

# Spectrum formation in Wolf-Rayet stars

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We highlight the basic physics that allows fundamental parameters, such as the effective temperature, luminosity, abundances, and mass-loss rate, of Wolf-Rayet (W-R) stars to be determined. Since the temperature deduced from the spectrum of a W-R star is an ionization temperature, a detailed discussion of the ionization structure of W-R winds, and how it is set, is given. We also provide an overview of line and continuum formation in W-R stars. Mechanisms that contribute to the strength of different emission lines, such as collisional excitation, radiative recombination, dielectronic recombination, and continuum fluorescence, are discussed.

## 1 Introduction

Wolf-Rayet (W-R) stars exhibit a wide variety of spectra – some show only emission lines in the optical while others can exhibit P Cygni profiles as well as photospheric absorption lines. It is generally accepted that the emission lines are formed in a dense wind with  $T(\text{wind}) < T_{\text{eff}}$ . In this review we will discuss some of the key physics behind W-R spectrum formation. Our discussion will be general, and since W-R stars exhibit a wide range of properties, these generalities should be applied carefully.

## 2 Overview

In brief, the spectrum of a Wolf-Rayet star is determined by:

1. The star's effective temperature.
2. The wind density.
3. Abundances.
4. Atomic physics.

Modeling reveals that similar spectra (i.e., with similar equivalent widths [EWs]) are produced when

$$\frac{\dot{M}}{V_{\infty} R_*^{1.5} \sqrt{f}} \quad (1)$$

is held constant (Schmutz et al. 1989).  $f$  is the volume filling factor – an equivalent quantity is the clumping factor which is related to  $f$  by  $D = 1/f$ . The invariance reflects the importance of density-squared processes in W-R atmospheres.

In their modeling, the Potsdam group utilizes this invariance by constructing a transformed radius ( $R_t$ ), defined by

$$R_t = R_* \left[ \frac{V_{\infty}}{2500 \text{ km s}^{-1}} \right]^{2/3} \left[ \frac{10^{-4} M_{\odot} \text{ yr}^{-1}}{\dot{M} \sqrt{D}} \right]^{2/3}$$

(e.g., Sander et al. 2012), to construct model grids for W-R stars. Grids, with  $T_{\text{eff}}$  and  $R_t$  as the independent variable, are constructed and these are used

to generate contour diagrams illustrating the functional dependence of the EWs of various lines. The grids have been used with great success to model W-R stars in the Galaxy (e.g., Sander et al. 2012) and the Magellanic Clouds (e.g., Hainich et al. 2014).

As a consequence of the degeneracy, it is not possible to deduce a star's distance from its spectrum. When comparing mass-loss rates derived for the same star it is important that the same distance is adopted. The degeneracy implies that  $\dot{M}$  scales as  $d^{1.5}$  (since  $R_*$  scales as  $d$ ).

Since electron scattering depends on the density, not the density squared, the strength of the electron scattering wings depends on the adopted value of  $f$ , and this, in principle, provides a means of determining  $f$ . However, while it is easy to show that  $f = 1$  produces electron scattering wings that are inconsistent with observation (Hamann & Koesterke 1998; Hillier & Miller 1999), it is difficult to determine  $f$  to better than a factor of two, especially since we typically use a very simple radial dependence for  $f$  –  $f$  is essentially constant in the line formation region.

W-R stars emit much of their flux in the unobservable EUV, shortward of 912Å. Since the observed fluxes represent only a small fraction of the emitted energy, and since it is also affected by the mass-loss rate, velocity law, and interstellar reddening, continuum fluxes cannot be used to constrain the star's  $T_{\text{eff}}$ . Instead, line ratios (such as the strength of He II lines relative to He I lines) are used to constrain the effective temperature.

One has to be careful as to which definition of  $T_{\text{eff}}$  is being used —  $\tau = 2/3$  is often located in the wind implying that the classical  $T_{\text{eff}}$  is influenced by  $\dot{M}$ ,  $V(r)$ , and clumping. A common practice is to define the effective temperature using the radius at the base of the stellar wind, and to hope that this can be meaningfully related to evolutionary calculations. However even this is not satisfactory. For stars with a dense wind, the velocity law in the inner wind is not constrained observationally, and the adopted velocity law can substantially influence the derived effective temperature. Fortunately, the derived luminosity is much less sensitive to the adopted velocity law.

Ideally we would calculate  $\dot{M}$  and the density structure from first principles, and incorporate them self-consistently into evolutionary calculations. Recent work by Groh et al. (2014) combining CMFGEN (for the atmospheres) and the Geneva evolutionary code have made progress towards calculating more reliable effective temperatures for evolved stars. There has also been work on trying to derive  $\dot{M}$  self-consistently (e.g., Gräfener & Hamann 2005, 2008; Gräfener & Vink 2013) which has highlighted the importance of the proximity of W-R stars to the Eddington limit. However there are complications that make this task very difficult. First, we cannot derive the inhomogeneous density structure from first principles. Second, our simple description of clumping using the volume filling factor is likely to be inadequate. Third, opacity bumps (e.g., due to the Fe peak near 200,000 K) can give rise to density inversions and cause the stars to “inflate” in size (by an order of magnitude!) (e.g., Ishii et al. 1999; Petrovic et al. 2006; Sanyal et al. 2015). The amount of inflation is uncertain since it is affected by the treatment of convection, the amount of mass loss (e.g., Sanyal et al. 2015), and the possible existence of radiation instabilities (which occur near the Eddington limit)(Shaviv 2001).

### 3 Continuum formation

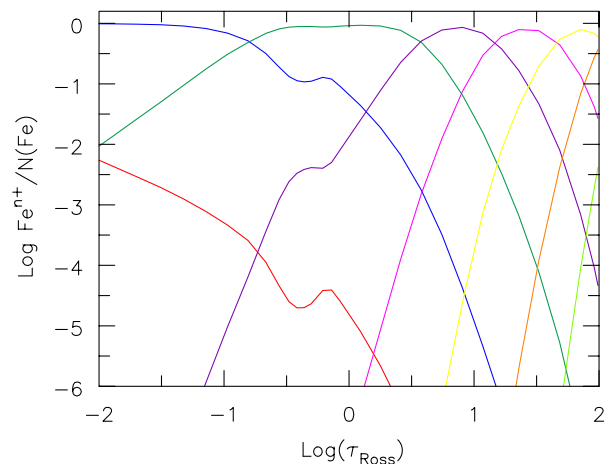
In the radio, the flux is (almost) independent of T (e.g., Wright & Barlow 1975), and we have the simple result that

$$F_\nu \propto \left( \frac{\dot{M}}{V_\infty \sqrt{f}} \right)^{4/3} \frac{1}{d^2} \quad (2)$$

where we have included the dependence of  $f$  using the result of Abbott et al. (1981). A similar result applies to the EW of optically thick recombination lines. Notice how this result is fully consistent with our degeneracy condition, and with our earlier result that  $\dot{M}$  scales as  $d^{1.5}$ . The infrared flux has a similar scaling to the radio, but the shape of the velocity law is also important (e.g., Lamers & Waters 1984). Shallower velocity laws (i.e., with a slower acceleration to terminal velocity) give higher fluxes. The visual flux of W-R stars is also affected by the mass-loss rate. In strong-lined WNE stars the optical continuum flux is directly affected by the mass-loss rate. However in weak-lined W-R stars, those most closely related to O stars, the optical continuum is much less sensitive to the mass-loss rate. While it is difficult to determine accurate mass-loss rates, photometry (particularly K band) allows variations in mass-loss rate of a few percent to be easily detected.

### 4 Ionization structure

A key property of Wolf-Rayet winds is that the ionization is not constant – the highest ionization stages (e.g.,  $\text{He}^{2+}$ ,  $\text{N}^{5+}$ ) are found in the inner wind while low ionization stages (e.g.,  $\text{He}^+$ ) are found in the outer wind (Fig.1). This property is believed to be crucial for understanding the radiative driving of the stellar wind, since lower ionization stages will have their lines (i.e., the distribution in wavelength) better matched to the flux distribution.



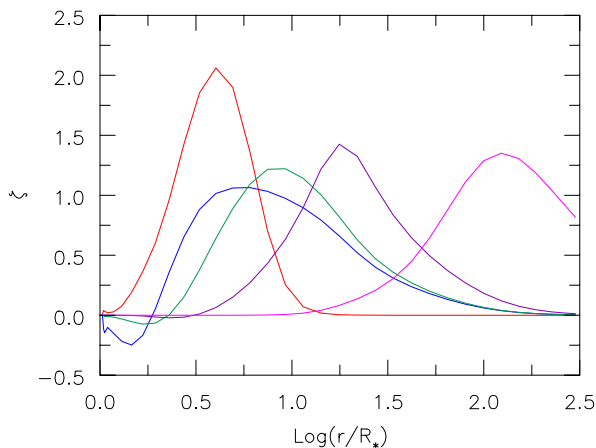
**Fig. 1:** Example of the ionization structure for a WN7-like model. Shown are the ionization fractions for  $\text{Fe}^{3+}$  (red) through  $\text{Fe}^{10+}$ .

Naively, one might have expected constant ionization in the wind. This follows if one assumes that ionizations occur from the ground state, since both the mean intensity and the density scale as  $1/r^2$ . However, in the dense winds of W-R stars ionization from excited states is important (e.g., Hillier 1987, 1989), and this helps to maintain the ionization. However, the population of the excited states generally falls off faster than  $1/r^2$ , causing the photoionization rate to decrease more rapidly than the recombination rate, and hence leading to a lowering of the ionization. Of course in the strongest-lined W-R stars the inner wind is optically thick, and changes in ionization occur simply because of changes in temperature.

Consider the case of He in WNE stars. The ionization of He (as  $\text{He}^{2+}$ ) in the wind is maintained by photoionizations out of the  $n = 2$  state of  $\text{He}^+$  (Hillier 1987). To first order, the population of the  $n = 2$  state is maintained by the radiation field in the He II Lyman resonance transition at 304Å. At larger radii, the population of the  $n = 2$  state falls, and  $\text{He}^+$  becomes the dominant ionization stage of He. For stars with a low  $T_{\text{eff}}$ , the transition will occur very close to the star, while for hotter stars the transition may occur (if at all) at 10 (or more)

stellar radii.

For O stars it was found that the inclusion of blanketing in models led to a decrease in  $T_{\text{eff}}$ . This occurred since blanketing led to back warming in the photosphere and hence a higher temperature and ionization (Martins et al. 2002, 2005). In order to maintain a fit to the spectrum, it was thus necessary to reduce  $T_{\text{eff}}$ . In contrast, the inclusion of line blanketing in W-R stars led to an upward revision of their effective temperature (e.g., Hillier & Miller 1999). In this case, blanketing reduces the radiation shortward of  $350\text{\AA}$ , thereby decreasing the  $n = 2$  population and hence reducing the ionization rate. To maintain the ionization at the observed value, as set by the observed ratio of He I to He II line strengths, a higher  $T_{\text{eff}}$  (and hence luminosity) is required.



**Fig. 2:** Illustration of where various lines originate in a WN4-like star. The integral of  $\zeta$  over  $\log r$  gives the EW which has been normalized to unity for illustration purposes (Hillier 1989). Shown (from left to right) are the origin of N V  $\lambda$  4620, He II 11-4, He II 7-4, He II 5-4 and He I  $\lambda$  10830.

## 5 Line formation in winds

In general a line can be in emission either because the line source function,  $S$ , is larger than in the continuum, or because the emitting surface is larger than in the adjacent continuum. For an optically thick line with constant source function and  $V(r) = \text{constant}$  it can be shown (using the Sobolev approximation) that

$$\text{Line Flux} \propto R_{\tau=1}^2 S .$$

For W-R stars, lines are generally in emission because the emitting surface is larger than that of the adjacent continuum. Of course  $S$  need not be constant, and will vary with the transition under consideration. In reality, a multitude of processes contribute to line formation in W-R winds. Further, the

ionization structure plays a key role in determining line strengths, and not all lines are optically thick (e.g., He I  $\lambda$  5876 in WNE stars). In detailed models each observed line originates over a range of radii, and different lines originate in different regions of the stellar wind (Fig. 2).

It is often assumed that the optically thick stellar core will cause an asymmetry in the observed line profile, since it blocks radiation from the receding hemisphere. However, this asymmetry need not be present since we also ‘lose’ line emission from material in front of the core. The Eddington-Barbier relation tells us that along a sight line we see the source function at  $\tau \sim 1$ . If  $S(\text{line}) \approx B(\text{continuum})$  we get a symmetric emission profile – there is no (excess) line contribution from directly in front of the star, and no line contribution from directly behind the star. When  $S(\text{line}) < B(\text{continuum})$  we get a P Cygni profile while  $S(\text{line}) > B(\text{continuum})$  can give rise to blue-shifted emission. In principle, it should be possible to see the influence of core occultation for an optically thin line. However, in practice optically thin lines (such as He I  $\lambda$  5876 in WNE stars) form at larger radii, and the effect of core occultation is small.

Below we consider various mechanisms that contribute to line emission in W-R stars. It is not always possible to identify a single dominant physical mechanism producing a given line – several processes may compete and the dominant process may depend on depth. Further, it does not necessarily make sense to determine the mechanism for a line whose levels are almost in LTE.

### 5.1 Resonance scattering

This process gives rise, for example, to P Cygni profiles in O stars. If resonant scattering is dominant, the absorption EW should be larger than the emission EW. This is typically observed for the N V and the C IV resonance doublets in O stars.

### 5.2 Collisional excitation

In dense winds, collisional excitation of low lying levels becomes important. For example, the N IV  $\lambda$  1486 and C III  $\lambda$  1909 intercombination lines are produced by collisional excitation. In W-R stars the UV resonance doublets of C IV and N V also have significant contributions by collisional excitation.

### 5.3 Recombination

The recombination process is important for H and He emission lines, and for most of the C lines in WC stars. Since the recombination line strength scale (crudely) with the abundance, recombination processes are primarily important for the most abundant species.

## 5.4 Dielectronic recombination

Low temperature dielectronic recombination (LTDR) refers to recombination via a doubly excited state with an energy just above the ionization limit (Nussbaumer & Storey 1983). For example, the C III  $2p\ 4d\ ^1F^\circ$  state lies just above the C III ionization limit. This state has a very high probability of autoionizing to  $C^{3+}$  plus a free electron, and the reverse process also occurs. Since the autoionization probability ( $\sim 10^{14}$ ) is much larger than radiative rates (typically  $10^8$  to  $10^9$ ), the state is in LTE with respect to the ground state of C IV. The state can lead to efficient recombination, and act as a selective excitation mechanism, if one of the two excited electrons has a significant transition probability to a lower state. LTDR autoionizing levels can be treated as bound states, or via resonances in the photoionization cross-sections.

High temperature dielectronic recombination (HTDR) refers to the general dielectronic process, and does not rely on the existence of specific states close to the ionization edge. For C III, HTDR refers to dielectronic recombination through states of the form  $2pnl$  where  $n$  is large (e.g., 100). For this process it is the decay of the  $2p$  electron (i.e.,  $2pnl \rightarrow 2s\ nl$ ) that provides the recombination route. Limited tests suggest that this process is not crucial in W-R stars.

## 5.5 Continuum fluorescence

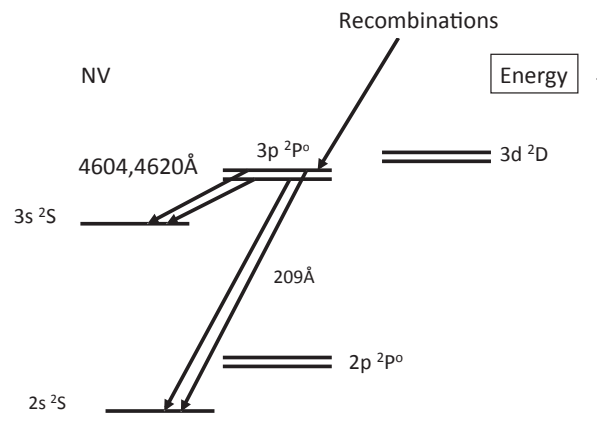
Continuum fluorescence, sometimes called the Swings mechanism, was originally discussed by Swings (1948) to explain the selective excitation of certain lines in various early-type emission-line stars. In this process a UV transition (e.g., C IV  $2s\ ^2S\ 3p\ ^2P^\circ$  at  $312\text{\AA}$ ) absorbs continuum radiation. If the transition is optically thick, the radiation will scatter many times in UV transition. However, eventually the upper level (e.g.,  $3p\ ^2P^\circ$ ) may decay to an alternative level (i.e.,  $3s\ ^2S$ ) giving rise to emission at a longer wavelength (e.g., the C IV  $5801, 5812\text{\AA}$  doublet) (Hillier 1988). For the process to be efficient the UV transition has to be optically thick – for the case discussed above the optically thin transition rate to the  $2s\ ^2S$  state is approximately 150 times the rate to the  $3s\ ^2S$  state.

Continuum fluorescence is important for C IV  $\lambda\lambda 5801, 5811$  and for some N IV lines in WN stars (e.g., Hillier 1988), as well as for many metal lines (Mg II, Si II, Fe II) in P Cygni-type stars.

## 5.6 Line interactions

Line transitions may overlap intrinsically (i.e., their intrinsic emission profiles overlap) or they may overlap because of the influence of the velocity field. As a consequence of such overlaps, line strengths can

be dramatically changed. Bowen fluorescence is an obvious example of the first case. Such overlap is automatically taken into account in line-blanketing codes, although unfortunately the wavelengths of many EUV lines are unknown, and thus important effects can be missed and/or treated incorrectly (since we include the line transitions but at the wrong wavelength). The use of super levels (see Hillier & Miller 1998) can also cause the effects of line overlap to be treated incorrectly.



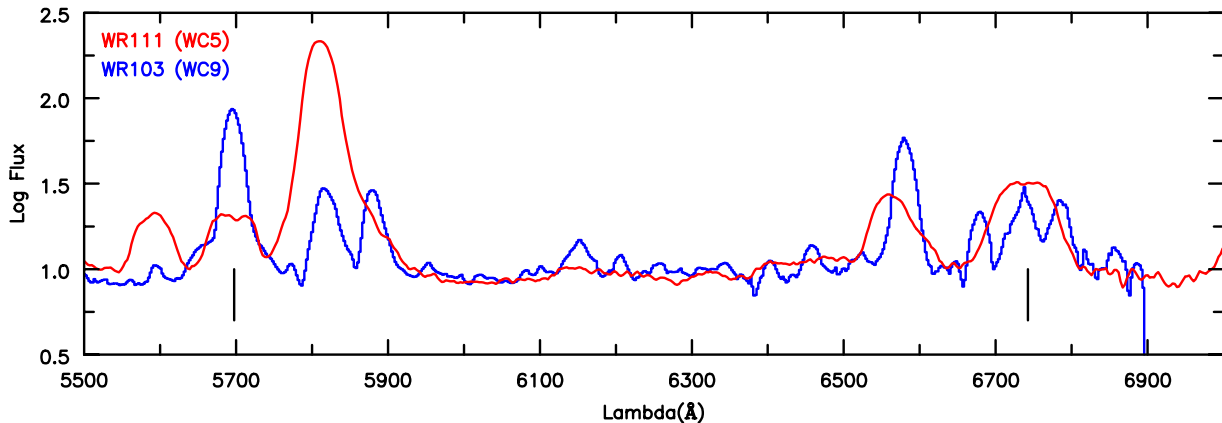
**Fig. 4:** Grotrian diagram for N v. A similar diagram applies for C IV except for a wavelength shift – the resonance transition is at  $312\text{\AA}$ , and the optical doublet is at  $\lambda\lambda 5801, 5812$ .

## 5.7 Atomic physics matters

The strengths of emission lines in W-R stars do not always behave in a simple manner. Some lines can be selectively excited (e.g., by continuum fluorescence), while other lines can be unusually weak. For WC stars, the ratio of C III  $5696\text{\AA}$  to C IV  $5805\text{\AA}$  is a principal classification ratio. In WC4 stars C III  $\lambda 5696$  is (almost) absent, while its strength increases rapidly toward later spectral types (Torres et al. 1986). Other C III lines do not show the same dramatic variation – lines such as C III  $\lambda 6740$ , are still present in WC4 stars (Fig. 3).

The behavior of C III  $\lambda 5696$  can be understood on the basis of the atomic structure of the C III atom (Torres et al. 1986; Hillier 1989). In WC4 stars the  $2s\ 3d\ ^1D$  state primarily decays to the  $2s\ 2p\ ^1P^\circ$  state. Emission in  $\lambda 5696$  ( $2s\ 3d\ ^1D$  to  $2s\ 2p\ ^1P^\circ$ ) increases in strength as the primary decay route, at  $574\text{\AA}$ , becomes optically thick. Further, the population in the  $2s\ 3p\ ^1P^\circ$  state is efficiently drained by transitions to  $2p^2\ ^1D$  and  $2p^2\ ^1S$ . Thus  $\lambda 5696$  can remain optically thin (and hence exhibit a flat-topped profile), even when it is strongly in emission.

Another interesting case occurs with the lithium-like ions C IV and N V (Fig. 4). In HD 50896 (WN5), for example, the C IV  $\lambda\lambda 5801, 5812$  and the N V



**Fig. 3:** Spectrum of HD 165763 (WC5, WR111)(red) and HD 164270 (WC9, WR103)(blue)(Torres-Dodgen & Massey 1999). Notice the different relative strengths of the two C III multiplets at  $\lambda 5696$  and  $\lambda 6742$  (indicated by vertical lines). In WC4 stars the  $\lambda 5696$  line is almost absent, while other C III multiplets are still prevalent.

$\lambda\lambda$  4604, 4620 transitions have “similar” strength (Hillier 1988) despite the fact that the N abundance is roughly 40 times the C abundance (e.g., Maeder & Meynet 1987). While part of this is attributable to a difference in ionization, it also occurs because of the different wavelengths of the respective EUV transitions. The 3p level in C IV is connected to the ground state by a transition at  $312\text{\AA}$ , and can be pumped efficiently by the EUV radiation field. In contrast, the N v transition lies in the optically thick He II continuum at  $208\text{\AA}$  where there is little UV flux, and hence it cannot be efficiently pumped. In WN stars the N v doublet primarily arises through recombination.

In the very hottest WN stars the N v  $\lambda\lambda 4604, 4620$  can actually weaken as  $T_{\text{eff}}$  increases, even while the strength of other N v (recombination) lines is increasing. This occurs because of the declining optical depth of the  $208\text{\AA}$  transition – electrons can then decay from the 3p state via this transition rather than via the optical transition.

Understanding line and continuum formation in W-R stars is not simply an academic exercise. With such information one is better able to discern the significance of discrepancies, and to better understand systematic errors associated with the modeling.

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D. J. Hillier

**Anthony (Tony) Moffat:** What do you mean when you say that WR winds are optically thick? As for the continuum, the wind is optically thick only below  $\tau_e \sim 2/3$ , then thin outside. If optically thick in lines, then does this mean for the whole domain in radius where the line is formed? What about weak vs. strong lines, or lines of different ionization formed at different radii?

**D. John Hillier:** When I say that W-R winds are optically thick, I mean that  $\tau \sim 2/3$  occurs in the wind, above the sonic point. However for W-R stars with “weak winds”, this may not be true at all wavelengths. The observed flux will typically arise from regions with tau somewhat above 2/3, and  $\tau$  below 2/3, but this statement is line/continuum dependent. Because of the extension caused by the wind, much of the flux will/can come from optically thin regions of the wind. In the case of the radio flux, for example, Wright and Barlow (1975) show that the observed radio flux is equivalent to receiving all of the emission coming from optical depths less than 0.244. For many optically thick lines (e.g., He II 4686 in WNE stars), it can be shown that roughly 50% of the flux arises from regions with a Sobolev optical depth  $> 1$ . For the case of He I 5876 in WNE stars, the line optical depth in the formation region

is less than 1, and this line can be considered to be “optically thin”. The strength of such lines is controlled primarily by the location in the wind where the dominant state of He changes from He<sup>++</sup> to He<sup>+</sup>.

**Gloria Koenigsberger:** How does one judge the goodness of the fit of a model to an observed spectrum? What guidance criteria should be used?

**D. John Hillier:** One needs to run sensitivity tests. That is, one varies the parameter (e.g., mass-loss rate, abundance etc) about the best fit value, and examines how observables changes. In some cases the sensitivity is quite strong (e.g., linear), and in other cases the sensitivity can be weak. When the strength of a feature is bracketed by the models, one can be fairly confident in the results. However, when the model feature is always discrepant with the observations the implications of the discrepancy can be difficult to determine. In other cases the strength of features can be controlled by several model parameters and this requires additional testing. In recent work it is becoming more common to perform systematic studies for a large number of parameters, and then to use, for example, chi squared minimization to deduce the best models. Even then, however, judgement calls are usually made to exclude certain features which cannot be fit.

