Wolf-Rayet Stars
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Hydrodynamic modeling of massive star atmospheres

A. Sander[1], W.-R. Hamann[1], R. Hainich[1], T. Shenar[1], & H. Todt[1]
[1] *Universität Potsdam, Institut für Physik und Astronomie, Potsdam, Germany*

In the last decades, stellar atmosphere codes have become a key tool in understanding massive stars, including precise calculations of stellar and wind parameters, such as temperature, mass-loss rate, and terminal wind velocity. Nevertheless, for these models the hydrodynamic equation is not solved in the wind. Motivated by the results of the CAK theory, the models typically use a beta velocity law, which however turns out not to be adequate for stars with very strong winds, and treat the mass-loss rate as a free parameter. In a new branch of the Potsdam Wolf-Rayet model atmosphere (PoWR) code, we solve the hydrodynamic equation consistently throughout the stellar atmosphere. The PoWR code performs the calculation of the radiative force without approximations (e.g. Sobolev). We show the impact of hydrodynamically consistent modelling on OB and WR stars in comparison to conventional models and discuss the obtained velocity fields and their impact on the observed spectral lines.

1 Introduction

Already since mass-loss is known to be common for hot and massive stars, the concept of line-driven winds, i.e. the momentum transfer from photons to metal ions by line absorption, has been suggested as the main mechanism to overcome gravity, backed by early calculations from Lucy & Solomon (1970) for UV resonance lines. A few years later, Castor, Abbott, & Klein (1975, hereafter CAK) developed a theoretical description for the radiative force on many lines using analytical approximations calibrated by fitting numerical calculations. Although their underlying assumptions in terms of which lines are important do not hold in the general case, their basic way of describing the radiative force, especially in the simplified way introduced by Abbott (1980), has been kept through all refinements of the CAK theory (e.g., Friend & Abbott 1986; Pauldrach et al. 1986; Kudritzki et al. 1989; Gayley 1995; Puls et al. 2000).

Backed by empirical mass-loss rates derived from the radio (Wright & Barlow 1975) or infrared (Barlow & Cohen 1977) regime, the modified CAK theory (mCAK) became the standard concept for describing the winds of hot stars, especially OB stars. However, the mCAK description fails for the winds of Wolf-Rayet (WR) stars for which much higher mass-loss rates $\dot{M}$ were deduced than what could be provided by mCAK. Lamers & Leitherer (1993) demonstrated that this shortcoming of the mCAK predictions occurs not only for WR stars, but instead for all hot, massive stars with dense winds.

Since the CAK-like theories use significant approximations, the question pops up whether the failure of the theory for certain objects stems from these approximations or if additional wind driving mechanisms need to be considered. To provide a reliable answer, radiation-driven wind models which do not rely on the CAK approximations are inevitable.

2 Obtaining the radiative acceleration

In order to study the influence of radiative driving, the hydrodynamic equation needs to be considered, since it describes the detailed acceleration balance of inward and outward forces. For a spherically-symmetric star with a stationary outflow, this equation can be written in the form

$$v\left(1-\frac{a^2}{v^2}\right)\frac{\mathrm{d}v}{\mathrm{d}r} = a_{\mathrm{rad}}(r) - g(r) + 2\frac{a^2}{r} - \frac{\mathrm{d}a^2}{\mathrm{d}r}. \quad (1)$$

Hereby the equation of state for an ideal gas $P=\rho a^2$ and the equation of continuity

$$\dot{M} = 4\pi r^2 \rho(r)\, v(r) \quad (2)$$

have been implied. The intricate quantity in Eq. (1) is, of course, the radiative acceleration a_{rad}, which is composed of

$$a_{\mathrm{rad}} = a_{\mathrm{lines}} + a_{\mathrm{thom}} + a_{\mathrm{true\ cont}}, \quad (3)$$

i.e., of line and continuum contributions with the latter being further split into the Thomson term and the so-called "true continuum". In the CAK-like approaches, the very last term is omitted, while the line term is approximated in the form

$$a_{\mathrm{lines,CAK}} \propto \left(\frac{r^2 v}{\dot{M}}\frac{\mathrm{d}v}{\mathrm{d}r}\right)^{\alpha}. \quad (4)$$

While this approach allows a fast calculation based on only a few parameters, its simplifications are not sufficient for more dense winds as mentioned above.

One method to obtain a_{rad} without requiring CAK-like approaches is to use Monte Carlo (MC) simulations (Abbott & Lucy 1985; de Koter et al. 1993, 1997; Vink et al. 1999, e.g.), where the radiative acceleration is calculated by following energy packets throughout the stellar atmosphere, allowing for multiple line transitions. However, such MC models need a prescribed wind velocity law and thus can obtain consistency only on a global scale.

For a more detailed understanding, wind models have to be locally consistent, i.e. the hydrodynamic equation must be fulfilled at every point throughout the stellar atmosphere. This can be obtained by using a comoving frame (CMF) approach for the radiative transfer calculation, which yields the radiative acceleration as a function of radius. The results are then used to derive a consistent velocity field via the hydrodynamic equation. The first attempt in this direction with the PoWR code has been performed by Gräfener & Hamann (2005, 2008), constructing a hydrodynamically consistent models for a WC star and several hydrogen-rich WNL stars. Unfortunately their approach turned out to have certain limitations in terms of applicability and was computationally extremely time-consuming.

Both issues have now been addressed in a new approach which is not only faster, but also applicable to a wide range of hot star atmosphere models, including O and B stars. In contrast to the approach from Gräfener & Hamann (2005), the calculation of a force multiplier parameter is not required. The hydrodynamic equation is integrated inwards and outwards from the sonic point. Furthermore, several code improvements have been performed in order to allow faster and more detailed calculations.

A side product of these developments is the implementation of a hydrodynamically consistent treatment of the quasi-hydrostatic, subsonic layers, which has now become a standard branch in the PoWR code and allows the calculation of models that can be used to precisely analyze O and B star spectra. Details of this improvement are described in Sander et al. (2015). These improvements are also crucial for Wolf-Rayet stars with lower mass-loss rates, as successfully shown in the analysis of the SMC single WR-stars by Hainich et al. (2015).

3 Hydrodynamically consistent models

The integration of the hydrodynamic equation (1) requires the values of $a(r)$ and $a_{\mathrm{rad}}(r)$ to be given. Since the latter one requires a CMF radiative transfer calculation, hydrodynamic models cannot be calculated from scratch, but instead a start approximation is necessary. The best models for this task are atmosphere models, which might not be hydrodynamically consistent at each depth point, but at least in terms of the global energy budget. To obtain this budget the hydrodynamic equation is written in the form

$$v\frac{\mathrm{d}v}{\mathrm{d}r} + \frac{GM_*}{r^2} = a_{\mathrm{rad}} - \frac{1}{\rho}\frac{\mathrm{d}P}{\mathrm{d}r} \tag{5}$$

and then integrated and multiplied with $\dot{M}$:

$$\dot{M}\int\left(v\frac{\mathrm{d}v}{\mathrm{d}r} + \frac{GM_*}{r^2}\right)\mathrm{d}r = \dot{M}\int\left(a_{\mathrm{rad}} - \frac{1}{\rho}\frac{\mathrm{d}P}{\mathrm{d}r}\right)\mathrm{d}r$$
$$L_{\mathrm{wind}} = W_{\mathrm{wind}} \tag{6}$$

This equation describes the balance between the modeled wind luminosity of the model L_{wind} and the provided work W_{wind}. Dividing Eq. (6) by L_{wind} yields the so-called *work ratio*

$$Q := \frac{W_{\mathrm{wind}}}{L_{\mathrm{wind}}}. \tag{7}$$

While stellar atmosphere models with $Q < 1$ do not provide a radiation field that is sufficient to drive the wind, models with $Q > 1$ possess a radiation field that could actually drive a stronger wind while models with $Q = 1$ exactly provide the energy that is required to drive the wind. Models with $Q \approx 1$ are therefore the best choice to start the hydrodynamic calculations.

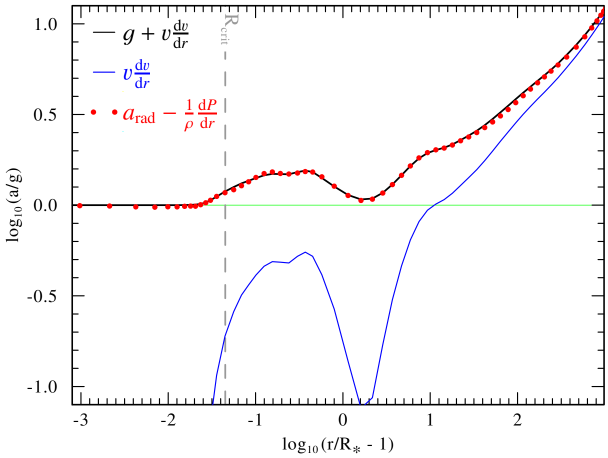

Fig. 1: Detailed acceleration balance for a hydrodynamically consistent WNE model. The black solid curve and the red dotted curve refer to the left and the right hand side of Eq. (5). The blue curve denotes the inertia term.

After a proper starting model is chosen, the velocity field is constantly updated by solving the hydrodynamic equation. Apart from this additional step, the atmosphere models are solved in the same way as models with a prescribed velocity field, i.e. until a consistent solution for the population numbers, the temperature structure, and the radiation field are obtained. Of course the update of the velocity field typically leads to larger corrections, thereby increasing the total calculation time of a hydrodynamically consistent model compared to a "standard" model. In contrast to the hydrostatic equation, the hydrodynamic equation (1) has a critical point, which is located at $v = a$, i.e. the sonic point. In order to have a finite velocity gradient at the critical point, the right-hand side of Eq. (1) must vanish at the sonic point. The new velocity field is obtained by integrating the hydrodynamic equation inwards and outwards from the current critical point r_c, i.e. where the righthand side of Eq. (1) vanishes, and setting $v(r_c) = a(r_c)$. If necessary, the mass-loss rate $\dot{M}$ is also adjusted, so that $Q \approx 1$ is always maintained during the iteration.

The result of a converged model with a hydrodynamically consistent stratification can be seen in Fig. 1, where the acceleration contributions for a consistent, early-type, hydrogen-free WN star model are shown. The total outward forces, radiation and gas pressure, now balance the total inward forces, gravity and inertia, at each depth point throughout the stellar atmosphere.

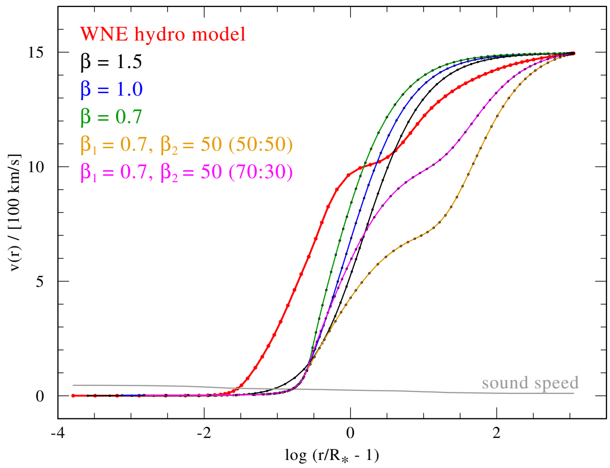

Fig. 2: The velocity field of the hydrodynamically consistent model (red curve) is compared to various prescribed velocity laws.

The velocity field obtained for the hydrodynamically consistent WNE star model is displayed in Fig. 2, where it is compared to several β-laws and two double-β-laws. The drop of the inertia term visible in Fig. 1 corresponds to the plateau in the velocity field seen here. Obviously a β-law is insufficient in describing the obtained velocity field in this example. In principle, the velocity field might be approximated by a double-β-law, but even though very few models have been calculated to far, it does not seem that there is something like a "typical" set of β-values which could be used to describe WR star velocity fields. For hydrogen-rich WR stars, Gräfener & Hamann (2008) showed that the hydrodynamic velocity fields could be approximated by a standard β-law. However, these stars are more like O stars and have less dense winds. Nevertheless, even for dense winds it might be hard to obtain "typical" values as a further model for WO star developed in this work did not show a similar plateau in the velocity field. Instead detailed calculations will likely need to be carried out at least for each spectral WR subtype. Assuming a β-law for dense winds in order to obtain the wind velocity at a certain distance can therefore only be a rough approximation, especially in the inner parts of the wind.

4 HD models as a diagnostic tool

In order to yield proper results for the mass-loss rates, the atmosphere models need to consider all elements which contribute significantly to the radiative acceleration due to their opacities. While this means that a large number of levels has to be accounted for, it also opens the perspective of using hydrodynamically consistent models as diagnostic tools, since detailed information about the depth-dependent contribution of each element is automatically obtained during the calculations of the CMF radiative transfer. These models will therefore allow a detailed study of the wind acceleration around the sonic point, including a check of the CAK approximations.

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Luca Grassitelli: Is the transition between the two velocity laws corresponding to a specific temperature or physical conditions?

Andreas Sander: In the hydrodynamically-consistent models throughout the atmosphere there is just one velocity law. If you refer to the transition from subsonic to supersonic, this is highly model-dependent and has to do with the opacities, mostly iron. There are just very few models so far, so I cannot really say anything about trends, but I doubt that you can assign a temperature to the transition which would hold for more than a particular model. In the standard models using a hydrodynamically-consistent solution in the quasi-hydrostatic regime and a beta-law outwards, we use the criterion of a constant velocity gradient for finding a connection point between the two laws.

Norbert Langer: Can line driving explain the WR winds even if the radii of WR stars are just $1\,R_\odot$ rather than $3\,R_\odot$?

Andreas Sander: So far, just a handful of models have been calculated, but the WNE and the WNO model shown in the talk both have $R_* < 1\,R_\odot$. More calculations have to be done in order to check if this also holds for a broader range of WR stars,

Jorick Vink: Did you need to adopt a clumping factor to achieve consistent models? And if so, what was the assumed value of D?

Andreas Sander: Yes, the models indeed assume a clumping factor, more precisely a depth-dependend one. The clumping in the test models so far starts around the sonic point and increases outwards with maximum values on the order of $D \approx 10$.

Götz Gräfener: Could you reproduce the observed terminal wind velocities with moderate clumping factors of ≈ 10?

Andreas Sander: The WNE star model is actually not adjusted to fit a particular observation, although its spectral appearance is not so far from a WN4 star. The WNE model uses a depth-dependent clumping with a maximum density contrast of $D = 20$. The WO model is based on the results of Tramper et al. (2015) and requires indeed only a maximum value of $D = 10$ to reproduce the terminal velocity of $\approx 5000\,\mathrm{km/s}$.