predecessors left off.
>
> (Freudenthal, 1983, p. ix)

Brousseau[33] (1998) also shares the same viewpoint when he creates and develops the *Theory of Didactical Situations in Mathematics*. He analyzes activities of mathematicians, mathematics teachers and students. Mathematicians put "knowledge into a communicable, decontextualized, depersonalized, detemporalized form", while mathematics teachers first create "the opposite action; a recontextualization and a repersonalization of knowledge" by providing their students with meaningful situations (Brousseau, 1998, p. 227). After responding to the situations, students often "have to redepersonalize and redecontextualize the knowledge" under guidance of their teachers (Brousseau, 1998, p. 227).

[32] Reality here means what is experienced real to the students.
[33] Prof. Guy Brousseau was awarded the first (2003) Felix Klein Medal of the International Commission for Mathematics Instruction (ICMI) for his contribution in development of mathematics education and effort in applying his studies in teacher training. His Theory of Didactical Situation in Mathematics was introduced in Vietnam in the 1990s (see, for instance, Nguyen Ba Kim, 2002, p. 204).

Freudenthal (1983, p. ix) discusses the notions of phenomenology and didactical phenomenology as follows:

> *Phenomenology* of a mathematical concept, structure, or idea means describing it in its relation to the phenomena for which it was created, and to which it has been extended in the learning process of mankind, and, as far as this description is concerned with the learning process of the young generation, it is *didactical phenomenology*, a way to show the teacher the places where the learner might step into the learning process of mankind.

Treffers believes that the idea of phenomenology is not entirely new, as it is often applied to find suitable mathematical applications in some other instruction approaches.[34] However, he stresses the essential difference of Freudenthal's viewpoint, that is, the idea of didactical phenomenology:

> The novelty in the realistic conception is that reality does not only function in applications but also serves as a source of concept formation, that is, in order to first develop intuitive notions […]. The emphasis is on laying a solid basis for learning rather than only on a *posteriori* applications at the end of the learning process.
>
> (Treffers, 1987, p. 246)

Freudenthal (1983) analyzes didactical phenomenology of mathematical structures with a variety of strands. Gravemeijer (1994, pp. 90-91) explains why and how "situations where a given mathematical topic is applied are to be investigated" according to the idea of didactical phenomenology. From phenomenological perspective, he offers an example about different phases of angles such as tangible or imaginary, static or dynamic, directional and positional indications (Gravemeijer, 1998, p. 60).

2.3 Meaning of 'realistic' in RME

The label 'realistic' in RME originated from Treffers's distinguishing four approaches to mathematics education in which he uses the criteria of horizontal and vertical mathematization. Therefore, the following sections first expound on the notions of mathematization, horizontal and vertical mathematization. Later, four different trends in mathematics education, including *mechanistic*, *structuralistic*, *empiristic* and *realistic* approach are analyzed. Finally, the authenticity characteristic is discussed.

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[34] Treffers (1987, p. 246) confirms that "such a didactical phenomenology is of course not new". It seems that he implied here phenomenology but not didactical phenomenology.

2.3.1 Mathematizing

As discussed previously (section 2.2.1.2 of this chapter), Freudenthal considers mathematizing primarily as an activity (Freudenthal, 1991, p. 30; see also Gravemeijer, 1994, p. 82). Freudenthal (1991, p. 31) explains that "[…] the origin of the term *mathematising* as an analogue to axiomatising, formalising, schematising." He also discusses aspects of mathematizing (Freudenthal, 1991, pp. 35-36). Gravemeijer (1994, p. 83) explains that:

> […] mathematizing mainly involves generalizing and formalizing. Formalizing embraces modelling, symbolizing, schematizing and defining, and generalizing is to be understood in a reflective sense.

De Lange defines mathematizing as "an organizing and structuring *activity* according to which acquired knowledge and skills are used to discover unknown regularities, relations and structures"(1987, p. 43).

The notions of *horizontal* and *vertical mathematization* are used in order to explain the differences between "transforming a problem field into a mathematical problem" and "processing within the mathematical system" (Treffers, 1987, p. 247). However, Treffers himself concedes that the distinction between the two types of mathematization is not necessarily readily apparent. According to Treffers (1987, p. 247), "[…] this distinction between horizontal and vertical components is a bit artificial given the fact that they may be strongly interrelated." This potential ambiguity is also confirmed by De Lange (1987, p. 44-45) and Freudenthal (1991, p. 132).

Following Treffers and Goffree, De Lange discusses horizontal and vertical components of mathematization. According to De Lange (1987, p. 43), horizontal components relates to "transferring the problem to a mathematically stated problem", and vertical components relates to "the mathematical processing and refurbishing of the real world problem transformed into mathematics" (De Lange, 1987, p. 43).[35]

At first, Freudenthal was not willingly to accept Treffers's distinction between horizontal and vertical mathematization:

> For a long time I have hesitated to accept this distinction. I was concerned about the theoretical equivalence of both kind of activities and, as a consequence, their equal status in practice, which I was afraid would be endangered by this distinction. How often haven't I been disappointed by mathematicians interested in education who narrowed

[35] Real world problems here should not be understood literally.

mathematising to its vertical component, as well as by educationalists turning to mathematics instruction who restricted it to the horizontal one […].

(Freudenthal, 1991, p. 41)

Freudenthal eventually approves of this distinction "because of its consequences for mathematics education, and in particular, for characterising educational styles." (1991, p. 41) Freudenthal (1991, pp. 41-42) distinguishes horizontal and vertical mathematization as follows:

> […] Horizontal mathematisation leads from the world of *life* to the world of *symbols*. In the world of life one lives, acts (and suffers); in the other one symbols are shaped, reshaped, and manipulated, mechanically, comprehendingly, reflectingly; this is vertical mathematisation.

Freudenthal also confirms the equal roles of horizontal and vertical mathematization and their presence in all mathematical activity levels (Van den Heuvel-Panhuizen, 1996, p. 11; Nguyen Thanh Thuy, 2005, p. 26).

De Lange (1987, p. 43) enumerates some activities containing strong horizontal components:

- identifying the specific mathematics in a general context;
- schematizing;
- formulating and visualizing a problem in different ways;
- discovering relations;
- discovering regularities;
- recognizing isomorphic aspects in different problems;
- transferring a real world problem to a mathematical problem;
- transferring a real world problem to a known mathematical model.

He also refers to some activity containing strong vertical components:

- representing a relation in a formula;
- providing regularities;
- refining and adjusting models;
- using different models;
- combining and integrating models;
- formulating a new mathematical concept;
- generalizing.

(De Lange, 1987, p. 44)

2.3.2 Different approaches to mathematics education

The 'realistic' term originated from Treffers's distinction of four different approaches to mathematics education: *mechanistic*, *structuralist*, *empiricist* and *realistic* by using the criteria of horizontal and vertical mathematization (De Lange, 1987, pp. 100-101; Treffers, 1987, pp. 250-252; Freudenthal, 1991, p. 132-136).

De Lange describes the mechanistic approach as follows:

> In the *mechanistic* approach mathematics is a system of rules. The rules are given to the students, they verify and apply them to problems similar to previous examples.
>
> (1987, p. 100)[36]

This approach fails to incorporate adequately not only application and methodology but also structure, interrelatedness and insight (De Lange, 1987, p. 100). The similar analyses are also found in Streefland (1991, pp. 16-17). Freudenthal (1991, p. 134) describes how a man is treated like "a computer-like instrument" in the mechanistic approach. Both horizontal mathematization and vertical mathematization are weak in this approach.

The structuralist approach considers mathematics as "an organized, closed deductive system" (De Lange, 1987, p. 93). Hence, this approach emphasizes mathematical structures in school. In the 1960s and 1970s, this approach, labeled the 'New Mathematics', widely influenced mathematics education (De Lange, 1987, p. 97). Its impacts as well as the criticisms are thoroughly analyzed (De Lange, 1987, pp. 97-98; see also Freudenthal, 1991, p. 135; Streefland, 1991, pp. 15-16). In the structualist approach, vertical mathematization is stressed excessively, whereas attention to horizontal mathematization is insufficient.

The empiricist approach used mainly in Great Britain (Streefland, 1991, p. 22; see also Freudenthal, 1991, p. 135) is described as follows:

> Provided with material from their living world, learners get the opportunity to acquire useful experiences, but they are not prompted to systematise and rationalise these experiences in order to break the barriers of the environment and to expand the reality they are familiar with.
>
> (Freudenthal, 1991, p. 135)

[36] 'Training for examination' approach in Vietnam is quite similar to the mechanistic approach.

In the empiricist approach, horizontal mathematization is emphasized, but vertical mathematization is weak.

On the contrary, the realistic approach fully incorporates both vertical and horizontal mathematization.

The following table is often used to illustrate the differences of the four mentioned mathematics education approaches under the criterion of horizontal and vertical mathematization (De Lange, 1987, p. 101; Treffers, 1987, p. 251; Freudenthal, 1991, p. 133):

	Horizontal mathematization	Vertical mathematization
Mechanistic	-	-
Empiricist	+	-
Structuralist	-	+
Realistic	+	+

Figure 2.1: Approaches in mathematics education

2.3.3 'Realistic' and 'authentic'

Jahnke (2001) discusses a notion of 'authentic', when introducing *productive exercises* for mathematics lessons (Produktive Aufgaben für Mathematikunterricht). A problem situation is authentic for learners if they "accept its actuality and involve it", and he urges that "productive exercises should be authentic" (Jahnke, 2001, p. 7; see also discussion in Jahnke, 2005). Authenticity, however, is not a compulsory characteristic in RME.

The word 'realistic' can therefore be slightly misleading. It has been often misunderstood, not only outside, but also inside the Netherlands, that the realistic approach focuses on reality or authenticity (Van den Heuvel-Panhuizen, 2000, p. 4 & 2003, pp. 9-10). Gravemeijer, Van den Heuvel-Panhuizen and Streefland (1990, p. VII) explains that 'realistic' "[…] not only means establishing the connection between reality and the mathematics to be learned, but also creating the possibility for the learners to construct a mathematical reality."

In fact, 'realistic', according to Van den Heuvel-Panhuizen (2003, pp. 9-10), was rooted in a Dutch verb 'zich realiseren' which means 'to imagine'. She explains further: "[…] the term 'realistic' refers more to the intention that students should be offered problem situations which they can *imagine* […] than that it refers to the 'realness' or authenticity of problems", and "The fantasy world of fairy tales and even the formal world of mathematics can be very suitable contexts for problems, as long as they are 'real' in the students' minds." (Van den Heuvel–Panhuizen, 2003, p. 10; see also Van den Heuvel–Panhuizen 2000) This is also confirmed when we consider mathematics materials which are presented at the beginning of every chapter of the Treffers's book about the Wiskobas Project- the initiation of RME (1987, pp. 1-7, 31-36, 75-82, 113-118, 159-166, 197-210 & 221-238) contain some mythologies, legends or "formal world of mathematics".

Gravemeijer (1994, p. 88) also confirms that:

> […] one cannot bring the reality into the classroom. Although students will be able to identify with well chosen contextual problems, these will never become real life problems.[37]

Gravemeijer provides an example about fair distribution of 18 bottles of cola for 24 students at a school party. Twelve-year-old low-achieving students are assigned this problem. However, some of them argue that: "Some students don't drink cola", and "They don't drink the same amount" (Gravemeijer, 1994, p. 89). There is an interesting problem about 'T-shirts and Sodas' which aims at encouraging students to find different informal strategies. However, this is not a real world problem because students do not encounter a similar situation in their lives (Van Reeuwijk, 1995, pp. 2-4; De Lange, 1996, p. 63-64; see also appendix B).

2.4 Tenets (principles) of RME

The following describes the developing of RME according to five tenets (principles) of RME: the use of contexts, the use of models, the use of students' own construction and production, the interactive principle and the intertwining of learning strands. These principles are created by combining the Van Hiele's levels of learning mathematics, Freudenthal's didactical phenomenology and Treffers's progress mathematization (Treffers, 1987).

[37] The author of this dissertation agrees with Gravemeijer. However, it is better if "these will never become real life problems" is replaced by "in some cases, these will not become real life problems".

2.4.1 The use of contexts

2.4.1.1 Context in RME

Borasi defines context as "[…] a situation in which the problem is embedded." (cited in Van den Heuvel-Panhuizen, 1996, p.118) In traditional mathematics textbooks, most problems are presented without context, and context "appears only in brief introductions or end-of-section story problems" (Meyer et al., 2001, p. 522). For this reason, students who use these textbooks often experience difficulties when they encounter a contextual problem because: "[…] they must first translate the problems into […] problems without context, before they can attempt to solve them." (Meyer et al., 2001, p. 522)

According to Gravemeijer and Doorman (1999, p. 111), context problems are "[…] problems of which the problem situation is experientially real to the student." Context problems include not only problems with reality contents, but also "pure mathematical" problems (Gravemeijer & Doorman, 1999, p. 111; see also Van den Heuvel-Panhuizen, 2000, p. 4).

The notion of context problems in RME is similar to a notion of problem situation in *Problem Posing and Solving* which is defined by Nguyen Ba Kim and Vu Duong Thuy (1997) has three following conditions:

- It contains a problem which students do not know any algorithm to solve this problem;
- Students understand the problem's relevance; and
- Although students cannot solve the problem immediately and do not know any algorithm to solve this problem, they have some knowledge and skills relating to the problem, and they believe that if they try the best they may solve the problem.

2.4.1.2 Roles of context

Freudenthal discusses how to use contexts properly when discussing mathematics application:

> If in traditional mathematical instruction the applications of mathematics are touched upon, it is always done according to the pattern of didactical inversion. Rather than departing from the concrete problem and investigating it by mathematical means, the mathematics comes first, while the concrete problem comes later as an "application".
>
> (Freudenthal, 1973, p. 132)

In RME, "[…] the use of realistic contexts became one of the determining characteristics" (Van den Heuvel-Panhuizen, 2003, p. 9), and "[…] context problems play a role from the start onwards." (Gravemeijer & Doorman, 1999, p. 111)

De Lange speaks of three levels of using context: the third, second and first order. The third order use is the "most significant" and "to introduce and develop a mathematical model or concept", and the second one "is less essential, but still very important" when student "find the relevant mathematics, to organize and structure" to deal with "real world problem", while the first one is "often found in traditional schoolbooks" when "[…] the mathematical operations are embedded in contexts […]", and a "[…] simple transition from the problem to a mathematical problem is sufficient […]" (De Lange, 1987, pp. 76-77).

Based on De Lange's three levels of context use, Meyer et al. (2001, p. 523) point out five different roles of context in teaching and learning mathematics which are often interactive:

- motivating students to explore new mathematics;
- offering students a chance to apply mathematics;
- serving as a source of new mathematics;
- suggesting a source of solution strategy;
- providing an anchor for mathematical understanding.

2.4.2 The use of models

2.4.2.1 Roots of models

Initial ideas of models "at a much more general didactical level" in RME are discussed in 1975 by Freudenthal (Van den Heuvel-Panhuizen, 2003, p. 15). The model which is determined by Freudenthal differs from the mathematical model.[38] Streefland further develops these ideas "within a micro-didactic context" to create the notions of "model of" and "model for" in 1985 (Van den Heuvel-Panhuizen, 2003, p. 15). Later they are

[38] In Freudenthal's words:

> According to my terminology, a model is just the- often indispensable- intermediary by which a complex reality or theory is idealised or simplified in order to become accessible to more formal mathematical treatment. I therefore do not like the term "mathematical model" in a context where it wrongly suggest that mathematics directly or almost directly applies to the environment. As a matter of fact, this only remained true as long as mathematics was tightly entangled with the environment. I lay out so much stress on the role of the model as an intermediary because people are all too often unaware of its indispensability. Much too often mathematical formulas are applied like recipes in a complex reality that lacks any intermediate model to justify their use.
>
> (Freudenthal, 1991, p. 34)

developed and elaborated by other researchers. According to Streefland (cited in Van den Heuvel-Panhuizen, 2003, p. 14), initially, a model is established and developed from a problem situation, and this context-based model which has close relationship with the problem situation called a *model of* (the specific problem situation); after this stage, the model of is developed and generalized which is independent of the problem situation defined as *model for* (not only the initial problem situation, but also other situations). “Model of” and “model for” are used as bridges to connect informal and formal knowledge. Typically, a problem situation is given to student. First, the student constructs context-based strategies to solve this problem with their informal knowledge. The student then develops these strategies into more general strategies, which can solve not only the given problem, but also other problems.

2.4.2.2 Self-developed (emergent) models

Gravemeijer (1994, pp. 100-101) elaborates disadvantages of using models in the information processing approach in which formal mathematics is a starting point, and concrete models, which he calls *didactical models*, are used to concretize this formal mathematical knowledge. The problem is that, “[…] although the models as such may be concrete- the mathematics *embedded in the models* is not concrete for the students.” (Gravemeijer, 1994, p. 77) Furthermore, the use of these models “does not really help students attain mathematical insight” (Gravemeijer, 1994, p. 77). He describes this approach as “top-down” approach, which means that the direction of the instruction is from formal mathematics to informal mathematics (Gravemeijer, 1994, p. 77).

Consequently, he stresses that in order to overcome the mentioned weakness, “[…] situated, informal knowledge and strategies should be the starting point for developing abstract mathematical knowledge.” (Gravemeijer, 1994, p. 77) In other words, one should reverse the direction from informal to formal mathematics in instruction process: from situations and informal knowledge, students develope themselves model-of, model-for and formal knowledge. He calls these “self-developed models”. Gravemeijer (1994, p. 77) uses the notion of “alternative” or “bottom-up” approach to imply the latter approach.

Gravemeijer (1994, p. 101) discerns four levels in RME, including situations, model of, model for and formal mathematics by using the following figure:

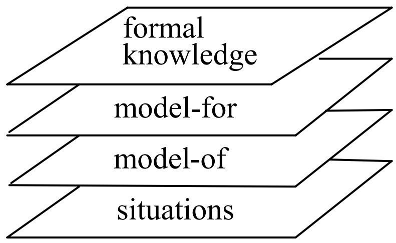


Figure 2.2: Self-developed models in RME

Gravemeijer (1994, p. 101) describes the levels in more general terms:

- the level of the situations, where domain specific, situational knowledge and strategies are used within the context of the situation (mainly out of school situations);
- a referential level, where models and strategies refer to the situation which is sketched in the problem (mostly posed in a school setting);
- a general level, where a mathematical focus on strategies dominates the reference to the context;
- the level of formal arithmetic, where one works with conventional procedures and notations.

Later Gravemeijer uses the notion of *emergent models* instead of self- developed models (Gravemeijer & Doorman, 1999; Gravemeijer, 2002 & 2004). He explains that "[…] 'emergent' refers both to the character of the process by which models emerge within RME, and to the process by which these models support the emergence of formal mathematical ways of knowing." (Gravemeijer, 2004, p. 98)

2.4.2.3 "Didactical modeling", "mathematical modeling" and "emergent modeling"

Gravemeijer elaborates three different types of modeling in mathematics education: "didactical modeling", "mathematical modeling" and "emergent modeling" (Gravemeijer, 2004, pp. 97-99). He also confirms that each modeling assumes a certain role in mathematics education (Gravemeijer, 2004, p. 97).

The weakness of using didactical models is discussed above (see section 2.4.2.2). In "mathematical modeling", "the mathematical model and the situation being modeled are treated as separate entities." (Gravemeijer, 2004, p. 97) Gravemeijer suggests that emergent modeling can be served as "a precursor to mathematical modeling" (2004, p. 97). His description about "emergent modeling", "mathematical modeling" and the relation between them can be expressed as the following figure: [39]

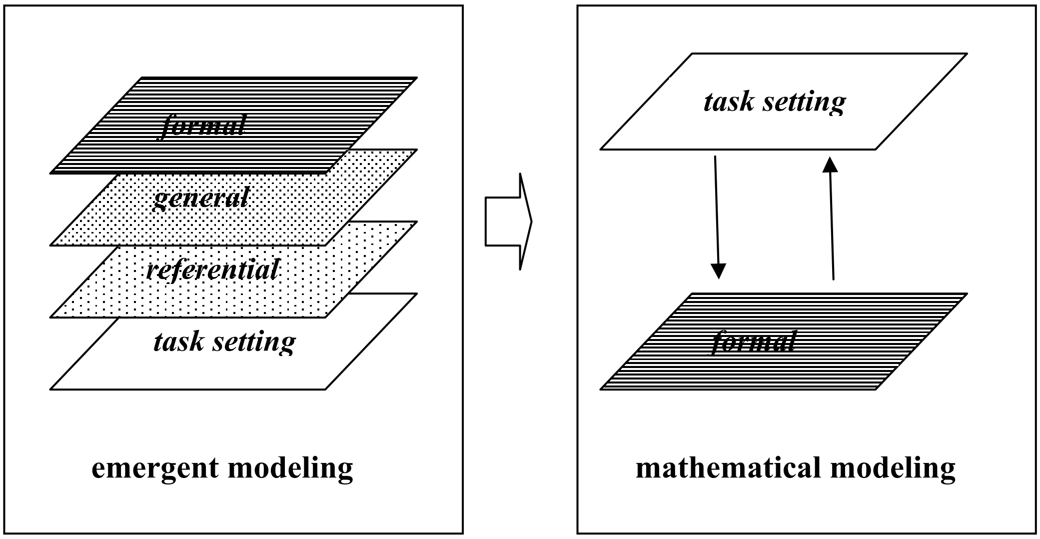


Figure 2.3: "Emergent modeling" and "mathematical modeling"

2.4.3 The students' own productions and constructions

Students should be encouraged and guided to reinvent mathematics (mainly by their teacher and learning materials, but also by other peers). In the learning process students should be encouraged to create their own productions and constructions, and then they can use those to approach latter tasks.

After confirming the role of the students' own constructions and productions as a key position in realistic instruction theory, Treffers (1987, p. 260) explains:

> When speaking about their own *constructions*, we stress their *actions*, while in speaking about *production* the stress is on *refection*. Of course, this is only a matter of more or less emphasis, as they are inseparable.

Treffers carefully analyzes and explains how students' own constructions and productions function in the teaching and learning process, and he also discusses the lack of attention to students' own constructions in other mathematics education approaches, especially in the structuralist one (Treffers, 1987, pp. 260-261).

Streefland (1990, pp. 33-50) explains the meaning of students' own production and its functions in the teaching and learning process and gives variety of illustrated examples.

[39] This figure is built by combining two Gravemeijer's figures about emergent modeling and mathematical modeling (Gravemeijer, 2004, p. 99).

2.4.4 The interactive principle

Treffers (1987, p. 261) discusses the role of interactive characteristic in realistic approach as follows:

> The pupils' own constructions and productions, as well as phenomenological exploration and modeling, can be efficient only if they are realised in interactive instruction, that is, instruction where there is the opportunity to consult, to participate, to negotiate, to cooperate, with review afterwards and where the teacher holds back from providing explanations.

It should be noted that although the interaction and cooperation between teacher and students and among students is emphasized, it does not mean that individual work is disregarded in the realistic approach (Treffers, 1987, p. 261).

Interactivity is not a distinguishing characteristic of RME. For example, Yackel and Cobb[40] (1996) refer to the notion of *sociomathematical norms* which is developed from their previous notion of (general classroom) *social norms.* The difference between sociomathematical norms and social norms is that the first ones mean "normative aspects of mathematical discussions that are specific to students' mathematical activity", whereas the second ones "apply to any subject matter area and are not unique to mathematic" (Yackel & Cobb, 1996). In their research paper, Yackel and Cobb (1996) describe how "sociomathematical norms are interactively constituted" and "these norms regulate mathematical argumentation and influence learning opportunities for both the students and the teacher". Discussion about social and sociomathematical norms are also found in Hershkowitz and Schwarz (1999) and McClain and Cobb (2001).

2.4.5 The intertwining of mathematical strands

This characteristic discusses relations among teaching mathematical strands (units). Freudenthal (1973, pp. 74-75) explains why it is necessary to intertwine mathematical strands as follows:

> In principle it is a healthy idea not to teach isolated pieces but coherent material. Connected matter is faster learned and longer retained.

He suggests that: "Algebra, trigonometry, analytic geometry, infinitive series, calculus should not be any longer considered as closed units." (Freudenthal, 1973, p.

[40] Prof. Dr. Paul Cobb was awarded the 2005 (second) ICMI Hans Freudenthal Medal for his contribution to mathematics education.

137)[41] He also offers some more examples about strands which should be intertwined such as ratio and fractions; functions, graphs and equations; negative numbers; vector algebra and geometry; linear graphs and functions; and plane and solid geometry (Freudenthal, 1991, pp. 118-119). He explains the role of intertwining learning strands as follows:

> Teachers who prefer systematic instruction are likely to accuse this approach of being chaotic. They forget that systematics is an *a posteriori* contraption. What looks like chaos may be well-organised didactically, while, on the other hand the system as such may be a subject of guided reinvention.
>
> (Freudenthal, 1991, p. 119)

Van den Heuvel-Panhuizen (2000, p. 8) thoroughly explains the relationship between different sections within a mathematical chapter and different mathematical chapters within in a textbook. Moreover, she underscores the necessity of exploiting different "mathematical tools and understandings" to solve rich context problem (Van den Heuvel-Panhuizen, 2000, p. 8).

2.5 Some examples

Numerous examples about RME can be found in De Lange (1987), Treffers (1987), Streefland (1991), Gravemeijer (1994), Van den Heuvel-Panhuizen (1996), Bakker (2004), Doorman (2005) and others. An example of 'T-shirts and Sodas' is used to interview some Vietnamese middle school mathematics teachers (appendix B). The problem in this example is assigned to middle school students so that they can develop different informal strategies to solve the given problem. Below, some additional

---

[41] Apart from the relations among mathematical strands and units, Freudenthal also mentioned and strongly emphasized the relation among mathematics and other disciplines such as physics, chemistry, and biology. He emphasized the latter coordination, especially between mathematics and physics: "Physics needs mathematics as an auxiliary discipline, but physics can also belong to the lived–through reality from which mathematics is provided with subject matters and suggestions for mathematics organization." (Freudenthal, 1973, p. 136) After having analyzed how mathematics is applied in physics textbooks, he concluded that:

> But meanwhile it remains a sad fact that physicists in their instruction use degraded mathematics. For the student this means that he learns de-mathematized physics, that he is not introduced to understanding how mathematics applies in physics, and that he experiences mathematics and physics as unrelated.
> (Freudenthal, 1973, p. 139)

Because of this, he recommended that "[…] integration or coordination of mathematics not *with* other disciplines but integration *around* mathematics, that is with mathematics as a nuclear discipline that attracts subject matter of other disciplines to have them worked on by the student as fields of mathematical organization." (Freudenthal, 1973, p. 139) We can call these two types of relation *internal* relations and *external* relations.

examples are included. The first one describes how students at ages of 8 and 9 who have only learned multiplications up until number 10 reinvent different informal strategies to the problem of long division. The second one depicts how the empty number line is modeled to help students add and subtract up to 100.

2.5.1 Developing long division

In this section, an example of developing long division is described. This example is presented following Gravemeijer (1994, pp. 83-84).

Children age 8 or 9 *who have learned only multiplication up until number 10* are given the following problem:

> Tonight 81 parents will be visiting our school. Six parents can be seated at each table. How many tables do we need?

(Gravemeijer, 1994, p. 83)

Then the teacher gives the students some suggestion by drawing the following figures:

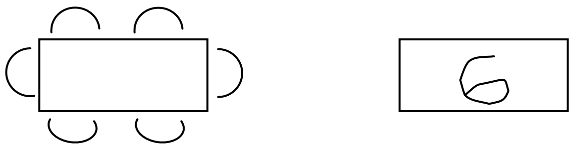


Figure 2.4: Guided figures (Gravemeijer, 1994, p. 84)

The students can find different solutions to the problem:

- *Repeated addition*:

Some draw tables for 6 persons until they get enough tables for 81 persons. Then they count a number of necessary tables:

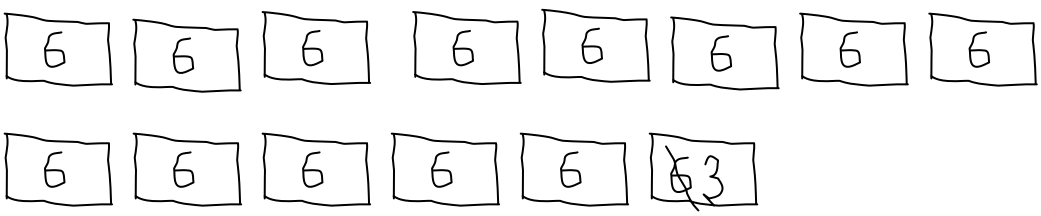


Figure 2.5: Using repeated addition

They can count 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78 and 84. Then they count a number of tables: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 and 14.

- *Using 10 × 6 = 60 as a starting point:*

Some students start by using the multiplication 10 × 6 = 60. Then they use repeated addition or multiplication:

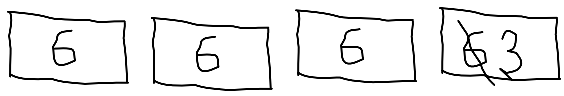


Figure 2.6: Using multiplication and repeated addition

- Using *6 × 6 =36 as a starting point:*

One student starts by using the multiplication *6 × 6 = 36.* Then this student doubles 36, adds 6 and adds 6 to find a number of necessary tables.

After that, students are encouraged to compare their solutions. Most of them agree that the solution starting by multiplication 10 × 6 = 60 is reasonable.

Finally, a similar problem is given to the students:

> One pot serves seven cups of coffee; each parent gets one cup. How many pots of coffee must be brewed for the 81 parents?
>
> (Gravemeijer, 1994, p. 84)

At this time, most of the student use "ten times" solution to deal with this problem, although their teacher has not told them to use it.

Van Galen and Feijs (1991, p. 187-190) describes a variety of third grade students' solutions when the videodisc of this lesson is used for in-service teacher education in the Netherlands. To find the solutions to this problem, Anita draws all (14) tables with chairs, but without numbers; Fatiha starts by drawing 4 tables (without chairs and numbers), then she uses numbers instead of tables; Osman starts by drawing 4 tables with chairs and numbers, then he switches to tables with numbers, and he jumps from 60 to 63 to finally reach 81. Noura starts out with two multiplications: 10 × 6 = 60 and 4 × 6 = 24, then she write down: 6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 + 3 = 81; etc. (Van Galen & Feijs, 1991).

2.5.2 Empty number line for additions and subtraction up to 100

Gravemeijer gives three reasons related to "phenomenological analysis of number", "informal solution procedures" and "level-raising qualities" for using the *empty number line* to add and subtract up to 100 (Gravemeijer, 1994, pp. 123-125).

Moreover, he also identified a disadvantage of using *full number line* which tempts the students to use “primitive counting strategies” when they dealt with the subtraction (Gravemeijer, 1994, pp. 123-125).

Then Gravemeijer suggests a way of introducing the empty number line. First, the students are given a context problem. Then, a bead string is introduced, and the students work with it. After that, an empty number line can be introduced as a model of the bead string.

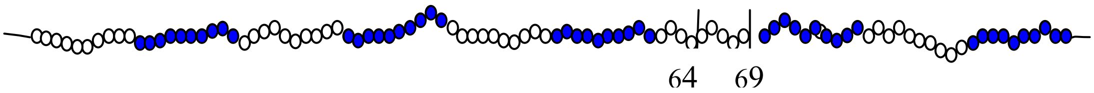


Figure 2.7: Making numbers of the bead string (Gravemeijer, 1994, p. 125)

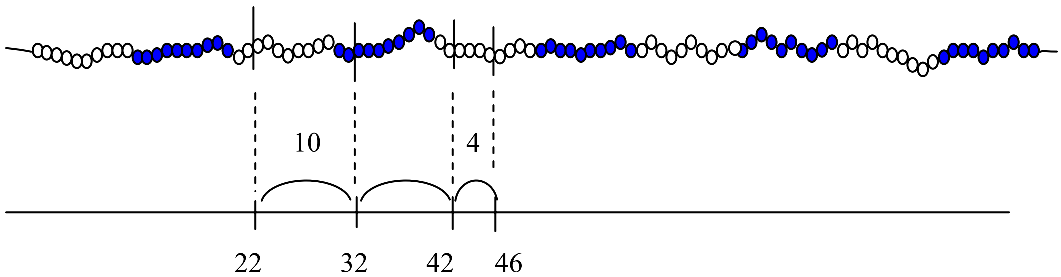


Figure 2.8: Modeling a bead string solution with an empty number line (Gravemeijer, 1994, p. 125)

Gravemeijer (1994, p. 125) explains that “[…] there are no marks on the number line, the student places the marks that he or she chooses.”

Then students can apply the empty number line to find different solution procedures for addition and subtraction up to 100. The following figure illustrates different student’s strategies when they deal with an addition, for instance: 27 + 38:

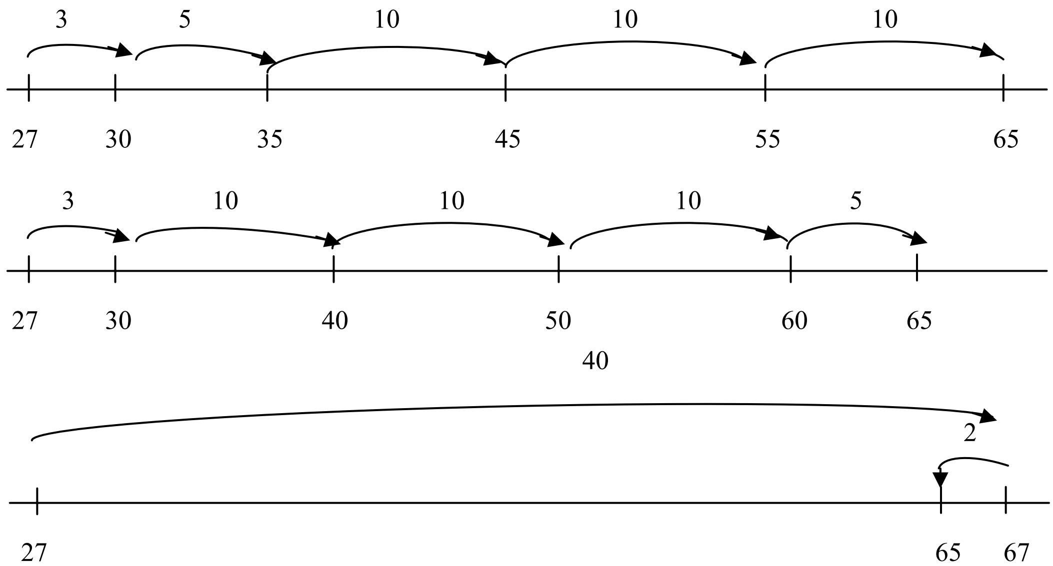


Figure 2.9: Different strategies for addition 27 +38 (Gravemeijer, 1994, p. 120)

Chapter 3 Vietnamese RME-based geometry lessons for grade 7

The aim of this chapter is to discuss how to design RME-based geometry lessons for grade 7 in Vietnamese middle school. Firstly, this chapter analyzes some characteristics of the Vietnamese geometry curricula in the 'reformed' period and at present. Recently, the 'reformed' mathematics textbooks have been replaced by new textbooks in Vietnam (In the school-year 2005-2006, the 'reformed' textbooks are still used for grade 5 (primary school) and all grades in high school (10 to 12), while the new (current) ones are utilized for the other grades (grades 1 to 4 in primary school and grades 6 to 9 in middle school)). For this reason, not only the current but also the 'reformed' middle school textbooks are discussed in section 3.1.2 of this chapter. Secondly, this chapter discusses abilities of applying RME in teaching in Vietnamese school. Thirdly, this chapter discusses foundations for designing RME-based geometry lessons. Finally, it describes a RME-based lesson of the Triangle Sum Theorem. Some other RME-based geometry lessons are presented in appendix C.

3.1 Middle school geometry curricula

First, this section briefly discusses an overview of geometry curricula. After that, it analyzes Vietnamese middle school geometry curricula and textbooks in the 'reformed' period (from the early 1980s until the early 2000s), as well as some changes at present.

3.1.1 An overview of middle school geometry curricula

According to Malaty (1999, pp. 231- 233), some landmarks in the (recent) history of school geometry curricula include:

> • until 1957 (in most countries, plane Euclidean geometry was taught in middle school, and analytic geometry and solid Euclidean geometry were taught in high school);
>
> • The Royaumont Seminar in 1959 (because of this seminar and especially Jean Dieudonnés's slogan "Euclid must go", Euclidean geometry virtually disappeared from mathematics curricula in most Western countries as one impact of the "new math" movement, which originated in the U.S.A);[42]
>
> • around the end of the 1960s (the "new math" movement started to affect mathematics education in the Third World countries);

[42] Germany is exceptional as the motion geometry, the simplification of Felix Klein's transformation geometry, had taken place of Euclidean geometry in some German schools before The Royaumont Seminar (Manlaty, 1999, p. 232).

• at the end of 1970s (the “new math” movement was replaced by the “Back-to-Basics” one after many weaknesses in the “new math” curriculum were found since it exposed many weaknesses).[43]

In the 1970s and 1980s, geometry curricula lacked direction, and geometry was not emphasized in mathematics curricula, in comparison with other branches of mathematics (Quadling, 1985, pp. 91-94).

Many controversies about middle school geometry have been discussed, such as the role of geometry in a school mathematics curriculum (Quadling, 1885, pp. 91-94; Costello, 1991, p. 53; Clements & Battista, 1992, p. 422; Clements, 2003, pp. 151-152), transformation versus congruence (equality) (Bender, 1982; Fischer, 1996; pp. 135-141; Holland, 2001; p. 16), two-dimensional (2-D) and three-dimensional (3-D) geometry in middle school, teaching deductive geometry in school, the use of information and communication technology (ICT) in teaching geometry (Hoyles et al., 2003 a, pp. 2-6; see also Hoyles, Küchemann & Foxman, 2003 b, pp. 36-40; Hoyles, 2005, p. 143) and the relation between geometry and real life (Bender & Schreiber, 1985; Wittmann, 1987).

It appears that the role of geometry in mathematics curricula is not emphasized in countries such as the United States and Britain. After showing U.S. students’ poor achievement in geometry, Clements and Battista (1992, p. 422) and Clements (2003, pp. 151-152) reveal its main causes: geometry plays an unessential role in U.S. mathematics curricula; requirements for students are low; and teachers are unwilling to teach it. The similar situation is also found in Britain: “[…] fears that geometry might be squeezed out of the curriculum are not new […]” (Costello, 1991, p. 53). In general, the International Association for the Evaluation of Educational Achievement (IEA) conclude, after conducting large international comparisons in mathematics education that geometry in school mathematics curricula is less important than some other strands (algebra and arithmetic) (Travers & Weinzweig, 1999, p. 25). However, geometry still plays an important role in mathematics curricula in countries such as China and Vietnam (Wang & Wu, 2002, pp. 107-108; section 3.1.2.1.1 of this chapter).

According to Holland (2001, p.16), geometric transformations are introduced in school mathematics curricula as a result of a modernist approach. Holland discusses

[43] However, it does not mean that geometry curricula returned to Euclidean geometry (Manlaty, 1999, p. 235).

transformations as a tool for studying geometry, as well as themselves subjects to be studied. In most states of Germany (Bundesländer), geometric transformations often appear as a tool for studying geometry, whereas the role of transformations as objects to be studied almost disappears in middle school mathematics curricula because of the unpopularity of the structuralist approach and the role of other new subjects such as statistics, probability and informatics in school (Holland, 2001, p. 16).

Fischer (1996, pp.135-144) discusses various aspects of congruent and transformational school geometry, including advantages, disadvantages and the historical roots of geometric transformations.

In middle school, 2-D geometry makes up the majority of the curriculum in some countries (e.g., England, Japan and France), while in others (e.g., the Netherlands) 2-D geometry is taught alongside 3-D geometry, and in others (e.g., Baden-Württemberg, a state in Germany) there is an emphasis on 3-D geometry (Hoyles et al., 2003 a, pp. 2-6).

According to Hoyles (2005, p. 143), in former times, the geometry curriculum in England and Wales emphasized deductive geometry, but recently, deductive geometry has nearly disappeared from the curriculum.[44] Similarly, Stigler and Hiebert (1999, p. 59) state that deductive reasoning often appears in mathematical proofs, but U.S. lessons have no mathematical proofs, while about 10 percent of German lessons and 53 percent of Japanese lessons contain mathematical proofs. Knuth (2002, pp. 61-62) likewise confirms that in U.S school, traditionally, only students who intend to go to college or university learn proofs-mainly Euclidean geometry proofs-but recently, the National Council of Teachers of Mathematics recommended that proofs should play a role in the curriculum for all students. After studying middle school mathematics teachers' conception of proof, Knuth concludes that these recommendations seem to challenge the teachers (2002, pp. 82-85). In Germany, proofs play an important role in mathematics curricula in school for high-achievement students (Gymnasien) and in some school for intermediate-achievement students (Realschulen) (Kaiser, 1999, p. 143). In Japan, students begin to learn deductive Euclidean geometry in grade 8, including proofs (Kunimune & Nagasaki, 1996). Similarly, Wang and Wu (2002, pp.

[44] It appears that deductive geometry is significantly reduced or even disappeared in geometry curricula in some Western countries because of 'Back-to-Basics' approach. However, it still played its role in geometry curricula in some Asian countries like Japan and Korea.

108-110) confirm that Chinese middle school geometry places great emphasis on formal mathematics such as logical reasoning and mathematical proof.

Recently, studies on the use of information and communication technology (ICT) in geometry lessons have flourished. A study of Hoyles et al. (2003 a, p. 5) has shown that the requirement to use ICT in geometry curricula varies from country to country: ICT is not mentioned in some geometry curricula (for example, in Poland) and mentioned sometimes in some others (for instance, in Japan and the Netherlands), while in other countries (for example, in Singapore and France) the use of ICT is explicitly stated in detail. Comparing the U.S. and Vietnamese middle school mathematics textbooks, Nguyen and Kulm (2002, p. 222) confirm that the U.S. textbooks place great emphasis on using technology in mathematics lessons, while the Vietnamese ones rarely mention it.[45]

3.1.2 Characteristics of Vietnamese middle school geometry

This section discusses the characteristics of the Vietnamese middle school geometry curricula and textbooks in the 'reformed' period and at present. It is necessary to repeat that typically, mathematics textbooks in Vietnam are unique (see chapter 1, section 1.3.1.1). Until the school-year 2005-2006, the 'reformed' textbooks are used in grade 5 (primary school) and in grades 10 to 12 (high school). These textbooks will be gradually replaced by the new ones in the following school years. Chapter one discussed some general characteristics of middle school geometry; this section discusses more details about the Vietnamese middle school geometry curricula and textbooks. These characteristics are analyzed in section 3.1.1 of this chapter, including the role of geometry in the mathematics curricula and textbooks, congruence and transformation, 2-D and 3-D geometry, teaching deductive geometry in school, using ICT in teaching and learning geometry and geometry and real life.

3.1.2.1 The 'reformed' period (from the early 1980s until the early 2000s)

3.1.2.1.1 The role of geometry in the middle school curriculum and textbooks

Unlike countries such as the United States and England, geometry always has been regarded as an essential part of the Vietnamese middle and high school curriculum, and this priority is reflected in the textbooks. In the 'reformed' period, for every grade from 7 to 9 there are two textbooks called *Geometry* and *Algebra*, and for grade 6, there are

[45] There are few researchers who compare mathematics education in Vietnam and other countries.

textbooks named *Mathematics 6: part 1* and *Mathematics 6: part 2*. Some basic concepts of geometry are first presented in the last chapter of *Mathematics 6: part 2* (Le Hai Chau, Nguyen Gia Coc & Pham Gia Duc, 1996, pp. 110-144).

3.1.2.1.2 Congruence and Transformation

In the 'reformed' period, Euclid-based geometry is a mainstay of the geometrical curriculum and textbooks. Middle school geometry introduces some basic shapes (a point and a line), relations (a point on a line and a point between two points) and measurements (measures of a segment and an angle); some axiomatic groups (incidence axioms, betweenness axioms, axioms about the measure of a segment, axioms about the measure of an angle, the axiom about two congruent triangles and the parallel axiom (the Euclidean axiom) (Pham Gia Duc, Nguyen Manh Cang, Bui Huy Ngoc & Vu Duong Thuy, 1998 b, pp. 124-126).

Geometric transformations are not regarded as "tools for proofs or solving a mathematical problem" and are not emphasized in the middle school curriculum and textbooks:[46] axial symmetry is introduced after the characteristics of a trapezoid (grade 8); central symmetry is introduced after the characteristics of a parallelogram (grade 8); rotation is introduced after the characteristics of a circle (grade 9); and translation is introduced in an extra, non-compulsory lesson (grade 8) (Pham Gia Duc et al., 1998 b, pp. 107-112).

3.1.2.1.3 2-D and 3-D geometry in the middle school curriculum

The middle school curriculum focuses on 2-D geometry. 3-D geometry is presented only in the last chapter in 'Geometry 9' textbook, which include four chapters (see some 'reformed' mathematics textbooks, for instance, Le Hai Chau et al., 1996; Nguyen Gia Coc & Pham Gia Duc, 1996; Nguyen Van Bang, 1997; Nguyen Ba Kim & Tran Kieu, 2002). In the middle school mathematics curriculum and textbooks, 2-D and 3-D geometry are regarded as two separate parts.

3.1.2.1.4 Geometry as a deductive system

In this period, deductive reasoning is strongly emphasized in the curriculum and textbooks. Most of the properties and theorems are deductively proven in the geometry

[46] Geometric transformations were also not regarded as tools for concepts' formation in the middle school curriculum and textbooks. However, 2- D geometry transformations, including axial and central symmetry, translation, homothetic transformation and similarity transformation were sufficiently presented in one chapter in *Geometry 10* (the first grade in high school) of the 'reformed' textbooks.

textbooks. Geometry textbooks look more like mathematics books than school textbooks. In other words, middle school geometry is regarded as a rigorous deductive geometry.

For instance, the intentions of the writers of the *Geometry 8* textbook (for 14-year-old students) are described in the teachers' book as follows:

> • Every concept which is widely used in the textbook is defined from known definitions and should not be intuitively described.
>
> • Every theorem must be proven from accepted characteristics and known theorems by using deductive proofs.
>
> (cited in Hoang Chung, 1999, p. 16)

The requirements of deductive reasoning in the mathematics curriculum and textbooks are extremely rigorous for most of middle school students (Hoang Chung, 1999, pp. 15-17; MoET, 2002 a, p. 2).

3.1.2.1.5 Using information and communication technology (ICT) in teaching and learning geometry

The middle school geometry curriculum and textbooks do not discuss using ICT; moreover, they do not even refer frequently to the use of calculators in teaching and learning mathematics (see the 'reformed' textbooks, for example, Le Hai Chau et al., 1996; Nguyen Van Bang, 1997; see also Nguyen & Kulm, 2002, p. 221).[47]

3.1.2.1.6 Geometry and real life

Generally, the 'reformed' geometry curriculum do not emphasize real life, and almost all geometry problems in the textbooks are pure mathematics problems (see the 'reformed' geometry textbooks, for instance, Nguyen Gia Coc & Pham Gia Duc, 1996; Nguyen Van Bang, 1997; see also Nguyen & Kulm, 2002, p. 221).

Following this paragraph, a grade-seven geometry lesson (for students at age of 13) from the 'reformed' textbook named *Geometry 7* is presented in order to illustrate this section. In *Geometry 7*, the theorem about two congruent triangles (side-angle-side) is presented without proof. Based on this theorem, the two other theorems about two congruent triangles (side-side-side and angle-side-angle) are fully proved in this textbook. First, this lesson presents the theorem about two congruent triangles (side-

[47] Nguyen and Kulm mention the shortest of using technology in the Vietnamese middle school textbooks.

side-side) and its long and complicated proof, with three cases: the textbook proves the first case and then asks the student to prove the others. Second, this theorem is applied to solve a mathematics problem.[48] More examples are presented in appendix A.

§ 16 Two congruent triangles, case side-side-side (s. s. s.)[49]

1. Theorem

If $\Delta\, ABC$ and $\Delta\, A'B'C'$ have $AB = A'B'$, $BC = B'C'$ and $AC = A'C'$ then these triangles are congruent.

Proof

Draw $\Delta\, B'DC'$ on the half plane that does not contain vertex A', which has edge $B'C'$ such that $m\angle C'B'D = m\angle B$, $B'D = BA$.[50] Clearly, $\Delta\, DB'C'$ and $\Delta\, ABC$ are congruent (case s. a. s.). Consequently, $DC' = CA$.

Point B' is equidistant from two points A' and D (because $\overline{DB'}$ and $\overline{B'A'}$ are congruent to $\overline{AB}$); point C' is equidistant from two points A' and D (because $\overline{DC'}$ and $\overline{C'A'}$ are congruent to $\overline{CA}$), so $\overleftrightarrow{B'C'}$ is the perpendicular bisector of $\overline{A'D}$. I is the intersection point of $\overline{B'C'}$ and $\overline{A'D}$. We consider three following cases which depend on positions of point I.

a) Point I is situated between B' and C' (figure 3.1). Right $\Delta\, IB'A'$ and $\Delta\, IB'D$ are congruent (because they have two pairs of two congruent sides). Consequently, $m\angle DB'I = m\angle A'B'I$.

Because $m\angle DB'I = m\angle B$ (from the way in which Δ DB'C' is drawn), $m\angle A'B'I = m\angle B$.

Because $\Delta\, ABC$ and $\Delta\, A'B'C'$ have $BA = B'A'$, $BC = B'C'$ (given) and $m\angle B = m\angle B'$ (is just proved), $\Delta\, ABC$ and $\Delta\, A'B'C'$ are congruent.

Figure 3.1: Figure 74 in *Mathematics 7*

[48] This proof is not found in the current mathematics textbook.
[49] This lesson is translated from the 'reformed' textbook called *Geometry 7*.
[50] In Vietnam, '$\angle C'B'D = \angle B$' sign is used instead of '$m\angle C'B'D = m\angle B$' and '$\angle C'B'D \cong \angle B$'.

b) Case in which point *I* is not situated between *B'* and *C'* (figure 3.2). Students prove it themselves.

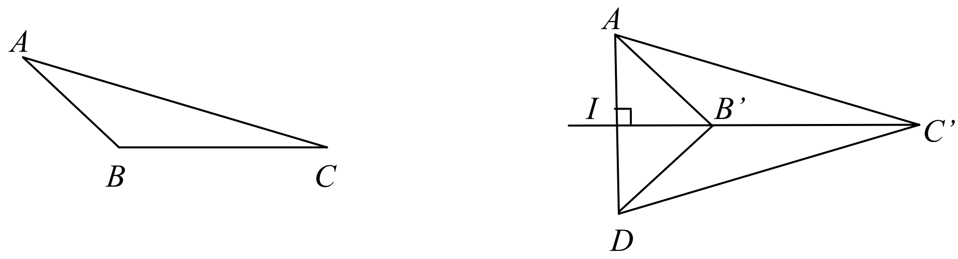


Figure 3.2: Figure 75 a in *Mathematics 7*

c) Case in which point *I* coincides with *B'* or *C'* (figures 3.3 a & 3.3 b). Students prove it themselves.

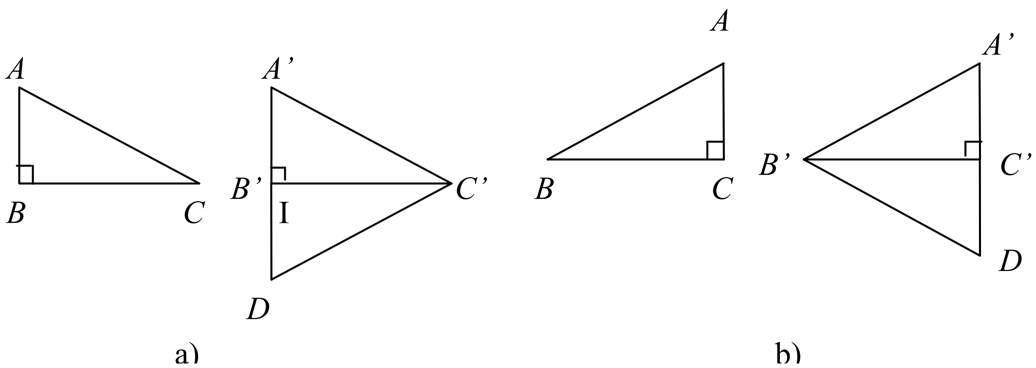


Figure 3.3: Figure 75 b & c in *Mathematics 7*

2. Circle

A shape that includes points that are equidistant from a point *O* is called a circle with center *O* and radius *r* and is signed *(O, r)*. In order to draw a circle, one uses a tool called a compass.

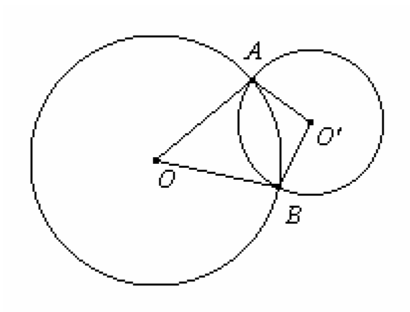


Figure 3.4: Figure 76 of *Mathematics 7*

Problem: Two circles with centers *O* and *O'* intersect each other at points *A* and *B*. Prove that Δ *AOO'* and Δ *BOO'* are equal.

Solution: Δ *AOO'* and Δ *BOO'* have a common side *OO'*, *OA* = *OB* (radius of circle with center *O*), *O'A* = *O'B* (radius of circle with center *O'*) (figure 3.4). Consequently, Δ *AOO'* and Δ *BOO'* are congruent.

(Nguyen Gia Coc & Pham Gia Duc, 1996, pp. 53-55)

3.1.2.2 Geometry in the current middle school curriculum and textbooks

3.1.2.2.1 An overview of changes in the middle school geometry curriculum

Recently, the middle school 'reformed' curriculum and textbooks were replaced by the new curriculum and textbooks with the following changes (MoET, 2002 a, p. 2; see also some geometry lessons of the current mathematics textbooks in appendix A):

> • Geometry is not represented as an absolutely pure deductive science (geometry is not represented by the way that theorems and characteristics are built by absolute deductive proofs from a system of axioms).
>
> • Proofs are significantly reduced, especially in grades 6 and 7.[51] However, requirements of reasoning and proof are gradually increased from grade 7 to grade 9. Results with many applications are earlier presented.
>
> • Spatial geometry *is not taught*, but students are taught to recognize some spatial objects so that some basic concepts of special geometry are maintained.
>
> • Greater emphasis is placed on mathematics applications and on using a calculator in learning mathematics[52].
>
> (MoET, 2002 a, p. 2; see also some geometry lessons in appendix A)

As discussed in chapter one (section 1.4.1.2), the prominent characteristic of the new curriculum is that the requirement of deductive reasoning is noticeably reduced in the middle school curriculum and textbooks, especially in grades 6 and 7.

Other aspects the current geometry curriculum and textbooks (see sections 3.1.2.1.1, 3.1.2.1.2 & 3.1.2.1.3) are nearly unchanged in comparison with the 'reformed' geometry curriculum and textbooks (see the 'reformed' geometry textbooks; current geometry curriculum and textbooks).

3.1.2.2.2 Geometrical contents in the middle school curriculum

The main geometrical contents of the middle school curriculum are presented in the following table (MoET, 2002 a):

[51] The same situation occurs with the Japanese geometry curriculum (Kunimune & Nagasaki, 1996). By contrast, the U.S mathematics educators tend to reconstitute the role of proof in their mathematics curriculum (Knuth, 2002, p. 62).

[52] Details of the way mathematics applications are presented in the current textbooks will be discussed in section 3.2.2.3 of this chapter. Although the roles of using calculator are paid attention, the current textbooks do not mention the use of computer in teaching mathematics.

Table 3.1: The main geometrical contents in the current middle school curriculum

Textbooks	Chapters	Contents
Mathematics 6*: *part 1	1. Segment	Point, line; three points in a straight line; a line is determined by two points; practice: plant trees in straight lines; ray; segment; measure of a segment; When does $AM + MB = AB$?; draw a segment with specific segment measure; midpoint of a segment.
Mathematics 6*: *part 2	2. Angle	Half of a plane; angle; measure of an angle; When does $m\angle xOy + m\angle yOz = m\angle xOz$? ; draw an angle with specific angle measure; bisector of an angle; practice: draw angles; circle; triangle.
Mathematics 7*: *part 1	1. Perpendicular lines. Parallel lines	Vertical angles; two perpendicular lines; angles at a line cut two other lines; two parallel lines; Euclidean axiom about parallel lines; from perpendicular to parallel; theorem.
	2. Triangle	Sum of three angles of a triangle; two congruent triangles; 1st case of two congruent triangles (side-side-side); 2nd case of congruent triangles (side-angle-side); 3rd case of congruent triangles (angle-side-angle); isosceles triangle; The Pythagorean theorem; Cases of two congruent right triangles.
Mathematics 7*: *part 2	3. Relation among factors of a triangle. Concurrent lines of a triangle	Relation between angles and their opposite sides in a triangle; relation among three sides of a triangle; characteristics of the three medians of a triangle; characteristics of the bisector of an angle; characteristics of the three bisectors of a triangles; characteristics of the perpendicular bisector of a segment; characteristics of the three perpendicular bisectors of a triangle; characteristics of the three altitudes of a triangle.
Mathematics 8:* *part 1	1.Quadrilateral	Quadrilateral; trapezoid; construction with ruler and compass; axial symmetry; parallelogram; central symmetry; rectangle; a line parallel to a given line; rhombus; square.
	2. Polygonal. Area of a polygonal.	Polygonal; regular polygonal; area of a rectangle; area of a triangle; area of a trapezoid; area of a rhombus; area of a polygon.
Mathematics 8:* *part 2	3. Similar triangles	Thalet's theorem for a triangle; the converse theorem and consequences of Thalet's theorem; characteristics of bisectors of a triangle; concept of two similar triangles; the first case of two similar triangles; the second case of two similar triangles; the third case of two similar triangles; more cases of two similar triangles.
***Mathematics 9*:**	1. Relations in	Some relations among sides and altitudes in a right triangle; trigonometric ratios of an acute angle; trigonometric table;

part 1	a right triangle	some relations among sides and angles in a right triangle; application of trigonometric ratios of an acute angle.
	2. Circle	Definition of a circle; symmetry characteristic of a circle; diameters and chords of a circle; relations among chords and distance from center to chords; relative positions of a line and a circle; tangents to a circle; the characteristics of two intersecting tangents; relative positions of two circles;
***Mathematics 9**:* *part 2*	3. Angles at a circle	Angle with the vertex at a center of a circle; relations among chords and arcs; angle with the vertex in a circle; angles with a tangent and a chords as sides; angle with the vertex inside a circle; angle with the vertex outside a circle; a quadrilateral inscribed within a circle; a circle circumscribed by a quadrilateral; a circle inscribed with a circle; length of a circle; length of an arc.
	4. Cylinder, Cone and Sphere	Cylinder, area and volume of a cylinder; cone, area and volume of a cone; sphere, area and volume of a sphere

3.1.2.3 Insufficiency of conditions for a Vietnamese RME curriculum

This section explains why it is impossible and unreasonable to design a RME curriculum that could be applied in teaching and learning in Vietnamese school at present.

Fauzan (2002) conducts a study on applying RME in teaching Indonesian primary school geometry. In this study, he designed and implemented the Indonesian Realistic Mathematics Education (IRME) Curriculum of topic Area and Perimeter for grade 4. In the Netherlands, teachers, at least in elementary level, are quite flexible in their teaching. The Dutch elementary schools can choose the suitable textbooks, and they can even design a curriculum for themselves; moreover, teachers can also change the timetable without school headmasters' permission if it is necessary (Van den Heuvel-Panhuizen, 2000, p. 10).

In the United States, some states recommend textbooks to the schools, while in other states, the schools themselves can choose suitable textbooks (Stevenson, 1999, p. 114). In Germany, before use in school, the textbooks must be accepted by a state committee and are often selected by grade-level committees; however, a teacher who finds the textbooks not suitable can develop his or her own teaching material (Stevenson, 1999, p. 114). In Japan, teacher committees select textbooks from those approved by the Ministry of Education, and the school administration or the individual teacher decides the timetable for and method of instruction (Stevenson, 1999, pp. 114).

The situation in Vietnam is quite different from the situation in these countries. There is a controversy surrounding the desire for multiple textbook series at the recent Vietnamese National Assembly meeting (Hoang Van Tu, Le Van Binh, Vu Anh Tuan, Vu Lan Anh & Phan Thanh Ha, 2005, pp. 27-28). Approvers appeal for different series so that teachers and students could choose the most suitable series for their conditions and instructional situation. There is much disapproval concerning low competence and underdeveloped methods of teachers and passive learning of students; the unevenness of teaching and learning among different schools; and difficulties in controlling the quality of education. Ultimately, the Vietnamese National Assembly struck down the proposal to use multiple series of textbooks.

According to Hoang Van Tu et al. (2005, pp. 77-78), "The Minister of Education and Training relies on the examinations of the National General Educational Curriculum and Textbook Examination Council to promulgate the general education curricula and ratify the textbooks for official unified stable use in teaching and learning at general educational establishments." According to this statement, in Vietnam, there is only one unified curriculum and one set of textbooks sanctioned for use in the classroom. Furthermore, school teachers often have to conform to the *curriculum distributions* which are set up by the local Education and Training Departments in a city or province (see, for example, Ho Chi Minh City's Education and Training Department, n.d.). These curriculum distributions set fixed timetables for mathematics lessons and tests and do not allow a teacher to modify the curriculum. In other words, mathematics lessons in each grade are taught the exact same way at every school throughout the country. If a teacher created his or her own mathematics curriculum, s/he would not be allowed to implement it in a Vietnamese school.

Moreover, both the 'reformed' and present mathematics curricula are quite different from the (Dutch) realistic curriculum (see, for instance, discussion about the Vietnam geometry curriculum and RME curriculum in section 3.2.2.3 of this chapter). Even though many in Vietnam advocate for the realistic curriculum, it would take much time to get it implemented in Vietnamese school.

For these reasons, this study does not discuss a design for a RME curriculum for Vietnamese school. However, it refers to what is called *RME-based lessons*. Bases for designing RME-based geometry lessons are discussed in section 3.2 of this chapter.

3.2 Foundations to design Vietnamese RME-based geometry lessons for grade 7

This section discusses how to design RME-based geometry lessons for grade 7. As discussed in section 3.1.2.2 of this chapter, deductive reasoning is first introduced in grade 7 of middle school.

This section discusses foundations for designing RME-based geometry lessons for grade 7 in Vietnamese middle school. In general, these principles can be also applied in designing other middle school geometry lessons. As discussed above (see chapter 3, section 3.1.2.3), because of the standardized national mathematics curriculum and textbook series, it is impossible to apply the entire realistic curriculum in teaching and learning mathematics in Vietnamese school. For this reason, this section first discusses the impact of the Vietnamese mathematics curriculum and textbooks on RME-based geometry lessons. It then considers studies on RME as an important base for creating RME-based geometry lessons. Aside from Freudenthal's ideas and the characteristics of RME discussed in chapter two, this part also delves into the details of the (Dutch) realistic geometric curriculum and analyzes geometry lessons in the current Vietnamese mathematics textbooks in the light of "didactical modeling", "emergent modeling" and "mathematical modeling". In addition, as discussed in chapter 1 (section 1.3.1.1), Vietnamese students' competence varies and often depends on their area of residence. To deal with such situation, in some RME-based lessons, different options are given so that teachers can choose the suitable ones for their students. To some extent, these selected situations can help teachers cope with the regimentation of teaching time. This part also briefly discusses making lessons specifically suitable for Vietnamese students. Next, the theory of Van Hiele's levels is discussed. Finally, this section explains why RME-based geometry lessons do not refer to the use of information and communication technology (ICT) although ICT, especially Dynamic Geometry Software (DGS), may enrich geometry instruction.

3.2.1 The Vietnamese mathematics curriculum, textbooks and curricular distributions

As discussed in chapter 1 (see section 1.3.1.1) and this chapter (section 3.1.2.3), the mathematics curriculum, textbooks and curricular distributions are rigidly standardized across Vietnam. Unlike in some other countries, typically, Vietnamese teachers are not allowed to change the orders and contents of lessons. Furthermore, they are bound to obey the curricular distributions, designed by the local departments of education and training, when they teach a discipline in school (see, for example, Ho Chi Minh City's Education and Training Department). For this reason, RME-based geometry lessons must be designed to fit within the order and contents prescribed by the official textbooks and curricular distributions. Because RME-based lessons often require more time than the usual mathematics lessons in Vietnam, the requirements related to curricular distributions are not always satisfactorily met (see section 4.6.3.1 of chapter 4).

In the case of the Triangle-Angle Sum Theorem, for instance, RME-based lesson uses the context of decorative figures with isosceles triangles on an ancient Greek vase to help the students discover the theorem and its proofs. However, in the current mathematics curriculum and textbook for grade 7, students do not learn the concept of an isosceles triangle until later. For this reason, RME-based lesson is altered, showing students a vase with arbitrary triangles instead of one with isosceles triangles. Moreover, it may save time if the figure with arbitrary triangles is used instead of one with isosceles triangles (see the Triangle-Angle Sum Theorem in appendix C for more details).

3.2.2 Studies on RME

3.2.2.1 The basic ideas of Freudenthal and the characteristics of RME

The basic ideas of Freudenthal and the characteristics of RME, which are discussed in sections 2.2 and 2.4 of chapter 2, are important foundations for designing RME-based geometry lessons.

- The basic ideas of Freudenthal (see discussion in chapter 2, section 2.2):
    - Mathematics as a human activity
    - Didactical phenomenology
    - Guided reinvention
- The characteristics of RME (see discussion in chapter 2, section 2.4):
    - The use of contexts
    - The use of models
    - The students' own production and constructions

• The interactive principle

• Intertwining mathematics strands

Generally, the Vietnamese RME-based geometry lessons are designed based on the Freudenthal's basic ideas and the characteristics of RME. Intertwining strands of mathematics is one of the characteristics of RME (see chapter 2, section 2.4.5); however, RME-based geometry lessons sometimes cannot reach this requirement because, as discussed above (see chapter 3, section 3.2.1), these lessons must be compatible with the Vietnamese grade 7 mathematics curriculum, which is certainly not a realistic curriculum. Of course, when a wholly realistic mathematics curriculum can finally be realized, the characteristic of intertwined strands should be emphasized (see chapter 2, section 2.4.5).

3.2.2.2 The characteristics of the (Dutch) realistic geometry curriculum

This section briefly introduces the Dutch realistic geometry curriculum. According to Gravemeijer (1990, p. 79), this type of curriculum is quite different from a deductive Euclidean geometry curriculum, and realistic geometry should replace formal geometry or at least be considered as preparation for it.

To create a realistic geometry curriculum, "[…] Van Hiele levels can help to establish the macro-structure of a course […]", and "micro-didactical structure […] originates from Freudenthal's (1983) didactical phenomenology and the reinvention principle […]" (Gravemeijer, 1990, p. 84). Freudenthal's didactical phenomenology principle lets an instructor know where an event appears so that s/he can design or create contextual geometry problems.

The following is considered as starting points for a realistic approach for children of ages 4 to 14:

• besides the study of number, mathematics also deals with the study of space;

• geometry instruction must start with and relate to real phenomena of the space that surrounds us;

• pupils of ages 4-12 are also entitled to geometry;

• the method for the introduction to geometry must have an intuitive character;

• formalisation will constitute the end point of a vertically planned curriculum.

(De Moor, 1991, p. 122)

There are various aspects of realistic geometry for children of ages 4 to 14, including "*sighting* and *projecting*, *locating* and *orienting*, *spatial reasoning*, *transforming*, *drawing* and *constructing*, *measuring* and *calculating*" (Treffers, de Moor & Feijscited cited in de Moor, 1991, p. 123).

Realistic geometry regards the realistic contexts and especially a variety of familiar phenomena in the real world (for example, the phenomena related to the lighting from the sun, the moon, a street-lamp and a light bulb and shadows of a people, trees, animals and posts; vision line; descriptions of routes, maps and graphs; constructions of blocks; overhead projectors and photocopiers and so forth) as sources for mathematics activities and as a starting point for instruction (De Moor, 1991, pp. 123-135; see also Gravemeijer, 1990, pp. 85-89; Goddijn, 1980, pp. 1-39).

Those who regard deductive geometry as the only legitimate geometry have reason to doubt the contextual problems of realistic geometry. More specifically, they question whether realistic context problems are mathematical and geometrical problems at all (Gravemeijer, 1990, p. 82). Gravemeijer (1990, pp. 82-84) convinces the doubters that realistic context problems are also mathematical and geometrical problems by using Freudenthal's idea of mathematics as a human activity and Van Hiele's theory of levels. Furthermore, borrowing the idea of mathematical literacy, Gravemeijer (1998, p. 48-49) confirms and explains that the aforementioned activities belonging to geometrical literacy. He tries to persuade the doubters, who often consider school geometry as a logic-deductive subject, of the necessary of the realistic context problems.

Phenomena, in a realistic curriculum, are often repeated at different levels, suitable for children of specific ages. De Moor offers several examples of using realistic contexts at different levels, related to sighting and projecting for different age groups of students (4-6, 6-10 and 10-14) (1991, pp. 123-124). In other words, the realistic geometric curriculum is spiral.

While trying to find answers to questions or to implement tasks, the children are gradually encouraged to reinvent informal geometry concepts and theorems. The realistic contexts help the children build their own informal geometric knowledge from their experiences and from what they obtained previously under the guidance of their teachers or the learning materials. (De Moor, 1991, pp. 123-135; Gravemeijer, 1990, pp. 85-89; Goddijn, 1980, pp.1-39)

Following Treffers, De Moor refers to and discusses bases for the justification of the study of geometry, including *applicability*, *preparatory value*, (*subject specific*) *value* and *personal value* (cited in de Moor, 1991, p. 136).

As discussed in chapter 2 (section 2.4.1.2), RME's viewpoint on teaching mathematics application is essentially different from the traditional viewpoint. In the traditional instruction, formal mathematics is taught, and applications of formal mathematics are afterwards introduced. Freudenthal often criticizes this by using the phase 'anti-didactical inversion'. By contrast, the realistic instruction takes the opposite direction. At the outset, realistic contexts are given to students. While working with these contexts, students can develop their own informal mathematical knowledge.

The value of realistic geometry as a preparatory tool is discussed at the beginning of this section (the idea of Gravemeijer about Dutch realistic geometric curriculum), as suitable starting points for instruction (Van Hiele's theory of levels). Realistic geometry can be considered a necessary, or at least useful, preparation for more formal geometry.

De Moor discusses the subject-specific value of geometry. He argues that geometry is one of the indispensable components in a mathematics curriculum because of its uses in exploring, aesthetically evaluating and conceptually modeling the spatial dimensions of the real world (De Moor, 1991, p. 137). In other words, geometry must play a role in any mathematics curriculum.

De Moor (1991, p. 137-138) emphasizes that geometry problems must have personal meaning to the student, i.e., a relevancy to his or her environment.

In conclusion, realistic geometry, based on Van Hiele' theory of levels as a macro-structure and Freudenthal's ideas of 'didactical phenomenology' and 'guided reinvention' as a micro-structure, is quite different from traditional Euclidean deductive geometry. It can perform a preparatory function for more formal geometry, or it can replace formal geometry. Phenomena in everyday situations are exploited to build realistic geometric contexts, which are the starting points and through which children can reinvent or rediscover informal geometrical knowledge. Following Freudenthal's idea of mathematics as an activity, mathematics includes not only definitions, theorems, regulations, etc. but also other components, which were discussed in previous parts of this section.

3.2.2.3 Differences between using manipulations and mathematics applications in the Vietnamese textbooks and realistic contexts in RME

As discussed in chapter 1 (section 1.4.1.2), in the present Vietnamese middle school mathematics textbooks, students often work with manipulations (drawing, measuring, folding, cutting and so forth) before formal geometry (definitions, theorems and regulations) is officially introduced. Furthermore, in comparison with the 'reformed' textbooks, the current textbooks include a significantly increased number of applications. In this section, the contexts of these manipulations in the Vietnamese mathematics textbooks are compared with realistic context in RME by using Gravemeijer's ideas of "didactical modeling", "mathematical modeling" and "emergent modeling" (see chapter 2, section 2.4.2.3). Gravemeijer (2004, pp. 97-100) argues that each of these types of modeling plays its role in mathematics education, and that "emergent modeling" (specific modeling in RME approach) should be the "precursor to mathematical modeling".

3.2.2.3.1 Didactical model and emergent model

In the present textbooks, before a theorem is officially presented, the students are often asked to do some activity such as drawing, measuring, cutting, folding and so forth, under the guidance of the textbooks. In other words, "didactical modeling" is utilized to concretize formal mathematics. Although studies of mathematics education confirm the use of "didactical modeling", they also note disadvantages to the practice (Gravemeijer, 2004; see also chapter 2, section 2.4.2). Detailed disadvantages are discussed in specific RME-based geometric lessons (see appendix C). On the one hand, the way in which the theorems are presented seems to be suitable for majority of the middle school students. On the other hand, as discussed in chapter 1 (see section 1.3.1.1), the use of a single, official textbook in every school in Vietnam cannot possibly cater to the competence of all students, especially since, as previously noted, the achievement of Vietnamese students often varies considerably from one area to another.

In the rest of this section, one example taken from the current mathematics textbook will illustrate how "didactical modeling" is used. More examples are found in appendix A.

Figure 3.5 represents how the concept of trapezoid is presented in the current textbook (Phan Duc Chinh et al., 2004 c, p. 69):

§ 2. Trapezoid

Do two sides *AB* and *CD* of the quadrilateral in figure 13 have any special characteristics?[53]

1. Definition

The quadrilateral in figure 13, in which *AB//CD,* is a trapezoid.

This approach, typical of the current mathematics textbooks, is an example of "didactical modeling".

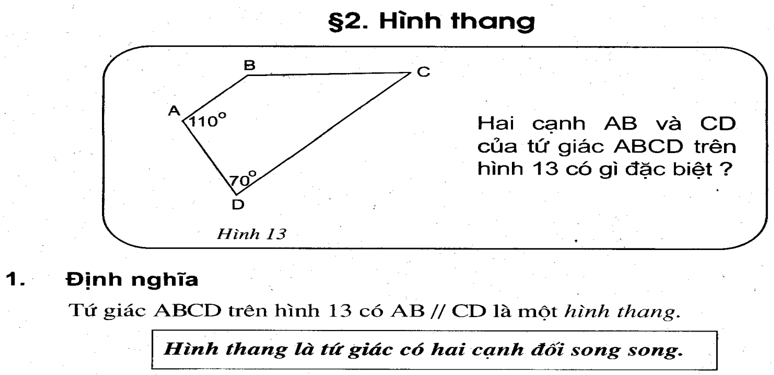


Figure 3.5: The presentation of the concept of a trapezoid in *Mathematics 8*

3.2.2.3.2 "Mathematical modeling" and "emergent modeling"

Although the number of mathematical applications has increased significantly in the Vietnamese middle school mathematics textbooks, the way in which the applications are presented in these textbooks is quite primitive (see Vietnamese middle school mathematics textbooks, for example Phan Duc Chinh et al., 2004 a, b & c). In other words, formal mathematics is taught first[54], and then formal mathematics is used to solve application-problems. Freudenthal and other RME researchers strongly oppose this way (see the details in chapter 2, sections 2.2.1.2 & 2.4.1.2). Freudenthal calls it "anti-didactical inversion" (1983, p. ix). It conforms to the concept of "mathematical modeling".

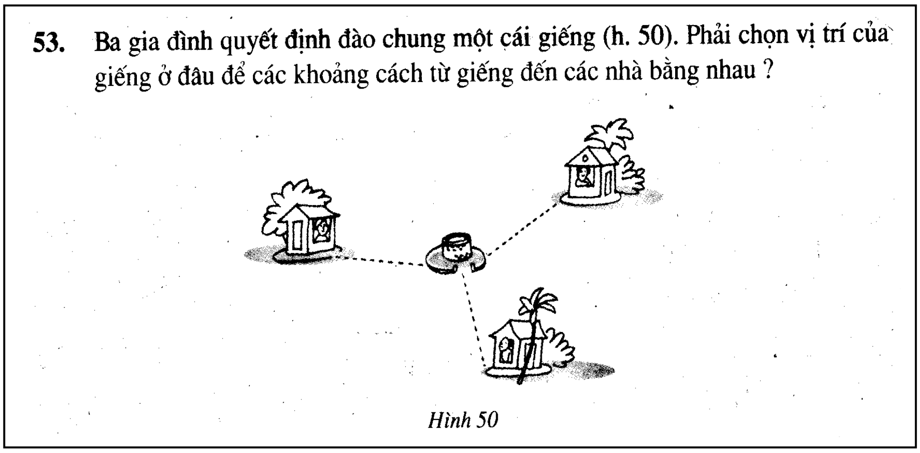
By contrast, realistic instruction assumes the opposite direction, as discussed in chapter two (see section 2.4.1 for details). That is, context problems are given to students first. Working with these contexts, students then gradually build up from informal to formal knowledge. This RME viewpoint on realistic contexts conforms to

[53] Besides the answer of the textbooks, teacher might expect other possible answers of students to the question:
- They are not equal;
- *CD* is two times longer than *AB*;
- They do not have any common vertex, or they are two opposite sides of quadrangle *ABCD*.

[54] As discussed in section 3. 2. 2. 3. 1, the present textbooks often use "didactical modeling" at the beginning of the lesson.

emergent modeling. Gravemeijer (2004, p. 97) has urges "emergent modeling as a precursor to mathematical modeling" in one of his research papers.

A copy of one exercise in *Mathematics 7: part 2* is presented in figure 3.6. After learning formal knowledge (the theorem about characteristics of the three perpendicular bisectors of a triangles and its formal proof), students are offered the exercise: "Three families decide to dig and build one common well. What position of the well should be selected such that the distances from the well to the families are equal?" (Phan Duc Chinh et al., 2004 b, p. 80). In this situation, students are expected to apply the formal mathematics theorem to solve this problem.


53. **Ba gia đình quyết định đào chung một cái giếng (h. 50). Phải chọn vị trí của giếng ở đâu để các khoảng cách từ giếng đến các nhà bằng nhau ?**

Hình 50

Figure 3.6: One mathematics application in *Mathematics 7: part 2*

3.2.2.4 Selected situations in RME-based geometry lessons

As discussed in chapter 1 (section 1.3.1.1), Vietnamese students' competence often depends on where they reside. Generally, students' achievement increases as one moves from the mountains and islands into rural areas and from there into urban areas. RME-based geometry lessons offer different situations so that middle school mathematics teachers can choose the suitable ones for their students and their teaching. If their students are highly competent, they can choose difficult situations. Otherwise, they can select situations with more or less guidance to help the students along.

Because of certain factors, contexts that are experientially real to students (Gravemeijer & Doorman, 1999, p. 111) in one country may not be experientially real to students in another country. For example, the bus (and bus-stop) context, which can function as emergent modeling to help students build "model of" and "model for" and understand “number sentences” (Streefland, cited in Van den Heuvel-Panhuizen, 2000, p. 6; Van den Brink, 1990, pp. 77-91; Van den Brink, cited in Gravemeijer, 1994, pp. 28-29 & 37-38), belongs to everyday life situations in the Netherlands and other developed countries. In other words, this context is “experientially real to the student” (Gravemeijer & Doorman, 1999, p. 111) or “real in the student’s mind” (Van den Heuvel-Panhuizen, 2000, p. 4). However, it might be unfamiliar to Vietnamese (primary) students, because at present there are only bus systems in certain busy provinces or cities in Vietnam. In other words, this context would be not experientially real to many Vietnamese students.

Contexts or situations in RME-based geometry lessons are selected such that they are, at least, familiar to the majority of Vietnamese students. In other words, they should be experientially real to most of the Vietnamese students.

3.2.3 Van Hiele’s levels of geometric thinking and phases of instruction

3.2.3.1 Van Hiele’s levels of geometric thinking and phases of instruction

This section briefly discusses Van Hiele’s levels of geometric thinking.

Van Hiele’s levels originally included five levels, but later on Van Hiele’s theory is sometimes also discussed as having three levels (Clements & Battista, 1992, p. 427). For instance, Clements and Battista (1992, pp. 427-428), Fuys, Geddes and Tischler (1988, p. 5), Pegg and Davey (1998, p. 111) and Clements (2003, pp. 152-155) refer to Van Hiele’s five-level-theory, while Gravemeijer (1990, pp. 83-84; 1998, pp. 54-55) and Costello (1991, pp. 52-53) mention Van Hiele’s three-level-theory.

Five Van Hiele’s levels:

> Level 0: The student identifies, names, compares and operates on geometric figures (e.g., triangles, angles, intersecting or parallel lines) according to their appearance.
>
> Level 1: The student analyzes figures in terms of their components and relationships among components and discovers properties/rules of a class of shapes empirically (e.g., by folding, measuring, using a grid or diagram).
>
> Level 2: The student logically interrelates previously discovered properties/ rules by giving or following informal arguments.
>
> Level 3: The student proves theorems deductively and establishes interrelationships among networks of theorems.

Level 4: The student establishes theorems in different postulational systems and analyzes/ compares these systems.

(Fuys, Geddes & Tischler, 1988, p. 5)

3.2.3.2 The characteristic of Van Hiele's theory of levels

Van Hiele's theory of levels has the following characteristics (cited in Clements & Battista, 1992, pp. 426-427; Fuys, Geddes & Tischler, 1988, pp. 5-8):

- Discontinuity of learning process;
- Sequencing and hierarchy of the levels;
- Relativity of the levels about implicitness and explicitness;
- Linguistics phase of each level;
- The level of instruction conforms to the level of students' understanding;
- Role of instruction in moving from one to the next level;
- Variety of phases in moving from one to the next level.

3.2.4 Using information and communication technology (ICT) in teaching and learning geometry

Recently, studies on using information and communication technology (ICT) in mathematics instruction have flourished around the world. Dynamic Geometry (DG) computer software such as Cabri Geometry, Geometer's Sketchpad, Euclide, Geonext and so forth are given special attention in teaching and learning geometry (see, for instance, King & Schattschneider, 1997; Nguyen Ba Kim, Dao Thai Lai & Trinh Thanh Hai, 2005; Hoyles, 2005; Laborde, 2005). Although it is unquestionable that DG has potential for enriching the teaching of geometry in school, RME-based geometry lessons do not consider applications of Dynamic Geometry because of insufficient resources in Vietnam. The current Vietnamese textbooks do not discuss using computer software in mathematics instruction. When comparing Vietnamese and the U.S. mathematics textbooks, Nguyen and Kulm (2002) find that Vietnamese textbooks concentrate on logical deductive reasoning and hardly ever refer to using computer software to help students learn mathematics, while the U.S. textbooks, on the other hand, pay special attention to utilizing computer resources but do not focus on logical deductive reasoning.

More recently, there has been a rapid increase of studies on the application of ICT to the teaching of mathematics in Vietnam. Building and using 'giáo án điện tử' (‘electronic lesson plans’), which are Power Point or Flash files containing lesson content, has become one of the prominent movements in mathematics instruction in Vietnam. However, a number of related questions which have arisen should be answered. In general, Vietnamese teachers have insufficient ICT knowledge.[55] In some schools, teachers who have just learned how to use Power Point or Flash and how to build and use ‘electronic lesson plans’. In some cases, ‘electronic lesson plans’ only function as ‘blackboards’ with sound and pictures. In addition, although Vietnamese schools have been gradually improving teaching equipment (computers, overhead projectors, beamers, etc.), there are still questions relating to the quantity, quality and appropriateness of these resources (see section 5.2.4.2 of chapter 5).

3.3 The Triangle Sum Theorem (Triangle-Angle Sum Theorem) as an example

This section describes how the ‘Triangle Sum Theorem’ lesson is created based on RME. Details of this RME-based lesson as well as other RME-based lessons are presented in appendix C.

3.3.1 The ‘triangle sum theorem’ lesson in the present textbooks

3.3.1.1 Advantages

In comparison with the ‘reformed’ textbooks (see appendix A), the theorem is presented in the present one (see appendix A) with the following improvements:

• The students have the chance to draw two triangles, measure the three angles of each triangle, calculate the sum of the angle measures for each triangle and comment on the results. In this situation, the students are encouraged to remember and use knowledge (the concepts of triangle, triangle’ angles, angle measure and sum) and skills (drawing, measuring and calculating). This knowledge and skill set belongs not only to geometry but also to algebra. Moreover, the students learn to understand the instructions of the textbook and do procedures following these instructions.

• Furthermore, the students have opportunities to manipulate a model triangle cut out of a board. This model functions as a didactical model (see chapter two, sections 2.4.2.2 & 2.4.2.3). In this situation, apart from the above-mentioned knowledge and skills, the

[55] In Vietnam, even some part of professors and associate professors do not have a habit of using IT. According to Do Tran Cat (cited in Kieu Oanh, 2005), about 30.3% of professors and 28.5% of associated professors do not use computers, and around 41.7% of professors and 53,3 associate professors use the internet. Le Hoang Mai (cited in Duc Hanh, 2005) shows the poor use of IT in teaching in primary schools in Hanoi: only 4 among 273 schools use IT in inteaching, and only 46 among 7172 teachers know how to use Winword and some educational software.

students must know and utilize other knowledge and skills such as the concept of adjacent angle, cutting, putting and predicting. Similar to the first activity, the students must understand the instructions and follow them. This time the instruction consists of not only the words but also the mathematical signs, including triangles, angles and measures of angles and the figures, including triangles and lines.

• The proof in the former textbook is quite long and complicated for the majority of grade-seven students. It requires a great deal knowledge of the concepts of segment, midpoint, ray, angle and triangle congruence, line parallel, opposite ray, alternate interior angle and ray between two rays. By contrast, the proof in the present textbook is shorter and simpler. The latter proof does not need the midpoint, ray, triangle congruence, opposite ray and ray between two rays. Moreover, before working with formal proof in the present textbook, the students work with its didactical model so that it is easier for them to understand the theorem and its proof.

3.3.1.2 Disadvantages

Despite the aforementioned advantages, there are also disadvantages to the present textbook. The following section analyzes and discusses these disadvantages:

• In *step ? 1*, the students may wonder why they should draw a triangle, measure its angles, and then calculate the sum of the angles' measures. Besides, some argue that the question may puzzle the pupils because it might be difficult for them to express any observations after simply drawing two triangles and calculating their angle measures' sums.

• In *step ? 2*, the students have to do some manipulations simply because their teacher or the textbook told them to do. In other words, they have to work by command.

• In the process of proving the theorem, the students usually observe their teacher's activities and try to understand the proof. The teacher often tries to help the pupils understand the proof by asking them some straightforward questions, for instance, "Why are $\angle ABC$ and $\angle A_1$ congruent?" and "Please give the reason why the sum of $m\angle BAC$, $m\angle A_1$ and $m\angle A_2$ is 180^0?" or by telling them to do some activities: "Please compare $m\angle ABC$ and $m\angle A_1$", "Similarly, please compare a pair of $m\angle ACB$ and $m\angle A_2$" and "From (1) and (2), please calculate the sum of $m\angle BAC$, $m\angle ABC$ and $m\angle ACB$" or "Please prove that the sum of $m\angle BAC$, $m\angle ABC$ and $m\angle ACB$ is equal to the sum of $m\angle BAC$, $m\angle A_1$ and $m\angle A_2$". In some circumstances these or similar types of suggestions and instructions are more or less necessary.

However, some argue that if the students do not have the chance truly to take charge of the proving process, they do not totally understand or will quickly forget the theorem and its proof- hence one Chinese proverb:

> Ich höre, und ich vergesse
> Ich sehe, und ich erinnere mich
> Ich tue, und ich verstehe

(Vollath, 1995, p. 18)

At first glance, the students seem to become active when they are instructed to follow the steps in *Mathematics 7: part 1* (they draw triangles, measure their angles and calculate the sum of the angles of each triangle in ***? 1*** or cut a triangle from a board, cut out $\angle ABC$ and put it as required, then do similarly with $\angle ACB$ in ***? 2***). But, actually, this lesson *constrains* them to do these manipulative activities, and all students with *different mathematical competence* have to *follow the completely fixed way* which was designed by the authors of the textbook. In other words, in this situation the student is, to some extent, reduced to the role of the actor, who must play his or her role within the confines of a prescribed script and follow the orders of a director. Similarly, the students have no choice but to follow completely the scenario in the textbook. A similar description is also fully discussed by Jahnke (2001, p. 5) when he analyzes a conversation between teachers and students in a 'normal' mathematics lesson („Normaler" Mathematikunterricht)

3.3.2 RME-based lesson

It is not easy to help students discover for themselves that the angle sum in a triangle is 180^0. This section briefly describes the situations in RME-based lessons.

The *first situation* helps students revise some related knowledge (triangle, angles of a triangle, angle measure, etc.)

In *situation 2*, the students are encouraged to discover that there is a relation among the angle measure of a triangle. The students work in pairs. One student randomly writes down three sets of three angle measures, and the second student tries to draw (three times) a triangle with the three given angle measures. After that, they change their position (see worksheet forms for the students in appendix D). From this situation, the students are aware of the fact that in some cases they can draw a triangle with three given angle measures, and in some other cases they cannot.

In the *third situation*, the students are offered a game for two players. The rule of the game is described sufficiently in the lesson (see appendix C). This situation encourages the students to use different knowledge and strategies and to reinvent the characteristics of the three angle measures in a triangle.

In *situation 4*, the students are first introduced to the figure in the ancient Greek vase and informed that the ancient Greeks used this figure to prove the relationship of the measures of the angles within a triangle. According to some books about the history of mathematics, the ancient Greeks used isosceles triangles to prove the theorem for the sum of the three angle measures in an isosceles triangle. However, in the present mathematics textbook, the concept of an isosceles triangle is not presented until later. For this reason, in RME-based lesson, instead of isosceles triangle, normal triangles are used in the model.

Then the students are asked to find the characteristic and prove it. The *situation* encourages the students to reinvent two different proofs of the theorem. In this *situation*, the students work in groups because the *situation* is more complicated than the two previous ones.

In the *fifth situation* (formal situation), the students are asked to prove the theorem. They are also asked to work in groups. *Situation 4* helps them find the proofs for *situation 5*.

Chapter 4 Analyzing feedback from experiment lessons

4.1 Introduction to feedback analysis

The purpose of this chapter is to analyze how RME-based geometry lessons, whose foundations are discussed in chapter 3, worked in teaching and learning middle school mathematics. The details of RME-based geometry lessons are presented in appendix C. The sources for the feedback analysis are the reports of the teachers, who used these lessons for their teaching and the worksheets of their students. Because the teachers believed that certain parts, especially the teaching time allowance, did not satisfy the requirements and conditions of teaching and learning in Vietnamese school, not all of RME-based geometry lessons were used (see section 3.2.1 of chapter 3; table 4.1 in this section). More specifically, lessons in Vietnamese mathematics textbooks often contain formal definitions and theorems with a specific timetable for each lesson determined by the local department of education and training. It should be noted that the unique nationwide series of curricula and textbooks is always considered as the official materials for teaching and learning in Vietnamese school (see sections 3.1.2.3 and 3.2.1 of chapter 3). In addition, informal mathematics is not usually accepted in teaching and learning mathematics in Vietnam (see the current Vietnamese mathematics curriculum and textbooks). On the contrary, the (Dutch) RME geometry curriculum often emphasizes ways to help students gradually amass informal knowledge. This curriculum, at least for primary and early grades of middle school, does not focus on formal knowledge such as mathematics definitions and theorems. Moreover, mathematics teachers in the Netherlands are quite flexible in their teaching (see section 1.3.1.1 of chapter 1).

Specifically, the detailed tasks of the feedback analysis are designed to answer the following questions:

- What difficulties did the teachers and their students encounter while teaching and learning with RME-based lessons?
- How could RME-based geometry lessons enrich teaching and learning mathematics in Vietnamese middle school?
- How did the teachers and their students respond to RME-based lessons?
- How should these RME-based lessons be adjusted so that they are more suitable for teaching and learning geometry in Vietnamese middle school?

Some RME-based geometry lessons were taught in the two-semester school-year 2005-2006 in classes 7A and 7B of Nguyen Luong Bang middle school (Thanh Mien district), a school in rural area of Hai Duong province and a national standard school.[56] Most of these lessons were taught in the second semester of the school-year (see table 4.1).

Table 4.1: The timetable for RME-based geometry lessons

Nr[57].	Semester	Teaching time	Class	Lesson
1	First Semester	31.10.2005	7 A	The Triangle Sum Theory
2	Second Semester	Not used[58]		Perpendicular line and slant line
3	Second Semester	Not used		The Triangle Inequality
4	Second Semester	15.4.2006	7 B	Characteristics of the bisector of an angle
5	Second Semester	Not used		Characteristics of the bisectors of a triangle
6	Second Semester	19.4.2006	7 A	Characteristics of the perpendicular bisector of a segment
7	Second Semester	26.4.2006	7 B	The 'train station' problem
8	Second Semester	Not used		Characteristics of the three perpendicular bisectors of a triangle

There were 37 students in class 7 A and 38 students in class 7 B. Ms. Do Lan Huong, a new teacher, taught the lesson 'The Triangle Sum Theorem', and Mr. Le Xuan Mui, who has been worked as a mathematics teacher for six years, taught the others (Characteristics of the perpendicular bisector of a segment, Characteristics of the bisector of an angle and the 'train station' problem).[59] The teachers did not have to

[56] A national standard (middle) school is a school which reached national standards stipulated by the Ministry of Education and Training (MoET). For example, national standards for a primary school are found in MoET (c, n.d.).
[57] These lessons are sorted on their orders in the current textbooks
[58] This lesson was not used by the teachers because of its time requirement.
[59] Ms. Huong graduated from Hai Duong College of Education, and Mr. Mui graduated from Hanoi University of Education with a Bachelor degree in Mathematics and obtained a Master degree in Education after an in-service training course.

follow *situations* in RME-based lessons strictly. Rather, they could alter the situations so that these lessons were more suitable for their teaching and learning conditions.

4.2 The sum of the measures of the interior angles in a triangle (lesson 1)

4.2.1 Introduction

Firstly, this section presents an overview of the *situation*s in RME-based geometry lesson (see the detailed lesson in appendix C). It then briefly describes teaching and learning process in which the teacher and their students were working with RME-based lesson.

This RME-based lesson includes 5 *situation*s whose purposes are described as follows:

• The purpose of the *first situation* is to help students revise some knowledge relating to the theorem: a triangle, the angles in a triangle, the measure of an angle and how to measure the angles of a triangle;
• The aim of the *second situation* is to help students realize that it is impossible to draw a triangle with three given arbitrary measures of angles;
• *Situation 3* aims at helping students discover that there is a relation between the three measures of angles in an arbitrary triangle;
• *Situation 4* is created to give students chance to reinvent the content of the theorem (the sum of the measures of the interior angles of a triangle) and its proofs;
• The purpose of *situation* 5 is to help student create formal proofs for the theorem.

As discussed in chapter 3 (section 3.2.2.4), in some cases, RME-based geometry lessons offer selected *situation*s so that teachers can choose the most suitable *situation*s for their teaching conditions such as the allowance teaching length of time and students' competence. This RME-based lesson also gives the teachers different choices of *situation*s.

Typically, a Vietnamese school teacher writes down a lesson title at the beginning of a lesson, while the teacher who used RME-based geometry lesson did not. First of all, the teacher, Mrs. Huong gave her students worksheets in which the students' tasks were explicitly stated and asked the students to work in pairs (*situation* 1, see the worksheet form in appendix D). After having completed the first worksheets, the students were given the second worksheets. Again, the students were instructed to work in pairs (*situation 2*, see the worksheet form in appendix D).

Each student worked twice with the same worksheet in different roles (one as the first student and the other as the second student) and with different partners (see forms of two worksheets in appendix D). Because there were 36 students in the class, the teacher received 36 worksheets with students' answers for the *first situation* and 36 worksheets for the *second situation*.

The teaching time pressure (see section 3.2.1 of chapter 3) did not leave the teacher sufficient time for *situation 3*. Instead, she proceeded to give her students *situation 4*. Some students were puzzled by the task of *situation 4* because they were unfamiliar with this situation. Normally, they would work with only concrete questions. Some students seemed to misunderstand *situation 4* and did not know how to deal with its instructions. Although the teacher had recognized her students' struggles, she did not have enough time to explain the detailed requirement of *situation 4* to them and let them work again with this *situation*. Therefore, she had to move to *situatio*n 5. The following sections analyze the feedback from the teacher and her students, who were working with RME-based lesson.

4.2.2 Analyzing the students' worksheets of *situation 1*

In the first worksheet, the first student was asked to draw three arbitrary triangles, and the second one was asked to measure the angles of each triangle and write down the results of her/his measures (see the form of the first worksheet in appendix D). They, however, were not asked to calculate the sum of the angles in each triangle[60].

Although the task in the *first situation* was not difficult for the students, there were 13 worksheets which contained at least one incorrect measure. Some worksheets showed a paradox: *in spite of incorrectness of the angle measures in a triangle, the sum of these measures was* 180^0.

It is believed that the sums of the three triangle's angles which the students had measured were *around* 180^0. However, it was surprising that nearly all of the students had the result of 180^0. *Did the student measure the angles so precisely?*

In 13 of 36 worksheets, the students did not measure angles precisely in at least one of three cases:

• Some students did not measure precisely, or if they measured precisely, they wrote down the wrong names of the angles (there are 5 cases).

[60] 'Sum of angles', which also means 'sum of measures of angles', is sometimes used in this chapter.

• One student wrote down 57^0, 60^0 and 117^0 after measuring the three angles of one drawn *acute* triangle and 90^0, 115^0 and 65^0 after measuring the three angles of one drawn *right* triangle (see first and second cases in figure 4.1). Another student had the results of 101^0, 66^0 and 35^0 from a drawn *obtuse* triangle. (There are 4 cases)

• It is surprising that: *although some students did not measure the angles properly, the sum of their three angles was 180^0*. For example, in the second case of the worksheet, V.V. Tu drew one obtuse $\Delta\ EGH$ ($\angle G$ is the obtuse angle); nevertheless, his partner N.T.N. Anh measured the three angles of this triangle and wrote down $m\angle E = 30^0$, $m\angle G = 61^0$ and $m\angle H = 89^0$ (see figure 4.2). Even though she did not measure these angles properly, the sum of the three N.T.N. Anh's angles was 180^0 ($30^0 + 61^0 + 89^0$). Similarly, H.T.H. Tam had the result: $m\angle GHI = 55^0$, $m\angle GIH = 39^0$ and $m\angle IGH = 86^0$ after measuring the angles of an obtuse triangle which had been drawn by his partner V.V. Dong in the third case of their worksheet (see figure 4.3). Interestingly, the sum of these angle measures was also 180^0. The same paradox was also found in the first and second case in the worksheet of D. T. Huong and P. T. Lanh (see figure 4.4). Their second case was similar to the mentioned cases of the pairs V.V. Tu-N.T.N. Anh and H.T.H. Tam-V.V. Dong. In the first case, D.T. Huong had drawn one acute triangle, however her partner-P.T. Lanh found that $m\angle A = 50^0$, $m\angle B = 110^0$ and $m\angle C = 20^0$ after measuring the thee angles of this acute triangle. Remarkably, the sum of the angle measures was also 180^0. (There are 4 cases in total)

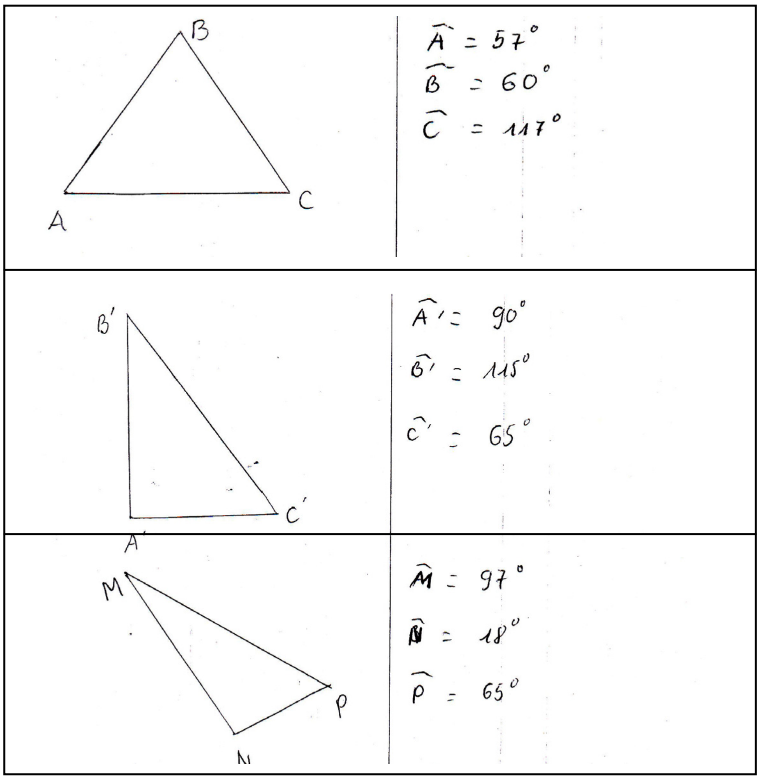


Figure 4.1: The worksheets of P.T. Lanh and T. T. Bich (L.1, S.1)[61]

From this paradox, the second question is posed: *Had the students known the content of the lesson before the lesson started?*

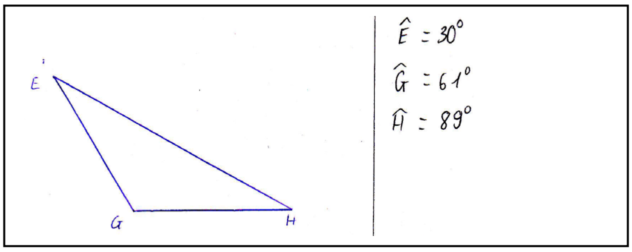


Figure 4.2: The worksheet of V.V. Tu and N.T.N. Anh (L.1, S.1)

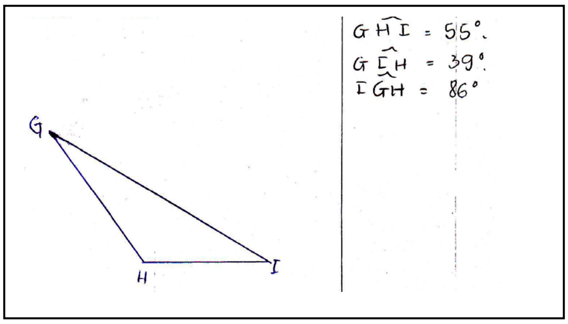


Figure 4.3: The third case of V.V. Dong and H.T.H. Tam (L.1, S.1)

[61] L.1, S.1 is an abbreviation of lesson 1, situation 1.

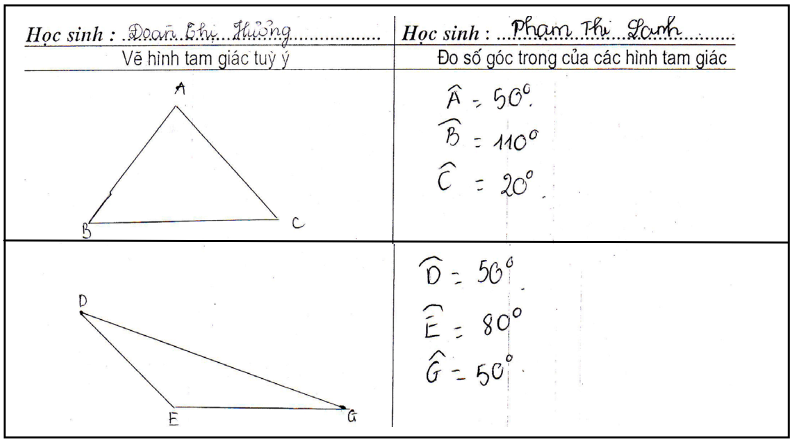


Figure 4.4: The first two cases of D.T. Huong and P.T. Lanh (L.1, S.1)

4.2.3 Analyzing the students' worksheets of *situation 2*

In the second worksheet, the first student was asked to write down three arbitrary measures of three angles, and the second one was asked to draw one triangle with these measures if it was possible. Again, they were asked to do this task three times.

• In 7 worksheets, *the students reasoned that it was impossible to draw a triangle because the sum of the three given measures of angles was not (greater or smaller than) 180^0*. For example, L.T. Ngoc gave three measures 80^0, 70^0 and 40^0; her partner- N.T.N. Anh calculated the sum of these three measures (190^0), and she reasoned that "It is impossible to draw [a triangle] because the sum of the three angle measures of a triangle equals 180^0" (figure 4.5). It implies that these students had known the content of the theorem already, and they also could reason that if three angle measures whose sum was not 180^0 could not be the three measures of the angles of a triangle.

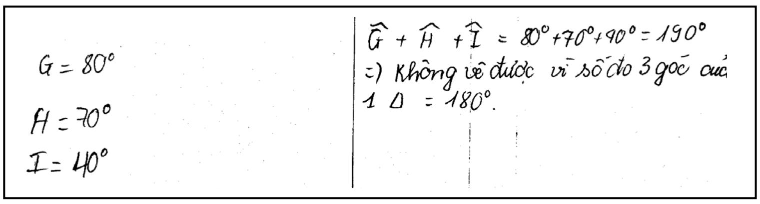


Figure 4.5: The third case of L.T. Ngoc and N.T.N. Anh (L.1, S.2)

• In 5 worksheets, the first students gave three cases, and in each case the sum of the given angle measures was 180^0 although they were asked to given three arbitrary angle measures. It appears that these students had also known the theorem already. For instance, N.T. Huong gave $m\angle A = 60^0$, $m\angle B = 63^0$ and $m\angle C = 57^0$ for the first case; $m\angle M = 54^0$, $m\angle N = 90^0$ and $m\angle G = 36^0$ for the second case; and $m\angle F =$

140^0, $m\angle K = 25^0$ and $m\angle H = 15^0$ for the third case (figure 4.6). In each of her given cases, the sum of the three angle measures was 180^0.

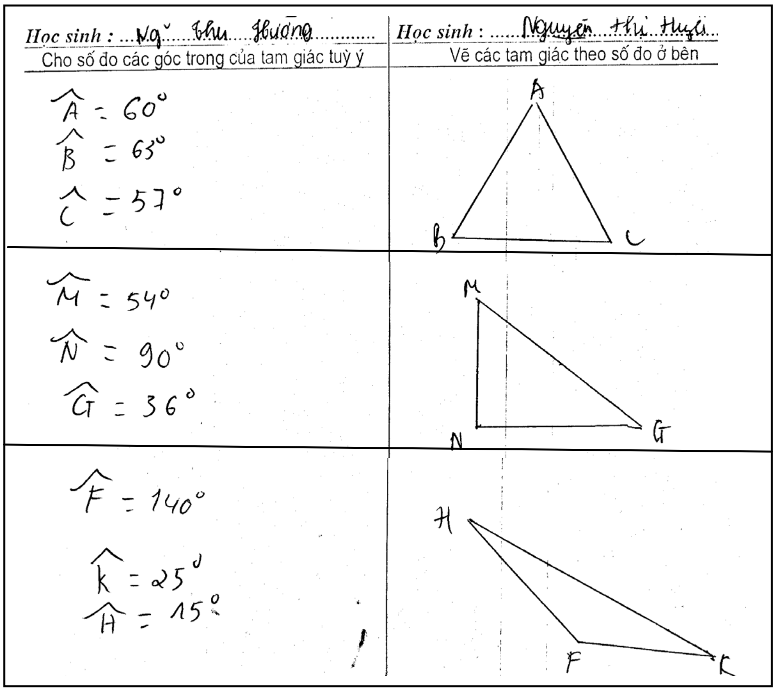


Figure 4.6: The worksheet of N.T. Huong and N.T. Huyen (L.1, S.2)

• In 5 worksheets, the students attempted to draw a triangle with the three angle measures whose sum was not 180^0. For instance, V.V. Dong gave three angle measures 10^0, 25^0 and 115^0. After failing to draw one triangle with these measures, his partner, P.T. Thanh concluded that "it is impossible to draw [a triangle with these measures of angles]" (figure 4.7).

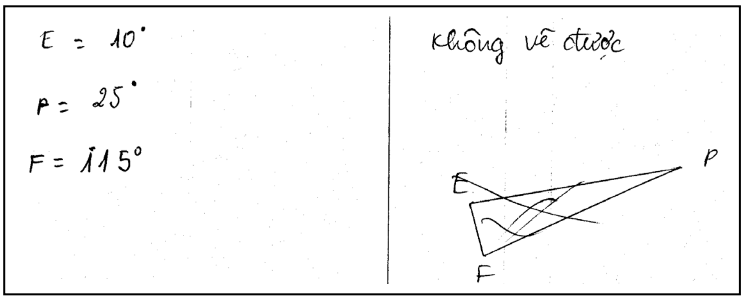


Figure 4.7: The second case of V.V. Dong and P.T. Thanh (L.1, S.2)

• In the other 15 worksheets, the students responded with answers, such as "it is impossible to draw" or "it is impossible to draw a triangle" without any additional explanation why they could not draw any triangle when the sum of the three measures of angles was not 180^0. For example, in the third case of the worksheet, V.M. Hai wrote down three angle measures 79^0, 81^0 and 69^0, and the partner, N.H. Duong only answered “it is impossible to draw [a triangle]” without further explanation. There are two possibilities. The first one was that N.H. Duong reached the conclusion after he did not succeed in drawing one triangle with these measures of angles in extra-pieces of paper. The second possibility was that he had known the sum of the angle measures is 180^0, and he had the conclusion after calculating the sum of the given angle measures (figure 4.8).

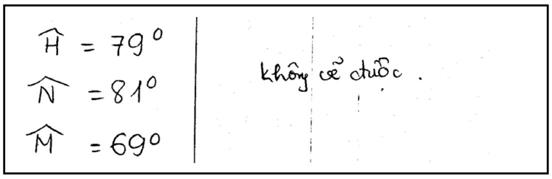


Figure 4.8: The third case of V. M. Hai and N. H. Duong (L.1, S.2)

From these situations, it can be deduced that some students had known the theorem before the teacher taught them. Maybe they had read the textbook, or they had learned the theorem before in an extra-lesson.

As discussed above, the students’ work with *situations 1* and *2* revealed that some students had known that the sum of the three angle measures in an arbitrary triangle is 180^0. While observing the students’ working, the teacher recognized this. Furthermore, she was pressured by the limited teaching time while using RME-based geometry lesson. Because some students already knew the content of the theorem, and the teacher was stifled by the teaching time pressure, she did not use the *third situation* which was designed to help students reinvent that there is a relation between the three angles’ measures in an arbitrary triangle. After the *first* and *second situations*, the teacher offered her students the *fourth situation*. It appeared that the students did not adapt to the similar situations. For this reason, some students did not fully comprehend the *situation*. Nevertheless, the teacher did not have enough time to explain the *situation* to them. She had to continue with the *fifth situation* in which she and her students were working with formal knowledge.

4.2.4 Teacher’s comments

According to the teacher’s report, some characteristics emerged during the RME-based lesson:

• With the students’ working in pairs or in groups, the classroom was noisier than usual. The louder noise level could disturb neighboring classes. Moreover, a few students did not work actively because they assumed the tasks were intended exclusively for high-performing students. Instead of focusing on the assignment, these students used the time for their own private work.

• RME-based lesson required more time than conventional lesson in the present textbooks. For this reason, in the lesson of the sum of the three angles in a triangle, the teacher could use *situations 1, 2, 4* and *5*. In addition, she could not thoroughly discuss *situation 4* with her students. She argued that a teacher can effectively use this RME-based lesson if s/he is not constrained by the pressure of limited teaching time and overwhelmed by the content amount.

• The students worked independently and actively with RME-based lesson. Despite significant education reforms, the students still have to follow strict instruction in the mathematics textbooks[62]. In contrast, while working with RME-based lesson, the students could naturally acquire knowledge.

• The teacher expended more energy and effort than usual because she had to maintain order, organizing and directing the students' activity. Moreover, she had to understand the philosophy of RME and the purposes of the *situations* in RME-based lesson. Nonetheless, while teaching RME-based lesson, the teacher did not have to lecture because the students were more independent and demonstrated the initiative to find and acquire the knowledge.

4.2.5 Findings

In general, the students became more active while working with RME-based lesson. They did not follow the extremely strict guidelines prescribed by the textbooks. The teacher confirmed that the *situation*s of RME-based lesson motivated her students.

As discussed in chapter 3 (section 3.2.1), Vietnamese school teachers must adhere to the curricular distribution created by a local department of education and training in a province or city. This regulation seemed to challenge the teacher while she was working with RME-based lesson. As explained above (sections 4.2.1 & 4.2.4, this chapter), she could not introduce *situation 3* and explain *situation 4* to her students because of the teaching time pressure. The teacher urged that she could apply RME-based lesson if she was not limited by the teaching time pressure.

---

[62] This is caused by the contradiction between the great quantity of knowledge and the limited length of teaching time (see discussion in section 1.2 of chapter 1).

The students learned actively with RME-based lesson. However, the teacher also 'complained' that she had to work harder than usual. First, she had to learn to understand RME approach and RME-based lesson. Second, she had to change her teaching habits. In addition, it was quite difficult for her to deal with the teaching time pressure. She should identify the reason why her students did not understand the task in *situation 4* and help them deal with it. In fact, she did not explain the task to them and turned to *situation 5*.

From analyzing the students' worksheets, two paradoxes emerged. Firstly, it appeared that most of the students had known the theorem's contents about the sum of the angles in an arbitrary triangle before the lesson began. In *situation 1*, some students incorrectly measured the three angles of a triangle; however, the sum of these angle measures was 180^0. Moreover, in most of the students' worksheets, the sum of the three angle measures was 180^0. In *situation 2*, most of the students wrote down three angle measures whose sum was equal to 180^0, when they were asked to write down three arbitrary angle measures. Moreover, some students even wrote down formal reasoning to conclude that it was impossible to draw a triangle with the three angle measures whose sum was not 180^0. There are two possible explanations for this occurrence: either, they had learned this lesson in an extra-class (see chapter 1, section 1.3.1.2), or they had read the lesson in the textbook. Tasks 1 and 2 in the textbook (see the Triangle Sum Theorem in the current textbook in appendix A), as well as *situations 1*, *2* and *3* in RME-based lesson (see appendix C), were not especially useful for the students who had known the theorem already. Secondly, some students did not know how to measure the angles in a triangle properly. As discussed in section 4.2.2 of this chapter, it is remarkable to note that although the students incorrectly measured the angles, the sum of the three angle measures was 180^0. This was caused by strong emphasis on formal reasoning and less attention to other activities such as drawing and measuring in teaching and learning geometry in Vietnamese school (see Hoang Chung, 1999, pp. 15-17).

From analyzing the students' worksheet and the paradoxes, it is clear that the students' competence was not homogenous. Some of them could successfully perform the tasks in *situations 1* and *2* and even knew how to reason deductively. Other students struggled with basic routine tasks. Inhomogeneity of students' competence existed not only among urban, rural and mountainous areas (see section 1.3.1.1 of chapter 1) but also within individual classes in a school and students in a class.

4.3 Characteristics of the bisector of an angle (lesson 2)

4.3.1 Introduction

Firstly, this section briefly introduces how the theorem was presented in the 'reformed' textbook called *Geometry 7* and is presented in the current textbook named *Mathematics 7: part 2*. After that, it discusses the aims of the *situation*s in RME-based lesson. Finally, some information about how the teacher, Mr. Le Xuan Mui organized his class is presented.

Geometry 7 of the 'reformed' textbooks presented only the content and one rigorous formal deductive proof of the theorem (see appendix A; see also discussion about the typical lessons of the 'reformed' textbooks in section 1.3.1.5 of chapter 1). In the current textbook, *Mathematics 7: part 2*, before the content and the proof of the theorem are presented, a short introduction is given:

> We consider the following problem:
> M is a given point insight $\angle xOy$ such that the distance from M to the two sides of this angle are equal. Is point M on the bisector ray (or is $\overrightarrow{OM}$ the bisector ray) of $\angle xOy$?

(Phan Duc Chinh et al., 2004 b, p. 69)

RME-based geometry lesson has three *situation*s (see details in appendix C). The first one, designed for individual work, introduces the context in which a farmer is working somewhere in an angle-shaped farm. The two 'sides' of the farm (the angle) are branches of a river. *Question a)* of this *situation* asks students to suggest a possible direction for the farmer when he wants to fetch some water from the river for his farm. *Questions b)* and *c)* ask students to point out some positions from which the farmer should go to branch 1 or branch 2 of the river. The *second situation* helps students rediscover that a point which is equidistant from the two sides of an angle is on the bisector of this angle. Students work with formal deductive geometry in the *last situation*. Students are asked to work in groups in the *second* and *third situations*.

The teacher, Mr. Le Xuan Mui, organized his students to work in groups (for *two first situations*). There were 37 students taking part in his class, and he divided his students into 6 groups of from six to seven members.[63] Mr. Mui gave his students the *two first situations* of RME-based lesson. For each *situation*, he gave one copy of the worksheet to all members of each group. He did not give detailed information about the third one.

4.3.2 Feedback analysis from the students' worksheets

a) *Situation 1*:

This section analyzes how the students worked with the *first situation* based on their worksheets.

Group 2 (Ngoc Anh, Bich, Huong, Ngoc Lan, Lanh and Tran Ngoc) reasoned that "the farmer should go to branch 2 because the distance from *A* to branch 2 is nearer than the distance from *A* to branch 1" and then identified some positions from which the farmer should go to branch 1 or 2 of the river. It appeared that this group used only informal knowledge to solve the problem. They arrived at the conclusion after seeing the figure in *this situation* and did not draw any additional lines (see figure 4.9). The students of this group used the same sign for two different points.

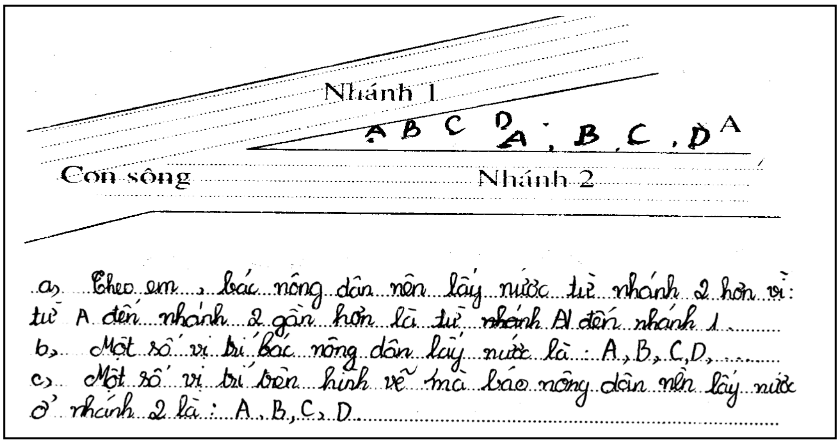


Figure 4.9: Worksheet 1 of group 2 (L.2, S.1)

Some other groups alternated between informal and formal knowledge to answer *question a)* of *this situation*. For example, initially, the students of group 3 (Nguyen Huyen, Giang Son, Thom, Nhan Ton, Le Ngoc and Loc) used informal

[63] The tasks of situations 1 and 2 are not complicated. He should divide his students into smaller groups in these situations. In situation 3, students work with the formal proof of the theorem. So he should divide his students into largerr groups.

knowledge to answer *question a)*, and then they applied formal knowledge to explain their conclusion (figure 4.10):

> According to our group, the farmer should fetch water from branch 2 of the river because the distance from his working place to branch 2 is nearer than the distance to branch 1. So he can save his time and health.
>
> Suppose that from position *A* we draw the two perpendicular lines to branches 1 and 2 at *K* and *H,* respectively.[64] We recognized that $AK > AH$, so the way from *A* to branch 2 is nearer.

The students of this group did not draw any additional lines in their worksheet. Moreover, they did not properly use Vietnamese: "Suppose that from position *A* we draw the two perpendicular lines to branches 1 and 2 at *K* and *H,* respectively."

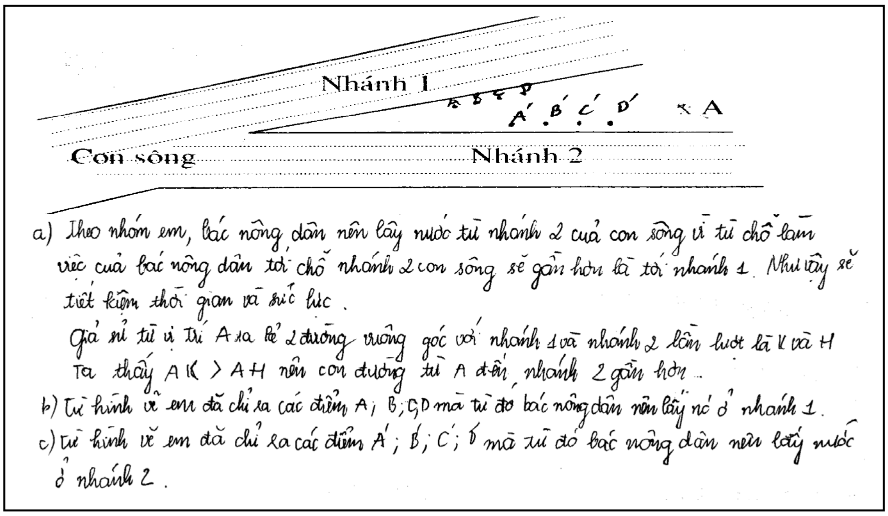


Figure 4.10: Worksheet 1 of group 3 (L.2, S.1)

In general, the other groups used more formal knowledge for making conclusions. All of them drew the perpendicular lines from *A* to branches 1 and 2 of the river and used the perpendicular sign. Most of them (4 groups) named the intersection points between the perpendicular lines and the sides of the angle (the branches of the river).

This is the answer of group 5 (figure 4.11):

> […] in order to save time, the farmer should fetch water from branch 2.
>
> *AH* is the perpendicular line to branch 2 (the shortest way from *A* to branch 2).
>
> *AK* is the perpendicular line to branch 1 (the shortest way from *A* to branch 1).
>
> We recognized that $AH < AK$.
>
> (Group 5: Muoi, Ngan, Le Ngoc, Ny, Quy Thanh and Thao)

[64] Their answer is translated into English from original one. They did not properly use mathematics language as well as Vietnamese. They did not draw $\overline{AH}$ and $\overline{AK}$ in their worksheet.

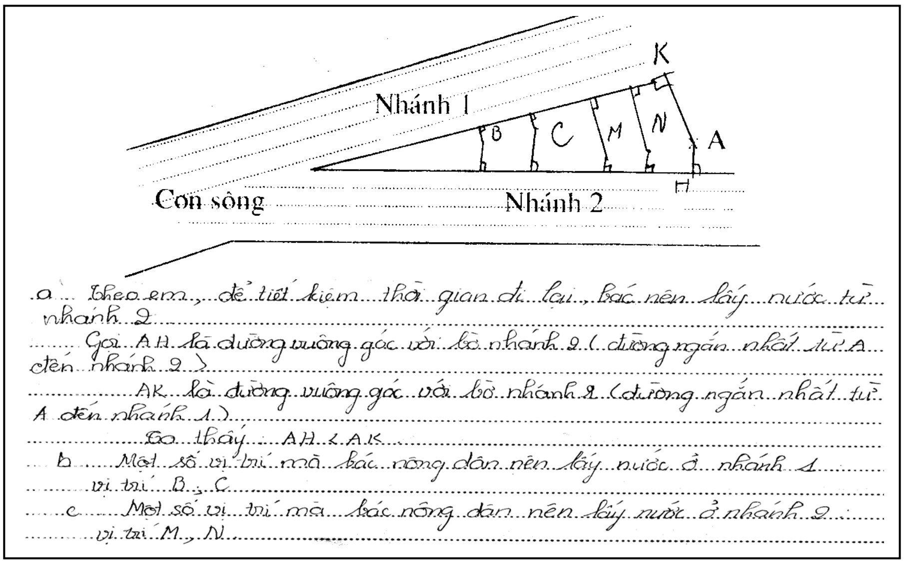


Figure 4.11: Worksheet 1 of group 5 (L.2, S.1)

Group 1 used quite formal knowledge to reason (figure 4.12):

According to our group, the farmer should go to branch 2 to take water. Because the segment from *A* to branch 2 which is perpendicular segment $\overline{AH}$ is the shortest way.
Because $AH < AK$.

(Group 1: Khanh Huyen, Hue, Hieu, Thanh Hien, Ngoc Tuan and Anh Tuan)

The students of this group also used improper sentences in Vietnamese. They had problem with the Vietnamese ‘because’ expression. Furthermore, they used lower-case letters (*a*, *b*, *c*, *d*, etc.) to sign points.

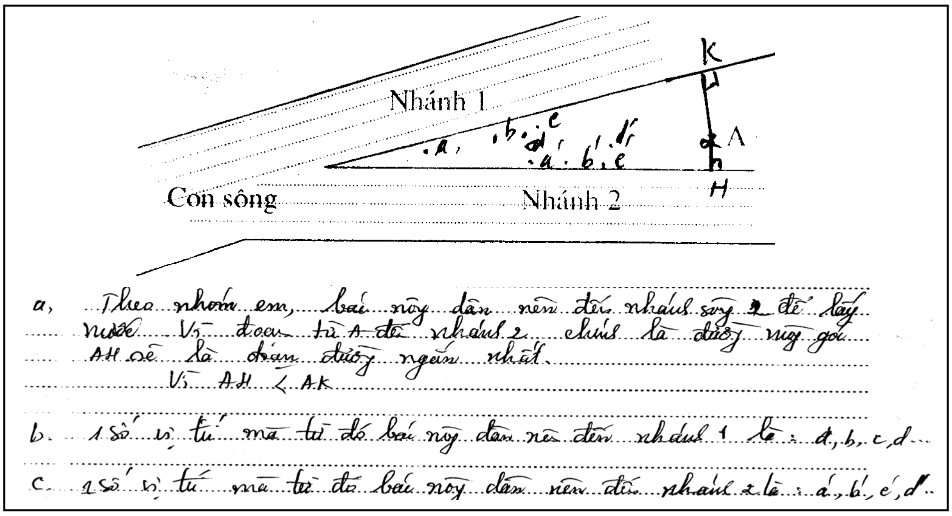


Figure 4.12: Worksheet 1 of group 1 (L.2, S.1)

This is the worksheet of group 4 (figure 4.1.3):

The farmer should fetch water from branch 2 of the river because the distance from point *A* to branch 2 of the river is shorter than the distance from point *A* to branch 1. Consequently, going to branch 2 will save time and effort.[65]

(Group 4: Thuy An, Cuong, Duong, Dong, Duc, Ha and Tam)

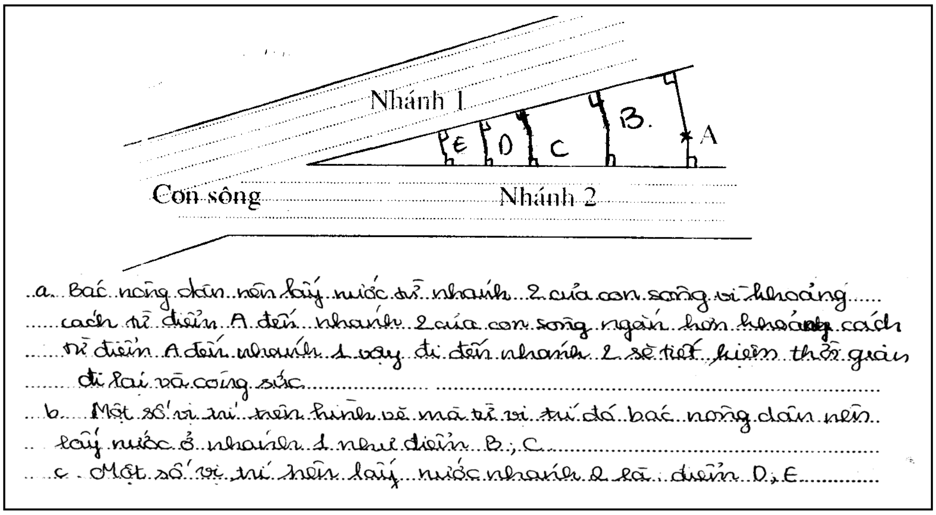


Figure 4.13: Worksheet 1 of group 4 (L.2, S.1)

Group 6 formally reasoned to answer the question (figure 4.14):

From *A* we draw the perpendicular lines to the two sides of the branches of the river at *B* and *C*. Seeing the figure, we recognized that *AB* > *AC*:
[…] the farmer should fetch water from branch 2.

(Group 6: Duong Hai, Vu Hai, Thanh, Theu, Ton and Tu)

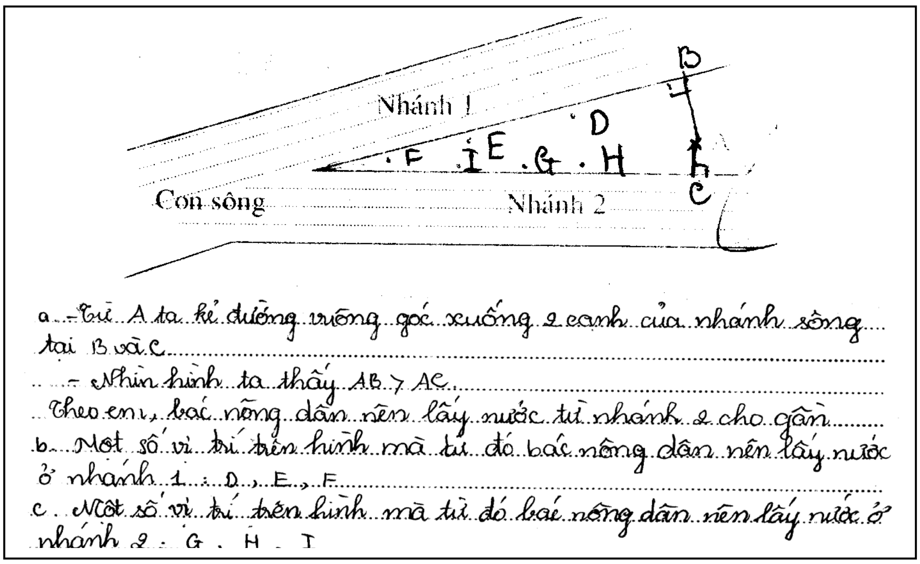


Figure 4.14: Worksheet of group 6 (L.2, S.1)

[65] In their worksheet, students did not properly use Vietnamese while writing the answer.

In conclusion, all groups reasoned that the farmer should go to the nearer branch. It appeared that both social norms and sociomathematical norms (Yackel & Cobb, 1996) emerged while the students were working in groups. The students discussed the tasks with each other to reason that the farmer should save his time and health (social norms) and find a shorter way from his position to the river (sociomathematical norms). However, they used different levels of formal reasoning to answer the first question. One group used only informal knowledge to answer *question a)*, while the other groups alternatively utilized informal and formal knowledge to reason. All groups could identify some positions (from 2 to 4 positions) from which the farmer should go to branch 1 or branch 2 of the river. Some groups only drew positions with names, while the others additionally the drew perpendicular lines from these points to the sides of the angles (the branches of the river). Most of the groups did not sufficiently explain why the distance from *A* to branch 2 was shorter than the distance from *A* to branch 1, often using the expression "we recognize that…". Exceptionally, the students of group 6 explained that they reached the conclusion by *seeing* the figure in the worksheet. It appeared that none of them measured the distances from *A* to the two branches of the river in order to compare these distances. In some cases, the students did not accurately use mathematics expression and Vietnamese language while writing their answers.

b) Situation 2:

The teacher did not reorganize the groups. In other words, in this *situation*, each group has the same members as in *situation 1*.

To answer *question a)*, each group identified from 2 to 4 positions. Two of them identified only the positions (points and their names) and did not connect these positions, while the others not only identified the positions but also connected these positions with each other and vertex *O* of $\angle\ xOy$. Group 6 (Duong Hai, Vu Hai, Thanh, Hieu, Ton and Tu) was an exceptional one because the students of this group applied the theorem of characteristic of the bisector of an angle to reply *question a)* (figure 4.15):

> Draw the bisector ray $\overrightarrow{Ot}$ of $\angle\ xOy$.
>
> On $\overrightarrow{Ot}$, we point out points *A*, *B* and *C*. These points are equidistant from the two branches of the river.

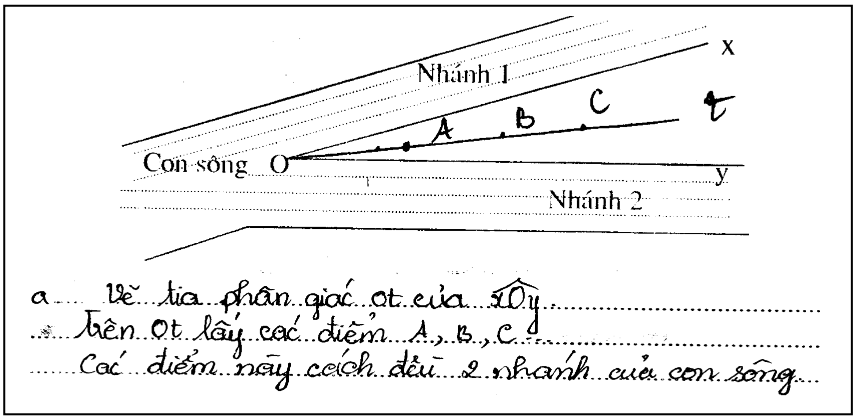


Figure 4.15: The worksheet of group 6 (L.2, S.2)

As discussed above, the students identified from 2 to 4 points for answering *question a)*. The students of groups 2 (Ngoc Anh, Bich, Huong, Ngoc Lan, Lanh and Tran Ngoc) and 3 (Nguyen Huyen, Giang Son, Thom, Nhan Ton, Le Ngoc and Loc) identified some points in their worksheets, but they did not draw the perpendicular lines to the two sides of the angles and connect these points (see, for example, the worksheet of group 2 in figure 4.16). Although the students of group 1 (Khanh Huyen, Hue, Hieu, Thanh Hien, Ngoc Tuan and Anh Tuan) did not draw the perpendicular lines from pointed points to the two sides of the angle, they connect these points and the vertex of the angle (see figure 4.17). The students of groups 4 and 5 not only drew the perpendicular lines but also connected the pointed points with the vertex of the angle (see, for instance, the worksheet of group 4 in figure 4.18).

The students of all groups could predict that positions mentioned in *question a)* are on the bisector ray of $\angle xOy$. Four groups used term "the bisector line" with the meaning of "the bisector ray". One group used term "the bisector ray". The others alternatively used both "the bisector line" and "the bisector ray".

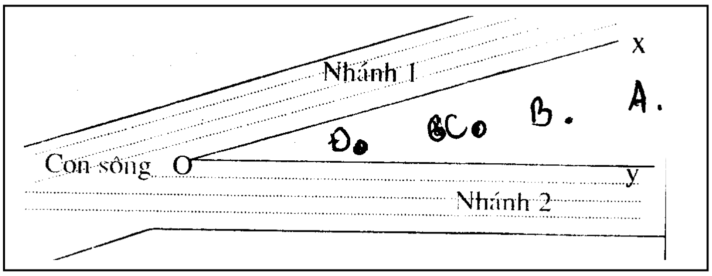


Figure 4.16: The worksheet of group 2 (L.2, S.2)

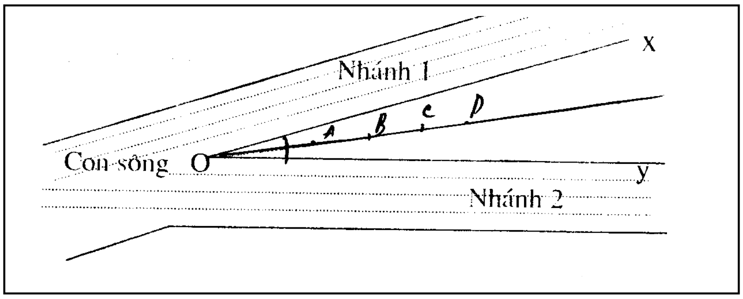


Figure 4.17: The worksheet of group 1 (L.2, S.2)

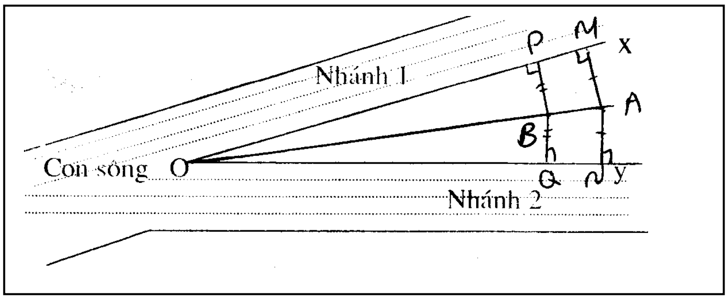
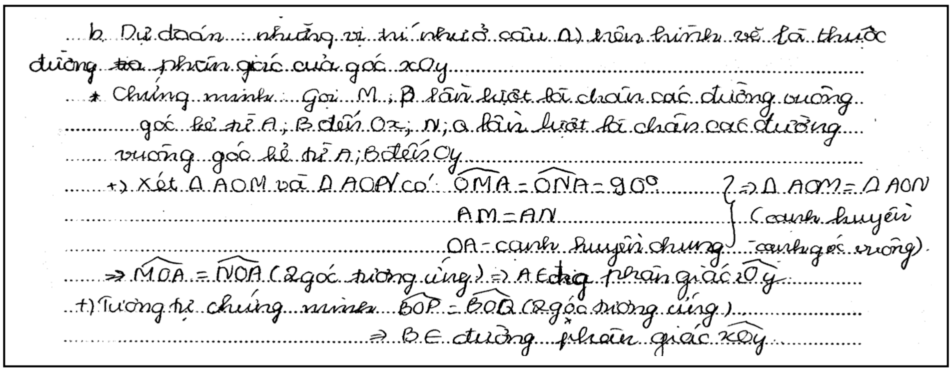


Figure 4.18: The worksheet of group 4 (L.2, S.2)

Two groups (4 and 5) even wrote down formal proofs (deductive proofs) for their prediction. For instance, the students of group 4 (Thuy An, Cuong, Duong, Dong, Duc, Ha and Tam) presented one deductive proof as follows (see figure 4.18 & 4.19):

> Prediction: Positions in *question a)* in the figure belong to the bisector line of $\angle\ xOy$.
> […] We consider $\Delta\ AOM$ and $\Delta\ AON$:
>
> $$\left.\begin{array}{r} \mathrm{m}\angle OMA = \mathrm{m}\angle ONA = 90^0 \\ \overline{AM} \cong \overline{AN} \\ OA \text{ is a common hypotenuse} \end{array}\right\} \Rightarrow \Delta AOM \cong \Delta AON \quad (\text{hypotenuse} - \text{side})$$
>
> $\Rightarrow\ \angle MOA \cong \angle NOA \Rightarrow A$ belongs to the bisector ray of $\angle\ xOy$
> Analogously, $\angle BOP \cong \angle BOQ \Rightarrow B$ belongs to the bisector ray of $\angle xOy$.



b. Dự đoán: những vị trí như ở câu a) trên hình vẽ là thuộc đường ~~tia~~ phân giác của góc xOy

* Chứng minh: Gọi M; P lần lượt là chân các đường vuông góc kẻ từ A; B đến Ox; N; Q lần lượt là chân các đường vuông góc kẻ từ A; B đến Oy

+) Xét Δ AOM và Δ AON có: $\widehat{OMA} = \widehat{ONA} = 90^o$; AM = AN; OA - cạnh huyền chung ⇒ Δ AOM = Δ AON (cạnh huyền - cạnh góc vuông)

⇒ $\widehat{MOA} = \widehat{NOA}$ (2 góc tương ứng) ⇒ A ∈ ~~tia~~ phân giác $\widehat{xOy}$

+) Tương tự chứng minh $\widehat{BOP} = \widehat{BOQ}$ (2 góc tương ứng)

⇒ B ∈ đường phân giác $\widehat{xOy}$

Figure 4.19: A formal proof of group 4 (L.2, S.2)

In conclusion, all groups of the students could predict that a point that is equidistant from the two sides of an angle is on the bisector ray of this angle. In comparison with *situation 1*, the students had fewer mistakes related to Vietnamese and mathematical language. Nevertheless, most of them improperly used the term "bisector ray". Two groups could create themselves formal proofs (deductive proofs) of this theorem.

4.3.3 The teacher's comments

a) About organizing the students' work in groups

While the students were working in groups, some weaknesses emerged:

• With the students' working in pairs or in groups, the classroom was noisier than usual. The greater volume of noise level could disturb neighboring classes. [66]

• The number of students in the class is quite high. Consequently, the teacher did not manage the students' activities while they were working in groups.[67]

• There were a few low-performing students who did not actively discuss and were not aware of necessary work. These students often thought that the tasks were for only high- performing students in their groups. Therefore, they often did some private work during the discussions.

The students' work in groups has the following advantages:

• The students discussed with each other to rediscover the knowledge: the distance from a point to a straight line which does not contain this point.

• The students had the chances of expressing their ideas and thought.

• Most of the students actively discussed and explained themselves to understand the problems and find the answers.

• The teachers did not have to spend much time lecturing.

b) The teaching time allowance

• Teaching by using RME-based lesson needed three times as much as teaching by using the current textbook.

[66] Generally, Vietnamese classrooms share walls, and their walls are not soundproof. Moreover, classrooms' windows and doors are often opened on not very cold days. Consequently, a noise which students make during their discussion may affect neighboring classes.

[67] In comparison with the average of a number of students in a class, the number of students in Mr. Mui's class was not high. It appeared that he lacked experiences of organizing students' group working (see discussion in section 4.3.4 of this chapter).

c) The teacher's activities

• The teacher concentrated on organizing, controlling and guiding the students to rediscover and confirm mathematics knowledge.

• The role of the teacher seemed to be 'less important' than his role when other 'traditional' methods are used. The teacher, however, had to spend more time preparing the lesson and used more tools for the teaching.

• The teacher did not have to present a ready-made knowledge, lecture and explain.

d) About the *situations* in RME-based lesson

• The students struggled with *situation*s in which requirements or questions were not directly presented.[68]

• It is possible to combine *situations 1* and *2* so that the teacher and their students could save time, and the students could expand their intelligence.

e) Other ideas about advantages and disadvantages of RME-based lesson

In general, the theory RME has some advantages:

• It provided the students more opportunities to work actively;

• The students could fully understand the lesson and could discover mathematics knowledge;

• The students naturally learned;

• The students could explain themselves so that they could understand the lesson;

• The teacher worked less during the lesson;

• The students did not mechanically learn under the commands and requirements of the teacher.

While working with RME-based lesson, the teacher and his students met the following difficulties:

• The teacher lacked teaching time while applying RME-based lesson.

• Some low-performing students did not take part in the discussions. Otherwise, they did their private work;

• The teacher had to prepare a great deal for the lesson. It was not suitable for the teacher's amount of work;

4.3.4 Findings:

The main difficulty the teacher faced was the teaching time allowance. He complained that he needed three times as much as using usual ways of teaching. Moreover, he

[68] This idea of the teacher is not clear because unlike the first lesson (the Triangle Sum Theorem), all questions and requirements are directly presented in this RME-based lesson.

complained that he had to spend much time in preparing the students' worksheets. It was not very suitable for him because it significantly increased his workload. In addition, he did not acclimate to teaching by using RME-based lesson as RME was quite new for him. He found that his role was 'less important' than usual since his students could discover mathematical knowledge. Furthermore, he complained that he could not manage his students' work because of the high number of students in his class.

While working with the *situations* in RME-based lesson, the students could rediscover the knowledge. They did not have to perform strict tasks under the commands and requirements of the teacher or the textbook.

Situations 1 and *2* were not difficult and complicated for the students. All groups could answer the questions and offered their own reasons. The students utilized different levels of formal knowledge to explain their ideas. In some cases, the students did not properly used Vietnamese and mathematical language while writing their answers. Moreover, the students' competence varied between groups. Some students of some groups could deductively reason and build formal proofs, while the students of some other groups were at a low level of reasoning.

While working in groups, the students often made a noise which could affect some neighboring classes. Furthermore, some low-performing students often did not actively discuss with other students in their groups to answer the questions, and they often did their private work. According to the teacher, these students thought that the tasks were for other high-performing classmates. In *two situations* of RME-based lesson, the teacher divided his students into groups of around 6 and 7 people. As discussed previously, it was not entirely reasonable to allow the students to work in groups in the *first situation* because this *situation* was not complicated. In addition, the number of students in each group was high. This could be one of the reasons why some of the low-achieving students did not actively participate in discussions with other members in their groups. Furthermore, it appeared that the teacher has little experience related to organizing students' working in groups. He gave one copy of a worksheet to all members of each group in each *situation*. It would have been better if each student had her/his own worksheet. Firstly, s/he should try to individually deal with the problems, and then the students could discuss their individual ideas in groups. After that the teacher could examine the students' work by asking a member of each group.

4.4 Characteristics of the perpendicular bisector of a segment (lesson 3)

4.4.1 Introduction

Firstly, this section briefly introduces how the theorem was presented in *Geometry 7* of the 'reformed' textbooks and is presented in *Mathematics 7: part 2* of the current textbooks. Secondly, it describes the aims of the *situation*s in RME-based lesson. Finally, it describes the way in which the teacher applied RME-based lesson in his teaching.

Geometry 7 of the 'reformed' textbooks presented only the content and the formal deductive proof of the theorem (see appendix A; see also the discussion about the typical presentation of the 'reformed' textbooks' lessons in section 1.3.1.5, chapter 1). In the current textbook called *Mathematics 7: part 2*, before the content and the proof of the theorem is presented, a brief introduction is presented:

> We consider point M which is equidistant from the two endpoints of $\overline{AB}$. Is point M on the perpendicular bisector of $\overline{AB}$?
> We have a theorem: […]

(Phan Duc Chinh et al., 2004 b, p. 75)

RME-based lesson includes three *situation*s (see details in appendix C). The first one which is designed for individual work introduces the context in which some explorers are walking somewhere in a dessert with two wells of water (A and B), and these explorers want to go to one well to take some water. *Question a)* of this *situation* asks students to give a suggestion of a possible direction for the explorers. *Questions b)* and c) help students identify some positions from which these explorers should go to well A or B. The aim of the *second situation* is to help students learn that a point which is equidistant from the two endpoints of a segment is on the perpendicular bisector of this segment. Students work with formal mathematics in the *last situation*. Students are asked to work in groups in the *second* and *third situations*.

The teacher, Mr. Le Xuan Mui, organized his students to work in groups for all *situation*s while applying RME-based lesson. There were 37 students participating in his class, and he divided his students into 6 groups of from five to seven people.[69] Mr.

[69] The tasks of situations 1 and 2 are not complicated. He should divide his students into smaller groups in these situations. In situation 3, students work with the formal proof of the theorem. So he should divide his students into larger groups.

Mui gave three different tasks to the three pairs of groups.[70] It appeared that he wanted to save time.

He asked the students of groups 1 and 2 to prove the following problem: “If distances from M to the two endpoints of $\overline{AB}$ are equal then M is on the perpendicular bisector of $\overline{AB}$.”

Groups 3 and 4 were asked to deal with the following task (figure 4.20):[71]

a. Determine point N such as distances from N to A and B are equal?

b. Predict the locus of point N?

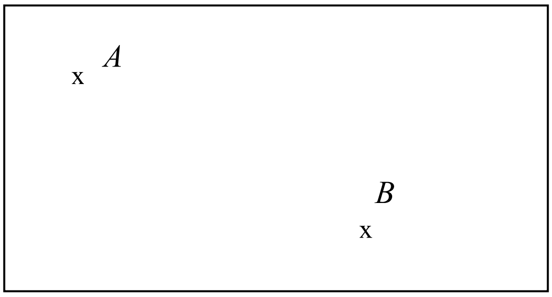


Figure 4.20: The questions for groups 3 and 4 (L.3)

Groups 5 and 6 were given the following task (figure 4.21):[72]

A farmer is at point M (see figure).

a. Where should the farmer go to take water?

b. Point out some positions in the figure from which the farmer go to well A.

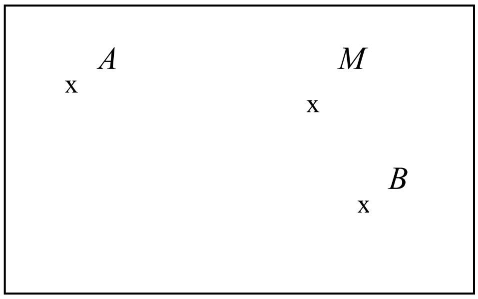


Figure 4.21: The task for groups 5 and 6 (L.3)

4.4.2 Analyzing the students’ worksheets of groups 1 and 2

It appeared that some students of groups 1 and 2 were high-performing students. They could devise different formal deductive proofs for the given problem although the solution of group 2 was not sufficient (see figure 4.22 and 4.23).

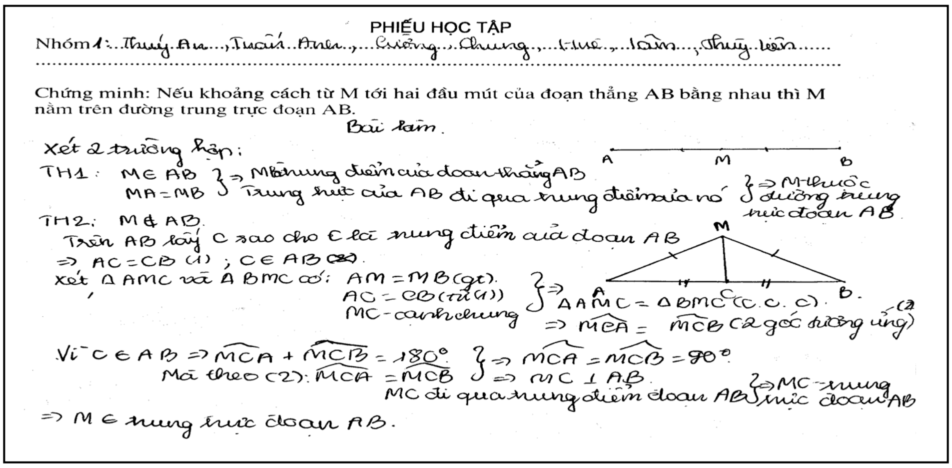
Group 1 (Thuy An, Tuan Anh, Cuong, Chung, Hue, Lam and Thuy Lien) proved the theorem by considering two cases: M is the midpoint of $\overline{AB}$, and M is not

[70] It is clear that this manner was not reasonable. More related discussions are found in section 4.6.3.1 of this chapter.

[71] This task may confuse students because of the word “determine”.

[72] This task is similar to the first situation of RME-based lesson. It appears that the teacher wanted to give groups 5 and 6 the ‘direct’ question and requirement by cutting down the first situation of RME-based lesson. It is clear that this task is quite short and not sufficient.

the midpoint of $\overline{AB}$ (see figure 4.22). It is easy to conclude that if *M* is the midpoint of $\overline{AB}$ then *M* is on the perpendicular bisector of this segment. Otherwise, they call *C* the midpoint of $\overline{AB}$ and prove that Δ *AMC* and Δ *BMC* are congruent. Consequently, $m\angle MCA = m\angle MCB$. Moreover, $m\angle MCA + m\angle MCB = 180^0$. Hence, $m\angle MCA = m\angle MCB = 90^0$. Therefore, *M* is on the perpendicular bisector of $\overline{AB}$.


PHIẾU HỌC TẬP

Nhóm 1: Thuý An, Tuấn Anh, Cường, Chung, Huế, Lâm, Thuỳ Liên

Chứng minh: Nếu khoảng cách từ M tới hai đầu mút của đoạn thẳng AB bằng nhau thì M nằm trên đường trung trực đoạn AB.

Bài làm.

Xét 2 trường hợp:

TH1: $M \in AB$, $MA = MB$ ⇒ M là trung điểm của đoạn thẳng AB; Trung trực của AB đi qua trung điểm của nó ⇒ M thuộc đường trung trực đoạn AB

TH2: $M \notin AB$.

Trên AB lấy C sao cho C là trung điểm của đoạn AB

⇒ $AC = CB$ (1); $C \in AB$ (2).

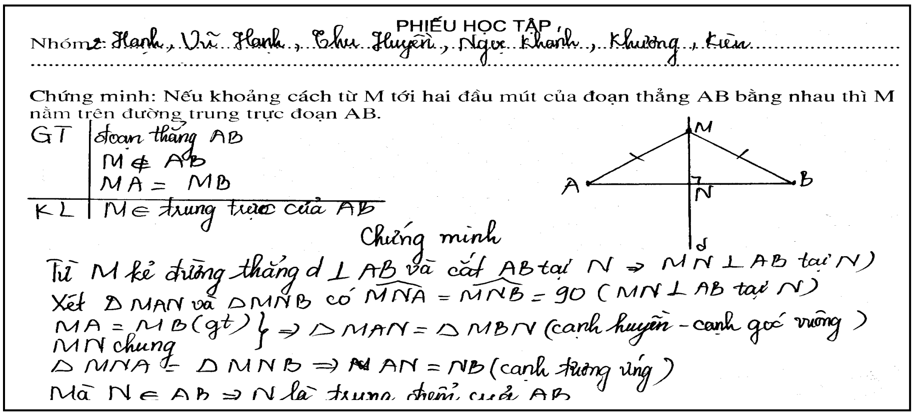
Xét ΔAMC và ΔBMC có: $AM = MB$ (gt), $AC = CB$ (từ (1)), MC - cạnh chung ⇒ $\Delta AMC = \Delta BMC$ (c.c.c) ⇒ $\widehat{MCA} = \widehat{MCB}$ (2 góc tương ứng) (2)

Vì $C \in AB \Rightarrow \widehat{MCA} + \widehat{MCB} = 180^\circ$; Mà theo (2): $\widehat{MCA} = \widehat{MCB}$ ⇒ $\widehat{MCA} = \widehat{MCB} = 90^\circ$ ⇒ $MC \perp AB$; MC đi qua trung điểm đoạn AB ⇒ MC - trung trực đoạn AB

⇒ $M \in$ trung trực đoạn AB.

Figure 4.22: The worksheets of group 1 (L.3)

Let *N* be the intersection of $\overline{AB}$ and a line which goes through point *M* and is perpendicular to $\overline{AB}$. Group 2 (Hanh, Vu Hanh, Thu Huyen, Ngoc Khanh, Khuong and Kien) proved that Δ *MNA* and Δ *MNB* are congruent. Then they proved that *N* is the midpoint of $\overline{AB}$. Consequently, *M* is on the perpendicular bisector of $\overline{AB}$ (figure 4.23).


PHIẾU HỌC TẬP

Nhóm 2: Hạnh, Vũ Hạnh, Thu Huyền, Ngọc Khánh, Khương, Kiên

Chứng minh: Nếu khoảng cách từ M tới hai đầu mút của đoạn thẳng AB bằng nhau thì M nằm trên đường trung trực đoạn AB.

GT: đoạn thẳng AB; $M \notin AB$; $MA = MB$

KL: $M \in$ trung trực của AB

Chứng minh

Từ M kẻ đường thẳng $d \perp AB$ và cắt AB tại N ⇒ $MN \perp AB$ tại N)

Xét ΔMAN và ΔMNB có $\widehat{MNA} = \widehat{MNB} = 90$ ($MN \perp AB$ tại N)

$MA = MB$ (gt), MN chung ⇒ $\Delta MAN = \Delta MBN$ (cạnh huyền - cạnh góc vuông)

$\Delta MNA = \Delta MNB \Rightarrow AN = NB$ (cạnh tương ứng)

Mà $N \in AB \Rightarrow N$ là trung điểm của AB

Figure 4.23: The worksheet of group 2 (L.3)

4.4.3 Analyzing the students' worksheets of groups 3 and 4

The students of group 3 (Pham Anh, Duc, Hoa, Yen, Tu, Tuoi and Son) did not draw any additional lines in the figure. They used formal knowledge to deal with their task (see figure 4.24):

> Draw *(A, r)* and *(B, r)* ($r \geq \frac{1}{2}AB$). The two circles intersect each other at *N*.[73]
>
> Proof: *M* belongs to *(A; r)* $\Rightarrow AN = r$ (1). *M* belongs to *(B; r)* $\Rightarrow BN = r$ (2). Consequently, $AN = BN$.
>
> b. Prediction: The locus of *N* is the perpendicular bisector of $\overline{AB}$.

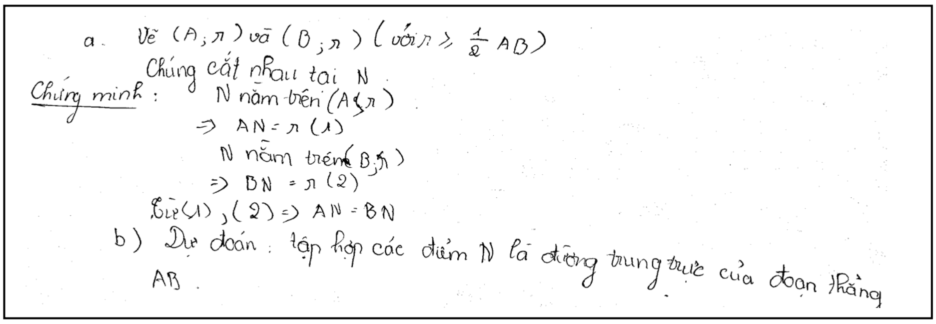

a. Vẽ (A; r) và (B; r) (với r ≥ 1/2 AB)
Chúng cắt nhau tại N
Chứng minh: N nằm trên (A; r)
⇒ AN = r (1)
N nằm trên (B; r)
⇒ BN = r (2)
Từ (1), (2) ⇒ AN = BN
b) Dự đoán: tập hợp các điểm N là đường trung trực của đoạn thẳng AB.

Figure 4.24: The answers of group 3 (L.3)

Similar to the students of groups 3, the students of group 4 (Thao, Hong Tien, Nga, Non, Tham, Thu and Thu Thuy) did not draw any additional lines either (see figure 4.25):

> […]
>
> a) Based on characteristics of the perpendicular bisector of a segment, we have: If *N* belongs to the perpendicular bisector of $\overline{AB}$ then $NA = NB$.
>
> b) Prediction: The locus of *N* is the perpendicular bisector of $\overline{AB}$.

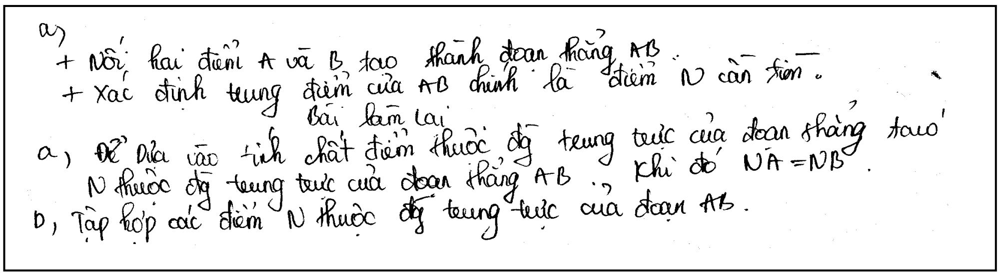

a)
+ Nối hai điểm A và B tạo thành đoạn thẳng AB.
+ Xác định trung điểm của AB chính là điểm N cần tìm.
Bài làm lại
a) ~~Để~~ Dựa vào tính chất điểm thuộc đg trung trực của đoạn thẳng tạo N thuộc đg trung trực của đoạn thẳng AB ., khi đó NA = NB.
b) Tập hợp các điểm N thuộc đg trung trực của đoạn AB.

Figure 4.25: The worksheet of group 4 (L.3)

[73] When $r > \frac{1}{2}AB$, the two circles intersect each other at two points.

In sum, as discussed in footnote of section 4.4.1, the teachers made his student quite confused because of the obscure task. The students of groups 3 and 4 understood that they had to use formal knowledge because of the word "determine" of the problem. For this reason, the solutions of these groups responded with formal reasoning. In general, this teacher's manner was not suitable.

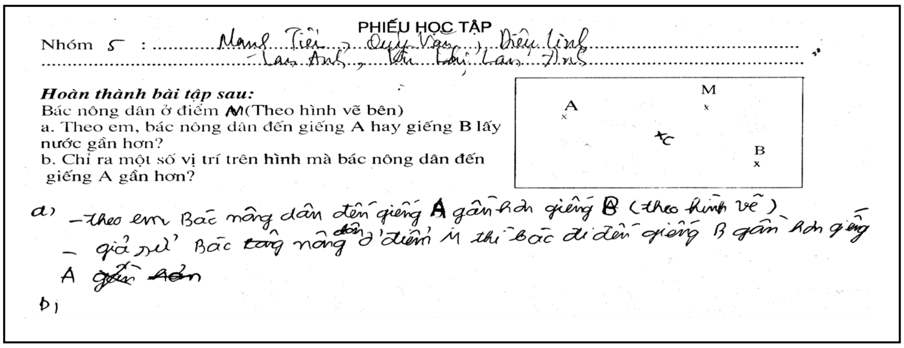
4.4.4 Analyzing the students' worksheets of groups 5 and 6

Although task 3 was not particularly complicated, it appeared that the students of group 5 (Manh Tien, Quy Van, Dieu Linh, Lan Anh and Vu Thi Lan Anh) could not deal with it. They did not answer *question b)*. These students appeared to be low-achieving students. Their answer of *question a)* was quite confusing (see figure 4.26):

• the farmer goes to well A which is nearer than well B (according to the figure)

• Suppose that the farmer at point M then he goes to well B which is nearer than well A.

These two sentences are contradictory. Furthermore, their second sentence is quite obscure.


PHIẾU HỌC TẬP

Nhóm 5 : Manh Tiến, Quy Vân, Diệu Linh, Lan Anh, Vũ Thị Lan Anh

Hoàn thành bài tập sau:
Bác nông dân ở điểm M(Theo hình vẽ bên)
a. Theo em, bác nông dân đến giếng A hay giếng B lấy nước gần hơn?
b. Chỉ ra một số vị trí trên hình mà bác nông dân đến giếng A gần hơn?

a) – theo em Bác nông dân đến giếng A gần hơn giếng B (theo hình vẽ)
– giả sử Bác nông dân ở điểm M thì Bác đi đến giếng B gần hơn giếng A ~~gần hơn~~
b)

Figure 4.26: The worksheet of group 5 (L.3)

The students of group 6 (Thuy Linh, Ly, Phong, Thang and Kiet) answered *question a)* by using formal and informal knowledge alternatively (see figure 4.27):

a. Connect AM, BM and AB. We call $\overleftrightarrow{d}$ the perpendicular bisector of $\overline{AB}$.

$\overline{AM}$ intersects $\overleftrightarrow{d}$ at N. Connect BN.

$$\left.\begin{array}{r}\left.\begin{array}{r} N \in \overleftrightarrow{d} \Rightarrow AN = BN \\ N \in \overline{AM} \Rightarrow AN + MN = AM \end{array}\right\} \Rightarrow AM = BN + MN \\ \text{In } \Delta \quad BMN, \text{we} \quad \text{have}: BN + MN > BM \end{array}\right\} \Rightarrow AM > BM.$$

Consequently, the farmer goes to well B, which is nearer.

b. We call a position from which distance to well A is nearer than well B L.

+ If L belongs to $\overleftrightarrow{d}$ then $AL = BL$ (not satisfy)

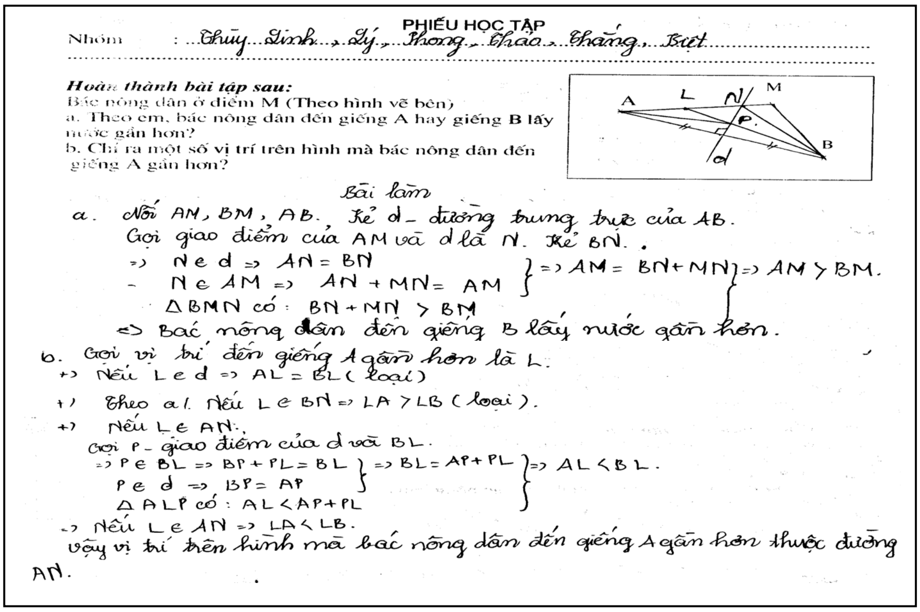
+ According to a), if L belongs to $\overline{BN}$ then $LA > LB$ (not satisfy)

+ If L belongs to $\overline{AN}$:

P is the intersect point of $\overleftrightarrow{d}$ and $\overline{BL}$.

$$\left.\begin{array}{r}\left.\begin{array}{r} P \in \overline{BL} \Rightarrow BP + PL = BL \\ P \in \overleftrightarrow{d} \Rightarrow BP = AP \end{array}\right\} \Rightarrow BL = AP + PL \\ \text{In} \quad \Delta\, ALP, \text{we} \quad \text{have}: AL < AP + PL \end{array}\right\} \Rightarrow \text{AL} < \text{BL}.$$

Consequently, positions in the figure from which the farmer should go to well A belongs to $\overleftrightarrow{AN}$ [74].


PHIẾU HỌC TẬP

Nhóm : Thủy, Linh, Lý, Phong, Thảo, Thắng, Biệt

Hoàn thành bài tập sau:
Bác nông dân ở điểm M (Theo hình vẽ bên)
a. Theo em, bác nông dân đến giếng A hay giếng B lấy nước gần hơn?
b. Chỉ ra một số vị trí trên hình mà bác nông dân đến giếng A gần hơn?

Bài làm

a. Nối AN, BM, AB. Kẻ d – đường trung trực của AB.
Gọi giao điểm của AM và d là N. Kẻ BN.
=> N ∈ d => AN = BN } => AM = BN + MN } => AM > BM.
– N ∈ AM => AN + MN = AM
ΔBMN có: BN + MN > BM
=> Bác nông dân đến giếng B lấy nước gần hơn.

b. Gọi vị trí đến giếng A gần hơn là L.
+> Nếu L ∈ d => AL = BL (loại)
+) Theo a/. Nếu L ∈ BN => LA > LB (loại).
+) Nếu L ∈ AN.
Gọi P – giao điểm của d và BL.
=> P ∈ BL => BP + PL = BL } => BL = AP + PL } => AL < BL.
P ∈ d => BP = AP
Δ ALP có: AL < AP + PL
=> Nếu L ∈ AN => LA < LB.
Vậy vị trí trên hình mà bác nông dân đến giếng A gần hơn thuộc đường AN.

Figure 4.27: The worksheet of group 6 (L.3)

In Vietnamese style, segment AM and segment BN means $\overline{AM}$ and $\overline{BN}$, respectively. However, these students omitted 'segment' in their writing (see figure 4.27).

It appears that deductive Euclidean geometry plays a prominent role in middle school geometry curriculum. Although the task was quite simple, the students of group 6 tried to answer it by exploiting deductive geometry. It appeared that some students of this group are high-achieving students.

[74] It appeared that the students did not properly use '$\overleftrightarrow{AN}$' (originally, they used 'line AN' in figure 4.27). Perhaps they wanted to refer to '$\overline{AN}$'.

From analyzing feedback from the students, it is clear that the competence levels of these students were quite different. Some of them could create formal deductive proofs, while the other could not deal with routine tasks.

4.4.5 Teacher's comments

a) Students' group working

Advantages:

• The students paid attention to the lesson. Most of them concentrated on activities which were guided by the teacher. The levels of the students' competence were quite similar.[75]

• The teacher did not have to work hard.

• Most of the students actively discussed to discover new knowledge. They had chances of discussing with each other and correcting their mistakes.

Disadvantages:

• A few low-performing students did not pay attention to the lesson and actively learn. They often relied on others. These students often did some private work during the discussions.

• There were some inactive students who often relied on others.

• The number of students in the class is quite high.[76] Consequently, it was quite difficult for the teacher to control and manage the students.

b) Teaching time allowance:

• RME-based lesson took about twice or three times as long as the normal lesson. It was not suitable for the teaching time allowance.

c) The students' activity

Advantages:

• The students had the opportunity to think, naturally discover knowledge and discuss and correct their mistakes.

Disadvantages:

• The students were noisier while discussing with each other in groups.[77] A few students did not take part in discussions.

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[75] This comment of the teacher seems not correct. In fact, it is clear that the students' competence is quite different.

[76] See footnote 67.

[77] See footnote 66.

d) The teacher's activity

Advantages:

The teacher needed to guide the students how to work. The teacher did not lecture.

Disadvantages:

• The teacher had to prepare tools for teaching.
• The teacher could not correct mistakes of the low-performing students.

e) About *situation*s in RME-based lesson

The *situation*s of RME-based lesson were naturally presented from easy to complicated one.

f) Other ideas

• RME theory was very new. It was quite difficult to apply this theory widely in teaching mathematics in Vietnam, in general and in rural areas, in particular.
• The students could naturally discover new knowledge;
• The teacher did not have to lecture and rigorously present knowledge;
• RME-based lesson could develop the students' thought, activeness and creativeness.
• The students could freely discuss. Consequently, they felt more confident.

g) Proposals for applying RME-based lesson

• A teacher would effectively apply this theory (RME) if s/he had few lessons per week.
• Teacher should be sufficiently equipped teaching resources.

4.4.6 Findings

Similar to the first lesson, the teacher met some difficulties while applying RME-based lesson. For instance, the teacher complained that RME-based lesson took him quite long time. In addition, some low-performing students did not take part in the discussions.

The current Vietnamese middle school geometry curriculum still places emphasis on deductive geometry. Although it is not necessary to use deductive geometry to answer the questions in tasks 2 and 3, the students often tried to apply deductive reasoning.

In some cases, the students did not properly use Vietnamese to express their ideas.

The competence of the students varied considerably. Some of them could build deductive formal proofs, while other could not deal with basic routine tasks. Perhaps the teachers did not read the worksheets of the student. For this reason, he commented that the students' competence was the same.

As discussed above, the teachers divided his students into 6 groups; each composed of about 6 to 7 members. He assigned the three different tasks to the three pairs of groups in order to save time. However, this manner was not effective because the requirement of these tasks is quite different. Groups 1 and 2 dealt with the *formal situation*, while groups 3 and 4 coped with the less *formal situation*, and the other groups (5 and 6) met the easier and *informal situation*. Moreover, this teacher's way of organizing his class counted for some of the low-achieving students' lack of participation in their groups' discussions.

As discussed in section 4.4.3 of this chapter, it is clear that the task of groups 3 and 4 confused the students. First, the teachers asked the students to "determine point *N* such that it is equidistant from two points *A* and *B*." The students tried to reason deductively in order to carry out this task because of the word "determine". Second, he asked the students to predict the locus of point *N*. It was not useful when the students predicted the locus from only one point. It appeared that deductive formal reasoning was emphasized in teaching middle school geometry. The students often tried to reason deductively to deal with problems, although in some cases, it was not necessary.

4.5 'Train station' problem (lesson 4)

4.5.1 Introduction

In *Geometry 7* (the 'reformed' textbook), this problem was presented within a series of exercises following the lesson 'The Triangle Inequality' (Nguyen Gia Coc & Pham Gia Duc, 1996, p. 73; see also appendix A). Students were taught formal knowledge (the theorem, its sequences and their proofs). Later, students were expected to apply this formal knowledge to solve a pure mathematical exercise:

> Let two points *A* and *B* be on a half plane with a straight-line edge *a* (*A* and *B* are not on $\overleftrightarrow{a}$); let *C* be a point which is on the other half plane such that $\overleftrightarrow{a}$ is the perpendicular bisector of $\overline{AC}$. Let *M* be an arbitrary point on $\overleftrightarrow{a}$; please compare *MA* + *MB* with *BC*. When has *MA* + *MB* the least value?

(Nguyen Gia Coc & Pham Gia Duc, 1996, p. 73)

Finally, students were given a problem:

> Two workstations are on the same side of a river. Where should a harbor be situated such that the sum of the distances from the harbor to the two workstations has least value?
>
> (Nguyen Gia Coc & Pham Gia Duc, 1996, p. 73)

This problem is similarly presented in the current textbooks entitled *Mathematics 7: part* 2 (Phan Duc Chinh et al., 2004 b, p. 77).

The way in which the 'reformed' and current mathematics textbooks present the problem is quite primitive and limited. Specifically, the order of teaching this mathematics application is: the formal mathematics (the theorem, its consequences and their proofs), the pure mathematics problem and the mathematics application. Freudenthal strongly objects to this manner which he calls *anti-didactical inversion* (1983, p. ix; see also section 2.2.1.2 of chapter 2). Gravemeijer defines this way as "mathematical modeling" which plays its role in mathematics education; however, he urges that "mathematical modeling" needs "emergent modeling", one of RME's tenets, as a precursor (Gravemeijer, 2004, p. 97; see also section 2.4.2.3 of chapter 2 and section 3.2.2.3 of chapter 3).

RME-based lesson gives students opportunities to reason.[78] Unlike the 'reformed' and present textbooks, students are not asked 'direct' questions in the *situations* of RME-based lesson and are encouraged to build up gradually from informal to formal mathematics (see details in appendix C). Furthermore, this lesson pays attention to developing students' social and sociomathematical norms.[79]

The teacher, Mr. Le Xuan Mui, organized his students' work in groups (see the discussion about the way the teacher organized his class in section 4.6.3.1 of this chapter). There were 37 students attending his class, and he divided these students into 6 groups of from five to seven people. Mr. Mui gave two different tasks to two triads of groups. More specifically, he gave the *first situation* of RME-based lesson to groups 1, 2 and 4 and *second situation* to groups 3, 5 and 6 (see details *these situation*s in appendix C).

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[78] Mr. Martin Kindt told the author of this dissertation about ideas which was written (in Dutch) in one of his research paper related to the 'train station' problem during the author's visit at Freudenthal Institute in October 2004. The author used and developed Mr. Kindt's ideas for this RME-based lesson.

[79] The notions of social and sociomathematics norms are defined and developed by Yackel and Cobb (1996).

4.5.2 Analyzing the students' worksheets of groups 1, 2 and 3

Group 1 (Thuy An, Tuan Anh, Chuong, Chung, Hue, Lam and Thuy Lien):

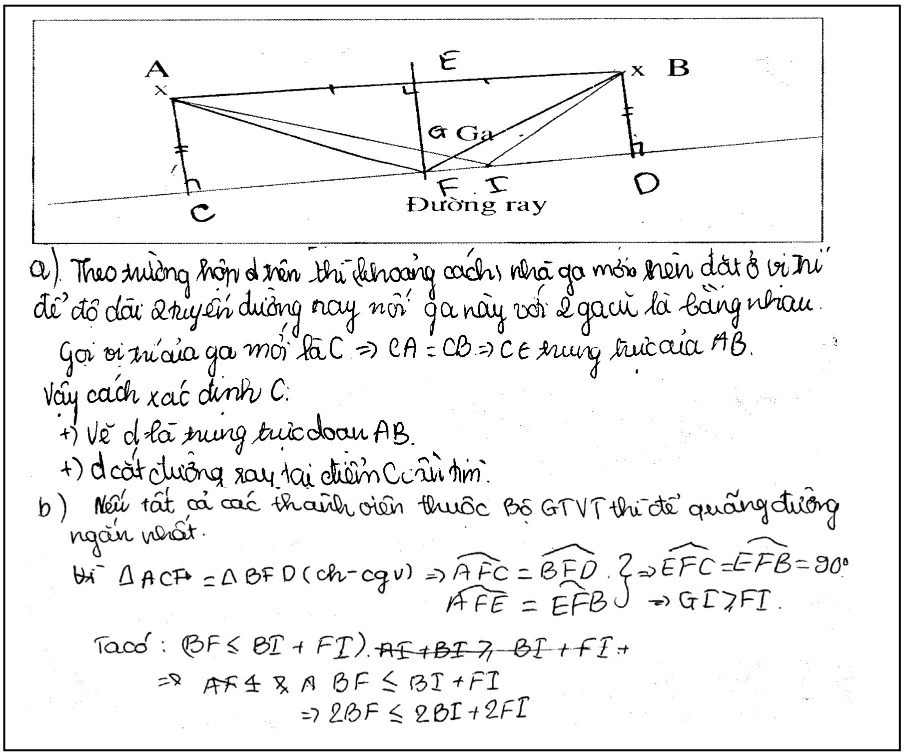
To answer the first question, firstly, the students used informal reason, and then they utilized formal knowledge to find a suitable position for the new railroad station (see figure 4.28):

> a) […] The new railway station should be situated in a position such that the track distances which connect this station with the two old stations are equal.
>
> We call the position for the new station C[80] $\Rightarrow CA = CB \Rightarrow C$ belongs to the perpendicular bisector of $\overline{AB}$. So the way to identify C is:
>
> + Draw perpendicular bisector d of $\overline{AB}$
>
> + d intersects the railroad at point C.
>
> b) If all members of the groups work for a ministry of transportation then the new station should be situated in a position such that the sum of the ways is shortest.[81]

Then they formally proved that $GI \geq FI$ and $BF \leq BI + FI$. They reasoned that the new station should be built in a position such that the sum of the distance from this station to the two old ones is shortest and tried to use formal knowledge to prove that $AF + FB \leq BI + FI$. They, however, did not succeed in proving this inequality.

[80] A role of point C in the figure of their worksheet is different from point C in their writing. It appears that point C in their writing should be replaced by point F in the figure.

[81] It appears that the students wanted to express something like this. However, they did not properly use Vietnamese grammar.



a) Theo trường hợp ở trên thì khoảng cách nhà ga mới nên đặt ở vị trí để độ dài 2 tuyến đường ray nối ga này với 2 ga cũ là bằng nhau.
Gọi vị trí của ga mới là C ⇒ CA = CB ⇒ C ∈ trung trực của AB.
Vậy cách xác định C:
+) Vẽ d là trung trực đoạn AB.
+) d cắt đường ray tại điểm C cần tìm.
b) Nếu tất cả các thành viên thuộc Bộ GTVT thì để quãng đường ngắn nhất.
Vì $\Delta ACF = \Delta BFD$ (ch-cgv) $\Rightarrow \widehat{AFC} = \widehat{BFD}$, $\widehat{AFE} = \widehat{EFB}$ $\Rightarrow \widehat{EFC} = \widehat{EFB} = 90°$ $\Rightarrow GI \geq FI$.
Ta có: $(BF \leq BI + FI)$. ~~AI + BI ≥ BI + FI +~~
$\Rightarrow$ ~~AF ≤~~ $BF \leq BI + FI$
$\Rightarrow 2BF \leq 2BI + 2FI$

Figure 4.28: The worksheet of group 1 (L.4)

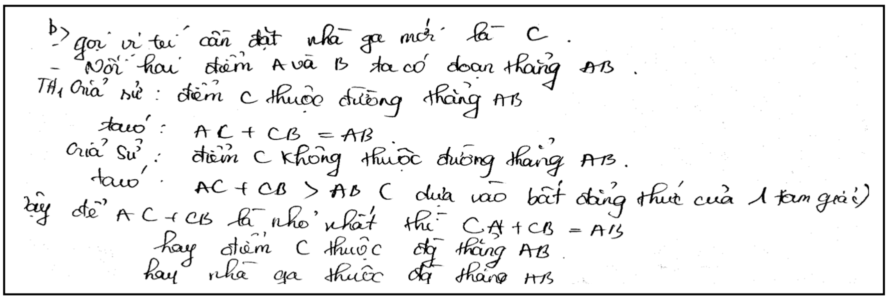
Group 4 (Thao, Hong Tien, Nga, Non, Tham, Thu and Thu Thuy):

The students of this group used formal knowledge to answer *question a)* (see figure 4.29):

> We connect two points A and B to form $\overline{AB}$.
>
> Draw the perpendicular bisector of $\overline{AB}$. The perpendicular bisector intersects the railway at a point.
>
> This point is a suitable position for the new station.

To answer *question b)*, the students utilized formal knowledge (the Triangle Inequality). They concluded that the new station should be build on $\overleftrightarrow{AB}$ (they forgot that the new station needed to be built near the rail) (see figure 4.29). Firstly, they considered the case in which point C on $\overleftrightarrow{AB}$ and prove that $AC + CB = AB$.[82] Secondly, they considered the case in which point C which is not on $\overleftrightarrow{AB}$ and prove that $AC + CB \geq AB$. Finally, they concluded that C should be on $\overleftrightarrow{AB}$. Their conclusion was incorrect.

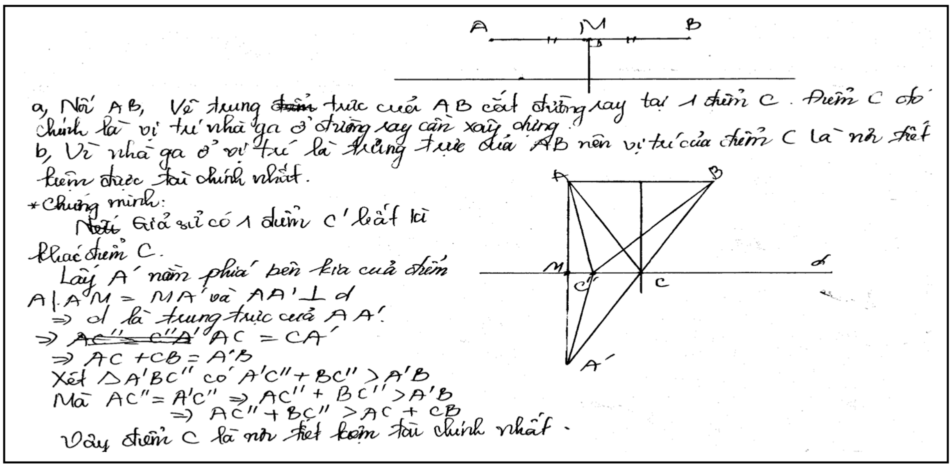
[82] This is incorrect deduction. $AC + CB = AB$ if and only if point C is somewhere on $\overline{AB}$.



b> gọi vị trí cần đặt nhà ga mới là C.
- Nối hai điểm A và B ta có đoạn thẳng AB.
TH1 Giả sử: điểm C thuộc đường thẳng AB
thì: AC + CB = AB
Giả sử: điểm C không thuộc đường thẳng AB.
thì: AC + CB > AB (dựa vào bất đẳng thức của Δ tam giác)
Vậy để AC + CB là nhỏ nhất thì CA + CB = AB
hay điểm C thuộc đg thẳng AB.
hay nhà ga thuộc đg thẳng AB

Figure 4.29: The answers for *question b)* of group 4 (L.4)

The students of group 2 (Hanh, Vu Hanh, Thu Huyen, Ngoc Khanh, Khuong and Kien) reasoned that the new station should be built in a position such that the distances from it to two points *A* and *B* was equal in case a) and the sum of the distances from it to two points *A* and *B* was shortest so that, "it is the financial saving position", and then they formally presented the way to identify a position (point *C*) for the new station in case a) and formally proved that this point was also the 'financial saving position' (see figure 4.30)



a, Nối AB, Vẽ trung ~~điểm~~ trực của AB cắt đường ray tại 1 điểm C. Điểm C đó chính là vị trí nhà ga ở đường ray cần xây dựng.
b, Vì nhà ga ở vị trí là đường trung trực của AB nên vị trí của điểm C là nơi tiết kiệm được tài chính nhất.
*Chứng minh:
~~Nối~~ Giả sử có 1 điểm C' bất kì khác điểm C.
Lấy A' nằm phía bên kia của điểm A | A'M = MA' và AA' ⊥ d
⇒ d là trung trực của AA'.
⇒ ~~AC'' = C''A'~~ AC = CA'
⇒ AC + CB = A'B
Xét ΔA'BC'' có A'C'' + BC'' > A'B
Mà AC'' = A'C'' ⇒ AC'' + BC'' > A'B
⇒ AC'' + BC'' > AC + CB
Vậy điểm C là nơi tiết kiệm tài chính nhất.

Figure 4.30: The answers of group 2 (L.4)

In conclusion, the students of groups 1, 2 and 4 reasoned that the new station should be built at a position such that the distances from this position to two points *A* and *B* were equal for *question a)* and the sum of the distances from this position to two points *A* and *B* was shortest for *question b)*. In other words, the *situation* motivated the students to discover that they should find an 'equal position' and a 'saving position' and try to find these positions although the *situation* did not mention 'the equality' and 'the least sum of distances'. All groups could find the equidistant points on the track by applying characteristics of the perpendicular bisector of a segment. They, however, used different levels of formal knowledge. In case b), all groups reasoned that they should find a ‘financial saving position’. It appeared that both group 1 and group 2 could project the ‘equal position’ is also the ‘financial saving position’ when the distances from two points *A* and *B* to the railroad are equal. However, only group 2 could write the formal proof for this prediction. The students from the last one, group 4, utilized formal mathematics to reach the conclusion: the new railroad station should be built on $\overleftrightarrow{AB}$. They did not draw any additional line in the figure of their worksheet. These students were not aware of the mistake because they mechanically used formal deductive knowledge which they did not correctly remember.

4.5.3 Analyzing the students’ worksheets of groups 4, 5 and 6

The students of group 5 (Manh Tien, Quy Van, Dieu Linh, Lan Anh and Vu Thi Lan Anh) used only informal knowledge to answer the question (see figure 4.31):

A station number 3 is on position *C* because it is in the middle of two centers *A* and *B*.

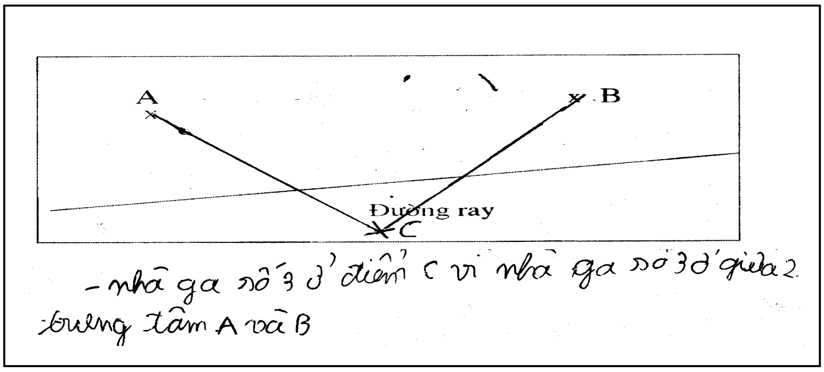


Figure 4.31: The answer of group 5 (L.4)

The students of group 3 (Pham Anh, Duc, Hoa, Yen, Tu, Tuoi and Son) changed from informal knowledge to formal knowledge to answer the question (see figure 4.32). Firstly, these students called the position for the new station *C*. Then they reasoned that “For convenience, *C* must be equidistant from two stations *A* and *B*”. After that, they used mathematical form ($AC = BC$) to express this relation. Later, they concluded that *C* was on the perpendicular bisector of $\overline{AB}$. Finally, they re-proved the theorem of the characteristics of the perpendicular bisector of a segment which they had learned before.

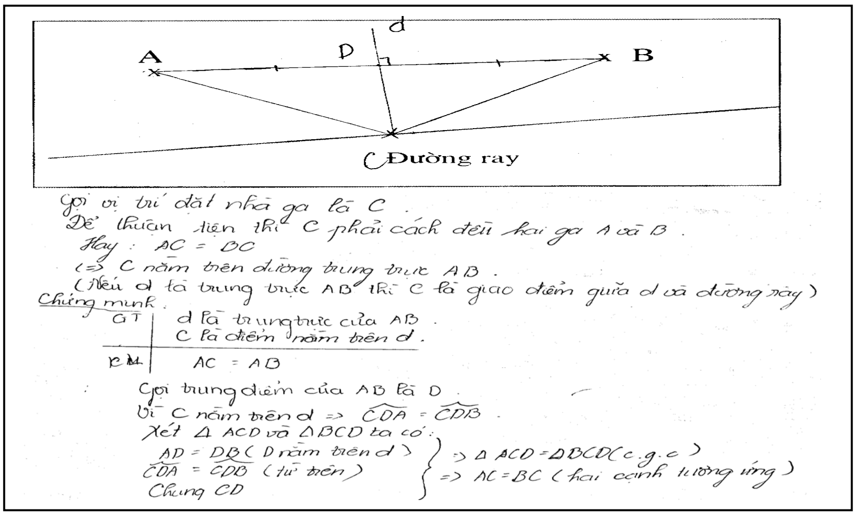


Figure 4.32: The answer of group 3 (L.4)

In contrast, the students of group 6, including Thuy Linh, Ly, Phong, Thang and Viet presented formal knowledge to answer the question:

> Let the position of the new train station be *M*.
>
> Connect *Ax* and *Bx*.[83] Let $\overleftrightarrow{d}$ be the perpendicular bisector of *AxBx*.
>
> Take *M* on $\overleftrightarrow{d}$. M is on $\overleftrightarrow{d}$, so *MAx* = *MBx*.
>
> Consequently, the new railroad station is situated on $\overleftrightarrow{d}$ (near the rail-way and equidistant from the two others in cities *A* and *B)*.

In conclusion, the students of groups 3, 5 and 6 reasoned that the new station should be built at the position such that the distances from this position to two points *A* and *B* are equal. All groups could find the 'equal position'. Like groups 1, 2 and 4, these groups also used different levels of formal knowledge. However, none of them referred to a 'financial saving point'. In other words, *situation 2* did not motivate the students to find a 'financial saving point' when they were not involved in *situation 1*. As discussed in section 4.5.1, the teacher was not able to give their students sufficient work opportunities for both *situation*s because of the teaching time pressure.

4.5.4 Teacher's comments

a) Students' working in groups

- Most of the students actively discussed and did the exercises.
- The students themselves could find the solutions to the exercises.
- Some weak-achieving students could not write down the exercises' solutions.

[83] There is a redundancy in their writing.

• The teacher worked less than usual.

• Most of the students could find the solutions and wrote down the solutions.

• Some students could not write down the solutions and could not totally understand the lesson.

b) The teacher's activity

• The teacher worked less than usual, could work as a referee for the students, and could guide the students to discovery knowledge.

• The teachers had to prepare many teaching aids. During the lessons, it was difficult for the teacher to maintain classroom discipline.

c) The students' activity

• The students quickly and thoroughly understood the lesson.

• The students work initiatively.

• The teacher did not have to work much during the lesson.

d) The *situation*s in RME-based lesson

• The *situation*s of RME-based lesson were presented from simple to complicated one. Some students could keep up with the *situation*s and quickly solved the problems.

• It is better if only from 1 to 2 situations were given to the students so that it was suitable for the facts in school. It was not necessary to give many situations because the role of the teachers was still very important for Vietnamese students.

f) Proposals

It is necessary to reduce amount of work for the teachers and reduce a number of students in a class. Moreover, a teacher should be equipped modern means for teaching.

4.5.5 Findings

In general, the students could work with 'indirect' questions. The students were motivated to discuss and find the 'equal position' and the 'financial saving position' in *situation 1*. However, the other students who were not involved in *situation 1* only mentioned the 'equal position' while working with *situation 2*. It should be noted that these *situation*s did not refer to 'the equality' and 'the least sum of distances'. Most of the groups could correctly find the 'equal position' in both two *situations*. In the *first situation*, only one group could find the 'saving position' and presented a formal deductive reasoning. Another group predicted that the 'saving position' was also the ‘equal position’ when the distance from two cities to the railway are equal; however they did not succeed in proving this conjecture formally. The other group failed in predicting the ‘saving position’ as they mechanically applied formal deductive knowledge when they did not sufficiently remember.

Although most of the students actively discussed the solution to discover knowledge, some of them who are low-achieving students did not attend the discussions to discover knowledge.

The teachers complained that RME-based lesson required a great deal of time. As a result, he gave the *first situation* to groups 1, 2 and 4 and the *second situation* to the other groups.

The teachers lectured less than usual. However, he had to prepare more than usual. In RME-based lesson, he divided his students into 6 groups and gave the *first situation* to the students of groups 1, 2, 4 and *second situations* to the other in order to save time. It is not good because both *two situations* should be given to students.

It appears that the middle school geometry curriculum still places emphasis on deductive geometry. The students often tried to reason deductively to answer the questions although in some cases, they were not asked to do so.

The students’ competence was quite different. Some of them could deductively reason and create formal proofs, while some other could not. It was not clear which individual worked well because the teachers gave only one copy of the worksheet to all member of each group.

4.6 Conclusions

The aim of this section is to answer the questions posed at the beginning of this chapter. More specifically, it discusses the difficulties the teachers and theirs students met while working with RME-based geometry lessons, RME’s potentials for enriching mathematics education in Vietnam, reactions of the teachers and students with RME-based geometry lessons, and some necessary adjustments for these RME-based geometry lessons.

4.6.1 Difficulties

This section analyzes difficulties which the teachers and their students met while working with RME-based lessons, including the teaching time pressure, inactiveness of low-achieving students, disadvantages of the students' group working, unfamiliarity of working with RME-based lessons.

Firstly, as discussed previously in this chapter (sections 4.2.4, 4.3.3, 4.4.5 and 4.5.4), the two teachers complained that RME-based lessons took them much more time than lessons in the current textbooks. Normally, Vietnamese mathematics teachers have to follow the mathematics curriculum, textbooks and a curricular distribution. Typically, lessons in a curricular distribution are divided into 'theoretical' and 'practical' lessons. Students are taught formal mathematics in the 'theoretical' lessons and expected to apply it in solving mathematics exercises, including pure mathematics and application problems in the 'practical' lessons. The teachers thought that RME-based lessons are similar to the 'theoretical' lessons in the current textbooks. However, in RME-based lessons, 'theoretical' and 'practical' lessons are not always clear-cut. It is argued that students do not need to solve many mathematics exercises to consolidate knowledge when RME-based lessons are properly used. In fact, *the situations* of RME-based lessons also contain implicitly these exercises.

Secondly, as discussed above in sections 4.2.4, 4.3.3, 4.4.5 and 4.5.4, some weak-achieving students did not attend the discussions when the students were asked to work in groups. These students often did their private work while the other discussing. They thought that the tasks were designed for high-performing students. Some details of possible causes and solutions are discussed in section 4.6.3.2 of this chapter.

Thirdly, as discussed in the teachers' feedback, the students often made a noise which affected other neighboring classes while working in groups or in pairs. Typically, Vietnamese school classes are not suitable for group working. In the past, this problem did not emerge since teachers did not use to organize students' group working.

Fourthly, the teachers complained that they could not manage the classes while their students were working in groups. They believed that the high number of students in classes is the cause of this problem. However, it should be noted that there were only about 37 and 38 students in these classes. It was believed that the teachers lacked experience for facilitating students' group-work. Possible solutions are discussed in section 4.6.3.2 of this chapter.

Moreover, the teachers complained that they had to spend much time preparing RME-based lessons. This was not suitable for them because they had to work more.

Moreover, they also stated that they did not become accustomed to teaching RME-based lessons.

In addition, as described in sections 4.2, 4.3, 4.4 and 4.5 of this chapter, the students often tried to reason deductively to answer the questions although in some cases, it was not necessary.

Finally, the way in which the teachers organized their classes were not entirely reasonable. For example, the number of students in each group was quite high, and the second teacher, Mr. Mui gave one copy to all members of each group. In addition, the teachers gave the groups different tasks with different levels.

4.6.2 Potentials of RME-based lessons

In general, RME-based lessons are suitable for the orientation of the teaching method reform in Vietnam (see section 1.4.2 of chapter 1).

Firstly, the *situation*s of RME-based lessons gave the students the chance to rediscover mathematics knowledge. The students did not have to follow completely strict tasks related to ready-made mathematics in the current textbooks.

Secondly, the students gradually got used to the 'indirect' tasks in the *situation*s of RME-based lessons. Not only *sociomathematical* but also *social norms* were carefully considered in these *situation*s. In contrast, both the 'reformed' and current mathematics textbooks only present 'bare' problems.

Thirdly, most students had chances to express and explain their ideas, approve and disapprove of ideas during RME-based lessons although few low-performing students did not actively take part in the discussions.

Fourthly, RME-based lessons promoted the students' independence, activeness, initiative and creativeness. As the second teacher, Mr. Mui, commented, in RME-based lessons, his students could discuss with each other to rediscover mathematics knowledge. For this reason, in comparison with the normal lessons, he felt that he 'did not have to work hard' and 'was less important'. It is not absolutely true because although he did not have to lecture as usual, he should have paid special attention to low-achieving students.

4.6.3 Adjustment of RME-based lessons

The teachers and students encountered several certain difficulties during RME-based lesson experiments (see section 4.6.1 of this chapter). This section discusses how to adjust these lessons so that they become more suitable for teaching and learning middle school geometry.

4.6.3.1 The teaching time pressure

As described previously (sections 4.2.4, 4.3.3, 4.4.5 and 4.5.4 of this chapter), the two teachers who applied RME-based geometry lessons in their teaching complained that these lessons took them much more time than usual. They tried to deal with this problem by different ways. Nevertheless, it appeared that their solutions were not effective. The section discusses the ways in which the teachers tried to cope with the teaching time pressure. In addition, this section explains that students do not have to spend much time consolidating knowledge when RME-based lessons are applied. Finally, it also discusses some necessary adjustment so that these lessons become more suitable for teaching and learning geometry in Vietnamese middle school.

In the 'Triangle Sum Theorem' lesson, the task of *situation 4* confused some students because they have not worked with such an 'indirect' task.[84] Nonetheless, the teacher, Ms. Huong, did not have enough time to expound this task to her students and let them work with it. She could not help resorting to the conventional teaching style because of the teaching time pressure. Unlike Ms. Huong, the second teacher, Mr. Mui, divided his students into large groups of around 6 to 7 members and gave different tasks to different groups in the third and fourth RME-based lessons. His solutions could not solve the problem because of many reasons. These tasks of varying levels of difficulty were intended to be taught in a specific sequence. Moreover, the groups were so large that some low-performing students had chances of working their private work and did not join the discussions. In addition, Mr. Mui did not receive the sufficient feedback from his students because in each situation, he gave a copy of worksheet to all members of each group.

Normally, as discussed in section 4.6.1 of this chapter, mathematics lessons are divided into 'theoretical' lessons in which students are taught mathematics concepts, formulae, regulations and theorems and 'practical' lessons in which students are guided

[84] The same situation was also found in Indonesian primary school when students dealt with realistic contexts in the first lessons. This students' state was described by Fauzan, Slettenhaar and Plomp (2002).

how to apply mathematical theory in solving exercises. In the 'reformed' period, most of the mathematics exercises were pure exercises. Recently, a number of application exercises have increased in the current middle school textbooks. However, the way in which application exercises are used is still quite conventional, primitive and limited. That is, students are expected to apply mathematical theory in solving application exercises. Freudenthal strongly opposed this viewpoint which he called 'anti-didactical inversion' on mathematics education. Conversely, according to RME's viewpoint, realistic contexts need to be firstly given to students and give them chances to reinvent mathematics. Unlike the usual lessons, 'theoretical' and 'practical' lessons are not clear-cut in RME-based geometry lesson. Although it appeared that these RME-based lessons took the teachers quite long time, students would need less time for consolidation if they fully grasp knowledge. The two teachers, who used RME-based lessons, were not aware of this.

However, it is possible to change the way of using the *situations* in RME-based lessons to deal with the teaching time pressure. More specifically, in some cases, group working can be replaced by whole class and individual working. Because most of the students were familiar with the content of the 'Triangle Sum Theorem' (the 'Triangle-Angle Sum Theorem) lesson before this lesson was officially presented (see section 4.2.5 of this chapter), a teacher does not sufficiently apply the first two *situation* so that s/he let her/his students work with the other *situation*s. In the other lessons, a teacher can also organize whole class or individual tasks for some first *situation*s. As discussed in chapter 1, typically, teacher-centered approach still has dominant in teaching and learning mathematics in Vietnam school. Furthermore, there are significant differences between the current mathematics curriculum and the (Dutch) RME one (see section 3.2.2.3 of chapter 3). In addition, the unique nationwide set of mathematics curriculum and textbooks is often regarded as the official materials for teaching and learning in Vietnam school; furthermore, teaching time is set up for a small unit by a curricular distribution which is designed by a local department of education and training in a province or city. These are the reasons why it is extremely hard to apply RME in teaching and learning in Vietnam. Using RME and the current approaches of teaching in Vietnam alternatively is also a feasible solution.

4.6.3.2 Encouraging low-performing students

According to the teachers, some low-performing students did some private work and did not participate in the discussions. They believed that the tasks were for only high-performing students. As presented in sections 4.3.4, 4.4.6 and 4.5.5 of this chapter, one of the reasons is that the second teacher, Mr. Mui, let his students work in large groups. Furthermore, for each situation, there was one worksheet for all students in each group. Consequently, the teacher could not collect enough feedback from his students. To remedy this problem, several solutions can be used. Firstly, a teacher should explain that every student should deal with the tasks of RME-based lessons. Furthermore, as discussed in section 4.6.3.2 of this chapter, different ways of organizing the class such as whole class discussion or individual working can be alternatively used. In addition, a teacher should divide students into smaller groups and distribute worksheets to every individual student so that s/he can control individual's work and obtain more feedback from the students' worksheets. Finally, a teacher should pay attention to low-performing students in groups during their discussions. Fully experienced with RME-based lessons, low-achieving students will notice this teaching and learning style is especially good for them because they have chance to discover from informal to formal mathematics.

Chapter 5 Proposals for applying RME-based geometry lessons in Vietnam

RME complies with the Vietnamese teaching method reform and, as such, has great potential for mathematics education in Vietnam. However, the mathematics teachers and students who worked with RME-based lesson plans encountered a variety of challenges and obstacles (see section 4.6.1 of chapter 4). This chapter discusses conditions and requirements for applying RME-based geometry lessons in Vietnamese middle school. It discusses the following factors: viewpoints on mathematics education, conditions for applying RME-based lessons (amount of content in mathematics curricula and textbooks, the teaching time pressure, teaching and learning equipments, etc.), teachers' education, students' ability and some additional factors. The materials for the proposals are based on analyzing the characteristics of mathematics education in Vietnam (see chapter 1), RME theory (see chapter 2), the bases for designing RME-based geometry lesson (see chapter 3), the feedback from the two teachers and their students (see chapter 4) and the feedback from the middle school mathematics teachers' survey (see the questionnaire form in appendix B).

Nguyen Thanh Thuy (2005) carefully analyzed the effects and process of learning to teach realistic mathematics. She discussed how student teachers developed during teaching methods course, micro-teaching and teaching practice. Because the high school mathematics teachers supervising the student teachers were active and familiar with new approaches in teaching and learning mathematics, including the student-centered learning (since they were involved in several projects related to the reform of teaching methods), Nguyen Thanh Thuy's subjects were quite flexible.

5.1. Viewpoints on mathematics education

As discussed in chapter 2 and section 3.2.2 of chapter 3, the Dutch RME curriculum differs from the Vietnamese mathematics curriculum, which is quite limited despite its recent significant changes. It is important to convince not only school mathematics teachers but also mathematics educators, curriculum creators and textbook writers to consider new perspectives on teaching and learning mathematics in school.

5.1.1 Mathematics as a ready-made product or mathematics as a human activity

As discussed in section 2.2.1 of chapter 2, one of the important Freudenthal's viewpoints on school mathematics is 'mathematics as a human activity'. The opposite viewpoint is ‘mathematics as a ready-made product’. School mathematics in Vietnam is similar to the second one; that is, mathematics is often regarded as a ready-made product. Typically, students are first taught formal mathematics (concepts, theorems, regulations and formulae) in ‘theoretical’ lessons. Later, they are taught how to apply this knowledge in solving mathematics exercises in ‘practical’ lessons. According to the surveyed middle school teachers, one of the important purposes of teaching and learning mathematics in school is to help students acquire formal mathematical concepts, theorems, regulations and formulae and apply them in solving mathematical exercises (see a statistics in section 5.1.2 of this chapter). The theoretical emphasis of Vietnamese school mathematics caused many students to struggle in their mathematics learning (see section 5.2.6 of this chapter). In middle school geometry parts in the current textbooks, formal deductive geometry is quickly introduced to students in each lesson although a number of complicated topics have been omitted. This approach forces students to learn formal deductive geometry which is not related to their common sense or informal strategies and solutions. Consequently, some students mechanically learned mathematics. The students learn formal geometry concepts, theorems, regulations, formulae and different problems’ forms, and then they apply them to solve geometry exercises (see also section 5.1.2 of this chapter). According to this way, Vietnamese middle school students must learn formal deductive geometry so early and so quickly that the majority of them often encounter many difficulties with this subject. According to the survey (the questionnaire form in appendix B and the statistics in section 5.2.6 of this chapter), many middle school teachers stated that their students often encountered obstacles when they learn geometry. Therefore, the ‘mathematics as a human activity’ viewpoint should be carefully considered in mathematics education in Vietnam.

5.1.2 Guided reinvention/rediscovery

This section discusses a role of guided reinvention/rediscovery in teaching and learning mathematics in Vietnam. One question in the questionnaire asked the middle school teachers what they should emphasize in their teaching. A number of the teachers’ ideas are presented in figure 5.1.

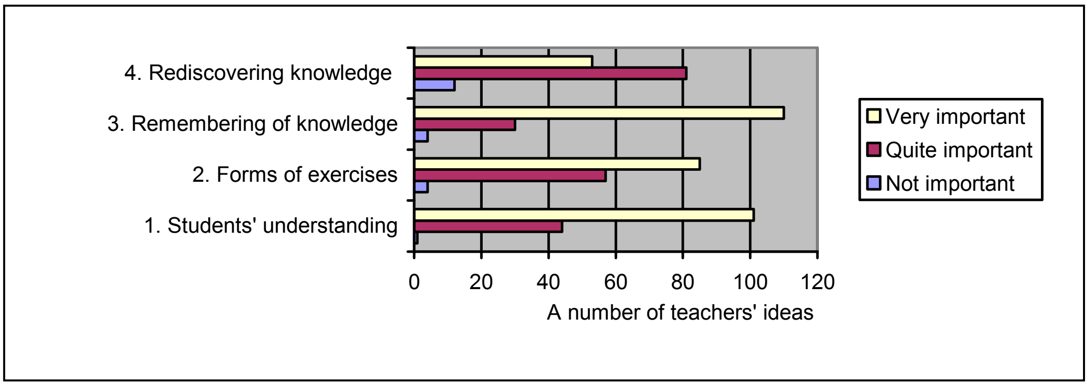


Figure 5.1: The teachers' ideas

According to the survey (see the questionnaire form in appendix B), the middle school teachers thought, above all, that the most important aspect of teaching is ensuring that students retain learned knowledge (e.g. formal concepts, theorems, formulae and regulations) and apply this knowledge to solve mathematics problems (see figure 5.1). On the contrary, only one third of the interviewed teachers believed that it is very important for students to be guided to rediscover mathematical knowledge. Guided reinvention/discovery, according to the interviewed middle school teachers, is less important than the other factors. However, they also recognized its role in teaching and learning mathematics, as more than a half of them believed that it is quite important (see figure 5.1). In general, the middle school teachers' viewpoints on mathematic education were still rather primitive and limited. Their primary objective was to help their students memorize formal concepts, theorems, formulae and regulations by assigning similar mathematics problems so that their students could apply them to solve formal mathematics problems. One possible cause of the teachers' focuses was examinations' pressure. The mechanistic approach still played a dominant role in teaching and learning mathematics in Vietnam (see discussion about four different approaches to mathematics education in section 2.3.2 of chapter 2).

As discussed in section 2.2 of chapter 2, one of the important ideas of Freudenthal for RME is 'guided reinvention'. It is necessary to convince Vietnamese school teachers that guided reinvention also plays an important role in teaching and learning mathematics.

5.1.3 Well-structured mathematics curriculum and textbooks

According to the feedback of the teachers' survey (see the questionnaire form in appendix B), most middle school mathematics teachers believed that good mathematics textbooks should be well-structured. In their opinions, mathematics should be scientifically, systematically and logically presented in school textbooks. They regarded this characteristic as one of the advantages of the 'reformed' mathematics textbooks. Generally, both the 'reformed' and current Vietnamese mathematics textbooks emphasize structures of mathematics, although the second series has slightly reduced requirements of mathematics structures. For example, the primary curriculum presents integer numbers with additions and subtractions up until 10 through 100, 1000 and 10 000 to millions. However, a structural characteristic is not a compulsory characteristic in a RME curriculum. According to Freudenthal (1991, p. 119), in some cases, education can benefit from things that "look like chaos".

5.1.4 Informal knowledge (strategies and solutions) in teaching and learning mathematics

As discussed in section 1.3.1 of chapter 1, Vietnamese school mathematics curricula and textbooks are quite formal. In general, informal mathematics (strategies or solutions) is not officially accepted (see MoET, 2002 a, pp.1-2). In contrast, RME approach encourages students to rediscover informal knowledge.

Generally, Vietnamese mathematics school teachers emphasize the way students present their solutions or strategies. One purpose of the mathematics curriculum is to help students practice how to express their ideals clearly (see MoET; 2002 a, p. 1). Consequently, students are able to practice correct usages of mathematical expression. However, its disadvantage is that in some cases, students have to follow fix their teachers' prescribed forms of presentation.

It appeared that the middle school mathematics teachers believed that a good solution must be logically presented and contain a detailed explanation. The middle school mathematics teachers were asked to give their feedback on six solutions of a problem known the 'T-shirts and Sodas' problem (see Van Reeuwijk, 1995, pp. 2-4; appendix B). More specifically, the teachers were asked to comment on six solutions of students. Five solutions based on five strategies in a paper of Van Reeuwijk (1995, pp. 2-4) were used. Aside from these five mentioned solutions, one very formal solution was added. In general, most of the teachers did not accept solutions 1 and 5. The teachers did not consider the first solution, known as *guess and check*, to be a mathematics solution, and generally thought the fifth solution was too complicated. Solutions 4 and 6 are very similar. Solution 6 is a formal style of solution 4. Most of them thought that the sixth solution, a very formal solution, was sufficient and correct, and the fourth solution was insufficient and incorrect. According to these teachers, formal solutions of mathematics problems were very important for students. Most of the teachers thought that solutions 2 and 3 were good; however, they complained that these solutions could not be generalized and applied for other similar problems. In general, these teachers preferred formal solutions. According to Van Reeuwijk (1995, p. 2), the 'T-shirts and Sodas' problem is given to students to encourage them discover different informal strategies. Some of the students could gradually build and use formal knowledge; however, it is not compulsory for them to use formal knowledge (Van Reeuwijk, 1995, p. 3). On the contrary, in some extents, formal knowledge is introduced too early and quickly to students, and understanding and knowing how to applying formal knowledge are compulsory in Vietnamese school.

As discussed in chapter 4, while working with RME-based lessons, the students tended to use formal mathematics to solve the problems, although in some cases, it was not necessary. It implies that deductive reasoning was typically regarded as a unique way for reasoning while Vietnamese students presented their solutions.

5.1.5 Teaching mathematics application

As discussed and analyzed in section 1.4.1 of chapter 1 and section 3.1.2 of chapter 3, a number of mathematics applications have increased significantly in the current middle school mathematics textbooks. However, the presentation of mathematics applications is quite conventional and rigid. Initially, formal mathematics (axioms, definitions, regulations and theorems) is taught, and then students are expected to use formal mathematics to solve application problems. Conversely, as discussed in section 2.4.1 of chapter 2 and section 3.2.2.3 of chapter 3, according to RME approach, context problems are presented first. While working with context problems, students gradually build their own informal knowledge. In some cases, it is not compulsory for students to work with formal knowledge (axioms, definitions, regulations and theorems) (see, for example, Van Reeuwijk, 1995 & 2001).

Moreover, although a number of mathematics applications have been significantly increased in the current middle school textbooks, teachers' application strategies are still in question. About 74% of the questioned middle school teachers thought that mathematics applications were adequately presented in the current mathematics curriculum. Nonetheless, there is a question about its actual role in teaching and learning mathematics. According to the feedback of the teachers' idea survey, although about 75% teachers often focused on mathematics applications in their teaching, only 23% usually used these applications in oral examinations, and around 38% frequently utilized those in written examinations (see figure 5.2).

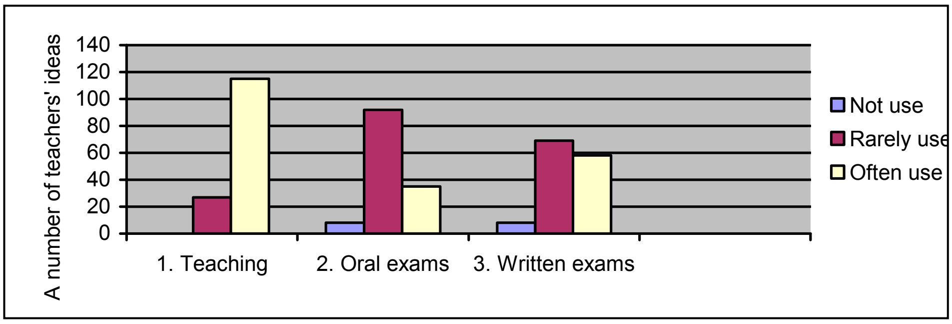


Figure 5.2: The frequent use of mathematics application in lessons

It should be noted that examinations are one of the factors which affect the way of teaching and learning mathematics in Vietnamese school. Because the role of mathematics applications is not emphasized in examination, this role in teaching and learning mathematics might be less essential than mathematics educators expect.

5.1.6 Emergent modeling

As discussed above in section 3.2.2.3 of chapter 3, "didactical modeling" and "mathematical modeling" are utilized in the geometry contents of the current Vietnamese middle school textbooks. Firstly, "didactical modeling" is used to concretize formal knowledge (concepts, theorems, formulae, etc.). Students are often asked to manipulate (draw, measure, fold, cut, etc.) according to the guidance of the mathematics textbooks. The purpose of this manipulation is to help students recognize geometrical characteristics. After that, formal mathematics (concepts, theorems, formulae, etc.) is presented. This way offers certain advantages. Students are introduced 'intuition pictures' before formal mathematics is officially presented. However, there are also some disadvantages. The first one is that students do not have the opportunity to reinvent knowledge. Rather, students rigidly adhere to specific instructions and require only simple comments. Moreover, formal knowledge is quickly purveyed to students because of the teaching time pressure. As discussed above in section 5.1.1 of this chapter, acquiring formal knowledge is compulsory tasks for all students in Vietnam. For this reason, some parts of students cannot understand the essence of events. Students often find geometry very complicated and abstract. Finally, it is better if some other different ways are used in the mathematics textbooks besides manipulations.

In the Vietnamese situation of mathematics education, it is impossible to apply a whole RME curriculum (see section 3.1.2.3 of chapter 3). In addition, the teachers and their students encountered some difficulties while working with RME-based lessons (see section 4.6.1 of chapter 4). These RME-based lessons were presented in order to help students rediscover knowledge. However, the teachers and students did not adjust to working with these lessons. Moreover, the teachers were always concerned about the teaching time allowance. The situation in Vietnam is quite different from that in the Netherlands. It is suggested that apart from "didactical modeling" and "mathematical modeling", "emergent modeling" should be used in teaching mathematics in Vietnamese middle school. In addition, emergent modeling can function as "a precursor to mathematical modeling" as Gravemeijer (2004) suggested.

5.2 Conditions for applying RME-based geometry lessons

Section 5.1 of this chapter discussed the viewpoints which mathematics educators, textbook authors and teachers should accept so that mathematics is taught in a meaningful way. This section discusses necessary conditions so that RME-based geometry lessons can be applied in teaching and learning in Vietnamese school. Specifically, it discusses teachers' quality and competence, amount of content in the mathematics textbooks, teaching time pressure, teachers' and students' difficulties, etc.

5.2.1 Teachers' quality and competence

As discussed in section 1.3.5 of chapter 1, due to historical factors, there is a rather high percentage of school teachers who do not reach the qualification standard in Vietnam. It is difficult for these mathematics teachers to change their teaching styles. They are often experienced teachers but not properly-trained. RME-based geometry lessons are not especially suitable for their teaching.

Recently, a national-wide study, implemented by the National Institute for Education Strategy and Curriculum Development (NIESAC) in two years (2004-2006), reveals that 60 % of primary and 70 % of middle school teachers have encountered difficulties when applying new methods of teaching and using modern teaching facilities in working with the current curricula and textbooks (Nhan Dan, 2006). This finding implies that, in general, quality and competence of Vietnamese teachers have not satisfied the requirements of the new curricula and textbooks. Consequently, there are related problems needed to be instantly solved, including training in-service and pre-service teachers and adjusting these curricula and textbooks.

Specifically, the two teachers using RME-based lessons encountered some difficulties during their teaching although Mr. Mui, the six-year teacher, got a master degree of education and Ms. Huong, the new teacher, graduated from a college of education. The way in which Mr. Mui used the *situation*s of RME-based lessons and organized his students' group working was not effective. In addition, both teachers complained that they could not manage their students during the groups' discussions. The detailed discussions are presented in section 4.6.1 of chapter 4. The possible solutions to this problem are discussed in section 4.6.3.2 of chapter 4.

5.2.2 Amount of content in the current mathematics textbooks

Although complicated and theoretical content has been significantly reduced in the current mathematics textbooks, it appears that the amount of content in these textbooks is not particularly suitable for the majority of students.

The current primary curriculum is quite rigorous for students. For this reason, the MoET (Ministry of Education and Training) decided to reduce 15% of the current primary curriculum from the school-year 2005-2006. However, primary teachers are rather perplexed because they have not received detailed instruction for this reduction. The study of The National Institute for Education Strategy and Curriculum Development (NIESAC) (see section 5.2.1) has also confirmed that mathematics curriculum and textbooks of grades 6 and 8 are quite challenging for students' competence and learning conditions, especially quantity of learning time (Nhan Dan, 2006).

Although most of the interviewed middle school teachers confirmed that the amount of mathematics contents has been decreased remarkably, some of them still argued that it was still difficult for most students in rural and remote areas to keep up with the current curriculum and textbooks (see the questionnaire form in appendix B and figure 5.3). Furthermore, half of the teachers believed that the current textbooks were not suitable for the teaching time allowance. More specifically, these teachers often argued that it was difficult for them to adjust methods of teaching because of the teaching time regulation.

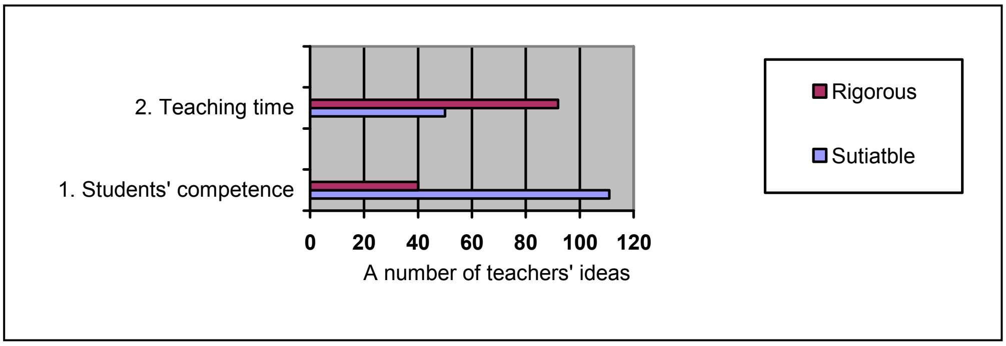


Figure 5.3: The teachers' opinions about the current mathematics curriculum and textbooks

5.2.3 Teaching time pressure

As discussed previously in section 3.2.1 of chapter 3, a curricular distribution which is designed by a local department of education and training often determines time for not only each chapter in a textbook, but also for each small unit in a chapter. More specifically, this material even prescribes to teachers what to teach within a 45-minute-lesson. This policy has both advantages and disadvantages. The main advantage is that a teacher knows what s/he has to teach, and how long s/he is allowed to teach. However, as discussed in section 1.3.1.1 of chapter 1, Vietnamese school students' competence varies not only by region- i.e. urban through rural to remote (mountainous and islandish) areas- but even within a same area or different students within a class students' individual abilities tend to differ. For this reason, a fixed curricular distribution cannot be appropriate for every grade. This section discusses how a curricular distribution should be adjusted for applying RME-based lessons as well as the renovation of teaching methods in Vietnam.

From the students' and teachers' feedback analysis, it was not easy for the teachers to apply RME-based lessons because RME-based lessons were quite new for them, and the teachers always suffered from the teaching time pressure.

As discussed in chapter 4 (sections 4.3, 4.4 & 4.5), Mr. Mui, who taught three lessons of RME-based lessons, dealt with the teaching time pressure by using the two solutions: dividing the students into large groups and giving different tasks to different groups.

With the first manner, he hoped that it was easier for the students to rediscover knowledge while they were working in groups of around 6 to 7 members. However, this way also had the disadvantage of some low-achieving students relying on the others and doing some private work during the discussions. Moreover, in each *situation*, he gave only one copy of the worksheet to each group. Hence, we did not get feedback of all students. Mr. Mui complained that he could hardly control the students while they were working in groups. Apparently, he does not have sufficient experience organizing students' group working. It would be better if he were to distribute a worksheet to every student in each group. At first, every student herself/himself should try to answer the questions; then the students discuss the questions in groups to find solutions. All worksheets of individual and groups should be collected so that a teacher can receive feedback from all students in a class.

The second solution of Mr. Mui was not particularly effective. Some groups worked with easy tasks, while others had to deal with more complicated and formal tasks. Furthermore, these tasks should be used following a specific order.

The reason for using the existing distribution curriculum is that it guides teachers what and how long should they teach. A local department of education and training wants to ensure that all contents which are set up by curricula and textbooks are taught in school. It is suggested that a curriculum distribution should have a general rules such as what and how long teachers should teach for a chapter in a textbook. However, it requires that teachers have to response for their work, and teachers are at a high competence.

5.2.4 Teachers' difficulties

5.2.4.1 Methods of teaching mathematics

The middle school teachers interviewed by the questionnaire (see appendix B) often used some methods or approaches: developing students' activeness[85], the student-centered approach, using suggested questions, problem solving and using visual means (see figure 5.4). There were some approaches or methods which they do not know or rarely used such as constructivism, the didactical situation in mathematics, teaching in activities and by activities and using ICT (see figure 5.4). The data of the figure 5.4 showed what methods or approaches (according to personal realization of the teachers) the teachers often or rarely used in their teaching; however, the questions about how

[85] It was easy to understand why 138 of 152 teachers answered that they often used a method called developing students' activeness because recently, it has become one motto of the Ministry of Education and Training (MoET) (see figure 5.4).

they use these methods or approaches have not been answered. According to Stigler and Hiebert (1999, p. 16), "It is difficult to know how accurately teachers describe their methods and what they mean by the words they use." Generally, 'new' approaches were not familiar to these teachers.

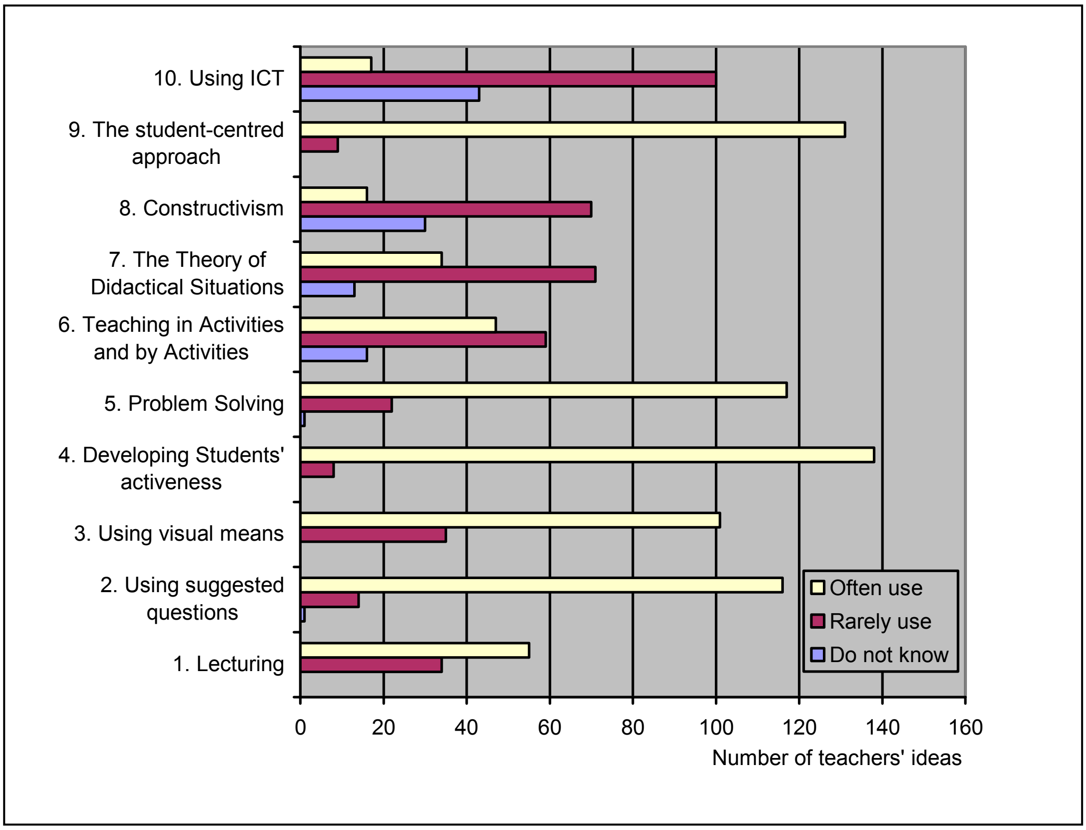


Figure 5.4: Methods of teaching mathematics

5.2.4.2 Teachers' difficulties for the reform of teaching methods

According to the feedback from the survey, the middle school mathematics teachers often encountered the following challenges (figure 5.5):

1. Teachers' difficulty with using new modern teaching equipment and teaching aids (computers, overhead projectors and beamers) (35 ideas);

2. Insufficient teaching equipment and teaching aids (computers, overhead projectors, beamers, transparent paper, etc.) (51 ideas);

3. Unsuitable quality of teaching facilities (24 ideas);

4. Students' low levels of awareness (especially those in rural areas) (44 ideas);

5. Different students' levels of awareness (43 ideas);

6. Rigorous amount of content (especially for students in rural areas) (43 ideas);

7. Insufficient teaching time and unsuitable curricular distribution (41 ideas);

8. High number of students in a class (47 ideas);
9. Insufficient guidance and reference material (30 ideas);
10. Miscellaneous difficulties (15 ideas).

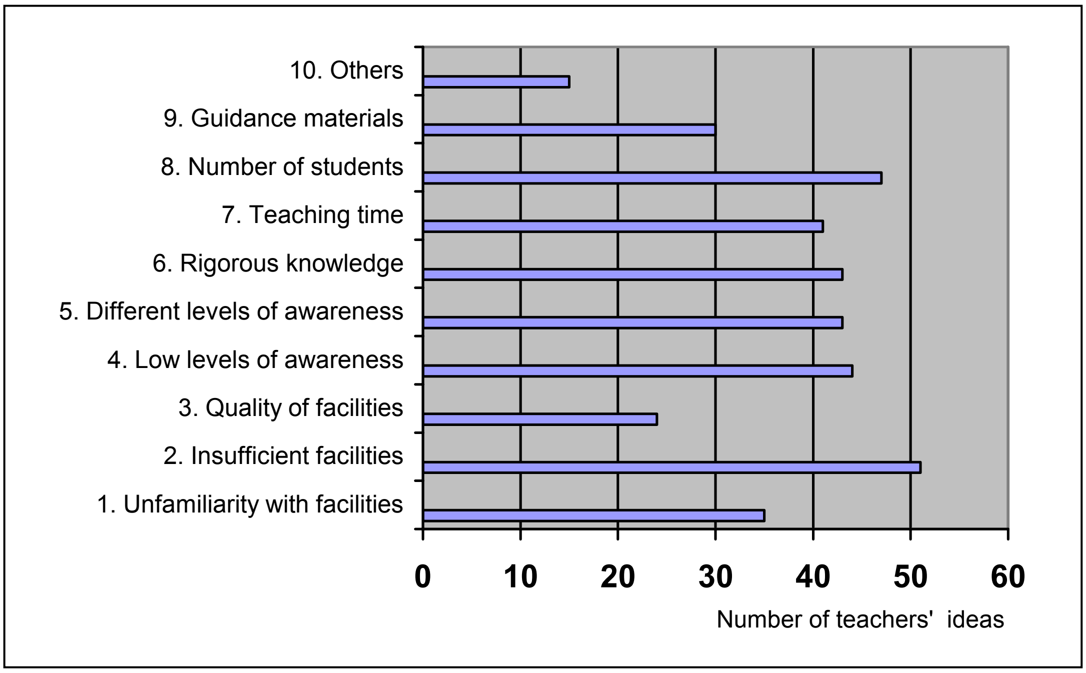


Figure 5.5: The teachers' difficulties in the reform of teaching methods

Most of the teachers complained that they experienced difficulties in teaching facilities: unfamiliarity with facilities (35 ideas), insufficiency of facilities (51 ideas), and unsuitable and low quality of facilities (24 ideas) (110 ideas in total). In addition, they had problem with students' awareness: low levels of awareness (44 ideas) and different levels of awareness (43 ideas) (87 ideas in total). More specifically, certain students in rural areas were at low levels of awareness, and students' competency levels were quite different in a class. Moreover, the teachers suffered from rigorous amount of content in the textbooks, insufficient teaching time, unsuitable curricular distributions, high number students per class and insufficient guidance and reference materials.

Although the teaching aids (computers, overhead projectors, beamers, photocopy machines, ect. and others) have been equipped for school, some of the surveyed middle school teachers complained about the quality of the teaching aids and the insufficient instructions. Some teachers did not acclimate to using the 'modern' teaching aids. In addition, some teachers also complained that it required too much time to design ‘electronic teaching plans’ by using some software such as PowerPoint. They found these additional preparation tasks inconvenient and frustrating.

5.2.5 Assessment and examinations

As discussed previously in section 1.3.3 of chapter 1, in Vietnam, teaching and learning mathematics are often affected by assessment and examinations which regard mathematics as a formal subject (see some examinations in Do Dat, 2000). RME-based lessons will be applied if informal strategies and knowledge are also accepted in mathematics examinations.

In general, assessment is rather rigid and conventional in Vietnam. Mathematics teachers often emphasize assessing how students understand and remember formal mathematics (concepts, theorems, formulae and regulations) and apply this knowledge in solving exercises. Students are often asked to solve problems in mathematics examinations. These exercises belong to either pure or applied mathematics problems. To solve applied mathematics problems, students often work with traditional mathematical modeling (as discussed in section 3.2.2.3 of chapter 3).

The ‘Polar Bear’ problem was used to interview the middle school teachers (see appendix B). The aim of the problem is to encourage students to predict an average weight of students and use informal strategies to deal with the problem. In Vietnam, the way of using similar problems is quite different. More specifically, students are initially taught formal knowledge (division with remainder). Then students are expected to utilize formal knowledge to solve problems. About 80% of the mathematics middle teachers interviewed confirmed that the mentioned problem did not have enough suppositions (see figure 5.6; see also appendix B). In contrast, only about one eighth of them thought that it was a good problem. Some of the teachers gave additional ideas about this problem (see figure 5.7). These teachers urged that this problem needed more suppositions such as an average weight, an age and a gender and a ‘same weight’ characteristic of students (50 ideas in total). In addition, some teachers believed that comparing the Polar Bear’ and students’ weight was useless (19 ideas). However, a few teachers thought that this problem was suitable for high-performing students because these students could predict or find an average weight of their classmates (6 ideas). It appears that Vietnamese mathematics teachers did not use ‘open’ problems which have different solutions. Generally, most Vietnamese school mathematics teachers did not accept similar problems. More specifically, Vietnamese students are firstly taught formal mathematics and are expected to apply it in solving mathematics problems which are either pure or applicable ones. It should be noted that most of applicable problems which are used in teaching and learning mathematics in Vietnam are bare problems. In other words, these problems are often situated in ideal conditions.

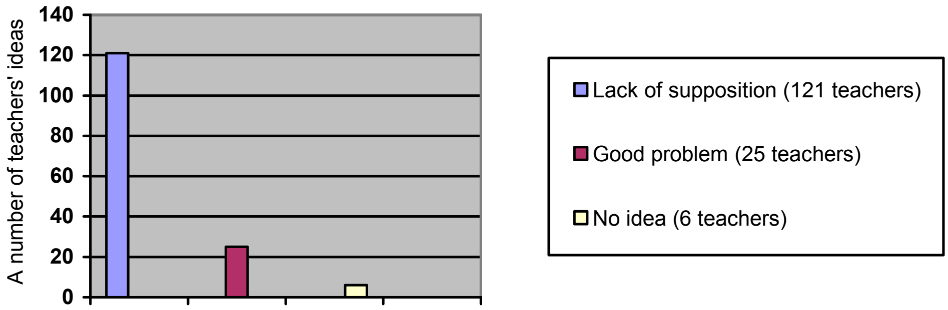


Figure 5.6: The teacher' opinions about the ‘Polar Bear’ problem

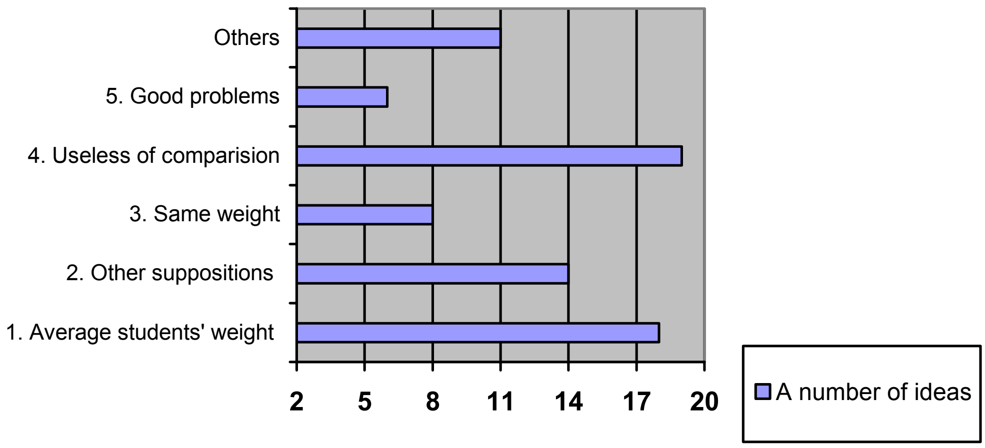


Figure 5.7: Additional ideas of teachers about the ‘Polar Bear’ problem

Recently, tests have been started to use in mathematics examinations in Vietnam. Forms of assessment have been renovated; however, content of mathematics examinations does not change.

It is argued that not only form, but also content of assessment and examinations should be improved. In the past, nearly all school mathematics examinations are written examinations. Recently, tests have been researched and experimented in school examinations. Special attention has been paid to forms of assessment and examination. Nevertheless, content of assessment and examinations need to be renovated.

5.2.6 Students' difficulties in learning mathematics

One question of the questionnaire asked the middle school teachers what difficulties their students often met while learning mathematics. Eighty-two among the 152 teachers answered this question. According to their answers, while learning mathematics, their students often meet the following difficulties (figure 5.8):

1. Students' dislike for mathematics and lack of amenities in learning mathematics (11 ideas);
2. Students' laziness of thinking, solving mathematics exercises, reading mathematics textbooks and learning mathematics (16 ideas);
3. Inactivity and uncreativeness of students in learning mathematics (17 ideas);
4. Students' carelessness in learning mathematics (15 ideas);
5. Poor students' presentation (students' difficulty in presenting a solution of a mathematics problem and using properly Vietnamese and mathematical languages) (30 ideas);
6. Some students' lack of basic mathematics knowledge of previous grades and lessons (32 ideas);
7. Students' weak ability of applying mathematics theory in solving mathematics exercises (25 ideas);
8. Students' difficulties in learning geometry (25 ideas);
9. Rigorous mathematics curriculum and textbooks (17 ideas);
10. Insufficient learning aids (12 ideas)
11. Abstract characteristics of school mathematics; lack of practical activity (19 ideas);
12. Low or different competence of students (15 ideas);
13. Lack of learning time (11 ideas)
14. Lack of orientation for solving a problem (16 ideas)
15. Weak ability of logical thought (17 ideas)
16. Others (37 ideas).

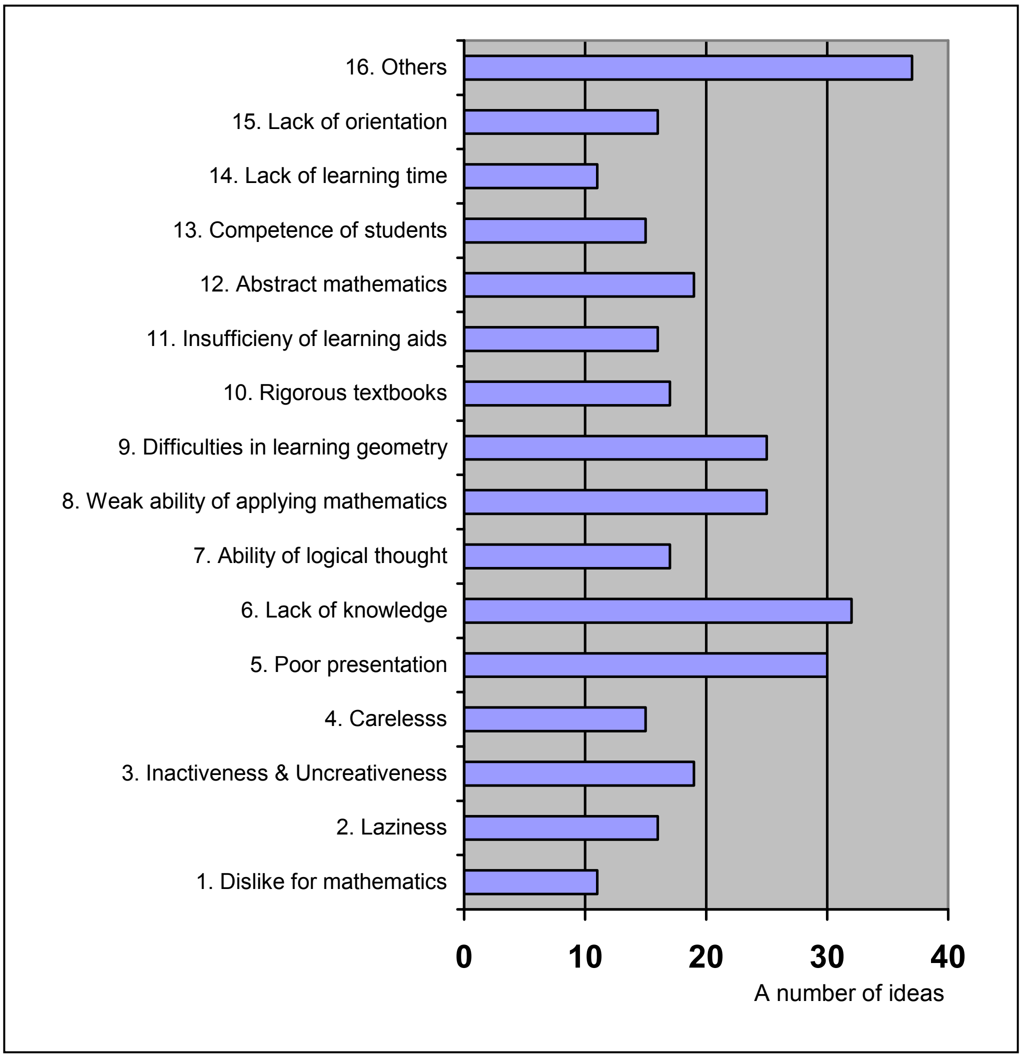


Figure 5.8: Students' difficulties in learning mathematics

According to these middle school mathematics teachers, their students often met various difficulties of learning mathematics. Nearly 40% of the teachers claimed that students' lacked of knowledge in previous lessons or grades and erratic learning patterns of acquiring knowledge also accounted for students' difficulties in learning mathematics. In addition, Vietnamese teachers often focus on ways in which their students present solutions to a mathematics exercises. They believed that it was very important for students to learn mathematics. Around 36% teachers argued that their students could not get high results of mathematics because of their poor presentation of mathematics solutions. These students did not know how to present properly solutions to mathematics problems although they could solve these problems. Furthermore, 30% teachers believed that their students often met difficulties in learning geometry. Similarly, the percentage of the teachers who thought that their students were not good at applying formal theorems (concepts, theorems, regulations and formulae) in solving mathematics problems is 30%.[86] Moreover, there were some difficulties such as abstract characteristic of school mathematics, students' uncreativeness and inactiveness, etc.

One of possible reasons for the mentioned students' difficulties was that Vietnamese mathematics curricula and textbooks place strong emphasis on formal mathematics. Consequently, it is rather difficult for students to learn mathematics. It is believed that working with RME-based lessons helps students overcome certain obstacles such as the student' lack knowledge in previous lessons, the student' difficulties with learning geometry, the abstract characteristic of school mathematics, the students' uncreativeness and unactivenss, etc.

5.3 Proposals for applying RME-based lessons in Vietnamese school

5.3.1 Applying RME-based lessons in Vietnamese middle school

This section reconsiders the difficulties the two teachers, Mr. Le Xuan Mui and Ms. Do Lan Huong, encountered while applying RME-based lessons in their teaching (see the detailed analysis of the feedback from the teachers and their students in chapter 4). To overcome similar obstacles, later, this section proposes possible solutions to help middle school teachers who will apply RME-based lessons.

5.3.1.1 The difficulties the teacher and students encountered

As discussed in chapter 4 (sections 4.2.5, 4.3.4, 4.4.6, 4.5.5 and 4.6.1), while teaching and learning with RME-based lessons, the teachers and their students encountered some challenges. The following paragraph reconsiders the most common difficulties the teachers and their students encountered:

- The teachers were constrained by the pressure of limited teaching time;
- When the teachers organized the students' group working, some obstacles emerged:
    - The teachers lacked experience for facilitating group work;
    - A few low-performing students did not actively participate in the discussions with other members in their groups;
    - The greater volume of noise from the students' discussions could disturb the neighboring classes;

[86] The National Institute for Education Strategy and Curriculum Development (NIESAC) (2006) has inquired into primary and middle school student' knowing, understanding, and applying. Its study also confirms that among these phases, students are worst at applying (Nhan Dan, 2006).

• The teachers had to understand the RME's philosophy and RME-based lessons, spend more time preparing these lessons and use more tools for their teaching;
• Formal mathematics was immoderately emphasized in Vietnamese middle school lessons;
• The students often encountered the 'indirect' tasks;

5.3.1.2 Proposals for applying RME-based geometry lessons

This section discusses possible proposals helping teachers and students solve the aforementioned problems (see section 4.6.1 of chapter 4) while applying RME-based lessons in their teaching and learning. More specifically, the primary aim of this section is to find the solutions for the teaching time pressure, the unfamiliarity with group working, RME, RME-based lessons and the 'indirect' tasks and the excessiveness of formal mathematics in middle school lessons.

5.3.1.2.1 The teaching time pressure:

As discussed in chapter 4 (sections 4.2.4, 4.3.3, 4.4.5, 4.5.4), the two teachers who applied RME-based lessons to their teaching, stated that the teaching time pressure is one of the main obstacles for them. This section discusses possible proposals to solve the problem of the teaching time pressure. More specifically, the following proposals are discussed:

• Explaining the difference between RME-based lessons and typical Vietnamese mathematics lessons;
• Adjusting RME-based lessons;
• Applying 'traditional' approach and RME-based lessons alternatively;
• Using RME-based lessons in selected mathematics lessons;

a) Explaining the difference between RME-based lessons and typical Vietnamese mathematics lessons

Chapters 3 and 4 (sections 3.2.2.3, 4.6.1 and 4.6.3.1) discuss the difference between the structure of RME-based geometry lessons and the structure of typical Vietnamese mathematics lessons. As discussed in section 3.2.2.3 of chapter 3, at the beginning of typical geometry lessons of the current mathematics textbooks, students are often asked to execute some manipulations (drawing, measuring, cutting, folding, etc.) which functions as "didactical modelling" (Gravemeijer, 2004, p. 98), followed by formal mathematics (geometric theorems, concepts and formulae). Later, students are expected to apply formal mathematics in solving mathematics application problems (see sections 3.2.2.3.2 of chapter 3 and 4.6.1 of chapter 4). Moreover, as discussed in section 4.6.1 of chapter 4, Vietnamese mathematics lessons are often divided into 'theoretical' lessons and 'practical' lessons.

By contrast, in RME-based lessons, students are offered situations which help them gradually discover from informal mathematics to formal mathematics. In fact, 'theoretical' and 'practical' lessons are not always explicitly distinguished from each other in RME-based lessons (see RME-based lessons in appendix C). Furthermore, when students fully grasp knowledge in RME-based lessons, there is no need to offer students many 'consolidation' exercises (see discussion in section 4.6.1 of chapter 4).

RME-based lessons (see appendix C) did not explicitly elucidate the difference between the structure of RME-based lessons and the typical Vietnamese mathematics lessons. The two teachers applying RME-based lessons did not realize this difference. For this reason, these teachers believed that similar to the typical Vietnamese lessons, they needed more 'practical' lessons to help their students consolidate mathematics knowledge. Consequently, the teachers were under the impression that RME-based lessons required a great deal of time (see sections 4.2.4, 4.3.3, 4.4.5 and 4.5.4 of chapter 4).

Middle school mathematics teachers who will apply RME-based lessons should understand the aforementioned difference in the structure of RME-based lessons and the typical lessons in the current Vietnamese mathematics textbooks. Teachers should fully comprehend this difference so that they will feel secure while applying RME-based lessons in their teaching.

b) Adjusting RME-based lessons

- Reducing some unnecessary situations

In some cases, it is possible to reduce usage of some situations in RME-based lessons such that teachers will not be overly concerned about the teaching time pressure. For example, in the Triangle Sum Theorem (Triangle-Angle Sum Theorem) (see RME-based lesson in appendix C), many students had known the content of the theorem already (see the students' worksheet analysis in sections 4.2.2 and 4.2.3 of chapter 4) before the lesson was officially taught. For this reason, *situations 1, 2 and 3* (see appendix C), designed to help students discover that the sum of the three angles in a triangle is 180^0, were not especially effective for these students. Therefore, when the content of this theorem is familiar to students, the new version of this RME-based lesson should offer students only two *last situations* (4 and 5) (see these situations in the Triangle Sum Theorem (Triangle-Angle Sum Theorem) in appendix C).

- *Applying group/pair working, individual working and the whole class discussion alternatively*

In the past, Vietnamese mathematics teachers rarely organized students' working in pairs or groups (see Do Dat, 2000, p. 4). Recently, they have been encouraged to use group/pair working. Consequently, the two teachers applying RME-based lessons in their teaching were not familiar with the instruction of the students' group/pair working (see discussion in section 4.6.1 of chapter 4). RME-based lessons (see appendix C) often suggested which type among group/pair working, individual working and the whole class discussion should be used in *each situation* of these lessons. However, the new version of RME-based lessons should let teachers choose the type they find most suitable for their teaching. Moreover, whereas group/pair working was primarily used in RME-based lessons (see appendix C), the new version of these lessons should offer students the chance to work individually and discuss with the whole class.

c) Applying 'traditional' approach and RME-based lessons alternatively

As discussed in section 3.1.2.3 of chapter 3, a RME curriculum cannot be applied in Vietnamese school for many reasons. This study discusses how to design RME-based geometry lessons and apply these lessons in Vietnamese middle school. However, it is also impossible for teachers to apply all RME-based lessons for a chapter in the current textbooks. In other words, some RME-based lessons could replace some lessons in the current textbooks. The feedback from the students' worksheets and the teachers' reports revealed that RME-based lessons were unfamiliar to them because of their different teaching and learning habits. To reduce the teaching time pressure, teachers, in some cases, can alternatively use 'traditional' methods for some specific situations in the first RME-based lessons when their students are not familiar with these lessons. More specifically, teachers can suggest their students when the students are puzzled by some tasks.

d) Applying RME-based lessons in selected mathematics lessons

Recently, apart from the official compulsory lessons, Vietnamese middle school students can choose *two selected forty-five-minute lessons* per week (see, for example, MoET, 2006, p. 3). In selected lessons, teachers are not constrained by the pressure of the limited teaching time. Therefore, mathematics teachers could apply RME-based geometry lessons in the selected mathematics lessons without having to worry about the teaching time pressure. RME-based geometry lessons were not applied in seventh-grade in the school-year 2005-2006 because this time, the selected lessons were available for students of eight-grade to twelfth-grade. The intended amount of time for RME-based lessons with the aforementioned adjustment is:

Table 5. 1: The proposals for the amount of time for RME-based lessons as the selected mathematics lessons

Nr.	**Name of lessons**	**The amount of time**
1	The Triangle Sum Theorem (*with situations 4 and 5*)	2 periods[87]
2	Perpendicular line and slant line	3-4 periods
3	The Triangle Inequality	2-3 periods
4	Characteristics of the bisector of an angle	2 periods
5	Characteristics of the three bisectors of a triangle	1 periods
6	Characteristics of the perpendicular bisector (mid-perpendicular) of a segment	2 periods
7	The ‘railway station’ problem	4-5 periods
8	Characteristics of the three perpendicular bisectors (mid-perpendiculars) of a triangle	2 periods

5.3.1.2.2 Guiding teachers through students’ group/pair working

As repeated in section 5.3.1.1 of this chapter, the teachers often encountered some difficulties while organizing their students in groups or pairs because they lacked experience. It appeared that the teachers, especially Mr. Mui, overly organized his students’ group working, but the ways he used were not suitable. More specifically,

[87] A teaching period lasts 45 minutes.

some of their unreasonable manners are (see the detailed discussions in section 4.3.4, 4.4.6 and 4.5.5 of chapter 4):

• Use students' group/pair working in some unnecessary cases;

• Apply students' group working improperly (the number of students per group, offering students' groups different-level tasks (situations) and distributing one worksheet to each group of about 6 or 7 members, etc.);

Some possible proposals which can help mathematics teachers overcome the aforementioned obstacles are:

• Considering which types among students' group/pair working, individual working and the whole class discussion should be use;

• Encouraging low-performing students to participate in discussions;

• Paying attention to low-performing students in each group;

• Dividing students into a reasonable number of groups based on tasks or situations in RME-based lessons;

• Paying attention to the orders of the situations in RME-based lessons;

• Distributing worksheets to every individual student in each group if necessary;

• Reminding students not to make noise during their discussions.

5.3.1.2.3 Additional proposals

As discussed at the instruction of this dissertation, some materials about RME's philosophy and characteristics, as well as RME-based geometry lessons, were translated into Vietnamese, and the two teachers applying RME-based lessons were offered these materials. However, according to the teachers' comments, the two teachers applying RME-based geometry lessons in their teaching requested more materials about RME. Therefore, more materials, especially more examples of RME, should be translated into Vietnamese and provided to teachers who will apply RME-based lessons in their teaching.

As analyzed in section 4.2, 4.3, 4.4 and 4.5 of chapter 4, it appeared that formal mathematics was emphasized in the Vietnamese mathematics curriculum and textbooks. In RME-base lessons, the students tended to exploit formal mathematics in order to solve the problems although in some cases, it was not necessary. The feedback from the students' worksheets revealed that the students sometimes made mistakes with formal deductive reasoning. In addition, in some cases, the students were puzzled because they often thought that they had to apply (formal) deductive reasoning in solving the problems, while some situations of RME-based lessons encouraged them to discover informal mathematics.

Teachers who will apply RME-based lessons can explain the requirement about using formal mathematics to their students. In each situation of RME-based lessons, students should know whether they need deductive reason or not so that the situation does not confuse students.

As discussed in section 4.2, 4.3, 4.4 and 4.5 of chapter 4, the students sometimes encountered difficulties with the situations of RME-based lessons because they are often offered mathematical exercises with explicit tasks. In some cases, the two teachers, applied RME-based lessons in their teaching, did not clearly explain the requirements of the situations to their students for their impression of the pressure of teaching time.

When students do not fully understand the tasks in RME-based lessons, teachers should explain the tasks to their students and let them try to solve the problems, instead of reverting to conventional teaching methods.

5.3.2 Mathematics teachers' education

5.3.2.1 Teachers with substandard training

As discussed in section 1.3.5 of chapter 1, some parts of school teachers have not reached the training standards because of certain historic factors. It is difficult for these teachers to reform methods of teaching. Most of them are experienced but not well-trained teachers. Because they are already older, it is impossible for many of these teachers to obtain higher teaching qualification. However, it is also impossible to disregard them, since they played important roles in difficult periods of Vietnamese education. The MoET are trying to deal with this problem. RME-based lessons are not suitable for these middle school teachers.

5.3.2.2 Introducing RME and RME-based geometry lessons to in-service mathematics teachers

In Vietnam, primary and middle school mathematics teachers who graduated from colleges are able to participate in in-service courses often taught by lecturers in universities to earn higher degrees. In-service mathematics teachers participate in some courses about methods of teaching mathematics. Generally, contents of these pre-service courses are quite flexible. Therefore, it is possible to introduce RME and RME-based geometry lessons to pre-service middle school mathematics teachers.

These teachers often participate at full-time courses in specific time of the year. In-service middle school mathematics teachers are often offered methodology courses with 90-120 teaching periods. It is possible to introduce in-service middle school mathematics teachers to RME-based lessons in the methodology courses. The following will propose a possible amount of time, main contents, as well as suggested references of the introduction to RME-based lessons about 15-20 teaching periods in the methodology courses for in-service middle school mathematics teachers.

A mount of time for the methodology courses: 90-120 periods (each period lasts 45 minutes).
A mount of time for an introduction to RME-based lessons: about 15 periods to 20 periods.

After the introduction to RME-based lessons, in-service mathematics teachers should have general ideas about RME and the application of RME-based lessons in Vietnam. Moreover, they should apply RME-based geometry lessons to their teaching if possible.

Contents:

0. Survey (see the questionnaire form in appendix B) (1-2 periods)

1. Overview of RME theory (0.5 periods)

    1.1 Brief introduction of the Wiskobas project

    1.1 Brief introduction of some other related projects/ studies

Suggested references: Treffers (1987), De Lange (1987), Gravemeijer (1994) and Van den Heuvel-Panhuizen (1996).

2. Basic ideas of Freudenthal for RME (mathematics as a human activity, guided reinvention, didactical phenomenology) (1-2 periods)

    2.1 Mathematics as a human activity

    2.2 Guided reinvention

    2.3 Didactical phenomenology

Suggested references: Freudenthal (1973, 1983 and 1991).

3. Meaning of 'Realistic' in RME (1 period)

3.1 Mathematizing

3.2 Different approaches to mathematics education

3.3 ‘Realistic’ and ‘authentic’

Suggested references: Treffers (1987), Freudenthal (1991) and Jahnke (2005).

4. Principles of RME (2-3 periods)

4.1 The use of contexts

4.2 The use of models

4.3 The students’ own production and construction

4.4 The interactivity principle

4.5 The intertwining of mathematics strands

Suggested references: Treffers (1987), De Lange (1987), Freudenthal (1991) and Gravemeijer (1994, 2004).

5. RME-based geometry lessons (4-5 periods)

5.1 Overview of geometry curriculum and Vietnamese geometry curricula

5.2 Insufficiency of conditions for a Vietnamese RME curriculum

5.3 Foundations to design Vietnamese RME-based geometry lessons

5.4 Introduction to RME-based lessons

Suggested references: chapter 3 and appendixes C and D of this dissertation.

6. Advantages and disadvantages of applying RME-based geometry lessons in Vietnamese school (2 periods)

Suggested references: Nguyen Thanh Thuy (2005) and chapter 4 of this dissertation.

7. Proposals for applying RME-based lessons in Vietnamese school (1.5-2 periods)

7.1 Viewpoints on mathematics education

7.2 Conditions for applying RME-based lessons

7.3 Proposals to help mathematics teachers overcome possible difficulties

Suggested references: chapter 5 of this dissertation.

8. Design and apply RME-based lessons to other mathematics strands (2-2.5 periods)

Suggested references: Treffers (1987), De Lange (1987), Gravemeijer (1994), Van den Heuvel-Panhuizen (1996), Bakker (2004), Doorman (2005) and Nguyen Thanh Thuy (2005).

5.3.2.3 Prospective mathematics teachers

In general, Vietnamese mathematics teachers' training curricula often favor theory over practice. Pedagogical students often study theories in universities, whereas they do pedagogical practice (practicum) in school under guidance and control of school mathematics teachers and/or university supervisors. In the past, university lecturers took part in guiding and controlling students during practical time. However, due to a confluence of factors, particularly financial problems, recently, prospective teachers in some colleges and universities have been instructed by only senior school teachers during practicing time.[88] Senior school mathematics teachers tend to prefer prospective teachers who use teaching methods most often used in teaching in school. Because teaching methods in Vietnamese school are quite conventional and limited, trainee teachers are not encouraged in practicing new methods or approaches of teaching which they have studies in colleges and universities during practicing time in school. Consequently, prospective teachers cannot help using conventional teaching styles instructed by senior teachers in practicum. For this reason, they tend to utilize these styles when they become real teachers and work in school. This problem can be called *a cycle of underdeveloped teaching methods*.

In general, there is little chance for mathematics student teachers who are instructed by only senior school teachers to practice applying RME-based lessons during practicum time even though they have been introduced RME approach in colleges and universities. However, prospective mathematics teachers have opportunity to use RME-based lessons if they are given a secure environment in which they are guided by senior school mathematics teachers and lecturers in colleges and universities and are encouraged experiments with new approaches, including RME (see Nguyen Thanh Thuy, 2005, pp.154 & 179).

This policy has both advantages and disadvantages. The disadvantage is that prospective students do not have the opportunity to apply and practice what they study in universities.

5.3.2.3.1 Introducing RME and RME-based lessons

Typically, Vietnamese college or university curricula are quite fixed. In other words, students of a same academic discipline have to study same subjects (compulsory

[88] Hanoi University of Education and Hue University of Education are examples (see Hanoi University of Education, n.d.; Le Cong Chiem, n.d., pp. 9-10).

subjects) which are set by colleges, universities, or MoET, and normally, they may choose one selected subject. It is possible that RME is introduced as a theme of a selected subject. Normally, teaching methods in Vietnamese colleges and universities are still conventional, rigid and limited. There are theoretical lessons and practical lessons for most of subjects. In the theoretical lessons, lectures often give a lecture for from 100 to 200 students. In the practical lessons, there are a few students per class, and the students are asked to solve exercises to consolidate theories they have studied in the theoretical lessons. Recently, several Vietnamese universities have implemented experiments of using *a system of credits*. In these universities, students have chance to select suitable courses to study in each semester. This section discusses an introduction to a selected course about RME and RME-based lessons for prospective teachers.

A mount of time for a course about RME and RME-based lessons: about 30 periods to 45 periods.

After the course, students should understand RME theory (Freudenthal' viewpoints for RME and the characteristics of RME), know how to design RME-based lessons and to apply these lessons to their teaching.

Contents:

0. Survey (see the questionnaire form in appendix B) (2-3 periods)

1. Characteristics of Vietnamese mathematics education (4-6 periods)

    1.1 Characteristics of mathematics education in the 'reformed' period

    1.2 A necessary of a reform for mathematics education

    1.3 Some changes of mathematics education at present

Suggested references: Nguyen Ba Kim (2002); Pham Gia Duc et al. (1998) and several research papers in The Educational Review and The Educational Research (in Vietnamese).

2. RME theory (10-15 periods)

    2.1. Overview of RME theory (1 period)

        2.1.1 Brief introduction of the Wiskobas project

        2.1.1 Brief introduction of some other related projects/ studies

Suggested references: Treffers (1987), De Lange (1987), Gravemeijer (1994) and Van den Heuvel-Panhuizen (1996).

2.2. Basic ideas of Freudenthal for RME (2-3 periods)

2.1 Mathematics as a human activity

2.2 Guided reinvention

2.3 Didactical phenomenology

Suggested references: Freudenthal (1973, 1983 and 1991).

2.3. Meaning of 'Realistic' in RME (1- 2 period)

2.3.1 Mathematizing

2.3.2 Different approaches to mathematics education

2.3.3 'Realistic' and 'authentic'

Suggested references: Treffers (1987), Freudenthal (1991) and Jahnke (2005).

2.4. Principles of RME (4-5 periods)

2.4.1 The use of contexts

2.4.2 The use of models

2.4.3 The students' own production and construction

2.4.4 The interactivity principle

2.4.5 The intertwining of mathematics strands

Suggested references: Treffers (1987), De Lange (1987), Freudenthal (1991) and Gravemeijer (1994, 2004).

2.5 Examples about RME (2-4 periods)

3. Applying RME to teaching and learning mathematics in Vietnam (14-21 periods)

3.1 A way of applying RME in Vietnam (3-5 periods)

3.1.1 Insufficiency of conditions for a Vietnamese RME curriculum

3.1.2 Foundations to design Vietnamese RME-based lessons

3.1.3 Introduction to RME-based lessons

Suggested references: chapter 3 and appendixes C and D of this dissertation.

3.2. Chances and difficulties of applying RME-based geometry lessons in Vietnamese school (3-4 periods)

Suggested references: Nguyen Thanh Thuy (2005) and chapter 4 of this dissertation.

3.3. Proposals for applying RME-based lessons in Vietnamese school (3-5 periods)

3.3.1 Viewpoints on mathematics education

3.3.2 Conditions for applying RME-based lessons

3.3.3 Proposals to help mathematics teachers overcome possible difficulties

Suggested references: chapter 5 of this dissertation.

3.4 Designing RME-based lessons (5-7 periods)

Suggested references: Treffers (1987), De Lange (1987), Gravemeijer (1994), Van den Heuvel-Panhuizen (1996), Bakker (2004), Doorman (2005) and Nguyen Thanh Thuy (2005).

5.3.2.3.2 Creating a secure environment for teachers-in-training during practicum

Although teachers-in-training have studied new approaches, they tend to used conventional ones as they wished for reliable structure (Nguyen Thanh Thuy, 2005, pp. 148-151). They could use RME in their teaching thanks to encouragement of their university supervisor, Mrs. Thuy. Moreover, Mrs. Thuy also asked their school supervisors to give the student teachers permission for applying RME. It could be implemented because the school supervisors understand and have experience with new approaches since they participated in some projects on renovation of teaching methods. In general, student teachers are affected by school supervisors during practicum time. It is quite difficult for them to apply new approaches which they have studied in colleges or universities during practicum time. Consequently, they tend to use conventional teaching styles for their practicum lessons.

Generally, curriculum for teacher's training is not flexible in Vietnam. More specifically, for instance, at Hanoi University of Education, nearly all of subjects are compulsory for every student who wants to become a mathematics teacher (a student can choose only one subject between several available subjects in the third year of study). The same situations are found in most colleges and universities in Vietnam. Recently, a few universities have started to reform curricula by using a system of credits. In the near future, all Vietnamese universities will apply this new type of curriculum. It also grants university lecturers and students more latitude in teaching and learning.

5.4 Conclusion

In sum, applying RME-based geometry lessons needs long-term solutions. First, viewpoints of Vietnamese mathematics teachers are still rather conventional although there have been considerable changes of mathematics education in Vietnam. Mathematics can be taught in a meaningful way if some new viewpoints are accepted in Vietnam (mathematics as a human activity, guided reinvention, teaching applications in appropriate ways, emergent modeling, etc.). Aside from these adjustments' viewpoints, applying RME-based lessons become especially effective if some conditions such as the teachers' quality and competence, the amount of content in the present textbooks, the teaching time pressure, the teaching and learning facilities, etc. are satisfied. Finally, this chapter also discusses some possible solutions for training mathematics teachers so that they can apply RME-based lessons in their teaching.

Chapter 6 Conclusions and suggestions

6.1 Conclusions

Vietnamese education has been influenced by countries such as China, France, the U.S.A. and the former U.S.S.R. (Mo, 2003; Nguyen Thanh Thuy, 2005, pp. 3-5). It appears that the educational system in the last mentioned country, 'the eldest brother' of the former socialist system, had a particularly strong impact on Vietnamese education in the 'reformed' period. Fraser showed that the majority of Vietnamese researchers have studied in the former U.S.S.R.; the Vietnamese National Institute of Educational Science (NIES) in Hanoi had a structure, function and relationship with universities and other institutions that was quite similar to those of the U.S.S.R. Academy of Pedagogical Sciences (APS) in Moscow; and viewpoints of well-known Soviet educational researchers influenced their Vietnamese counterparts (1984, pp. 78 & 80-81). Moreover, education in Vietnam was influenced by consequences of the wars (Pham Minh Hac, 2002; Bui Minh Hien, 2005; Nguyen Thanh Thuy, p. 3). Vietnam's 'open' economic policy, started in 1986, has also affected its education system (Nguyen Thanh Thuy, 2005, p. 3). Finally, cultural factors have made a significant impact on Vietnamese education.

Recently, according to the former Minister of Education and Training, Nguyen Minh Hien, Vietnamese education has faced some serious problems, such as low quality of education, abuse of extra-classes, 'abuse of achievements'[89] and negatives in education (see, for instance, Thanh Ha, 2005). To some extent, mathematics education has also been affected by these factors.

This section discusses some conclusions related to the current situation of mathematics education, specifically the teaching and learning of geometry and the proposals for applying RME-based lessons.

6.1.1 Vietnamese mathematics education reconsidered

Recently, in comparison with the 'reformed' period, along with the education reform, mathematics education has significantly changed in all levels from primary through high school. As discussed in the introduction to this dissertation, the new series of mathematics textbooks has been used for all grades in primary and middle school. In

[89] 'Abuse of achievements' means an excessive attaching much important to *good* data of students' results. In other words, data in reports on education are extremely better than the fact.

addition, a new grade-ten mathematics textbook has been used since the school-year 2006-2007. In general, mathematics education has undergone noticeable improvements; however, it has faced many challenges. Generally, the structruralist and the mechanistic approaches still dominate mathematics education in Vietnam (see the distinction of the four different approaches in mathematics education in Treffers, 1987, p. 251 and section 2.3.2 of chapter 2). In other words, mathematics education in Vietnam is still quite conventional and rigid.

6.1.1.1 The current mathematics curricula and textbooks

The current mathematics curricula and textbooks still place strong emphasis on structures of mathematics and formal mathematics, although they do demonstrate some adjustment in the material's presentation order, as well as some deletion of complicated content. 'Uniqueness' is a characteristic of the current mathematics textbooks. That is, beyond the uniqueness of the curricula and textbooks as a whole, typically, a unique situation with a unique instruction or strategy is introduced to students at the beginning of each lesson. In comparison with the 'reformed' mathematics curricula and textbooks, the present ones pay more attention to pedagogic phase. More specifically, many long complicated contents are deleted in the current mathematics curricula and textbooks. Secondly, requirements of mathematical structures are slightly reduced. Thirdly, students are often introduced tasks at the beginning of each lesson before the formal mathematics (definitions, theorems, regulations and formulae). In addition, mathematics applications are paid special attention in the current textbooks. Finally, they have been designed to look more attractive to students, with figures, pictures, stories, events and so forth.

However, there are still many challenges for the current mathematics curricula and textbooks. The following paragraphs reconsider some of them.

Firstly, as discussed previously in section 1.3.1.1 of chapter 1, Vietnamese students' competence varies from urban to remote areas for many reasons, especially teaching and learning conditions and quality of teachers. According to the feedback from the teachers' survey, some teachers in rural areas complained that some material in the current mathematics textbooks are still difficult for their students (see the questionnaire form in appendix B)[90] although they admitted to a noticeable reduction of complicated theoretical content in these textbooks. This is also confirmed by a

[90] Almost no middle school mathematics teachers from remote areas were interviewed.

recent national study about the present primary and middle school curricula and textbooks of the National Institute for Education Strategy and Curriculum Development (NIESAC) (Nhan Dan, 2006). Consequently, some questions should be answered:

• Are the unique mathematics curricula and textbooks suitable for teaching and learning conditions in different areas in Vietnam?
• How to deal with the aforementioned problem if it still exists?

Secondly, as discussed above in section 1.3.1.2 of chapter one, it appears that there is the imbalance between the amount of content in the present mathematics textbooks and the quantity of time teachers and students have in school. If teachers and students suffer from the excessive time pressure, it is quite difficult for them to change their conventional teaching and learning styles. Therefore, it is worth researching the relation between the factors of allotted time versus the degree of instructional adjustment.

Thirdly, in comparison with the ‘reformed’ mathematics curricula and textbooks, the present ones have made many improvements; however, Vietnam may still lack the necessary philosophical outlook, scientific foundation and pedagogical theory necessary to further improve mathematics curricula and textbooks. Furthermore, there remain some questions related to situations in which students are expected to discover formal mathematics knowledge and the way mathematics applications are presented in the current textbooks (see, for example, section of 3.2.2.3 chapter 3). Finally, questions about the effect of tasks with *unique* instruction in these textbooks have yet to be answered (see some mathematics lessons in appendix A). More specifically, the question must be answered, are students bored with such unnatural repeated situations, in which they are required to follow word-for-word instructions in the textbooks in order to achieve results?

6.1.1.2 Teaching styles

In general, aside from the ‘teacher explains, students listen and write down’ style sufficiently described in section 1.3.2.1 (chapter 1), the mechanistic approach is still a favorite style in teaching and learning mathematics (see, for example, MoET, 2000 and section 5.1.2 of chapter 5). More specifically, mathematics teachers often try to help their students memorize formal knowledge (concepts, theorems, regulations and formulae) so that they can then apply this knowledge to solve mathematical problems.

Moreover, students are frequently given different forms of mathematics problems with solutions; they are then expected to remember these forms and solve similar problems with previously given ones. They are often equipped with sufficient tricks to pass and get good results on mathematics examinations, rather than actually taught the material. It appears that the pressure of examinations has resulted in this teaching style in Vietnam.

6.1.1.3 Examinations and assessment

In the 'reform' period, the main tool used to assess students' learning was written examination. Recently, tests have been researched and gradually used in mathematics examinations. Lately, a number of studies related to assessment have slightly increased (see, for instance, Nguyen Ba Kim, 2002, pp. 301-333; Institute for Education Research, 2004; Tran Kiem Minh, 2006, pp. 20-22). The MoET (2001, 2002 a & b) also discussed an innovation on examinations and assessment in mathematics. However, more studies on examinations and assessment should be conducted, and it is necessary to apply these studies on teaching and learning mathematics in Vietnam. In addition, it appears that although there have been studies on forms of examinations and assessment, there is lack of research about the contents of examinations and assessment. More specifically, assessments use different forms (e.g. written examination, tests, etc.) to assess how students remember and apply formal mathematics.

6.1.1.4 Competence of teachers

As discussed in section 1.3.5 of chapter 1, some parts of Vietnamese teachers are still fall below the training standards. In general, it is rather difficult for these teachers to implement the teaching method innovation which have been launched by the MoET. In addition, it does not mean that the standard-trained teachers satisfy the requirements of working with the current curricula and textbooks. A recent study of the National Institute for Education Strategy and Curriculum Development (NIESAC) reveals that about 60% of primary teachers and 70% of middle school teachers have encountered difficulties with the present curriculum in working with the present curricula and textbooks (Nhan Dan, 2006). Generally, there are still some questions related to competence of teachers in Vietnamese school, especially those in rural and remote areas.

6.1.1.5 Some other factors

Although the MoET and the Vietnamese government have implemented many policies to upgrade conditions of teaching and learning in Vietnam, there are still many challenges for the education innovation in Vietnam. Aside from the factors in sections 6.1.1.1, 6.1.1.2, 6.1.1.3 and 6.1.1.4, this section discusses some other factors related to the renovation of teaching methods in Vietnam (see characteristics of mathematics education in Vietnam in chapter one; analyzing the teachers' comments and students' worksheets in chapter four; the feedback of the interviewed teachers, Chapman & Adams, 1998, pp. 645, 648 & 658; Nguyen Thanh Thuy, 2005, pp. 6-8; Nhan Dan, 2006):

- Large quantity of teachers' work;
- Low salary for teachers;
- Insufficient or poor quality of teaching facilities;
- Teachers' difficulties with using modern teaching equipment such as overhead projectors, beamers and computers;
- Insufficient teacher guidebooks;
- High number of students in a class;
- Low and/ or varying levels of students' competence[91];
- Unaccommodating classrooms' structures for renovation of teaching and learning styles;
- Pressure of examinations and high result expectation.

6.1.2 The way of applying RME in teaching and learning mathematics in Vietnam (answers to sub-questions 1 and 5)

This section discusses a possible way for applying RME in teaching and learning mathematics in Vietnamese school, in general and in middle school, in particularly. It answers to sub-questions 1 and 5 in sections 1.5.2.1 and 1.5.2.5, chapter 1.

Sub-question 1: *How can RME be applied in teaching and learning grade-seven geometry in Vietnamese school?*

[91] Recently, there have been some newspapers' articles about a few students who finished grades 4, 5, 6, and 8 in some different areas in Vietnam cannot read, write and carry out very simple mathematics operations (see, for example, Duong The Hung, 2005; Minh Giang, 2005; Tran Dang, 2005; Ngoc Long, 2006). This fact probably is the consequence of what is called a 'disease of results' (see the brief description at the beginning of section 6.1 of this chapter).

Sub-question 5 is: *How often should RME be implemented in teaching and learning in Vietnamese middle school?*

As discussed in section 3.1.2.3 of chapter 3, it is impossible, at least in the near feature, to apply a RME curriculum in Vietnamese school because of the characteristics of mathematics education and the regulations relating to teaching and learning in Vietnam. However, RME-based lessons can be potentially applied in teaching and learning in school. The foundations for designing these RME-based lessons were discussed in section 3.2 of chapter 3. In addition, mathematics teachers should, themselves, understand the philosophy and the basic principles of RME if they wish to apply it to their teaching. More materials about RME should be introduced in Vietnam.

Based on the situation of Vietnamese mathematics education (chapter one) and the feedback analysis from the experiment of RME-based geometry lessons (chapter four), it is not reasonable to expect teachers to replace all normal lessons in the current mathematics textbooks by RME-based ones.

6.1.3 The potential of RME for mathematics education in Vietnam (answers to sub-question 3)

In general, RME is a promising approach for Vietnamese mathematics education. Its philosophy is appropriate to the orientation for the teaching method innovation discussed in section 1.4.2 of chapter 1. This section reconsiders some potentiality of RME for mathematics education in Vietnam to answer sub-question 3 in section 1.5.2.3 of chapter 1.

Sub-question 3 is: *What is the potential of RME, and how can this potential help mathematics education in Vietnam overcome its shortcomings?*

As discussed in section 1.3.2 of chapter one, generally, teaching and learning styles have been rather conventional, basic and rigid in Vietnamese school. For this reason, a reform of teaching methods has become necessary. The orientation of the renovation was described by MoET (2002 a) and Nguyen Ba Kim (2002, pp. 110-120). The philosophy of RME is quite suitable for this orientation. There is a need for long-term solutions for applying RME in mathematics education in Vietnam.

However, as discussed in section 3.1.2.3 of chapter 3, it is impossible to apply a RME curriculum to current teaching and learning in Vietnam because of the reasons related to its present regulations and facts of education as well as mathematics education. As described in section 1.3.1 of chapter 1, the pedagogical phase was not sufficiently emphasized in the 'reformed' mathematics textbooks. Recently, both the form and content of the present mathematics textbooks have noticeably changed (section 1.4.1 of chapter 1). Particularly, in middle school geometry lessons, students are asked to do some manipulations (drawing, cutting, measuring, folding, etc.) to recognize geometrical characteristics before formal mathematics is presented. In addition, there is a variety of mathematics application problems in the present mathematics textbooks. Nonetheless, the way in which manipulations and mathematics applications are used is still quite primitive and conventional and has some disadvantages (see section 3.2.2.3 of chapter 3). Moreover, there are some questions related to the effect of a *unique* manipulation situation at the beginning of a lesson (see section 6.1.1.1 of this chapter). According to Gravemeijer's notions of modeling (2004), the use of manipulations and applications in the current mathematics textbooks belongs to "didactical modeling" and "mathematical modeling", respectively. Although he confirms the role of each modeling type in mathematics education, he argues that emergent modeling should be "a precursor to mathematical modeling" (Gravemeijer, 2004, p. 97). Although a RME curriculum, at least in its current form, is not applicable to Vietnamese mathematics education, lessons called RME-based ones, in line with emergent modeling, can be used in teaching and learning mathematics in Vietnam. Chapter three discussed the foundations for creating (Vietnamese) RME-based geometry lessons. Chapter 4 analyzed the feedback of the teachers and students while they were working with these lessons.

Although the teachers and students still struggled with RME-based geometry lessons, there were still some positive outcomes of using RME-based lessons. These positive outcomes could potentially outweigh the potential disadvantages of the lessons from the current textbooks. Firstly, unlike the lessons in the present textbooks, students do not completely follow the strict instructions in RME-based ones. On the contrary, they are encouraged to build up gradually from informal to formal knowledge. Secondly, students' activeness and creativeness are developed during RME-based lessons. Thirdly, the situations in RME-based lessons allow students to rediscover mathematics knowledge. Finally, most students actively participated in their discussions. They had the opportunity to express and explain their ideas, accept or reject some other ideas.

6.1.4 The difficulties of applying RME-based lessons in teaching and learning in Vietnam (answers to sub-question 2)

Teachers and their students may encounter certain difficulties with RME-based lessons because of several reasons. Some of these reasons are related to the conventional habit of teaching and learning mathematics in Vietnam may result in challenges teachers face while working with not only RME-based lessons but also other non-traditional approaches. The following paragraphs reconsider certain factors such as the teaching time pressure, the students' working in groups, the teachers' competence and others to answer sub-question 2 in section 1.5.2.2 of chapter 1.

Sub-question 2 is: *What difficulties do teachers and students meet while RME is applied in teaching and learning middle school geometry?*

As discussed in section 3.2.1 of chapter 3, length of time and timetables for each mathematics lesson are often stipulated by a curricular distribution of the local department of education and training in a province or a city. It is important to note that students are firstly taught formal mathematics (concepts, theorems, regulations and formulae) and are expected to apply it in solving either pure or applicable mathematics problems. In contrast, in RME-based lessons, students are situated in the situations which encourage them to rediscover from informal to formal mathematics. While working with RME-based lessons, teachers may assume that these lessons require much more time than conventional lessons. Contrary to these teachers' belief, however, teachers can reduce mathematics application problems when their students acquire knowledge with RME-based lessons.

In addition, traditionally, Vietnamese students rarely ever worked in groups. Recently, the MoET has encouraged teachers to use and promote group-work in their lessons. Due to a number of factors, such as that the unsuitability of classrooms' and student-tables' structures, the students' behavior (i.e. noise-levels), the inactivity of low-performing students, the high number of students in a class, the teachers' uncontrollability, the inappropriate organizing students' work in groups (see section 4.6.1 of chapter 4), it is difficult for these teachers to implement this method successfully.

Finally, teachers and students may face some other obstacles with RME-based lessons. Firstly, teachers and students do acclimate to these lessons. Teachers often tend to introduce formal mathematics quickly to their students. Students usually try to utilize formal mathematics to solve problems even though it is not necessary because of over-emphases on deductive formal mathematics in mathematics curricula and textbooks in Vietnam. Secondly, teachers often hesitate to reform their teaching styles because of their unwillingness to change their habit and high quantity of their work. For example, the teachers who used RME-based geometry lessons complained that they had to expend too much time and energy preparing these lessons. Moreover, students may encounter difficulties with some first lessons because they are not accustomed to working with 'indirect' tasks.

6.1.5 The proposals for applying RME-based lessons (answers to sub-question 4)

As discussed previously in section 6.1.1 of this chapter, in general, mathematics education is still rather conventional and rigid in Vietnam. Applying RME as well as RME-based lessons needs long-term solutions although RME is a promising approach for mathematics education in Vietnam. This section reconsiders the proposals for applying RME-based lessons about the viewpoints on mathematics education, the balance between amount of content and quantity of time, the training mathematics teachers, the materials about RME and RME-based lessons and several other factors to answer to sub-question 4 in section 1.5.2.4 of chapter 1.

Sub-question 4 is: *What and how proposals should be made so that RME can be applied in teaching and learning in Vietnam?*

Firstly, non-traditional, more progressive viewpoints should be accepted and advocated by mathematics educators, curriculum creators, textbook authors and teachers in Vietnam. Some of RME's outlooks, such as *mathematics as a human activity, guided reinvention, didactical phenomenology, using contexts in mathematics lessons versus conventional way of teaching mathematics applications, emergent modeling, using informal mathematics*, and other RME outlooks can help Vietnamese mathematics education overcome its weakness (see section 5.1 of chapter 5).

Secondly, the balance between the amount of content in mathematics textbooks and the quantity of time for teaching and learning should be established, and mathematics teachers should be given necessary flexibility. It is quite difficult for teachers to innovate their conventional teaching styles if they are always suffering from the teaching time pressure. In addition, a regulation of a current mathematics curricular distribution appears inappropriate. Although a regulation of the current mathematics curricular distribution is necessary, it should not strictly fix length of time for each mathematics lesson. However, mathematics teachers should be highly competent and responsible for their work if the flexibility is allowed to them.

Thirdly, one of the necessary tasks for applying RME-based lessons is training pre-service and in-service mathematics teachers. These lessons can be introduced in methodology courses for prospective mathematics teachers. In Vietnam, there are two different training types in universities and colleges. In most of them, students have to no choice but to adhere rigidly to the fixed curricula. In contrast, a few of colleges and universities use a system of credits, and their students have a chance to choose suitable subjects they want to study in each semester. Recently, the MoET have encouraged universities and colleges to change from the first style to the second one. In general, it is easier to introduce new approaches, in general and RME-based lessons, in particularly to those who use a system of credits. For universities or colleges whose student teachers are supervised by only seniors teachers in school, it seems to be quite difficult for student teachers to practice teaching with non-traditional approaches, in general and with RME-based lessons, in particularly (see discussion about 'a cycle of underdeveloped teaching methods' in section 5.3.2.3 of chapter 5). For the others, trainee teachers should be given a 'secure environment' when they try to apply non-traditional approaches in teaching (see Nguyen Thanh Thuy, 2005, pp. 154 & 179). In addition, RME-based lessons can be introduced in-service mathematics teachers when they attend in-service course to get higher degrees.

Finally, some other conditions related to teaching and learning facilities, a number of students in a class, a quantity of teachers' work, teachers' reference books, etc. should be gradually improved.

6.2 Suggestions

This section, firstly, discusses some restrictions of a range of the dissertation. Next, it considers some suggestions for further studies related to Vietnamese mathematics education. As discussed previously, it is argued that more studies should be conducted so that RME can be applied widely in teaching and learning mathematics in

Vietnamese school. For this reason, finally, this section gives some general orientations for further studies on applying RME in Vietnam.

6.2.1 Restrictions of the dissertation

This dissertation mainly emphasizes teaching and learning geometry in Vietnamese middle school, especially grade 7. In addition, some RME-based geometry lessons were taught by only the two young mathematics teachers. Furthermore, these teachers applied these lessons in their teaching once in the school-year 2005-2006. The following paragraphs give more related discussions and explanations.

Firstly, it is necessary that there is a restriction for the dissertation. Typically, deductive formal Euclidean geometry is mainly taught in middle school (grades 6 to 9) in Vietnam. Grade-seven students begin learning formal geometry theorems and their proofs. However, formal geometry is taught so early, quickly, formally and rigorously that it becomes the most difficult topic for middle school students. Recognizing this problem, mathematics curriculum creators and textbook authors have reduced significantly complicated theoretical contents in the current series of middle mathematics curriculum and textbooks. Furthermore, students are expected to work with manipulations before formal geometry is formally introduced. However, the real effect of manipulation uses in teaching and learning geometry is still not entirely clear (see section 3.2.2.3 of chapter 3). For this reason, the author of this dissertation restricted himself a study on teaching and learning middle school geometry. More particularly, grade-seven geometry is emphasized in the research.

Secondly, RME-based geometry lessons could not offer different contexts like the (Dutch) geometry curriculum because of the significant differences between Vietnamese mathematics lesson mechanism and Dutch RME curricula. More specifically, as reconsidered in section 6.1.3 of this chapter, the current Vietnamese mathematics lessons often introduce a unique situation with a unique instruction to students at the beginning of each lesson; after that formal mathematics is quickly presented. In general, the current mathematics textbooks often impose these situations on students. On the contrary, in RME curriculum, students are often introduced different realistic contexts which can be built based on Freudenthal idea of didactical phenomenology. These realistic contexts allow students to rediscover informal strategies or solutions. Moreover, the scope of this dissertation does not expand to include assessment within RME-based lessons.

Furthermore, although RME-based lessons were offered to several mathematics teachers in several middle schools, only the two young teachers were willing to use them as an experiment, for several reasons. First, some senior ones did not want to change their teaching habit. They complained that they had a work overload, and they had no time to apply new approaches in their teaching. Next, some teachers rejected the proposals since they were not allowed by their headmasters, or they did not want to 'break' the teaching regulations. Finally, some of them argued that these RME-based lessons would require too much time. Fortunately, the two junior teachers with one- and six-year experience agreed to use these lessons as an experiment. They are quite eager to learn new approaches. The author of the dissertation met them several times to discuss RME and RME-based lessons during his visit to Vietnam in 2005. Nevertheless, both teachers complained that they were so occupied that they could concentrate on a RME-based lesson only a few days before it was implemented. The author asked them to maintain contact via email. However, Mr. Mui, a six-year experience teacher, did not have an email address and therefore never used email. Although the other teacher, Ms. Huong, has her own an email address, she rarely used it.

Finally, the two teachers used RME-based geometry lessons once. Although the authors of this dissertation wanted them to apply these lessons at least once more time with adjusted RME-based lessons, it was not implemented. The first version of RME-based geometry lessons could not create earlier as RME was quite new for not only mathematics educators in Vietnam but also the author of this dissertation. Moreover, applying RME-based lessons also depended on a timetable of teaching and learning in a school-year in Vietnam. In addition, the author of the dissertation was granted a three-year scholarship from the government of Vietnam. For this reason, it was impossible for him to organize the second experiment.

6.2.2 Mathematics education in Vietnam

Firstly, in general, students' competence varies significantly among not only different areas (urban, rural, mountainous and islandish areas) but also among different schools in the same area, different classes in a school and different students in a class. In addition, conditions for teaching and learning (infrastructures and teachers' competence) in urban areas are better than those in rural areas and much better than those in remote areas. It should be stressed that there is the unique series of mathematics textbooks in Vietnam. Although recently, many complicated contents have been removed from the current mathematics textbooks, their suitability for the majority of Vietnamese students is still questionable. Consequently, it is worth researching the appropriateness of the present mathematics curricula and textbooks for students' competence.

Secondly, it appears that there continues to be an imbalance between the quantity of content in the present mathematics curricula and textbooks and the quantity of time teachers and students can spend in school. For this reason, studies on the relationship between the quantity of time and the quantity of content should be conducted.

Thirdly, according to the feedback from the survey, students often encountered difficulties with drawing figures, logically reasoning and deductive proving of geometry. The current mathematics textbooks still emphasize deductive formal geometry. One question is posed: is deductive formal geometry in the textbooks suitable for most middle school students, especially those in rural, mountainous and islandish areas? There is a clear need for further research on the relation between geometry content in the current textbooks and levels of students' awareness. Moreover, the scope of this dissertation does not include a sufficient discussion of teaching formal geometry, particularly deductive proofs.

Fourthly, more studies should be conducted on teaching and learning mathematics in Vietnamese school, mathematics teachers' difficulties in innovation of teaching styles, mathematics teachers' competence and solutions for renovation of teaching methods.

Finally, although a number of studies on examinations and assessment have slightly increased, there is a need for assessing students' activities in a mathematics forming process. That is, aside from formal mathematics, informal mathematics also should also be seriously considered in assessment.

6.2.3 Realistic Mathematics Education

The application of RME in Vietnamese school requires long-term solutions as RME is quite new for mathematics educators, curriculum creators, textbooks authors and teachers, and teaching and learning mathematics are still rather rigid in Vietnam. This section suggests some general orientations for further studies on applying RME to teaching and learning mathematics in Vietnam.

Firstly, RME-based geometry lessons should be tested several times and upgraded so that they can be widely applied to teaching and learning mathematics in Vietnam although these lessons revealed some potential for enriching Vietnamese mathematics education, and several proposals were discussed to help mathematics teachers encounter difficulties with these lessons.

Secondly, there is a need for further research on creating RME-based lessons with other content (arithmetic, algebra, analysis, statistics, probability, analytic geometry, etc.) in different grades (primary, middle and high school) in Vietnam. This dissertation is restricted to some RME-based lessons for grade 7 because of its range and the limited time, the regulations of teaching and learning in Vietnam, and the unfamiliarity of these lessons to Vietnamese mathematics teachers.

Thirdly, studies related to assessment with RME-based lesson should be implemented. Typically, assessment in Vietnamese school often focuses on assessing the students' abilities of memorizing, understanding, and applying formal mathematics. Recently, the number of studies on assessment has significantly increased in Vietnam. However, it appears that most of these studies usually emphasize the form of assessment. There are a few studies which assess the students' abilities of developing from informal to formal mathematics.

In addition, it is worth studying further the application of Freudenthal's viewpoints on RME, the characteristics of RME and the studies about RME in creating Vietnamese mathematics curricula and textbooks. It appears that there is a lack of philosophies, scientific foundations, and theories for designing mathematic curricula and textbooks in Vietnam. RME may become a promising theory which can be utilized to design mathematics curricula and textbooks.

Next, further studies about the application of RME-based lessons and RME in training mathematics teachers-in training and in-service teachers should be conducted. This study mentioned and discussed several possible courses of RME-based lessons which can be offered mathematics prospective teachers and in-service teachers. However, these courses should be tested, and feedback from prospective teachers and in-service teachers should be carefully analyzed.

Finally, studies on designing and applying RME-based lessons for low-performing students are worth conducting. As discussed previously, achievements of Vietnamese students vary significantly. Typically, students from urban regions have higher achievements than those from rural and remote regions. RME-based lessons for low-performing students can be designed and used because RME-based lessons often offer students the chance to use informal mathematics. By contrast, usual lessons introduced formal mathematics so early and quickly that low-achieving students often encounter difficulties with these lessons.

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Appendix

Appendix A: Some seventh- grade geometry lessons (for 12-year-old students) in the 'reformed' and current textbooks

Nr.	Name of lesson
1	The Triangle Sum Theorem
2	Characteristics of the bisector of an angle
3	Characteristics (property) of the perpendicular bisector (mid-perpendicular) of a segment
4	The Triangle Inequality

A.1 The Triangle Sum Theorem

A.1.1 The Triangle Sum Theorem (the Triangle-Angle Sum Theorem) in the 'reformed' textbook

This lesson is translated from the 'reformed' textbook entitled *Geometry 7* (Nguyen Gia Coc & Pham Gia Duc, 1996, pp. 36-37).

1. The Theorem

The sum of the measures of the three interior angles of a triangle is 180^0.

Given: ΔABC

Prove: $m\angle A + m\angle B + m\angle C = 180^0$.

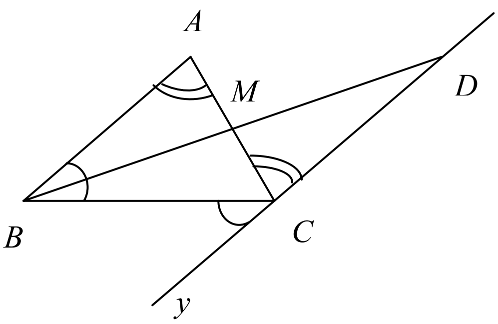


Figure A.1.1

*Proof (*figure A.1.1*)*:

Suppose that M is the midpoint of $\overline{AC}$; D is a point on $\overrightarrow{BM}$ such that M is the midpoint of $\overline{BD}$.

$\Delta\ ABM$ and $\Delta\ CDM$ are congruent (SAS Congruence Postulate). From this, it follows that $m\angle A = m\angle ACD$. Hence, $\overleftrightarrow{AB}$ and $\overleftrightarrow{DC}$ are parallel.

Suppose that $\overrightarrow{Cy}$ is the opposite ray of $\overrightarrow{CD}$. We have $m\angle B = m\angle BCy$ (Two alternate interior angles).

However, $m\angle BCy + m\angle BCD = 180^0$ (1).

Moreover, since $\overrightarrow{CA}$ is somewhere between $\overrightarrow{CB}$ and $\overrightarrow{CD}$, $m\angle BCD = m\angle BCA + m\angle ACD$ (2).

From (1) and (2), we obtain: $m\angle BCy + m\angle BCA + m\angle ACD = 180^0$.

Since $m\angle BCy = m\angle B$ and $m\angle ACD = m\angle A$, $m\angle A + m\angle B + m\angle C = 180^0$.

2. Problem: Prove that every triangle cannot have two right angles, or one right angle and one obtuse angle, or two obtuse angles.

Proof:

If a triangle has two right angles, or one right angle and one obtuse angle, or two obtuse angles, then the sum of its three angles is greater than 180^0. This contradicts the above theorem.

3. Consequence

For every right triangle, the sum of the two acute angles is 90^0.

Given: $\Delta\ ABC$ and $m\angle A = 90^0$

Prove: $m\angle B + m\angle C = 90^0$.

Proof:

We know that $m\angle A + m\angle B + m\angle C = 180^0$. Since $m\angle A = 90^0$, $90^0 + m\angle B + m\angle C = 180^0$. Hence, $m\angle B + m\angle C = 180^0 - 90^0 = 90^0$.

Note: Two angles whose measures' sum is 90^0 are called complementary angles.

4. An exterior angle of a triangle

In figure A.1.2, $\angle ACD$ and $\angle C$ of $\Delta\ ABC$ are linear pair of angles. $\angle ACD$ is called an *exterior angle at vertex* C of $\Delta\ ABC$.

$\angle\ A$ and $\angle B$ are called *the interior angles that are not adjacent to the exterior angle*.

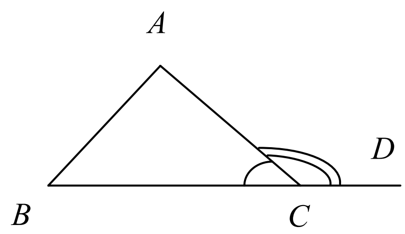


Figure A.1.2

Theorem

Each exterior angle of a triangle is equal to the sum of two interior angles that are not adjacent to this exterior angle.

Proof:

$m\ \angle ACD + m\angle C = 180^0$ (1) (a linear pair of angles).

$m\angle A + m\angle B + m\angle C = 180^0$ (2) (the sum of the three angles in a triangle).

From (1) and (2), it follows that $m\angle ACD = m\angle A + m\angle B$.

Analogously, it can be proved for the exterior angles at vertexes *A* and *B*.

5. Theorem

If two sides of one angle are perpendicular, respectively, to two sides of another angle, then: a) The angles are equal if both of them are the obtuse angles or the acute angles b) The angles are supplementary angles if one of them is an obtuse angle and the other is the acute angle.

A.1.2 The triangle sum in the present textbook

This lesson is translated from the current textbook entitled *Mathematics 7: part 1* (Phan Duc Chinh, Ton Than, Vu Huu Binh, Pham Gia Duc & Tran Luan, 2004 a, pp. 106-107).

1. The sum of the three angle measures in a triangle

1. Theorem

? 1

Draw two triangles, measure the three angles and calculate the sum of the angles' measures for each triangle. Have you got any remark on the results?

? 2 Practice: *Use a board to cut Δ ABC. Cut out $\angle ABC$ and put it such that it becomes an adjacent angle of $\angle BAC$; then cut out $\angle ACB$ and put it such that it is an adjacent angle of $\angle BAC$ (figure A.1.3). Predict the sum of $m\angle BAC$, $m\angle ABC$ and $m\angle ACB$.*

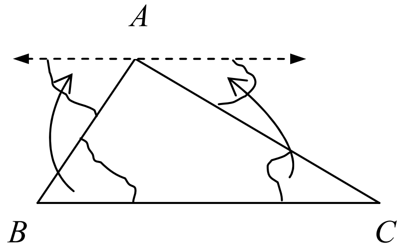


Figure A.1.3

We have the following theorem: *The* sum *of the measures of the three interior angles of a triangle is 180^0.*

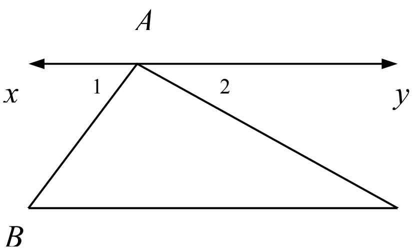


Figure A.1.4

Given	$\Delta\ ABC$
Prove	$\angle BAC + \angle ABC + \angle ACB = 180^0$

Proof: (figure A.1.4)

Let $\overleftrightarrow{xy}$ be a line that contains point A and is paralleled to $\overline{BC}$.

$m\angle ABC = m\angle A_1$ (1) (Two alternate interior angles).

$m\angle ACB = m\angle A_2$ (2) (Two alternate interior angles).

From (1) and (2), we have:

$m\angle BAC + m\angle ABC + m\angle ACB = m\angle BAC + m\angle A_1 + m\angle A_2 = 180^0$.

Note: Conventionally, we call sum of two angle measures sum of two angles. Analogously, we call difference of two angle measures difference of two angles.

2. Application in a right triangle

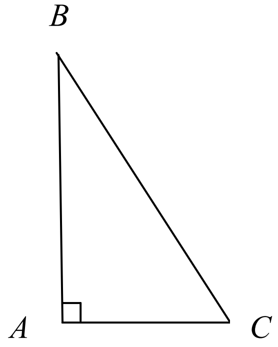


Figure A.1.5

Definition: A right triangle is a triangle that has one right angle.

In figure A.1.5, ΔABC has $m\angle A = 90^0$. We speak: right ΔABC at A; $\overline{AB}$ and $\overline{AC}$ are called the legs; and $\overline{BC}$ is called the hypotenuse of this right triangle.

? 3 Given a right triangle at A. Calculate $m\angle B + m\angle C$.

We have a theorem:

In a right triangle, the two acute angles are complementary angles.

ΔBAC, $m\angle A = 90^0 \Rightarrow m\angle B + m\angle C = 90^0$.

3. An exterior angle of a triangle

Definition: *An exterior angle of a triangle is an angle that and one angle of the triangle are a linear pair of angles.*

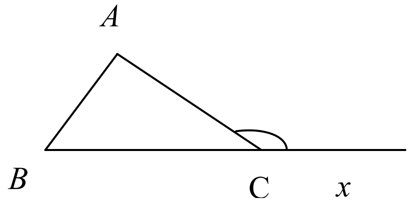


Figure A.1.6

In figure A.1.6, $\angle ACx$ is an exterior angle at vertex C of ΔABC. $\angle A$, $\angle B$, and $\angle C$ are called the interior angles of ΔABC:

? 4 Fill in the following blanks (…), and compare $m\angle ACx$ and $m\angle A + m\angle B$.

Since the sum of the three angle measures of ΔABC is 180^0, $m\angle A + m\angle B = 180^0$ -...

Because $\angle ACx$ is the exterior angle of the triangle, $m\angle ACx = 180^0$ -...

We have a theorem about a characteristic of an exterior angle of a triangle:

Each exterior angle of a triangle is equal to the sum of two interior angles that are not adjacent to the exterior angle.

Comment: In a triangle, the measure of an exterior angle is greater than the measure of each interior angle that is not adjacent to this exterior angle.

$$m\angle ACx > m\angle A,\ m\angle ACx > m\angle B \text{ (figure A.1.6).}$$

A.2 Characteristics of the bisector of an angle

A.2.1 In the 'reformed' textbook

This lesson is translated from the 'reformed' textbook entitled *Geometry 7* (Nguyen Gia Coc & Pham Gia Duc, 1996, pp. 76-77).

1. Characteristic of the bisector of an angle

Theorem

A point that on the bisector of an angle is equidistant from the two sides of the angle

Given: M is on the bisector ray of $\angle\ xOy$

$\overline{MA} \perp \overrightarrow{Ox}$, $\overline{MB} \perp \overrightarrow{Oy}$

Prove: $MA = MB$

Proof: Since right $\Delta\ MOA$ and $\Delta\ MOB$ have the common hypotenuse $\overline{OM}$ and $m\angle MOA = m\ \angle MOB$, these triangles are congruent. Hence, $MA = MB$.

Theorem

If a point that is in the interior of $\angle xOy$ is equidistant from its two sides, then this point is on the bisector ray of the angle.

Given: M is in the interior of $\angle xOy$; $\overline{MA} \perp \overrightarrow{Ox}$; $\overline{MB} \perp \overrightarrow{Oy}$; $MA = MB$.

Prove: $m\angle MOA = m\angle MOB$

Proof: (figure A.2.1)

Since two right $\Delta\ MOA$ and $\Delta\ MOB$ have common hypotenuse $\overline{OM}$ and $MA = MB$, these triangles are congruent. Therefore, m$\angle MOA$ = $m\angle MO$B.

This is what must be proved.

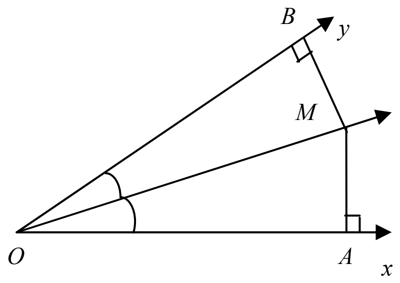


Figure A.2.1

The bisector ray of an angle is the locus of all points that are in the interior of this angle and equidistant from its two sides.

A.2.2 In the present textbook

This lesson is translated from the current textbook entitled *Mathematics 7: part 2* (Phan Duc Chinh et al., 2004 b, pp. 68- 69).

1. The theorem

a) Practice

Using paper, cut $\angle xOy$. Fold $\angle xOy$ such that $\overrightarrow{Ox}$ lies on $\overrightarrow{Oy}$ in order to determine its bisector ray $\overrightarrow{Oz}$ (figure A.2.2).

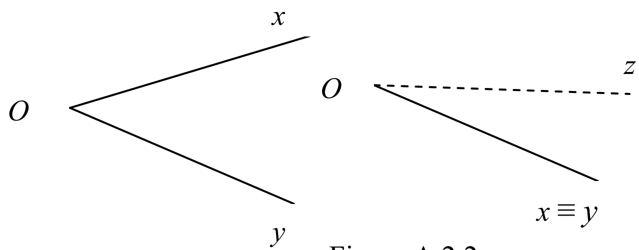


Figure A.2.2

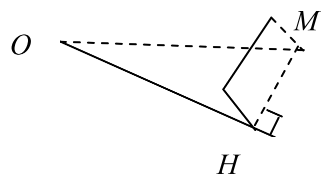


Figure A.2.3

M is any point that lies on $\overrightarrow{Oz}$. Fold $\overline{MH}$ such that $\overline{MH}$ is perpendicular to $\overrightarrow{Ox}$ ($\equiv \overrightarrow{Oy}$) (figure A.2.3). The measure of $\overline{MH}$ is the distance from M to $\overrightarrow{Ox}$ and $\overrightarrow{Oy}$. Based on the folding way, compare the distances from M to $\overrightarrow{Ox}$ and $\overrightarrow{Oy}$."

b) Theorem 1

?2 *Based on figure A.2.4, write the 'given' information and conclusion of theorem 1.*

A point that is on the bisector of an angle is equidistant from the two sides of the angle.

Proof: (Figure A.2.4)

We consider ΔMOA and ΔMOB:

$OM = OM$ (reflexive Property)

$m\angle MOA = m\angle\ MOB$ (given)

$m\angle OAM = m\angle OBM$ ($= 90^0$)

Thus, ΔMOA and ΔMOB are congruent (ASA Congruence Postulate). Therefore, $MA = MB$.

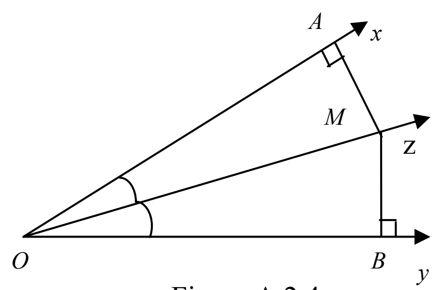


Figure A.2.4

2. The converse theorem

We consider the following problem:

M is a given points in the interior of $\angle xOy$ such that the distance from M to the two sides of this angle are equal (figure A.2.5). Is point M on the bisector ray (or is $\overrightarrow{OM}$ the bisector ray) of $\angle xOy$?

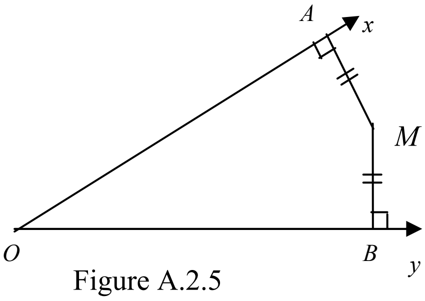


Figure A.2.5

We have the following theorem:

Theorem 2 (The converse theorem)

> If a point that is in the interior of an angle and equidistant from the two sides of the angle then it is on the bisector ray of this angle.

Based on the figure A.2.5, write down the given information (supposition) and conclusion of the theorem.

Guide to prove: (see figure A.2.5)

- Draw $\overrightarrow{OM}$
- Prove that ΔMOA and ΔMOB are congruent.

From this, it follows that $m\angle MOA = m\angle MOB$. In other words, $\overrightarrow{OM}$ is the bisector ray of $\angle xOy$.

Comment:

From theorems 1 and 2, we have: *The bisector ray of an angle is the locus of all points that are in the interior of this angle and equidistant from its two sides.*

A.3 Characteristics of the perpendicular bisector of a segment

A.3.1 In the 'reformed' textbooks

This lesson is translated from the 'reformed' textbook entitled *Geometry 7* (Nguyen Gia Coc & Pham Gia Duc, 1996, pp. 40-42).

1. Definition

> *A line that contains the midpoint of a segment and is perpendicular to the segment is called the perpendicular bisector of this segment.*

In figure A.3.1, $\overleftrightarrow{a}$ that contains the midpoint I of $\overline{AB}$ and is perpendicular to $\overline{AB}$ is the perpendicular bisector of $\overline{AB}$.

When $\overline{AB}$ is a side of $\Delta\ ABC$, the perpendicular bisector of $\overline{AB}$ is called the perpendicular bisector of $\Delta\ ABC$ (figure A.3.2).

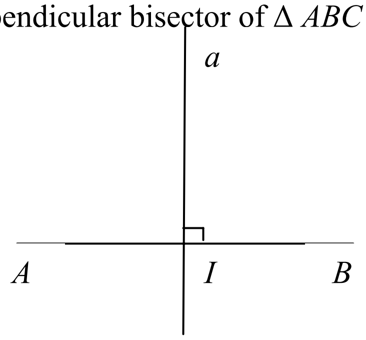


Figure A.3.1

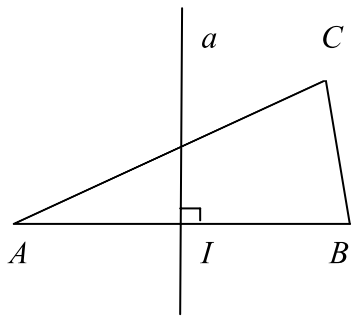


Figure A.3.2

2. Theorem

> *If a point M is on the perpendicular bisector of $\overline{AB}$, then the distances from M to the endpoints of $\overline{AB}$ are equal.*

Given: M is on the perpendicular bisector of $\overline{AB}$

Prove: $MA = MB$

Suppose that I is the midpoint of $\overline{AB}$.

If $M \equiv I$, then the theorem is obviously true.

If M does not coincide with I:

We consider $\Delta\ MIA$ and $\Delta\ MIB$:

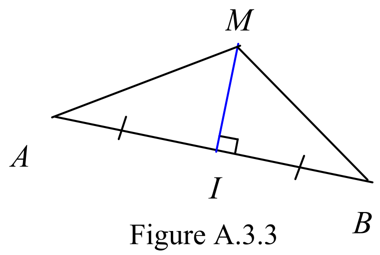


Figure A.3.3

$IA = IB$ ($\overleftrightarrow{MI}$ is the mid-perpendicular of $\overline{AB}$)

$m\angle MIA = m\angle MIB$ ($=90^0$)

$MI = MI$ (Reflexive Property)

Consequently, ΔMIA and ΔMIB are congruent (SAS Congruence Postulate). Therefore, $MA = MB$.

2. Theorem

If there is a point M such that the distances from M to A and B are equal, then M is on the perpendicular bisector of $\overline{AB}$.

Given: $MA = MB$

Prove: M is on the perpendicular bisector of $\overline{AB}$

Proof: (figure A.3.4)

Suppose that I is the midpoint of $\overline{AB}$.

If $M \equiv I$ then the theorem is obviously true.

If M does not coincide with I, we call $\overline{MI}$ the bisector at vertex M of Δ ABC. We consider Δ MIA and Δ MIB:

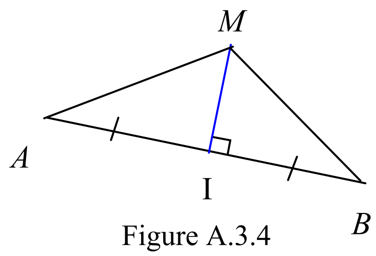


Figure A.3.4

$m\angle AMI = m\angle BMI$

MI is the common side of both two triangles

$MA = MB$ (Supposition)

Consequently, ΔMIA and ΔMIB are congruent (SAS Congruence Postulate). From this, it follows that:

$IA = IB$ (1)

$m\angle MIA = m\angle MIB$

A pair of $\angle MIA$ and $\angle MIB$ is a linear pair of angles. Furthermore, they are congruent. Consequently, $m\angle MIA = 90^0$ (2).

From (1) and (2), it follows that M is on the perpendicular bisector of $\overline{AB}$.

Combing theorems 1 and 2, we state:

The set of points whose each point is equidistant from the endpoints of $\overline{AB}$ is the perpendicular bisector of $\overline{AB}$.

$MA = MB \Leftrightarrow M$ is on the perpendicular bisector of $\overline{AB}$.

A.3.2 In the present textbooks

This lesson is translated from the present textbook entitled *Mathematics 7: part 2* (Phan Duc Chinh et al., 2004 b, pp. 74-75).

1. The theorem about characteristic of the mid-perpendicular of a segment

a) Practice

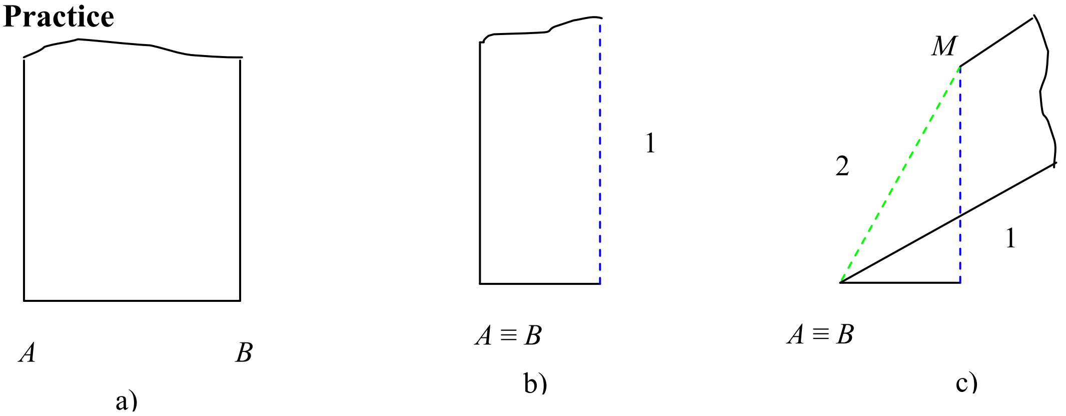


Figure A.3.5

• Cut a piece of paper, whose cutting edge is AB (figure A.3.5.a)

• Fold this piece of paper such that point A lies on point B (figure A.3.5.b). Folding crease 1 is the mid-perpendicular of $\overline{AB}$.

• M is a point on folding crease 1, fold $\overline{MA}$ (or $\overline{MB}$), we have folding crease 2 (figure A.3.5.c). The measure of folding crease 2 is the distance from M to A (or B respectively). From this, we recognize that $MA = MB$.

We have the following theorem:

b) Theorem 1

A point that is on the perpendicular bisector of a segment is equidistant from the two endpoints of this segment.

Specifically, if a point M is on the perpendicular bisector of $\overline{AB}$, then $MA = MB$ (The students self-prove this theorem).

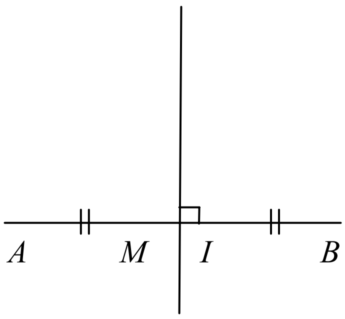
2. Converse theorem

We consider a point M that is equidistant from the two endpoints of $\overline{AB}$. Is point M on the perpendicular bisector of $\overline{AB}$?

Theorem 2 (The converse theorem)

If a point that is equidistant from the two endpoints of a segment, then it is on the perpendicular bisector of this segment.

Please write down the given information and the conclusion of the theorem



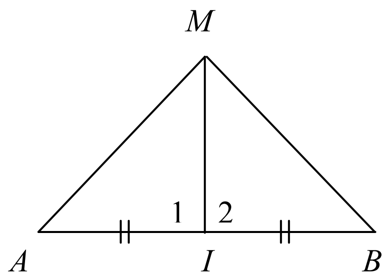


Figure A.3.6

Proof:

We consider two cases:

$M \in AB$ (figure A.3.6.a):

Since $MA = MB$, M is the midpoint of $\overline{AB}$. Consequently, M is on the perpendicular bisector of $\overline{AB}$.

$M \notin AB$ (figure A.3.6.b):

Draw a segment that connects M with midpoint I of $\overline{AB}$.

We have: ΔMIA and ΔMIB are congruent (SSS Congruence Postulate). Thus, $m\angle I_1 = m\angle I_2$. Moreover, $m\angle I_1 + m\angle I_2 = 180^0$. Thus, $m\angle I_1 = m\angle I_2 = 90^0$. Therefore, $\overline{MI}$ is the perpendicular bisector of $\overline{AB}$.

Comment:

From the theorem and its converse theorem, we have: *The perpendicular bisector (mid-perpendicular) of a segment is the locus of all points that are equidistant from its endpoints.*

A.4 The Triangle Inequality

A.4.1 In the 'reformed' textbook

This lesson is translated from the 'reformed' textbook entitled *Geometry 7* (Nguyen Gia Coc & Pham Gia Duc, 1996, pp. 70-71).

1. Theorem

In a triangle, the sum of the length of any two sides is greater than the length of the other side.

Given: ΔABC

Prove: $AC + BC > AB$, $AB + BC > AC$, $AB + AC > BC$.

Proof:

On the opposite ray of $\overrightarrow{CA}$, point D is identified such that $CD = CB$ (figure A.4.1). Since $\overrightarrow{BC}$ is somewhere between $\overrightarrow{BA}$ and $\overrightarrow{BD}$, $m\angle ABD > m\angle\ CBD$ (1).

According the way in which point D is identified, ΔBCD is the isosceles triangle with base BD; therefore, $m\angle CBD = m\angle D$ (2)

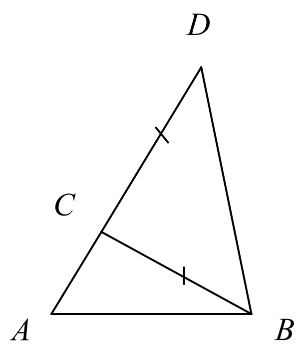


Figure A.4.1

From (1) and (2), we have: $m\angle ABD > m\angle D$.

In ΔABD, since $m\angle ABD > m\angle D$, $AD > AB$. We know AD = AC + CD = AC + CB. Thus, AC + CB > AB.

Analogously, we have can prove that $AB + BC > AC$, $AB + AC > BC$.

Consequence

In a triangle, the difference of two sides is shorter than the other one.

Given: ΔABC

Prove: $BC - AC < AB$ ($AB - AC < BC$, $BC - AB < AC$)

Proof:

Suppose that $AC < BC$. According to the above theorem, in ΔABC, we have: $BC < AB + AC$.

From this, it follows that $BC - AC < AB$ (1)

If $AC \geq BC$, then inequality (1) is obviously true.

Analogously, we have:

$$AB - AC < BC$$

$$BC - AB < AC$$

Note: Combining the theorem and its consequence, we have:

$$BC - AC < AB < BC + AC$$

In a triangle, each side is longer than the difference and shorter than the sum of the other sides

The above inequality is called the *Triangle Inequality*.

A.4.2 In the present textbooks

This lesson is translated from the present textbook *Mathematics 7: part 2* (Phan Duc Chinh et al., 2004 b, pp. 61-63).

"? 1 *Try to draw a triangle with the lengths of side* 1cm, 2cm and 4 cm*. Can you do it?*

Theorem

In a triangle, the sum of the two sides is longer than the other side

With ΔABC, we have the following inequality:

· $AB + AC > BC$
· $AB + BC > AC$
· $AC + BC > AB$

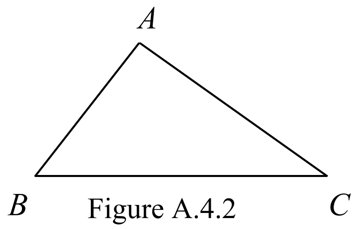


Figure A.4.2

? 2 *Based on figure A.4.2, write down the given information (supposition) and conclusion of the theorem*

Proof:

We will prove the first inequality. The last two inequalities can be proved analogously.

Let D be a point on $\overrightarrow{BA}$ such that $AD = AC$ (figure A.4.3).

In ΔBCD, we will compare BD with BC.

Since $\overrightarrow{CA}$ is somewhere between $\overrightarrow{CB}$ and $\overrightarrow{CD}$, $m\angle BCD > m\angle ACD$ (1).

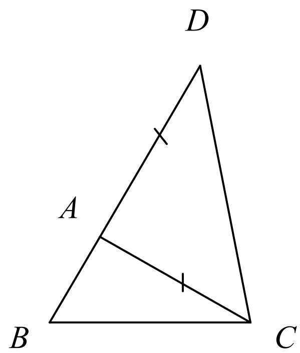


Figure A.4.3

One the other hand, ΔACD is the isosceles triangle.

Consequently, $m\angle ACD = m\angle ADC = m\angle BDC$ (2)

From (1) and (2), we obtain: $m\angle BCD > m\angle BDC$. Thus, $BD > BC$.

Hence, $AB + AC > BC$.

Appendix B: Questionnaire Form

Mr. Le Tuan Anh

The Institute for Mathematics

The University of Potsdam

Questionnaire

(For middle school mathematics teachers)

This questionnaire is intended to collect information about mathematics instruction in Vietnamese school. Please answer the following questions to the best of your ability:

I- <u>Your personal information</u>

1- Name:……………………….………… Birth date: …………Sex: (male) (female)

2- Name of the school where you teach: ………………………………………

- District (town):…..……………………………..
- Province (city):………………………

3- Personal qualifications:…………………………………….

4- Number of years you have taught: ………………………………..

II- <u>Mathematics curricula and textbooks</u>

1- Your thoughts on the curriculum and textbooks that you are currently using at your school (check the appropriate boxes). The curriculum and textbooks are:

a- suitable for students' competence	☐
b- rigorous for students' competence	☐
c- suitable for the time allotted for teaching	☐

d- rigorous for the time allotted for teaching

e- attach special importance to the application of mathematics in everyday life or to its application in other school disciplines

f- do not attach special importance to the application of mathematics in everyday life or to its application in other school disciplines

2- Additional thoughts about the current mathematics curriculum and textbooks:

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3- If you used the ‘reformed’ textbooks, please comment upon both the good and bad aspects you encountered with this curriculum and these textbooks:

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III- <u>Methods of teaching mathematics in school</u>

1-What methods/ approaches do you often use in teaching mathematics? Please check the appropriate boxes:

	I do not know this method	I rarely use this method	I often use this method
a- Lecturing			

b- Using suggested questions			
c- Using visual means of teaching			
d- Developing Students' Activeness			
e- Problem Posing and Solving			
f- Teaching through Activities			
g- The Theory of Didactical Situations in Mathematics			
h- Constructivism			
i- The Student as the Center of Teaching and Learning Process			
J-Applying ICT in teaching			

2- Apart from the methods mentioned directly above, what additional methods of/approaches to teaching do you know or use?

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3- What difficulties do you usually experience when you implement the teaching method reforms? (For instance, difficulties related to the amount or quality of mathematics content in the curriculum or textbooks, the allotted teaching time, the number of students per class, the teaching guide materials, the teaching tools, different students' levels of competence, etc.)

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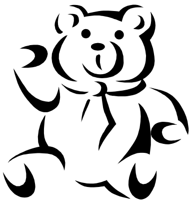
IV- <u>Your thoughts on the following problem and solutions</u>

1- Here is a problem (Van den Heuvel-Panhuizen, 1996, p. 95):[92]



students

One bear is 500 Kg. How many students in your class weigh as heavy as this bear? Please write the solution in the blank and explain why you get this solution.

You can use the scratch paper provided.

Your ideas about this problem:

a- this problem does not have enough given information

b- this problem is a good problem with realistic sufficient content

c- Additional ideas:

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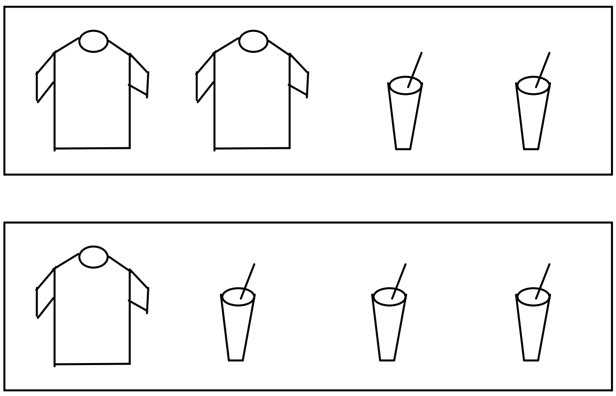
2- Here is a problem called 'T-shirts and Sodas' (Van Reeuwijk, 1995, pp. 2-4):



44 000đ

30 000đ

How much does a T-shirt cost? How much does a cup of soda cost? Explain why?

[92] Some words of the original problem are changed to make a problem more suitable for Vietnamese students.

These are some students' solutions:

<u>The first solution (using the guess and check strategy)</u>:

A = 11 000 and C = 11000 (Does not satisfy)

A = 12 000 and C = 10000 (Does not satisfy)

A = 13 000 and C = 9000 (Do not satisfy)

A = 14 000 and C = 8000 (Does not satisfy)

A = 15 000 and C = 7000 (Does not satisfy)

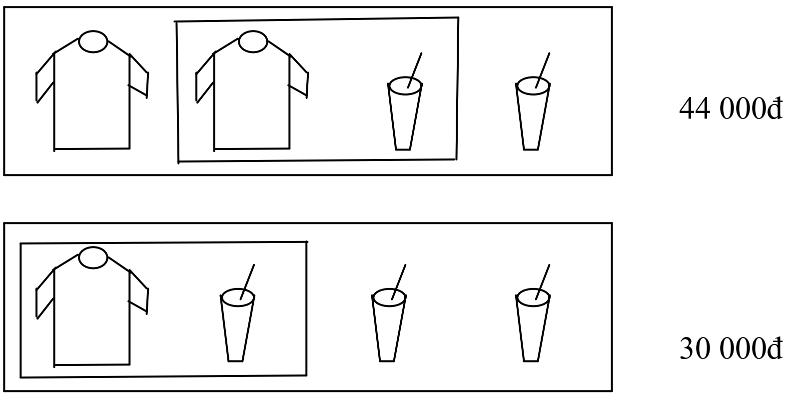
A = 16 000 and C = 6000 (Does not satisfy)

A = 17 000 and C = 5000 (Does not satisfy)

A = 18 000 and C = 4000 (Satisfies)

Consequently, one T-shirt costs 18 000 đ, and one soda costs 4 000 đ.

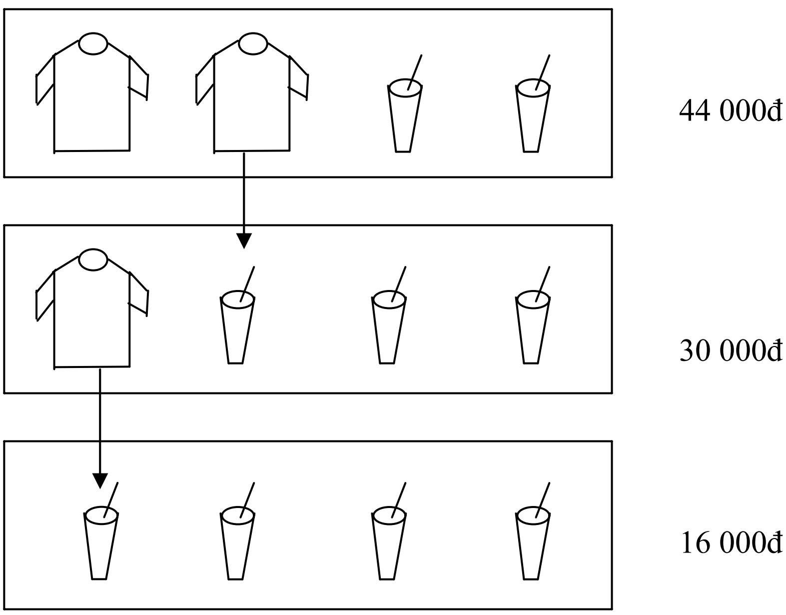
<u>The second solution</u>:



One T-shirt and one soda cost 22 000 đ.

Two cups of soda cost: 30 000 - 22 000 = 8 000 (đ). Hence, one cup of soda costs: 8000:2= 4 000 đ. Therefore, one T- shirt costs: 22 000 đ – 4 000 đ = 18 000 đ.

<u>The third solution:</u>



When I replace one T-shirt by one cup of soda, the total price is reduced: 44 000 – 30 000 = 14 000 đ. Again, when I replace one T-shirt by one cup of soda, the total price reduces 14 000 đ too. Consequently, the price of 4 cups of soda is: 30 000 - 14 000 = 16 000 đ. Therefore, the price of one cup of soda is: 16 000: 4 = 4 000 đ. Hence, one T- shirt costs: 3 000 - 3 x 4 000 = 18 000đ.

<u>The fourth solution:</u>

$2A + 2C = 44\,000$ đ

$1A + 3C = 30\,000$ đ

$2A + 6C = 60\,000$ đ

$0A + 4C = 16\,000$ đ

$C = 4\,000$ đ

$A = 18\,000$ đ

One T-shirt costs 18 000 đ. One cup of soda costs 4 000 đ.

<u>The fifth solution:</u>

	T-shirt	Soda	Price
1	2	2	44 000 đ
2	1	3	30 000đ
3	2	6	60 000đ
4	0	4	16 000đ
5	0	1	4 000đ
6	1	0	18 000đ

One T-shirt costs 18 000 đ. One cup of soda costs 4 000 đ.

<u>The sixth solution:</u>

• *Step 1: Select unknown*

Let x be the price of one T-shirt and y be the price of one cup of soda ($x, y > 0$).

• *Step 2: Set up the system of equations*

Since two T-shirts and two cups of soda cost 44 000 đ, $2x + 2y = 44\,000$ (đ) (1).

Since one T-shirt and three cups of soda cost 30 000 đ, $3x + y = 30\,000$ (đ) (2).

From (1) and (2), we have the following system of equations:

$$\begin{cases} 2x + 2y = 44000 & (1) \\ x + 3y = 30000 & (2) \end{cases} \text{(I)}$$

• *Step 3: Solve the system of equations*

$$\Rightarrow \begin{cases} 2x + 2y = 44000 \\ 2x + 6y = 60000 \end{cases}$$

$\Rightarrow$ $(2x + 6y) - (2x + 2y) = 6000 - 44000$

$\Rightarrow$ $2x + 6y - 2x - 2y = 6000 - 44000$

$\Rightarrow$ $4y = 16\ 000$

$\Rightarrow$ $y = 16\ 000 : 4$

$\Rightarrow$ $y = 4\ 000$

Substituting $y = 4\ 000$ into equation (1), we have

$2x + 8\ 000 = 44\ 000$.

Hence, $2x = 36\ 000$.

Therefore, $x = 18\ 000$.

• *Step 4: Check and answer*

18 000 and 4 000 satisfy the suppositions.

One T-shirt costs 18 000 đ. One cup of soda costs 4 000 đ.

Please comment on the above solutions (including your thoughts on their precision, sufficiency, characteristics, form of presentation, etc.).

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V- <u>Additional questions</u>

1- What difficulties do your students meet when they learn mathematics?

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2- What should teachers emphasize when teaching mathematics? Please rate the following from 0 to 2, with 0 = not very important, 1 = important and 2 = very important).

	Levels of importance
a- Teach students so that it is easy for them to understand lessons	
b- Help students understand the various forms of mathematics problems so that they can solve similar problems in the future	
c- Help students memorize mathematic principles (such as concepts, rules, formulae and theories) so that they can use this knowledge to solve mathematical problems.	
d- Help student re-invent or re-discover mathematics principles	

3- Do you often use mathematics problems relating to real life or the real world? Please check the appropriate boxes:

	I do not use	I rarely use	I often use
a- in the teaching process			
b- oral examinations (often at the beginning of each lesson)			
c- in examinations (15 minutes, 45 minutes, etc.)			

4- Do you often let your students work in groups or work individually (Please check appropriate blank)?

	Often	Sometimes	Rarely
a- work individually			
b- work in pairs or in groups			

VI- Your additional ideas about teaching and learning mathematics in school:
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Thank you very much for answering these questions.

Please send any further comments to:

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D- 14 415 Potsdam

Deutschland

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Appendix C: RME-based geometry lessons

Nr.	Name of lesson
1	The Triangle Sum Theorem (The Triangle-Angle Sum Theorem)[93]
2	Perpendicular line and slant line[94]
3	The Triangle Inequality
4	Characteristics of the bisector of an angle
5	Characteristics of the three bisectors of a triangle
6	Characteristics of the perpendicular bisector (mid-perpendicular) of a segment
7	The 'Railway station' problem[95]
8	Characteristics of the three perpendicular bisectors (mid-perpendiculars) of a triangle

[93] Sometimes the sum of angles is understood as the sum of measures of angles.
[94] This RME-based lesson is suitable for high-achieving students.
[95] This RME-based lesson is suitable for high-achieving students.

Notes:

• Teachers should select *situation*s from RME-based lessons that are suitable for their teaching practices, students' level of competence and allotted teaching time. They do not have to use all given aspects of each lesson. Moreover, teachers can add suggestions and guidances in order to aid students when necessary. For instance, at first sight, the first lesson looks long and complicated. However, this lesson offers some selected situations which teachers can choose to use.

• Because some of these lessons can be quite long and complicated for students (for example, lessons 4 and 7), teachers have the option not to use them.

• In these lessons, students are expected to build on informal knowledge by acquiring formal mathematical knowledge. Some situations in these lessons function as a bridge which connects informal with formal mathematical knowledge. It should be noted that Dutch RME geometry, at least in middle school, emphasizes informal knowledge. But in the current Vietnamese middle school textbooks, it is frequently formal knowledge (mathematics definitions, theorems and rules) that is emphasized.

• In RME-based lessons, the figures are not drawn with their real measures.

1. The Triangle Sum Theorem (Triangle-Angle Sum Theorem)

Tools: students need rulers and protractors.

Situation 1 (*Worksheet one, students work in pairs*)
Each student draws a triangle. Then each student measures the three angles of the triangle drawn by her/his partner.

Situation 2[96] *(Worksheet two, students work in pairs)*
Use a ruler and a protractor to draw a series of triangles with the following measures of angles if possible:

a) 60^0, 60^0 and 60^0

b) 30^0, 50^0 and 70^0

c) 40^0, 80^0 and 100^0

d) 35^0, 45^0 and 100^0

The purpose of this situation is to help students realize that there are cases (e.g. *a* and *d*) in which students can draw a triangle with the three given angle measures and the other cases (e.g. *b* and *c*) where a triangle cannot be drawn with the given measures of angles.

Situation 3[97] *(Worksheet three)*
1) (*Students work in pairs*)

Two students play a game in turn (approximately 5 to 10 turns)

The rules of the game:

• Each player thinks of and writes down three measures of angles. If s/he can draw a triangle with the three measures of angles s/he has written down then s/he receives one point. Otherwise, s/he obtains no point.

• A player can use a ruler and a protractor during the game. However, in each turn, a player can use only the protractor to measure angles *once*. But a player can use both tools to draw a triangle.

---

[96] When the teachers used this lesson for their teaching, they added one additional situation and made change in situation 1. They did not use situation 3 and question a) of situation 4 because of the teaching time pressure. Situations which the teachers utilized in the classes in October 2005 are presented at the end of this lesson.
[97] This is a 'voluntary' situation. It means that teachers should decide themselves whether they should use this situation or not.

• The winner is the player who receives the more points. If the two players receive the same number of points, then the game is a draw.

2) (*Students* work *in groups*)

Answer the following question in group discussions: What are some strategies you can use in order to obtain more points?

The purpose of this situation

• At first, a player tends to think of three random angle measures and tries to draw a triangle with these three measures. After some failures, s/he may try to justify her/ his strategies. S/he may discover that "*perhaps there is a relationship among the three angle measures in a triangle*", or at least "*in some cases a triangle cannot be drawn with three random angle measures".*

• *The situation suggests* that each player (student) has to change her/his strategy to receive more points. During the game, students may discover that there is a relationship between the three angles of a triangle (*The teacher does not have to interfere in the game*). By contrast, in the situations in the textbook entitled *Mathematics 7: part 1, students have to manipulate under the textbook's instructions* (see appendix A).

• As discussed above, all students have to work the same way despite their different levels of mathematical capability when *Mathematics 7: part 1* is used. Hopefully, this situation encourages them to use a variety of strategies during the game.

• *It is ideal* when students can rediscover that the sum of the three angle measures of a triangle is 180^0 after playing the game. However, this is not the primary aim of this situation.

1) Some possible strategies students may use in the first situation (expected strategies)

a) 'Random' strategy:

Some players may play the game without paying attention to the three angle measures they think of. In other words, they may think of three random measures of angles. Later, they will try to draw a triangle with these measures of angle. Obviously, these players tend to lose the game.

b) 'Familiar triangle' strategy (figure C.1.1):

For example, a player can recall familiar triangles they have already learned about such as *an equilateral triangle* with angle measures 60^0, 60^0 and 60^0; *a right isosceles triangle* with angle measures 90^0, 45^0 and 45^0; and *a right triangle with one angle that measures 60^0*. (S/he can find this triangle by dividing an equilateral triangle into halves)

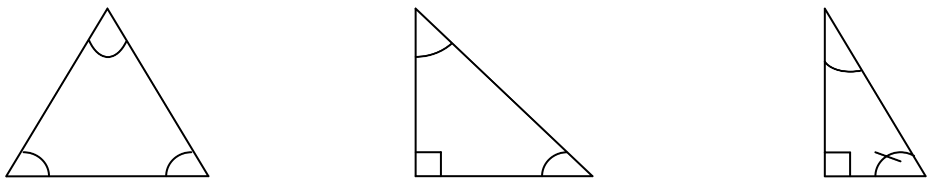


Figure C. 1.1

c) 'Measurement' strategy:

At first, a player can think of three random measures of angles, for instance, 30^0, 40^0 and 60^0. But s/he cannot draw a triangle with these three measures of angles (figure C.1.2). After this turn, s/he may keep two measures of angles, for example, 30^0 and 40^0, and draw a triangle with these two measures of angles and then measure the last angle (110^0) (figure C.1.3). Afterwards, this player can draw a triangle with the three angle measures 30^0, 40^0 and 110^0.

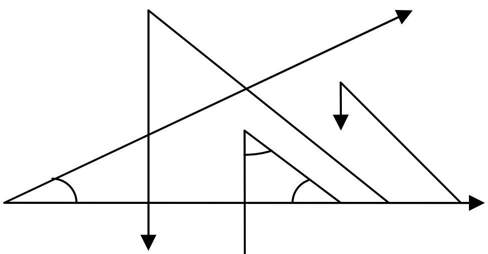


Figure C. 1.2

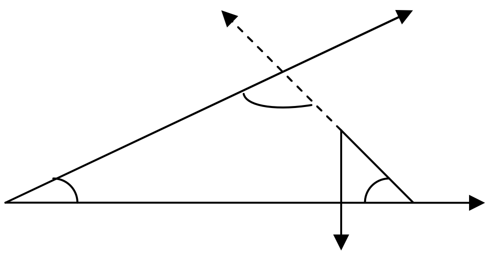
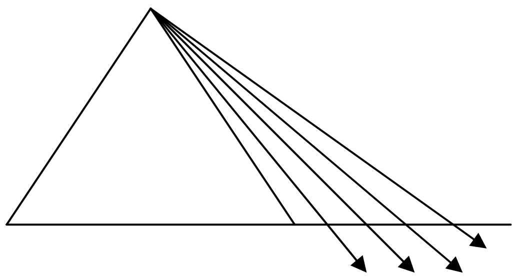


Figure C.1.3

d) 'One constant angle' strategy

After succeeding to draw an equilateral triangle with the angle measures 60^0, 60^0 and 60^0, a player can keep the first angle measure at 60^0 and change the second angle to a different number. In the following turn, this player can increase the measure of the second angle from 60^0 to 70^0 and measure the last one (50^0). Then s/he can increase the measure of the second angle from 70^0 to 80^0, measure the last one and find 40^0, etc. (figure C.1.4).



1^{st} angle	2^{nd} angle	3^{rd} angle
60^0	60^0	60^0
60^0	70^0	50^0
60^0	80^0	40^0
60^0	90^0	30^0
60^0	100^0	20^0
…	…	…

Figure C.1.4

e) 'Bisector' strategy

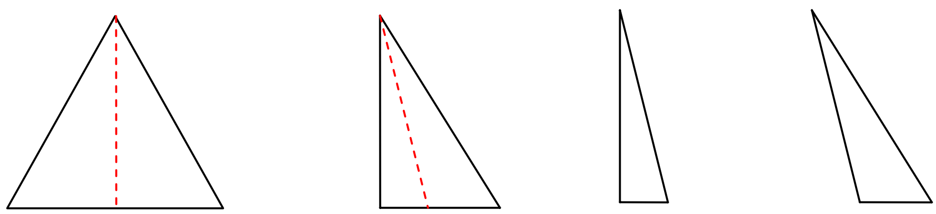


Figure C.1.5

A player can draw an equilateral triangle and one bisector of the triangle in order to divide this triangle into two smaller triangles. In the next turn, s/he draws a triangle with angle measures 90^0, 60^0 and 30^0 after measuring the unknown-measure angle of one of the two smaller triangles. Again, s/he draws one bisector of the right triangle in order to divide this right triangle into two smaller triangles. Because s/he knows the measures of the two angles of each triangle, in the following turns, s/he can measure the other angle so that s/he can draw a triangle, etc. (figure C.1.5).

<u>Note</u>

• A player not only can use one of the above strategies but also can *alternate or combine strategies*. For example, first, a player can use the *'random' strategy,* and then switches to either the '*familiar triangle*', '*measurement*', '*one constant angle*', or the *'bisector'* strategy. Furthermore, a player can apply different strategies during the game.

• Apart from the mentioned strategies, students may *use other strategies*.

• The game offers students *opportunities to use different kinds of mathematical knowledge*. Although there might be students who use only the *'random' strategy*, these players need to know the concepts of an angle, a triangle and the measure of an angle, as well as know how to draw angles.

• This situation is most useful when most of the students do not know the content of the theorem before playing the game.

2) Some possible answers

• After the game is over, a student can realize that *in some cases, it is impossible draw a triangle with three arbitrary angles' measures*. Students are not told about this principle beforehand (by their teacher), but instead discover it when playing the game.

• Some students may notice that if *two angles of a triangle are identified, then the last one is also identified.*

• If students use the strategy of 'one constant angle' *they may realize that if the measure of the second angle increases, then that of the third angle reduces. They may even discover that if the second angle increases, for example 10^0, then the last angle decreases 10^0.*

• Some students may discover that *there might be a relationship among the three angles in a triangle.*

<u>Situation 4</u> (*Worksheet four, students* work *in groups*)

Notice the decorative form in figure C.1.6; where $\overleftrightarrow{g}$ and $\overleftrightarrow{h}$ are parallel; $\overline{AB}$ and $\overline{CD}$ are parallel; and $\overline{AC}$ and $\overline{DE}$ are parallel

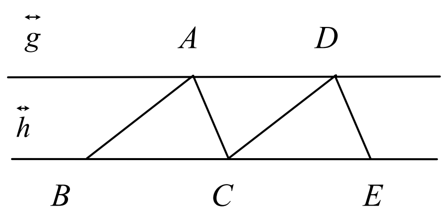


Figure C.1.6

a)[98] Suppose that you want to give some precise instructions (on the telephone) so that one of your friends can draw figure C.1.6. How can you instruct your friend, using different strategies?

[98] This is a 'voluntary' situation. It means that teachers should decide themselves whether they should use this situation or not.

In each group, one student plays the role of the listener while the others are the instructors. One of the instructors should write down guidance steps. Note that the listener cannot see the figure and must try to draw this figure only with the instructions of the other members. The instructors are not allowed to see the listener's work.

b) The ancient Greeks used a figure similar to this figure to prove one characteristic of the three angles of a triangle. Based on figure C.1.7, please find this characteristic of ΔABC and prove it.

Expected solutions

Strategy 1: (figure C.1.7)

- Draw ΔABC; draw $\overleftrightarrow{h}$ through B and C;
- Draw $\overleftrightarrow{g}$ through A and is parallel to $\overline{BC}$;
- Determine point D on $\overleftrightarrow{g}$ such that $\overline{CD}$ and $\overline{AB}$ are parallel; Connect C to D;
- Determine point E on $\overleftrightarrow{h}$ such that $\overline{DE}$ is parallel to $\overline{AC}$; connect D to E.

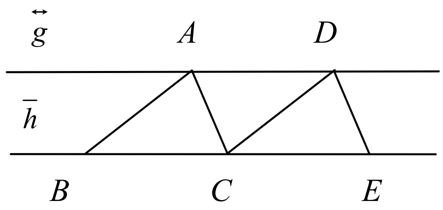


Figure C.1.7

Strategy 2 (figure C.1.8)

- Draw $\overleftrightarrow{g}$ and $\overleftrightarrow{h}$ such that they are parallel;
- Determine points A and B on $\overleftrightarrow{g}$ and $\overleftrightarrow{h}$, respectively; connect A to B;
- Determine point D on $\overleftrightarrow{g}$;

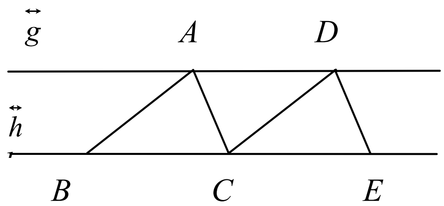


Figure C.1.8

- Determine point C on $\overleftrightarrow{h}$ such that $\overline{CD}$ and $\overline{AB}$ are parallel; connect C to D; connect C to A;
- Determine point E on $\overleftrightarrow{h}$ such that $\overline{DE}$ and $\overline{AC}$ are parallel; connect D to E.

Note:

- Students may use an alternative strategy
- Apart from the two mentioned strategies, students may use other strategies

b)

Strategy 1 (figure C.1.9)

We have:

$m\angle ABC = m\angle A_1$ (Two alternate interior angles) (1)

$m\angle ACB = m\angle A_2$ (Two alternate interior angles) (2)

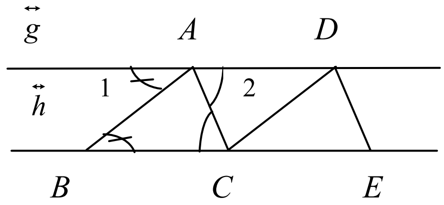


Figure C.1.9

From (1) and (2) we have:

$m\angle BAC + m\angle ABC + m\angle ACB = m\angle BAC + m\angle A_1 + m\angle A_2 = 180^0$.

Therefore, the sum of the three angle measures is 180^0.

Strategy 2 (figure C.1.10)

We have:

$m\angle ABC = m\angle C_2$ (Two corresponding angles) (1)

$m\angle BAC = m\angle C_1$ (Two alternate interior angles) (2)

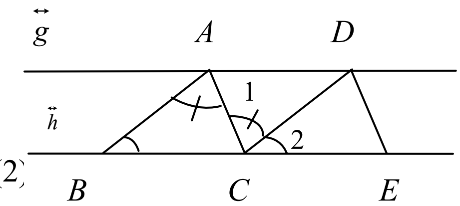


Figure C.1.10

From (1) and (2), we have:

$m\angle BAC + m\angle ABC + m\angle ACB$

$= m\angle C_1 + m\angle C_2 + m\angle ACB = 180^0$

Thus, the sum of the three angle measures is 180^0.

<u>Situation 5</u> (*Formal, students work in groups*)

Prove the following theorem: *The sum of the measures of the three angles of a triangle is 180^0*

<u>Expected solution</u> Students can prove the general theorem in situation 5 by applying the proofs from situation 4 b).

Students may discover that not all segments, lines and triangles in figures C.1.11 and C.1.12 are necessary for the proofs in situation 4 b) and situation 5 (Dashed lines (- - -) in figures C.1.11 and C.1.12 are not necessary for the proofs).

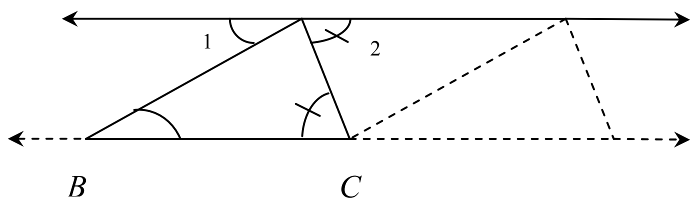


Figure C.1.11

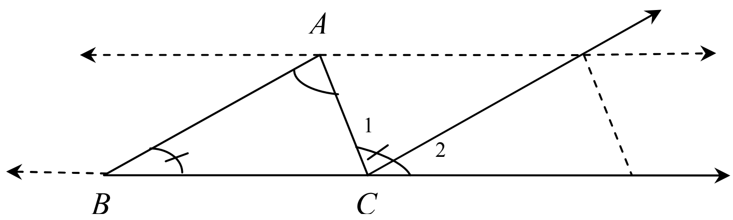


Figure C.1.12

Tasks the teacher, Ms. Huong, intended to use in her teaching:

Task 1: In the first worksheet, the first student was asked to draw three arbitrary triangles, and the second student was asked to measure the thee angles of each triangle and write down the results of her/his measuring (see also the form of the first worksheet for the students in appendix D).

Task 2: In the second worksheet, the first student was asked to write down three arbitrary measures of three angles, and the second student was asked to draw one triangle with these measures of angle, if possible. They were also asked to do this work three times (see also the form of the second worksheet for the students in appendix D).

Task 3: *Situation 3 b)* in RME-based lesson

Task 4: *Situation 4* in RME-based lesson

Task 5: *Situation 5* in RME-based lesson

2. Perpendicular line and slant line[99]

Situation 1 (*Worksheet one, student works individually*)

A game is organized in a stadium. Suppose that a player stands at point *A*. There is a ball in each point *B*, *C*, *D*, *E*, *F*, *G* and *H*.

The rules of the game: A player tries to run as fast as s/he can from point *A* to another point among *B*, *C*, *D*, *E*, *F*, *G* and *H* and intercept the ball at this point. The player who wins is the one who spends the least time running from point *A* to her/his destination. In order to win, which point should a player choose to reach and why? Assume that the velocity of the wind at the stadium is not a factor (figure C.2.1 is a simple map of the stadium with positions *A*, *B*, *C*, *D*, *E*, *F*, *G* and *H*).

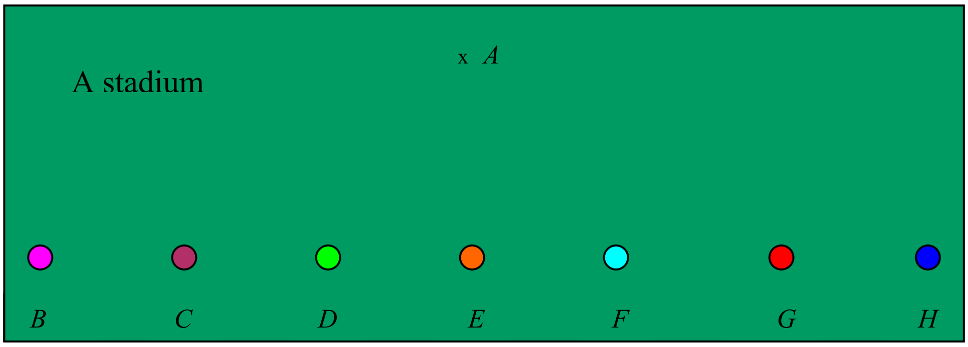


Figure C.2.1

Expected solution

A player needs to find the shortest path in order to win the game. Students can measure and compare the lengths of $\overline{AB}$, $\overline{AC}$, $\overline{AD}$, $\overline{AE}$, $\overline{AF}$, $\overline{AG}$ and $\overline{AH}$. When they do, they find that *AE* is the shortest way. Therefore, a player should run 'directly' from *A* to *E*.

Situation 2 (*Worksheet two, students work in groups*)

A game is organized in a swimming pool (see figure C.2.2). Suppose that a player stands at point *A,* and there is a ball at each point *C*, *D*, *E*, *F* and *G*; $m\angle AFE = m\angle ADE = 60^0$, $m\angle AGE = m\angle ACE = 40^0$, and $m\angle AEF = 90^0$. We assume that the velocities of the water and the wind at the swimming pool are not significant factors.

[99] This is a 'voluntary' lesson. It means that teachers should decide themselves whether they should use this lesson or not.

The rules of the game: A player tries to swim as fast as s/he can from point *A* to another point among *C*, *D*, *E*, *F* and *G* and take the ball at this point. The player who wins is the one who spends the least time swimming from point *A* to her/his destination.

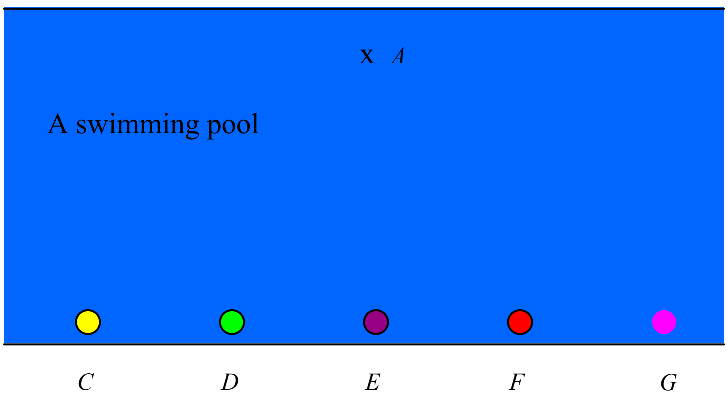


Figure C.2.2

In a group, discuss which end point is the best choice and explain why.

<u>Expected solution</u>

Note:

• *Students cannot measure distances AC, AD, AE, AF and AG because this time the game is organized at a swimming pool.*

• *In fact, conditions* $m\angle AFE = m\angle ADE = 60^0$ *and* $m\angle AGE = m\angle ACE = 40^0$ *are not necessary. However, they are given in this situation so that middle school students have* the *chance to reinvent theorems and their proofs.*

<u>Orientation 1 (From pupils' experiences)</u>

This time, the students have to compare lengths without measuring $\overline{AC}$, $\overline{AD}$, $\overline{AE}$, $\overline{AF}$, and $\overline{AG}$.

At first glance and from the previous experience, students may recognize that AE is the shortest. As a consequence, the situation motivates them to prove that *AE* is the shortest length among *AC*, *AD*, *AE*, *AF* and *AG*.

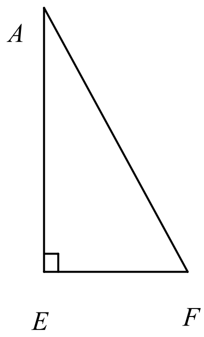


Figure C.2.3

Firstly, students should prove that AE is shorter than AF (figure C.2.3). Analogously, AE is shorter than AD, AC and AG.

Students may use informal strategies to compare the lengths of two segments.

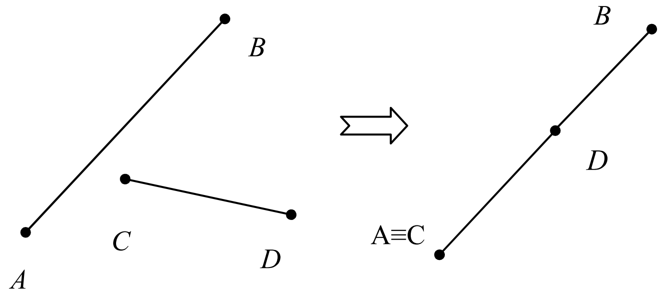


Figure C.2.4

For example, in order to compare AB and CD, they can put two $\overline{AB}$ and $\overline{CD}$ together as figure C.2.4. D is on $\overline{AB}$. Therefore, $AB > CD$.

Some possible strategies:

Strategy 1: "Guess and check" (figure C.2.5)

The students 'find' a point E' on $\overline{AF}$ such that $AE' = AE$. From this supposition, they know $m\angle AEF$ and $m\angle AFE$ are 90^0 and 60^0, respectively. It is suggested that they can find the point E' on $\overline{AF}$ such that $m\angle AEE' = m\angle AE'E$. The 'guess and check' strategy can be used:

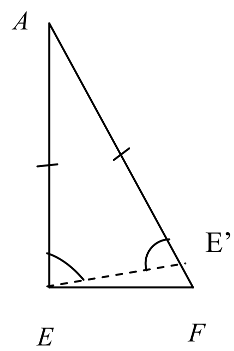


Figure C.2.5

$m\angle AEE'$	$m\angle FEE' = 90^0 - m\angle AEE'$	$m\angle AE'E = 60^0 + \angle FEE'$
50^0	40^0	100^0
60^0	30^0	90^0
70^0	20^0	80^0
80^0	10^0	70^0
$\mathbf{75^0}$	15^0	$\mathbf{75^0}$

We draw $m\angle AEE' = 75^0$. Consequently, $m\angle FEE' = 90^0 - m\angle AEE' = 90^0 - 75^0 = 15^0$. Thus, $m\angle AE'E = m\angle E'EF + m\angle E'FE = 15^0 + 60^0 = 75^0$. Since $m\angle AEE' = m\angle AE'E$ ($= 75^0$), $\Delta AEE'$ is the isosceles triangle. Thus, $AE = AE'$. Consequently, $AF > AE$.

Strategy 2

($m\angle AEF$ (90^0) subtracts $m\angle FEE'$) is equal to $m\angle AEE'$ (1)

($m\angle AFE$ (60^0) adds $m\angle FEE'$) is equal to $m\angle AE'E$ (Exterior angle theorem) (2)

Thus, (m$\angle AEF$ (90^0) subtracts m$\angle FEE'$) is equal to (m$\angle AFE$ (60^0) adds m$\angle FEE'$)

Students can relate to a similar problem they had seen in primary school: Hoa has 5 pencils and his brother, Tuan, has 9 pencils. Tuan wants to give Hoa some pencils such that afterwards Hoa has the same number of pencils as he has.

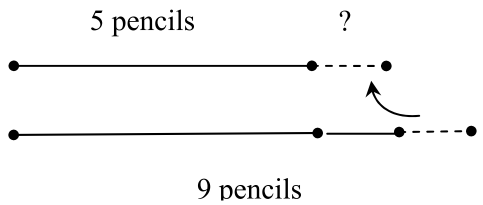


Figure C.2.6

How many pencils should Tuan give Hoa? *(*figure C. 2.6*)*

$\angle FEE'$= 15^0 satisfies condition (3) (figure C.2.7). We draw $\overrightarrow{Ex}$ such that m$\angle$FEE' is 15^0 (E' is the intersect point of $\overrightarrow{Ex}$ and $\overline{AF}$).

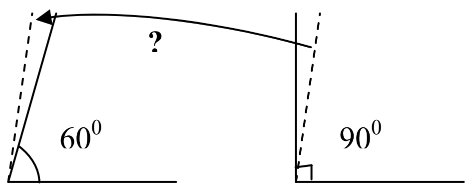


Figure C.2.7

Since $m\angle AEE' = 90^0 - m\angle FEE'$, m$\angle AEE'$ is 75^0.

Then we can prove that $AE' = AE$ (see strategy 1). Thus, $AF > AE$.

Strategy 3 (*Formal strategy*)

After drawing m $\angle AEE' = 75^0$, we can prove that $AE = AE'$. We have:

m$\angle$ FEE' = m$\angle AEF$- m$\angle$ $AEE' = 90^0$- m$\angle AEE' = 15^0$ (1).

m$\angle AE'E$ = m$\angle AFE$ + m$\angle FEE' = 60^0$ + m$\angle FEE' = 60^0 + 15^0 = 75^0$ (2).

Since (1) and (2), m$\angle AEE'$ = m$\angle AE'E$.

Thus, $AE = AE'$.

Therefore, $AE < AF$.

With a *similar strategy*, students can prove that AE is shorter than AD, AC and AG. Therefore, AE is the shortest. Thus, the player should choose to swim to point E.

Orientation 2 (Using the Pythagorean Theorem)

ΔABC is a right triangle with the base $\overline{AF}$ (figure C.2.8). Therefore, $AE^2 + EF^2 = AF^2$ (The Pythagorean Theorem).

Thus, $AF^2 > AE^2$. Hence, $AF > AE$.

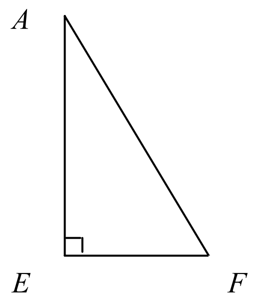


Figure C.2.8

Although the proof using the *Pythagorean Theorem* seems to be simple, students may not know that this theorem should be applied in this situation (unless they are told by their teacher or the textbook) because they most likely learned it quite a bit earlier.

<u>Situation 3</u> (*Worksheet three, students work in groups*)

A game is organized at a swimming pool (figure C.2.9).

The rules for the game: There is one ball at each point *B*, *C*, *D*, *E*, *F*, *G* and *H* ($BC = CD = DE = EF = FG = GH$). Each player has a specific length of time to swim, for example 5 minutes.

<u>Step 1</u>: A player starts from *A* and swims to another point among *B*, *C*, *D*, *E*, *F*, *G* and *H*, picks up the ball at this point and swims back to *A*.

<u>Step 2</u>: If a player has time left, s/he can repeat step 1 until time runs out.

The winner is the player who collects the highest number of balls. In groups, decide what would be a suitable strategy for a player to use in order to win.

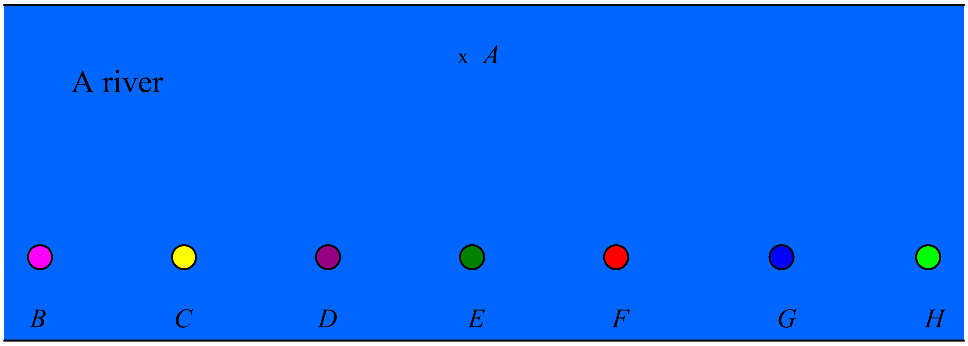


Figure C.2.9

Let's assume that the velocities of the water and the wind are not significant factors.

<u>Expected solution</u>

This time, students have to compare the measures of $\overline{AB}$, $\overline{AC}$, $\overline{AD}$, $\overline{AE}$, $\overline{AF}$, $\overline{AG}$ and $\overline{AH}$ without measuring them. In this situation, they cannot use the '*guess and check*' strategy because they do not know the measure of each angle.

Hopefully, they discover that:

- $\overline{AE}$ is the shortest segment among $\overline{AB}$, $\overline{AC}$, $\overline{AD}$, $\overline{AE}$, $\overline{AF}$, $\overline{AG}$ and $\overline{AH}$.
- If *F* is 'nearer' to *E* than *G*, then *AF* is shorter than *AG*.

- If H is farther from E than G, then AH is longer than AG.

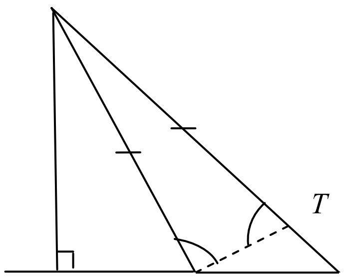


Figure C.2.10

Orientation 1 (figure C.2.10)

$m\angle AFG = m\angle AEF + m\angle EAF = 90^0 + m\angle EAF > 90^0$.

In $\Delta\ AFG$, since $m\angle AFG > 90^0$, $m\angle AFG > m\angle AGF$.

By generalizing, we find a point T on $\overline{AG}$ such that $AT = AF$ by drawing $m\angle GFT = \dfrac{m\angle AFG - m\angle AGF}{2}$.

From this, we can prove that $AG > AF$.

Students may discover that *in every triangle ABC, if* $m\angle ABC > m\angle ACB$*, then* $AC > AB$.

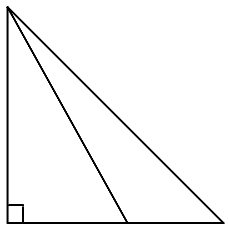


Figure C.2.11

Orietation 2 **(Using the Pythagorean Theorem)** (figure C.2.11)

Because ΔAEF and ΔAEG are right triangles, we have:

$AE^2 + EF^2 = AF^2$ (Pythagorean Theorem) (1)

$AE^2 + EG^2 = AG^2$ (Pythagorean Theorem) (2)

$EG > EF$ (given).

Consequently, $EG^2 > EF^2$ (3).

Hence, $AG^2 > AF^2$ (From (1), (2) and (3)). Thus, $AG > AF$.

In this lesson, we suppose that points B, C, D, E, F, G and H are on a straight line which is perpendicular to $\overline{AE}$.

3. The Triangle Inequality

Situation 1 *(Student works individually)*

The distance from Hoa's house to her school is 3 km. Suppose that Hoa's school is at position *S* (figure C.3.1).

a) Point out some possible positions of Hoa's house (In a worksheet or notebook, students can replace 3 km by 3 cm).

Figure C.3.1

b) Do you have any ideas about what the possible positions of Hoa's house could be?

Expected solution

a) Students point out some positions for Hoa's house (for example, positions H_1, H_2, H_3, and H_4 in figure C.3.2).

b) These positions are on the circle with center *S* and radius 3 km.

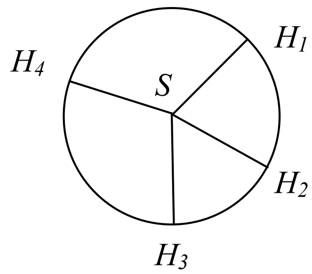


Figure C.3.2

Situation 2 (*Students work in groups*)

The distances from An's house and Hoa's house to their school are 8 km and 3 km, respectively.

a) An estimates that: "my house is 2 km from Hoa's". Is his estimation true or false?

b) What is the shortest distance from An's house to Hoa's house?

Expected solutions

*Strategy 1 (Informal)**:***

Suppose that An's house is at position *A* and the school is at position *S*. Hoa's house is on the circle with center *S* and radius 3 km (figure C.3.3). This circle intersects $\overline{SA}$ at *M*.

MA is the shortest distance from An's house to Hoa's house. Since *SA* = 8 km and *SM* = 3 km, *MA* = 5 km. Therefore, the shortest distance is 5 km. Consequently, An's estimation is false.

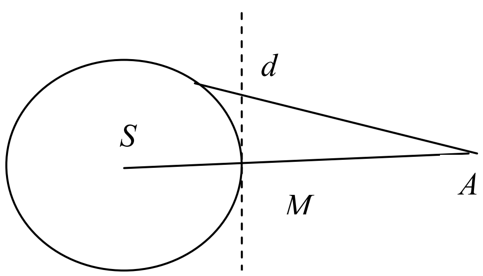


Figure C.3.3

Note:

Students can draw a line that is perpendicular to $\overline{SA}$ through M.

Strategy 2 (Informal):

The Hoa's house is on the circle with center S and radius 3 km. Analogously, Hoa's house is on the circle with center A and radius 2 km (Figure C.3.4). However, these two circles have no common point.

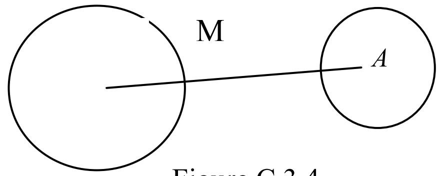


Figure C.3.4

Suppose that the circle (S) intersects $\overline{SA}$ at point M. From this figure, we can conclude that the shortest distance from Hoa's house to An's house is $MA = 5$ km.

Strategy 3 (Formal):

We prove that $HA > MA$ (figure C.3.5).
Because the triangle SMH is the isosceles triangle, $\angle SMH$ is the acute angle.
Consequently, $\angle AMH$ is the obtuse angle.

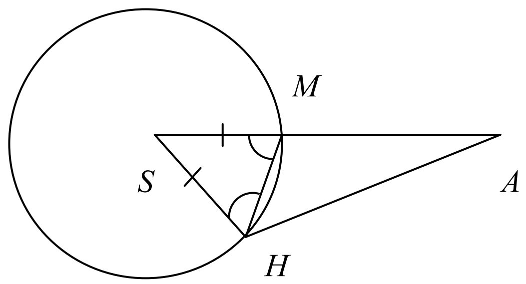


Figure C.3.5

We consider triangle MHA. In this triangle, we have $m\angle AMH > m\angle AHM$. Therefore, $HA > MA$.

Situation 3 (*Students work in groups*)

Prove that in every triangle, the length of a side is longer than the difference of the other sides' lengths.

Similarly, the following situations are introduced to students to help them discover that in every triangle, the sum of the lengths of two sides is longer than the length of the other side.

Situation 4 (*Worksheet three, students work in groups*)

The distance from Hoa's house (H) to a central station (C) is 8 km and to her school (S) is 3 km.

a) She estimates that: "My school is about 12 km from the central station." Is this estimation true or false and why?

b) What is the maximum possible distance from the central station to the school? Can you prove it?

__Situation 5__ (*Generalization, student work in groups*)

Suppose that *ABC* is a triangle. What is the relationship between the three lengths of three sides *AB*, *AC* and *BC* of the triangle? Prove your statement.

4. Characteristics of the bisector of an angle

RME-based lesson

Situation 1 *(Worksheet one-*Student *work individually)*
Suppose that there is a farmer working at point *A*. He wants to go to the river to collect some water for his farm (figure C.4.1).

a) In what direction do you suggest that the farmer chooses to travel?

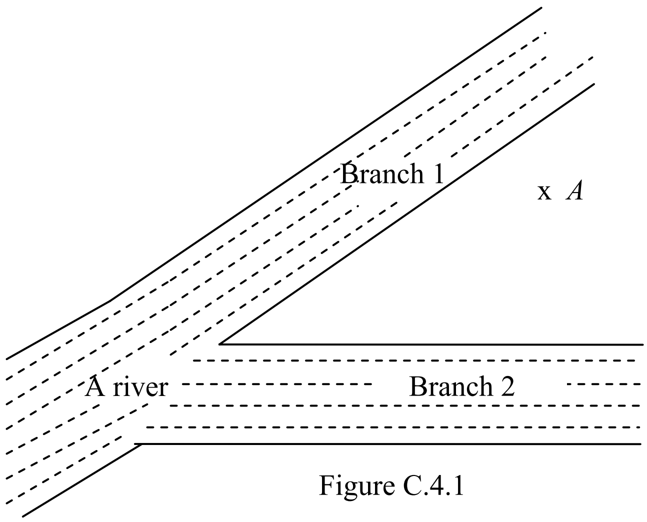


Figure C.4.1

b) Point out some positions from which the farmer should go to *branch 1* of the river.

c) Point out some positions from which the farmer should go to *branch 2* of the river.

Expected solution (figure C.4.2)

a) Students should realize that the distance from *A* to *branch 1* is shorter than the distance from *A* to branch 2. Therefore, the farmer should go to *branch 1* of the river.

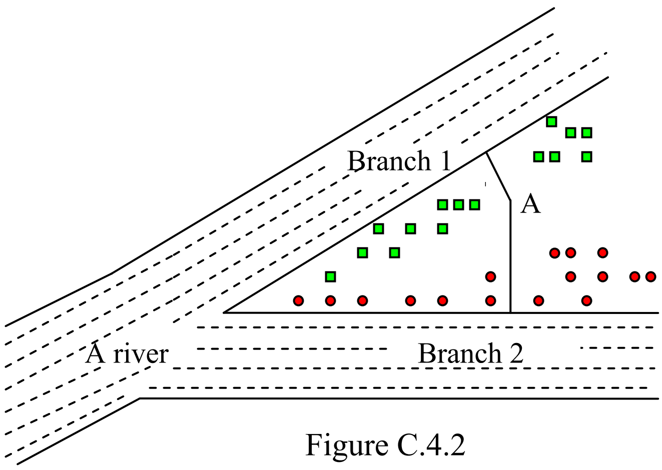


Figure C.4.2

b) Students should point out some positions from which the farmer should travel to *branch 1* on the river.

c) Students should point out some positions from which the farmer should travel to *branch 2* on the river.

Situation 2 (*Worksheet two-Pupils work in groups*)

Suppose that a farmer is working somewhere in the interior of $\angle xOy$ (figure C.4.3).

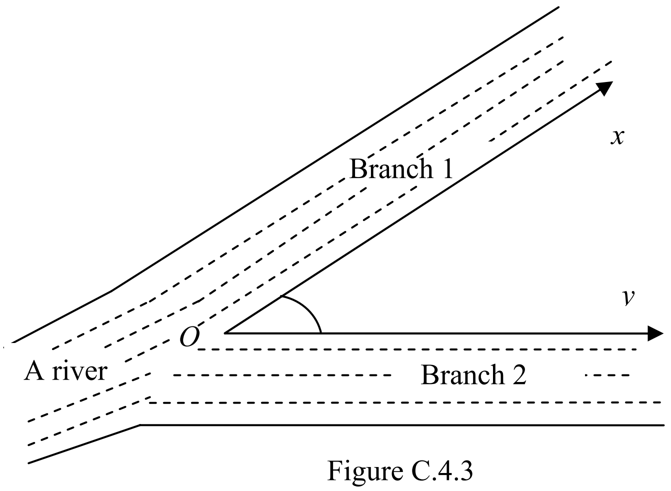


Figure C.4.3

a) Find all positions from which he should go to branch 1 and find all positions from which he should go to *branch 2* of the river.

b) Are there any positions from among these that the farmer can choose in order to go to *branch* either *1* or *2*? If these positions exist, do you know where they are?

Expected solution

a) For every position in $\angle xOy$, the students can measure the distance from it to each branch of the river.

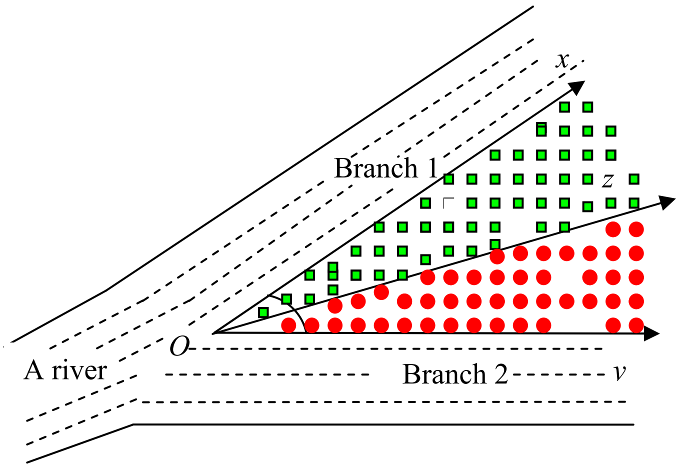


Figure C.4.4

If the distance from this position to *branch 1* is shorter than the distance from this to branch 2, then the farmer should go to *branch 1* (small square-positions).

Otherwise, the farmer should go to *branch 2* (small circle-positions) (figure C.4.4).

b) Students may discover that maybe there is a border between zone one and zone two, and this border is a straight line which divides $\angle xOy$ into two equal angles. Thus, the border of the two zones is the bisector of $\angle xOy$.

Note: The following situation can be used instead of 2 b): Identify some positions in the interior of $\angle xOy$ such that the distances from each position to the two sides of the angle are equal. What have you discovered about these positions?

Situation 3 *(Worksheet three,* student*s work in groups)*

a) Prove that if point M is on the bisector $\overrightarrow{Oz}$ of $\angle xOy$, then the distance from M to $\overrightarrow{Ox}$ is equal to the distance from M to $\overrightarrow{Oy}$.

b) Find a set of points in the interior of $\angle$ xOy such that the distance from each point of the set to $\overrightarrow{Ox}$ is equal to the distance from it to $\overrightarrow{Oy}$.

Solution (figure C.4.5)

a) We consider ΔMOH and ΔMOK:

$OM = OM$ (Reflexive property)

$m\angle MOH = m\angle\ MOK$ ($\overrightarrow{Oz}$ is the bisector of $\angle$ xOy)

$m\angle OHM = m\angle OKM$ ($= 90^0$)

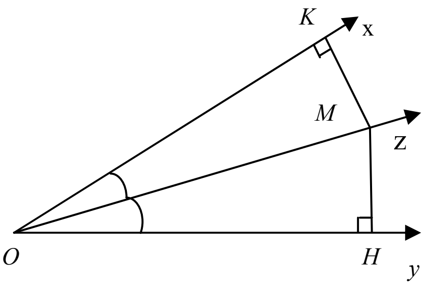


Figure C.4.5

Therefore, ΔMOH and ΔMOK are congruent (ASA Congruence Postulate). Thus, $MH = MK$.

b) (figure C.4.6)

$OH^2 = OM^2 - MH^2$ (Pythagorean Theorem)

$= OM^2 - MK^2$ (because $MH = MK$)

$= OK^2$ (Pythagorean Theorem)

Consequently, $OH = OK$ (1).

We consider ΔMOH and ΔMOK:

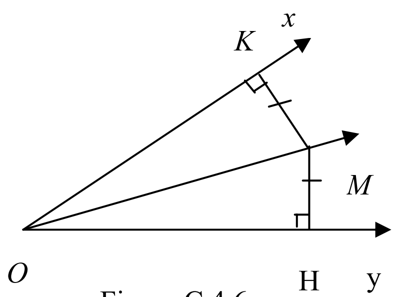


Figure C.4.6

$OM = OM$ (Reflexive property)

$MH = MK$ (given)

$OH = OK$ (From (1))

Therefore, ΔMOH and ΔMOK are congruent (SSS Congruence Postulate). Thus, $m\angle MOH = m\angle\ MOK$. In other words, M is on the bisector of $\angle\ xOy$.

From (1) and (2), the set of points is the bisector of $\angle\ xOy$.

Note: Apart from this solution, a contradiction proof can also be used to prove this theorem.

5. Characteristics of three bisectors of a triangle

Situation (figure C.5.1)

Suppose that a farmer is working somewhere in the interior of ΔABC where $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$ are banks of rivers.

a) Divide ΔABC into three zones. Zones A, B and C include positions in the triangle from which the farmer should go to banks $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$, respectively.

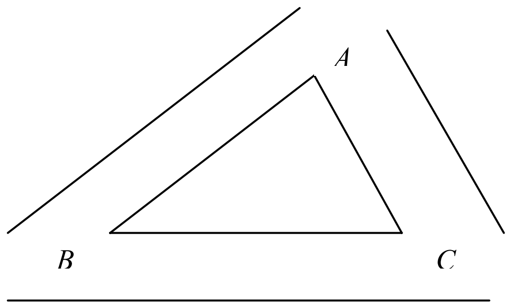


Figure C.5.1

b) Are there any common part(s) of the three zones? What kind of shape is it (are they)? Can you prove this?

Expected solution (figure C.5.2)

a) After the discussion, students may predict that the three bisectors (three borders of zones A, B and C) meet at one single point.

b) Suppose that the bisectors $\overline{AH}$ and $\overline{BK}$ intersect each other at point M.

From b) in the situation 3 (lesson 1), we have:

The distances from M to $\overline{AB}$ and $\overline{AC}$ are equal ($\overline{AH}$ is the bisector of $\angle BAC$) (1)

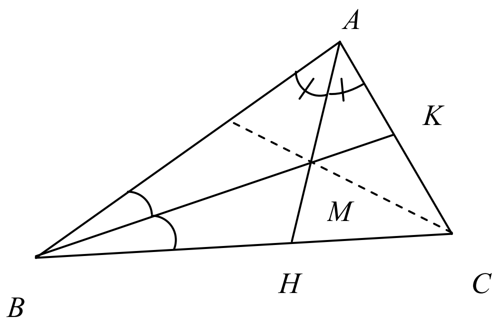


Figure C.5.2

The distances from M to $\overline{BA}$ and $\overline{BC}$ are also equal ($\overline{BK}$ is the bisector of $\angle ABC$) (2)

Hence, the distance from M to $\overline{AC}$ is equal the distance from M to $\overline{BC}$ (From (1) and (2)). Therefore, M is on the bisector of $\angle ACB$ (Situation 3-b in lesson 4).

In other words, the three bisectors of ΔABC are concurrent lines.

6. Characteristics of the perpendicular bisector (mid-perpendicular) of a segment

RME-based lesson

Ideas from *Vorodoi-diagrams* are used in lessons 6 and 8 (Characteristics of the perpendicular bisector of a segment and characteristics of the perpendicular bisectors of a triangle) (Meyer, 1999, pp. 57-59; Goddijn, Kindt & Reuter, 2004, pp. I-5-16). Meyer uses a 'fire stations-large city-fire points' context, whereas Goddijn, Kindt and Reuter refer to a 'wells of water-desert-standing positions' context. The latter is chosen in these two lessons because it sounds more reasonable. However, less than five wells are mentioned to create situations that are suitable for middle school students.

Situation 1 (*Worksheet one,* student *work individually*)

Suppose that some explorers are traveling in a desert, and they are at position *M*. There are two wells of water at positions *A* and *B*. Figure C.6.1 is a simple map of the desert. The explorers are thirsty and want a drink of water.

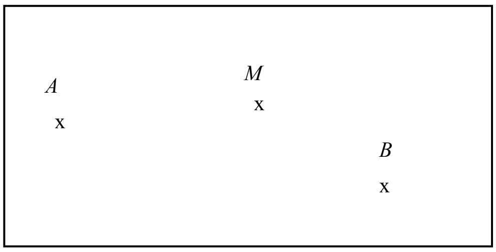


Figure C.6.1

a) Which well should they choose to travel to and why?

b) Point out some positions from which they should go to well *A*

c) Point out some positions from which they should go to well *B*

Expected solution

a) The explorers should choose to go to a nearer well. Students can measure the distances from *M* to *A* and *B* and compare the measures of $\overline{MA}$ and $\overline{MB}$. In this case, $\overline{MB}$ is shorter than $\overline{MA}$. Hence, the explorers should go to well *B*.

b) Students can point out some positions from which the explorers should go to well *A*.

c) Students can point out some other positions from which the explorers should go to well *B*.

Situation 2 (*Worksheet two, students work in groups*)

Let's assume that some explorers are traveling somewhere in a desert. There are two wells of water *A* and *B* (figure C.6.2). They are thirsty and wants to get a drink of water from one of these two wells.

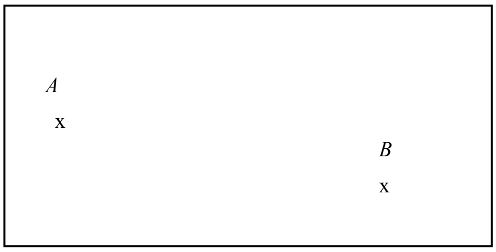


Figure C.6.2

a) Divide the map into three parts: Zone *A* includes places from which they should go to well *A*; zone *B* includes places from which they should go to well *B*; and zone *A-B* includes positions from which they can go to either well *A* or *B*.

b) How can you identify exactly the zone *A-B* when you know points *A* and *B* already?

Note: Similar to lesson about characteristics of the bisector of an angle, the following situation can be used instead of situation 2:

Please point out some positions from which the explorers can go to either well *A* or *B*. What do you notice about these positions?

Expected solution

a) (figure C.6.3)

Based on the experience from situation 1, the students can work in groups in order to find zone *A,* including places from which the explorers should go to well *A* (small circle positions) and zone *B,* including places from which they should go to well *B* (small square positions).

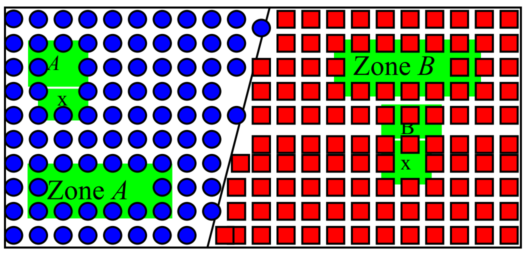


Figure C.6.3

b) Hopefully, the students discover that the characteristics of zone *A-B* (figure C.6.4):

· zone *A- B* is $\vec{d}$;

· $\vec{d}$ is perpendicular to $\overline{AB}$; and

· $\vec{d}$ divides $\overline{AB}$ into two equal segments.

In other words, d is the mid-perpendicular of $\overline{AB}$.

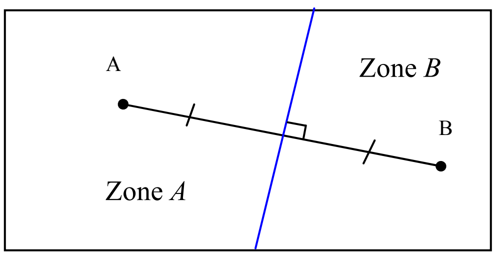


Figure C.6.4

Situation 3 (*Worksheet three, students work in groups*)

Prove that:

a) If M is a point on the mid-perpendicular of $\overline{AB}$, then the distances from M to A and B are equal

b) If there is a point M such that the distances from M to A and B are equal, then M is on the mid-perpendicular of $\overline{AB}$.

7. 'Railway station' problem[100]

7.1 Typical use of the problem in the current textbooks

• This problem may appear in different forms. For instance, there are similar problems in Meyer (1999, pp. 54-55) and Goddijn, Kindt and Reuter (2004, pp. I-82-83). This problem also appears in Vietnamese mathematics textbooks, including the current textbook entitled *Mathematics 7: part 2* (Phan Duc Chinh et al., 2004 b, p. 77):

> *Exercise 48*. Two points M and N lie on a half plane with edge $\overleftrightarrow{xy}$. L is the symmetry point of M through $\overleftrightarrow{xy}$. I is an any point on $\overleftrightarrow{xy}$. Compare $IM + IN$ and MN.
>
> *Exercise 49*.
>
> Two factories are built at positions A and B near a bank of a river (figure C.7.1). Find a position for a mechanical water-pump at the river bank such that the sum of the lengths of the water-pipes from two factories to the machine for pumping water is shortest.

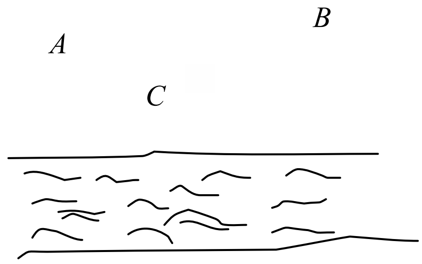


Figure C.7.1

Exercise 49 is difficult for students if they do not have the suggestions given in exercise 48.

• Ideas from Goddijn, Kindt and Reuter (2004, pp. I-82-83) are used in RME-based lesson. However, one condition is added in the first situation (*The distances from A and B to the railway are equal*) so that this situation may offer middle school students the chance to discover mathematical principles on their own.

[100] This is a 'voluntary' lesson. It means that teachers should decide themselves whether they should use this lesson or not.

7.2 RME-based lesson

Situation 1 (*Worksheet one*- Student*s work in groups*):

Suppose that A and B are two cities. The distances from A and B to a railway are equal. A new station will be built. Where should this station be built? (figure C.7.2 is a simple map of the area)

In groups, discuss the following cases:

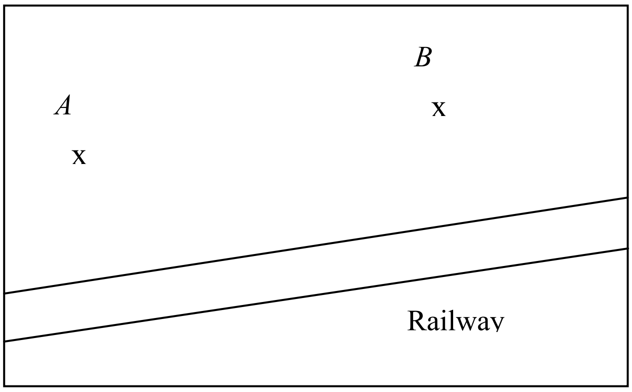


Figure C.7.2

a) Some members of each group live in city A and the other members live in city B

b) All members of each group work for the Ministry of Transportation

Expected solution (figure C.7.3)

a) The citizens in city A want to find one location for the new station such that they can travel comfortably. They want the new station to be within the nearest proximity to city A as possible

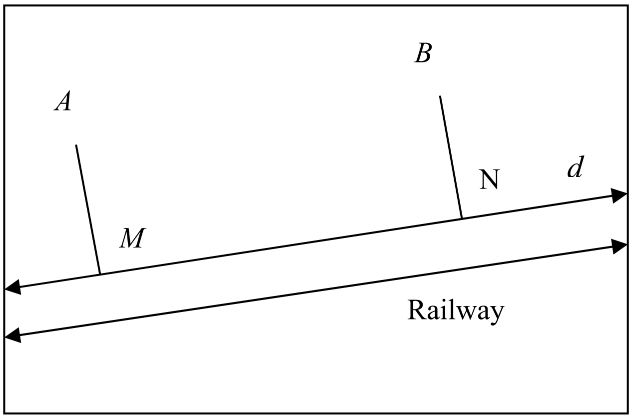


Figure C.7.3

Therefore, they tend to propose position M such that $\overline{AM} \perp \vec{d}$.

Similarly, the citizens of city B want the new railway station to be built at the position N such that $\overline{BN} \perp \vec{d}$.

b) When the students play the role of staff persons for the Ministry of Transportation, they may think that (after discussion in a)) the new railway station should be built at the 'fair place'.

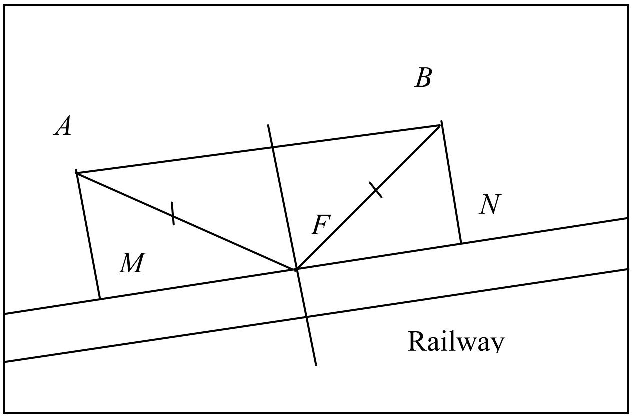


Figure C. 7.4

Of course, it should not be built at position M because this position is not comfortable for people from city B to travel. Similarly, position N is also not chosen.

Consequently, they may think that they need to find a position F such that the distances from F to A and B are equal (figure C.7.4) (They can apply characteristics of the mid-perpendicular of a segment to find F).

However, students might think that it is better if "*we can save money and time for our citizens*". So the new railway station should be built at a position S such that the sum of the distances from S to A and B has the least value.

They may compare $MA + MB$, $NA + NB$ and $FA + FB$ by measuring and find that $MA + MB = NA + NB > FA + FB$ (figure C.7.4).

At first, perhaps students think that point F is the most suitable for building the new railway station because $FA = FB$ (F is the '*fair position*'), and $FA + FB$ is the minimum (F is the '*saving position*'). However, the sum $FA + FB$ is only the minimum sum among $FA + FB$, $MA + MB$ and $NA + NB$.

Some students may believe that perhaps there is another point Q such that sum QA + QB is less than sum $FA + FB$.

Students can choose some positions for Q; measure QA and QB and compare $FA + FB$ and $QA + QB$. However, the number of points on a line is unlimited. Thus, this way cannot solve that $FA + FB$ is the minimum. As a result, they should prove that $FA + FB$ is always less than $QA + QB$.

They may use their experiences of distances' comparison of two segments. Therefore, they may think they should solve the problem as in figure C.7.5, where *A'* and *A"* lies on $\overrightarrow{BF}$ and $\overrightarrow{BQ}$ respectively such that *FA* = *FA*' and *QA* = *QA"*.

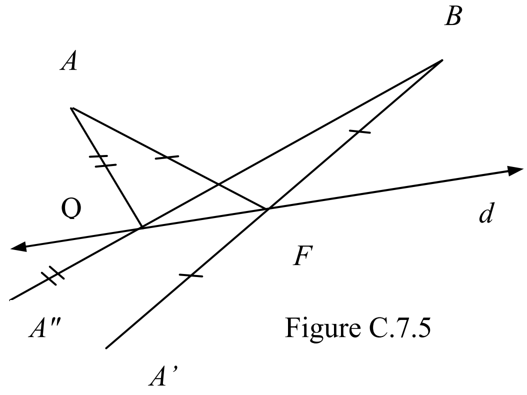


Figure C.7.5

However, this way does not work because it is difficult to compare *BA'* and *BA"*.

If it is necessary, the teacher may give students some guidance such that (figure C.7.6):

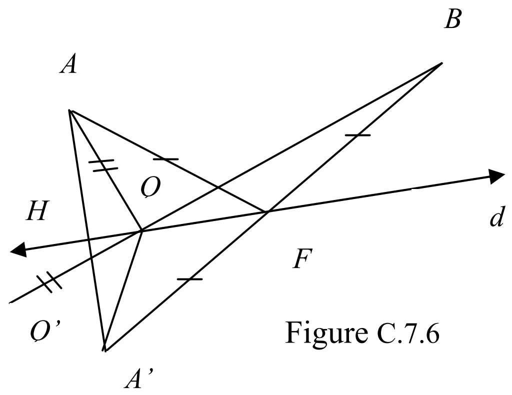


Figure C.7.6

- What characteristic does Δ *FAA*' have?

Student may realize that Δ *FAA'* is the isosceles triangle.

- This suggests that Δ *QAA'* is also the isosceles triangle. From this students can find the solution to the problem.

Δ *AFA'* is the isosceles triangle because *AF* = *A'F*. Consequently, $\vec{d}$ is the mid-perpendicular of $\overline{AA'}$. Since Δ *AFA'* is the isosceles triangle, $\overline{HF}$ is not only the mid-perpendicular but also the altitude of Δ*AFA'*.

Since $\overline{HQ}$ is also not only the mid-perpendicular but also the altitude of Δ*AQA*', *QA* = *QA'* (1).

Therefore, *QA* + *QB* = *QA'* + *QB* (2)

> *BA'* (The theorem of Triangles Inequality)

= *FA* + *FB*

Hence, *QA* + *QB* > *FA* + *FB*

In conclusion, the new railway station should be built at position *F* because of two following reasons:

- It is the ‘*fair position*’. This means that the distances from cities *A* and *B* to *F* are equal.

- It is the ‘*saving position*’. This means that the sum of the distances from *F* to cities *A* and *B* is the shortest sum among the sums of the distances from any point on $\overleftrightarrow{d}$ to two cities *A* and *B*.

Situation 2 (*Worksheet two*- Student*s work in groups*)

Suppose that *A* and *B* are two cities. The distance from *A* to the railway is longer than the distance from *B* to the railway. A new station will be built. (Figure C.7.7 is a simple map)

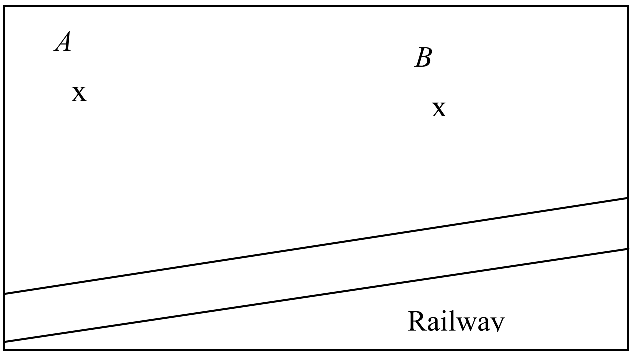


Figure C.7.7

In groups, answer the question “where should the new station be built?” (figure C.7.7)

Expected solution

From the experience in situation 1, students may want to find the ‘fair position’ and the ‘save position’

It is not difficult for them to find that the ‘fair position’ *F* is the intersection point of the mid-perpendicular of $\overline{AB}$ and $\vec{d}$ (figure C.7.8).

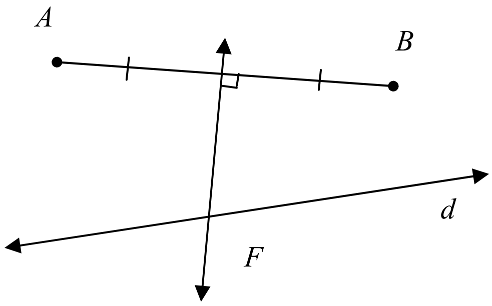


Figure C. 7.8

Normally, pupils think that *F* is also the ‘saving point’. To try to prove this, they tend to use a similar strategy to situation 1. However, the strategy in situation 1 is not applied in this situation because $\Delta QAA'$ is not the isosceles triangle (figure C.7.9).

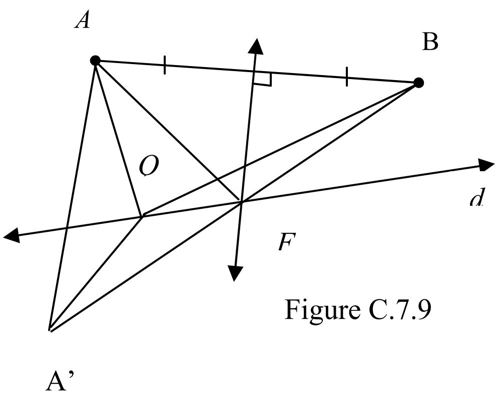


Figure C.7.9

They may think that they should find a point *S* on $\overleftrightarrow{d}$ such that Δ *QAA'* is an isosceles triangle for every point *Q* on $\overleftrightarrow{d}$.

This suggest that $\overleftrightarrow{d}$ is the mid- perpendicular of $\overline{AA'}$. In other words, *A'* is the picture of *A* through the symmetry transformation S_d . From this, they can find the solution to situation 2 (figure C.7.10).

Δ *QAA'* is the isosceles triangle because *Q* belongs to the mid-perpendicular $\overleftrightarrow{d}$ of $\overline{AA'}$.

Consequently, *QA* = *QA'* (1)

QA + *QB* = *QA'* + *QB* (From (1))

> *BA'* (The Theorem of Triangle Inequality) (2)

Suppose that $\overline{BA'}$ intersects $\overleftrightarrow{d}$ at *S*.

Therefore, *SA* = *SA'* (3)

BA' = *BS* + *SA'*

= *BS* + *SA* (From (3))

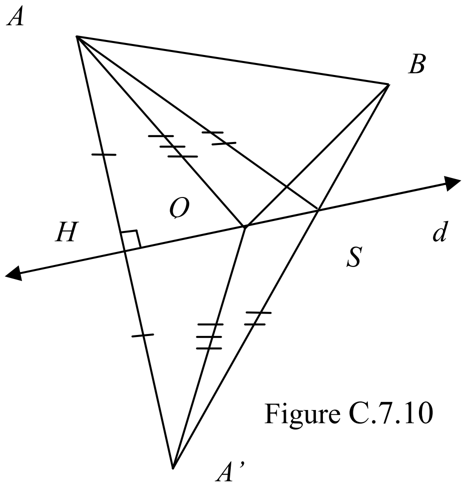


Figure C.7.10

From (2) and (3), we have *QA* + *QB* > *SA* + *SB* for every point *Q* on $\overleftrightarrow{d}$

As a consequence, *S*, the intersect point of $\overline{BA'}$ and $\overleftrightarrow{d}$, is the '*saving position*'.

This situation helps students distinguish '*fair position*' *F* and '*saving position*' *S.*

<u>Note</u>

Apart from situation 1 and 2, the following problem also can be introduced to student*s in high school*:

• The population of city *A* is twice as much as the population of city *B*. Therefore, we should find the least value of sum 2 *AM* + *BM* where *M* is a point on $\overleftrightarrow{d}$.

• Generally, find a point *M* on $\overleftrightarrow{d}$ such that sum of *m MA* + *n MB* has the least value where *A* and *B* are one side of $\overleftrightarrow{d}$; m and n are two positive real numbers.

These problems can be solved by the *analysis method*.

8. Characteristics of three perpendicular bisectors of a triangle

In the present textbook, this theorem is presented as follows:

Students are asked to do this task:

> By using compass and a ruler, construct the three mid-perpendiculars of a triangle. Do you recognize that these three lines are concurrent lines?

(Phan Duc Chinh et al., 2004 b, p. 78)

Later, the theorem and its proof are presented.

RME-based lesson

Situation (*Worksheet one, student work in groups*):

Suppose that some explorers are traveling in a desert. There are three wells of water at points *A*, *B* and *C*. The explorers are thirsty and want to get some water from one of these wells.

a) Find all positions from which they should go to:

· well *A*

· well *B*

· well *C*

Are there any position (s) from which they can go to any of wells *A*, *B* and *C*?

b) Make a statement and prove this statement about characteristic of the three mid-perpendiculars of $\overline{AB}$, $\overline{AC}$ and $\overline{BC}$.

Please consider two cases:

• *A*, *B* and *C* are on a straight line (figure C.8.1.a),

• *A*, *B* and *C* are the three vertexes of a triangle (figure C.8.1.b))

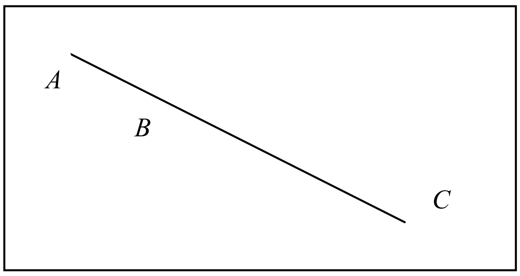


Figure C.8.1.a

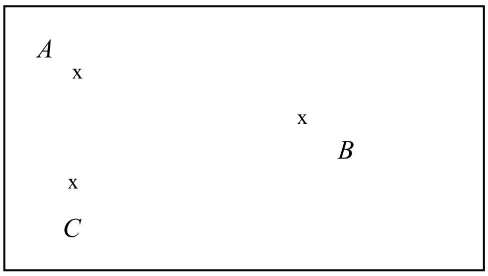


Figure C.8.1.b

Expected solution

• *A*, *B* and *C* are collinear points (figure C.8.2)

a) Students may use their experience from lesson 6 (characteristics of the perpendicular bisector of a segment) for this situation.

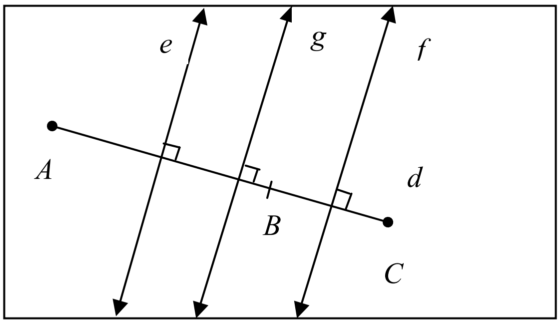


Figure C.8.2

Suppose that $\overleftrightarrow{e}$, $\overleftrightarrow{f}$ and $\overleftrightarrow{g}$ are the mid-perpendiculars of $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$, respectively.

Suppose that zones *A*, *B* and *C* include positions from which they should go to wells *A*, *B* and *C*, respectively.

They might find that zone *A* is the half plane which includes *A* with border $\overleftrightarrow{e}$. Similarly, zone *C* is the half plane which includes *C* with border $\overleftrightarrow{f}$. Zone *B* is a part of the plane which includes *B* and is confined by $\overleftrightarrow{e}$ and $\overleftrightarrow{f}$.

In this case there isn't any point from which they can go to any of wells *A*, *B* and *C* because $\overleftrightarrow{e}$, $\overleftrightarrow{f}$ and $\overleftrightarrow{g}$ do not have any common point. This also suggests that these lines are parallel.

b) Students can realize and prove that $\overleftrightarrow{e}$, $\overleftrightarrow{f}$ and $\overleftrightarrow{g}$ are parallel lines.

• *A*, *B* and *C* are not collinear points (*A*, *B* and *C* are three vertexes of a triangle) (figure C.8.3)

a) Hopefully, when comparing group members' figures with the three mid-perpendiculars of a triangle, students will realize that mid-perpendiculars of $\overline{AB}$, $\overline{AC}$ and $\overline{BC}$ "meet each other at a single point". Later, they can prove this supposition.

b) Suppose that $\vec{e}$, $\overleftrightarrow{f}$ and $\vec{g}$ are the perpendicular bisectors of $\overline{AB}$, $\overline{AC}$ and $\overline{BC}$, respectively; $\vec{e}$ and $\overleftrightarrow{f}$ intersect each other at M (figure C.8.3).

Since M is on $\vec{e}$, $MA = MB$ (1)

Analogously, $MA = MC$ (2)

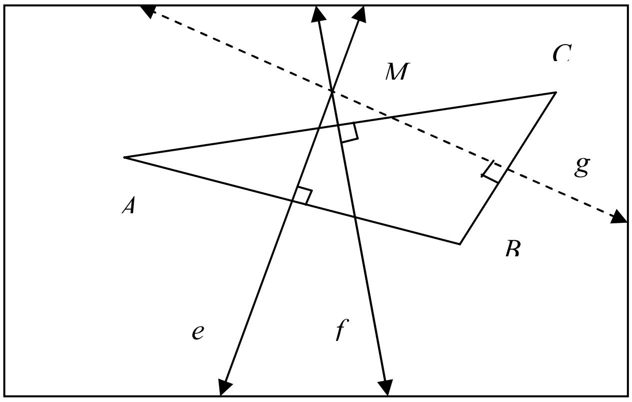


Figure C.8.3

Consequently, $MA = MC$ ($= MB$) ((1) and (2)). Thus, M lies on $\vec{g}$. In other words, $\vec{e}$, $\overleftrightarrow{f}$ and $\vec{g}$ concurrent lines

There is unique point M from which they can go to any of wells A, B and C.

Note

The *Voronoi-diagrams' ideas can be used to teach content of condition for a cyclic quadrilateral ABCD (grade 9 in Vietnam).*

Appendix D: The forms of the students' worksheets (lesson 1)[101]

Table D.1: The first worksheet

THE LEARNING WORKSHEET (1st time) Grade 7 A *Nguyen Luong Bang middle school, Thanh Mien district, Hai Duong province*	
Name of student: ……………………	*Name of student*: …………………….
Please draw an arbitrary triangle	Please measure the angles of the left triangle

Table D.2: The second worksheet

THE STUDENTS' WORKSHEET (2nd time) Grade 7 A *Nguyen Luong Bang middle school, Thanh Mien district, Hai Duong province*	
Name of student: ……………………	*Name of student*: …………………….
Please write down three arbitrary angle measures	Please draw a triangle with the three angle measures on the left (if it is possible)

[101] These worksheet forms were designed by the first teacher, Ms. Do Lan Huong.

CURRICULUM VITAE

Le Tuan Anh was born in 1973 in Hoa Binh, Vietnam and was raised in Hai Duong. He began to study in the Faculty of Mathematics and Informatics at Hanoi University of Education in 1991. In 1995, he graduated from Hanoi University of Education with his bachelor's degree in mathematics. Between 1995 and 1996, he attended a course for training informatics teachers at the same university. After that, he attended a master course and received a master's degree in mathematics education at Hanoi University of Education in 1998. He then worked as a lecturer in the Department of Teaching Methods at the Faculty of Mathematics and Informatics, Hanoi University of Education. During this time, he was mainly involved in training high school mathematics teachers. He also taught elementary mathematics courses as well as mathematics education courses for in-service middle school mathematics teachers in many provinces in the Northern Vietnam. Moreover, he worked part-time as a high school mathematics teacher in several schools and centers in Hanoi. In 2001, he passed examinations for candidates who intend to study overseas and obtained a three-year scholarship sponsored by the Vietnamese Government. In October 2003, he became a PhD student in the Department of Didactics for Mathematics, the Institute of Mathematics at Potsdam University, Germany.