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Ralf E. Becker

Revisiting Public Investment – Consumption Equivalent Public Capital and the Social Discount Rate

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Ralf E. Becker
Magdeburg (Germany)
Tel.: (+49) 0391 555 86-30; Fax: ...-29
E-mail: *ralf_e_becker@t-online.de*

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Ralf E. Becker [1]
Magdeburg (Germany)

The consumption equivalence method is the theoretical basis of public cost-benefit analysis. Consumption equivalence public capital prices are explicitly introduces in order to sufficiently care for the opportunity cost of public expenditure. This can solve the dispute about the social rate of discount within public cost-benefit analysis witch was generated on a criterion looking similar to the capital value formula, known as Lind's approach. The social rate of discount is liberated from opportunity costs considerations and the discounting away of the effects for future welfare vanishes. The corresponding question whether one should accept a positive value of the pure rate of social time preference is an old issue. Its current state between the prescriptive and descriptive view can also be interpreted as a consequence of the oversimplification of standard cost–benefit analysis. But apart from an economic self-process the pure rate of social time preference is also defined as a business-as-usual value of social distance discounting. Hence, a political choice has to be made about this rate which is free in principal.

JEL: H43, H63

[1] P.: +49(0)391 55586-29 (fax: …-30); ralf_e_becker@t-online.de; Ralf E. Becker © (Magdeburg, Germany)

1. Introduction

What is the rate that should be used to discount the effects of a public investment project? For example, should we use the market rate of interest and discount damages of climate change in two hundred years down to zero? Should we alternatively ignore the opportunity costs of capital used for public investment and use a rate which is much lower referring to a pessimistic expectation of future wellbeing? This *O-problem* of orientation is still unsolved.[2]

Another corresponding old dispute about the *pure* social rate of time preference already started in 1928 with Ramsey's clear rejection of a positive rate on ethically grounds.[3] This rate can be found in intertemporal welfare integrals and co-determinates the optimal growth paths. Hence, we call it the *G-problem*. The corresponding Ramsey rule states that the private rate of return is equal to the *overall* social rate of time preference whenever the growth paths is intertemporally optimal. Today, the most important attempt to justify a positive rate is excellently expressed by Nordhaus (1994), called descriptive approach. In the view of Nordhaus it is not essential how one generates the value (path) of the overall rate of social time preference as long as it fits to the data of the private rate of return based on the most probable extrapolation into the future.

The state of the art is that we do not know whether we should use a high or low social rate of discount, whether we should accept a pure rate of social time preference, and what the logical link between these issues is (e.g., cf. Portney and Weyant, 1999). Below we try to answer these three questions on firm grounds of standard theory. David Bradford (1975) had subsumed the contributions about public cost–benefit analysis (pCBA) resulting in the consumption equivalence method (CEM) which is still the currently accepted bases of discussion by the protagonists as well as the antagonists of a high discount rate.[4] With the consumption equivalent public capital method (CE*PC*M) we present a refinement of the CEM. First, the so-called generalized Arrow–Kurz assumption which was implemented by Lind (1982) in his approach of pCBA is eliminated. It was thought to hold for special project categories (e.g., Lind 1982, p. 46: "standard cost-effectiveness analysis").[5] But more important was the covered property of Bradford's (1975) two period example to be fully tax-financed. In contrast, his general multi-period decision criterion formally leaves the question of credit or tax financing unspecified. Because we refine the criterion given by Bradford (1975) it is the unavoidable start of our investigation:

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2 E. g., cf. Nakićenović et al. (eds., 1994), Arrow et al. (1996) or Portney and Weyant (eds, 1999).

3 Cf. Ramsey (1928, p. 543). Similar to this is the prescriptive approach (e.g., cf. Cline, 1992).

4 Cf. Cline (1992), Nordhaus (1994) or the exposition of the state of discussion in Arrow et al. (1999).

5 Portney and Weyant (1999, p. 3) called *Lind's approach* an „apparent compromise".

2. Public Cost–Benefit analysis

2.1. Consumption equivalence method (CEM; Bradford, 1975)

Suppose that there is only one good that can be consumed or invested. Let e_t represent cost less benefit of a public investment which affect the public budget in period t (budget-CB). In contrast, b_t comprises benefit less cost which do not affect it (non-budget-BC). Both budget- as well as non-budget positions are measured in current consumption units. But some shares of cost and benefit will also influence private investment. Hence, these shares must be transformed into consumption equivalents (CE). For a public investment resources must be deprived from the other economic sectors. A withdrawal of one unit via the public budget replaces current private consumption of $1 - a_t$ and private investments of a_t ($0 \leq a_t \leq 1$). Private investments a_t would produce a future consumption stream of $a_t v_t$ measured in consumption equivalents of time t where the shadow price of private capital v_t transforms a unit of private investment into its consumption equivalent value at time t.[6] Altogether the burden expressed in a consumption equivalent t-current value is $1 - a_t + a_t v_t$ for each unit of e_t. Accordingly, $1 - \alpha_t$ of a unit of non-public-budget output will immediately be consumed and α_t privately reinvested. This leads to additional consumption in current consumption equivalents of $1 - \alpha_t + \alpha_t v'_t$. Hence, the following factors β_t and ε_t break down the respective effects on private consumption and private investment where the latter is evaluated using shadow prices of private capital:

$$\begin{aligned} \varepsilon_t &= 1 - a_t + a_t v_t \\ \beta_t &= 1 - \alpha_t + \alpha_t v'_t \end{aligned} \tag{1}$$

These *sharing factors* represent the overall current value of all the private consumption which is changed by one unit of cost or output of the project expressed in consumption units of time t. The social rate of time preference (SRTP) i for simplicity (here) is presumed to be constant and represents the rate at which society is willing to exchange consumption now with consumption in the future. David Bradford's (1975, p. 896) consumption equivalence method (CEM) of public cost-benefit analysis (pCBA) results in his criterion (here with a constant SRTP) for public investment decisions:

$$B \equiv \sum_{t=0}^{T} \frac{b_t \beta_t - e_t \varepsilon_t}{(1+i)^t} \geq 0 \tag{2}$$

What are the fundamental characteristics of the CEM? First, the CEM distinguishes two types of costs and benefits: budget CBs and non-budget BCs. Second, the current value numeraire of all these positions is private consumption. Hence, effects on private investment are transformed into current CE-units which subsume the value of the whole stream of additional consumption which is due to each unit of this private investment. Third, the present value is calculated using the SRTP as discount rate.

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[6] Already Mendelsohn (1981, 239) gave the catchy description of the intention of shadow prices of private capital: "Investments are treated as merely stream of future consumption." For different concepts see Bradford (1975, 893/4), Lind (1982, 48–55), or Cline (1992, *Annex 6A*, 270–274). For all of them the value increases with a higher marginal rate of return of private capital r or a higher saving rate s. But it decreases if the social rate of time preference i is increased. For $r = i$ all concepts produce $v_t = 1$.

Bradford's (1975) empirical view was that the marginal rate of return on private investment r would have a similar value as the social rate of time preference i. This implies for shadow prices of private capital generally $\nu_t \approx 1$ ($\nu'_t \approx 1$). Hence, in turn also the decomposing factors are close to one ($\beta_t \approx 1 \approx \varepsilon_t$). The result was Bradford's (1975, p. 898) rule of thumb (cf. (3)below): "*Maximize present value of net dollar flows (including dollar equivalents of nonmarketed effects), discounting at the social rate of time preference.*"

2.2. Lind's approach

As already Mendelsohn (1981) also Lind (1982, p. 50) criticizes this empirical view of Bradford. But he arrived at the same rule by the stipulation of a widespread relevance of the so-called generalized Arrow–Kurz assumption $\beta_t = \beta = \varepsilon = \varepsilon_t$, which states that all costs and benefits of a public investment project show the same and therefore also constant sharing effects with respect to private consumption and private investment. For both of the two justifications by Bradford and Lind the *rule of thumb* for public CBA is simply:

$$L \equiv \sum_{t=0}^{T} \frac{b_t - e_t}{(1+i)^t} \geq 0 \qquad (3)$$

The *Ramsey rule* below represents a necessary condition for growth paths to be optimal based on an intertemporal social value function. The rule is beyond any dispute within the framework of optimal growth theory.[7]

$$r = i \qquad (4)$$

The SRTP i comprises a growth dependent discount rate ($\alpha \hat{c}$) and the pure social rate of time preference (P-SRTP) δ where α is the absolute value of the elasticity of the marginal intertemporal social value with respect to per-capita consumption with $\hat{c}$ as the respective growth rate of this consumption:

$$i \equiv \alpha \hat{c} + \delta \qquad (5)$$

If we insert Ramsey rule (4) in Lind's approach (3) then this has two consequences: First, public project evaluation is thought to be very similar to the private capital value method because the insertion of the Ramsey rule (4) in the rule of thumb (3) leads to rule (6) witch one could call *mock capital value* (MCV). The name shall draw attention to the fact that in contrast to the normal capital value method the b_t-positions in this MCV do not represent public cash flow:

$$MCV \equiv \sum_{t=0}^{T} \frac{b_t - e_t}{(1+r)^t} \geq 0 \qquad (6)$$

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[7] Cf. Chakravarty (1969, p. 65/107), Dasgupta (1982, p. 274–277: graphic), Nordhaus (1994, p. 124), Arrow et al. (1996, p. 130/134). "The condition tells us that the marginal rate of transformation between consumption at dates t and t+1 must equal the marginal rate of indifferent substitution." (Dasgupta, 1982, p. 277)

Second, the Ramsey rule (4) implies that the P-SRTP δ contributes to the overall value of the discount rate in the mock capital value(6). Beyond the general mathematical acceptance it is assumed that economic reality can be expressed by the Ramsey rule. In the so-called descriptive approach represented by Nordhaus (1994) under some common assumptions via market arguments the level of the Ramsey rule (4) is drawn from observed or usefully extrapolated data about the market rate of interests. The application of the MCV using observable market rates of interest is the mainstream approach of public cost–benefit analysis. Nordhaus (1994) leaves open, why he determined his special shares of parameters α and δ. This suggests a degree of freedom.

2.3. The *G*-problem and the *O*-problem

A first traditional dispute within optimal growth theory (*G-problem*) is whether one should accept a positive P-SRTP δ or not.[8] (The decision implies, whether i or $\alpha\hat{c}$ is used in pCBA.) In the later case the descriptive positions (e.g., cf. Nordhaus, 1994) would argue that one would have to choose a higher α in order to guarantee that the right hand side of the Ramsey rule remains equal to the left hand side according to data of the private rate of return given a certain path of $\hat{c}$. But there is a major hind to reach this emergency exit from the old δ-dispute: Both parameters, α and δ, have an economic meaning. For example, Schelling (1994) interprets the P-SRTP as the intertemporal version of a more general concept of "distance discounting" with time as a dimension of distance similar, say, to the regional one. In addition, a part of δ could also be interpreted as the natural outcome of the market economy with variables like bequest affinity, intertemporal private market externalities, tax systems or life time expectancies. Hence, the so-called pure rate of social time "preference" in part may not represent social self preferences but also a *social-economic self process*. Intuitively based on this obstacle for an easy out way and in contrast to Nordhaus, the prescriptive approach (e.g., cf. Cline, 1992) rejects the application of (a part of) a positive δ also for pCBA in accordance already to Ramsey (1928).[9]

A second problem is that the MCV (6) generates an orientation conflict (*O-problem*) of the discount rate between the implementation of opportunity costs of public capital and intertemporal consumption weighting. The major representative of this problem is the evaluation of projects to avoid damages of climate change far into the future at high costs for the public budget today.[10] The descriptive view has a problem with the application of a discount rate lower than market rates ($< r$) within pCBA implied by a rejection of a positive δ. Because budget-CBs e_t represent public cash flow it is thought that the application of $\alpha\hat{c} < r$ would not sufficiently implement the opportunity costs of public capital. Hence, they vote for pCBA à la(6). In contrast, the other group of economists think that the usage of r (inclusive a positive δ) discriminates future consumption at the burden of the intertemporal social value sum. On the basis of (3) this implies some kind of a *prescriptive rule of thumb*

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[8] For an overview about the classical P-SRTP dispute see Becker (2002, pp. 11–21).

[9] The conflict between the descriptive and prescriptive approach is explicitly stated in Arrow et al. (1996).

[10] The conflict had far reaching consequenes. For example, Solow (1999, p. vii) within his foreword to Portney and Weyant (1999) distinguishes between the plausibility for discount rates for costs and discount rates for benefits. Especially for costs the market rate of interest in many cases would be the proper value. Discounting benefits would contradict to common sense due to the fact that market rates would discount even very great future damage back to peanuts.

$$C \equiv \sum_{t=0}^{T} \frac{b_t - e_t}{(1+\alpha\hat{c})^t} \geq 0 \tag{7}$$

Obviously, the *O*-problem is robust to any answer to the *G*-problem. Hence, there must exist a CBA-inherent reason for the first problem.

3. Consumption Equivalent Public Capital

Now we solve the *O*-problem. One side of this problem was that a discount rate lower than r was thought to miss a sufficient implementation of opportunity costs of public capital. The major local result is that this is questionable within a CE framework because these opportunity costs must be considered by additional cost positions. They are not automatically considered by the most obvious discount rate which has the purpose of social intertemporal consumption weighting. The reason is that in CEM the standard alternative opportunity is consumption and not capital in contrast to the capital value method. The goal of this section is to find the trap of standard public CBA. First of all, we eliminate the generalized Arrow–Kurz assumption which is an interesting point for reflection but a rather questionable assumption even for special project categories. In addition, we use the original i. In short, we start with (2) in 1975. Money is not only a veil but also an essential economic phenomenon:

3.1. The CEPC-criterion

Let k be the share of public debt financing and $\varepsilon^{CR} = 1 - a^{CR} + a^{CR}\ v^{CR}$ the sharing effect analog to (1) of a withdrawal of one money unit by an increase of public debt. Then $1 - k$ is financed by taxation with sharing effect $\varepsilon^S = 1 - a^S + a^S\ v^S$. Hence, the initial burden (*F1*) of financing one unit of public investment is $\varepsilon_I = (1 - k)\ \varepsilon^S + k\ \varepsilon^{CR}$ (*initial factor*). Suppose that public loans have a fixed duration of N years with r_d as interest rate of public debt. After N periods government has to pay interest and redemption per unit of public investment of $k\,(1 + r_d)^N$. In turn this amount will be financing by taxation and public debt representing a withdrawal of monetary resources witch can be expressed in current CE-units by $k\,(1 + r_d)^N\,((1 - k)\ \varepsilon^S + k\ \varepsilon^{CR}) = k\,(1 + r_d)^N\ \varepsilon_I$. The additional multiplication with d^N where $d = 1/(1 + i)$ transforms this into present CE-values (*F2*). But the same amount $k\,(1 + r_d)^N$ also represents a revenue (*F3*) for the private sector which leads to the present CE-value of $k\,(1 + r_d)^N\ \varepsilon^{CP}\ d^N$ where $\varepsilon^{CP} = 1 - a^{CP} + a^{CP}\ v^{CP}$ is the sharing effect of repayment for public debt. Along this line we can generate an infinite chain of *F2* and *F3* effects:

Period	Effect	CE-effects in t-present values
t	*F1*	$(1 - k)\ \varepsilon^S + k\ \varepsilon^{CR}$
$t + N$	*F2*	$k\,(1 + r_d)^N\,((1 - k)\ \varepsilon^S + k\ \varepsilon^{CR})\ d^N$
	F3	$-\,k\,(1 + r_d)^N\ \varepsilon^{CP}\ d^N$
$t + 2N$	*F2*	$k^2\,(1 + r_d)^{2N}\,((1 - k)\ \varepsilon^S + k\ \varepsilon^{CR})\ d^{2N}$
	F3	$-\,k^2\,(1 + r_d)^{2N}\ \varepsilon^{CP}\ d^{2N}$
	⋮	⋮ ⋮ ⋮

$t + P{\cdot}N$	$F2$	$k^P\,(1 + r_d)^{P\cdot N}\,((1 - k)\,\varepsilon^S + k\,\varepsilon^{CR})\,d^{\,P\cdot N}$
	$F3$	$-\,k^P\,(1 + r_d)^{\,P\cdot N}\,\varepsilon^{CP}\,d^{\,P\cdot N}$
⋮	⋮	⋮ ⋮

Using abbreviations $\gamma = (1 + r_d)/(1 + i)$ and $\varepsilon_D = (1 - k)\ \varepsilon^S + k\ \varepsilon^{CR} - \varepsilon^{CP}$ (*difference factor*) we can simplify the sum of expressions *F2* and *F3* (*F2* + *F3*) for each $P \geq 1$ by:

$$k^P\,(1 + r_d)^{P\cdot N}\,((1 - k)\,\varepsilon^S + k\,\varepsilon^{CR})\,d^{\,P\cdot N} - k^P\,(1 + r_d)^{\,P\cdot N}\,\varepsilon^{CP}\,d^{\,P\cdot N} \;=\; (k\gamma^N)^P\,\varepsilon_D$$

This leads to the overall infinite summation

$$\mu \equiv \varepsilon_I + \varepsilon_D \sum_{p=1}^{\infty}\left(k\gamma^N\right)^p = \varepsilon_D \sum_{p=0}^{\infty}\left(k\gamma^N\right)^p + \varepsilon^{CP}$$

For $k\gamma^N \geq 1$ and $\varepsilon_D \neq 0$ the *consumption equivalent public capital* (CEPC) *price* μ diverges to $\mathrm{sign}(\varepsilon_D)\cdot\infty$. But for $k\gamma^N < 1$ the geometric series converges and we can write:

$$\mu = \varepsilon_D \frac{1}{1-k\gamma^N} + \varepsilon^{CP} = \varepsilon_D\left(\frac{1}{1-k\gamma^N} - 1\right) + \varepsilon_I \tag{8}$$

With the additional abbreviation for the *substance factor*

$$f \equiv \frac{1}{1-k\gamma^N} - 1 \tag{9}$$

we arrive at the following *CEPC-price* formula:

$$\mu = f\,\varepsilon_D + \varepsilon_I \qquad (\text{for } k\gamma^N < 1) \tag{10}$$

The CEPC-price sufficiently expresses the current CE-value of the burden witch is generated by the normally infinite stream of debt financing effects. The relevance of the difference effect ε_D depends on the substance factor *f*. Now the government has the money unit in hand. But still the influence of the *purchasing effect* (*K*) has to be considered by

$$K \equiv \varepsilon^{-I} - \beta^{-I} \tag{11}$$

where ε^{-I} represents the sharing factor of the public expenditure as a negative budget cost position (negative share of e_t) which is actually a transfer to the private economy (*project invoice payment effect*). A burden is generated by the corresponding binding of resources for the production of the equipment of the public investment (*production resource binding effect*).[11] Because the components of

[11] The non-budget cost effect of the binding of private production resources could also be considered separately within a b_t-element which would respect a strict distinction between budget CBs and non-budget BCs.

K depend on the affected sector of the private economy it is useful to separate the cash transfer due to the payment for the public project equipment from the effects *F1–F3* above.

Formally the budget-CB positions e_t comprise all public cash flows. But we already included the future debt effects of type *F2* and *F3* implicitly by the calculation of μ. Hence, we can concentrate of the initial cash flows e^*_t at the start of the chain of any expenditure for the public project. With the shortcut

$$\varepsilon^* \equiv \mu - K \tag{12}$$

the initial-budget-CBs e^*_t define the *CEPC-method*:

$$B^{**} \equiv \sum_{t=0}^{T} \frac{b_t \beta_t - e_t^* \varepsilon_t^*}{(1+i)^t} \geq 0 \tag{13}$$

Note that the CEPC-criterion (13) and the original (2) by Bradford are equivalent ($B^{**} \geq 0 \Leftrightarrow B \geq 0$). The concentration on initial public cash flows e^*_t is compensated by the value of ε^*_t which already includes the shadow price of public capital μ_t which counts for the infinite stream of burdens due to the difference effect ε_D which is subsumed in the substance factor f. The stimulating purchasing effect K_t incorporates ε^{-1}_t which is financed by public revenue associated with ε_I. But if CEPC-criterion (13) is only a rewriting of Bradford's criterion (2) then what is the advantage?

3.2. The liberalization of the social rate of discount (*O-solution*)

The advantage of B^{**} over B is that $\varepsilon^*_t \equiv \mu_t + K_t$ in general is much higher as ε_t because the shadow price of public capital μ_t subsumes all future credit effects of a unit of money taken in the hand by the public investor. This potentially higher burden in comparison to B is compensated by the concentration on initial public budget payments, e_t^*. An essential point is that even if this stands in contrast to the formal interpretation e_t it nevertheless coincides with the praxis of the application of Bradford's formula describe by:

$$B^{*} \equiv \sum_{t=0}^{T} \frac{b_t \beta_t - e_t^* \varepsilon_t}{(1+i)^t} \geq 0 \tag{14}$$

This expression misses to compensate the usage of e_t^* by the usage of ε^*_t. Both the descriptive *and* the prescriptive approach applied to pCBA are based on the shortcoming of insufficient opportunity costs consideration:

$$MCV^{*} \equiv \sum_{t=0}^{T} \frac{b_t - e_t^*}{(1+r)^t} \geq 0 \quad \text{or} \quad C^{*} \equiv \sum_{t=0}^{T} \frac{b_t - e_t^*}{(1+\alpha\,\hat{c})^t} \tag{15}$$

Here, the conflict between the care for opportunity costs and the care for the future is obvious. One can only decide between two mistakes. If you lower r as a methodically defective compensation for the miss of ε^*_t then opportunity costs proponents do think that you ignore public budget considerations not knowing that even if r is high the same is true. The dispute about the proper social rate of discount within pCBA is not generated by B but by MCV^* (Lind's approach). The advantage of B^{**} is that it reopens the discussion about the usefulness of a pure rate of social time preference (*G-problem*) because it ensures the implementation of opportunity cost of public capital by the usage of shadow prices of public capital μ_t. The *O*-problem is solved.

3.3. A vote for both descriptive and prescriptive approaches (*G-solution*)

The term $B*$ of CEPC-criterion (13) liberalizes the overall social rate of time preference i from considerations about opportunity cost of public capital because the shadow price of public capital μ defined in (10) ensures a massive implementation of a potentially infinite chain of social burden. Now we are free to discuss δ as a part of i (cf.(5)). The power of Nordhaus' descriptive approach in favor for the usage of r is essentially rooted in the imagination that this would be necessary for a sufficient implementation of public budget-cost. But this is based on a degenerated mock capital value method and even in this framework not acceptable. If we abstract from the share of the *social self process* that contributes to the δ-value and which cannot be influenced directly by public investment behavior then the δ of Nordhaus must comprise a business-as-usual (BAU) value of Schelling's distance (here: time) discounting. To think that this is a matter of fact disregards the degree of freedom which is given by the option, that we can deviate form BAU. Hence, the descriptive approach is a useful references line and corresponds to optimal growth theory under the restriction of a BAU-value of the pure rate of social time preference, i.e., it is BAU-observable growth theory. But the maximum of the intertemporal welfare –in accordance to the old Ramsey–Pigou view– is closer by setting the part of public behavior in the P-SRTP equal to zero. This corresponds to prescriptive reasoning à la Cline (1992) et al. The actual value of the P-SRTP used in public CBA will be framed between these values as an outcome of political choice. But only prescriptive reasoning has a genuine economic character of intertemporal optimization because BAU self-preferences are suboptimal per definition. Hence, our rule for public investment, hence, is:

PROPOSITION 1: *Maximize the consumption equivalent present value of initial public dollar flows multiplied with the shadow price of public capital and of non-marketed effects, discounting with the political value of the social rate of time preference chosen between the prescriptive (optimal) and descriptive (business-as-usual) level.*[12]

4. Plausibility

The main local result is rule (13) of public cost–benefit analysis above. In addition, we can generate and organize economic arguments with respect to public financing:

[12] There is a Ramsey rule paradox: mathematics and reality with complementary P-SRTP definition says that the rule is valid, but public CBA is based on the fact that the rule does not hold. (Otherwise shadow prices of private capital would collapse to one and Lind's empirical view would be impossible as well as his generalized Arrow–Kurz assumption superfluous.) In addition, the rule's economic interpretation implies that society is indifferent between saving and investment. The solution to this paradox may be a mixed version with BAU data for r including a pure rate of social time preference witch can be used as degree of freedom and cause a mixed application of Ramsey rule levels resulting in its practical invalidity for public CBA.

4.1. The crowding out break even (paradox)[13]

Does the shadow price of public capital μ fulfill the expectations of common economic sense? Let us start with the reaction of μ towards the extend of influence witch the withdrawal (or transfer per reduction) of one monetary unit by taxation upon private investment and saving in current consumptions equivalent values has: $\partial\mu / \partial\varepsilon^S > 0$ for $0 < k < 1$ implies that a higher tax effect factor ε^S increases the social costs of taxation for a money unit in public hand.[14] But now we turn to more interesting investigations. At a first glance there is a surprise:

PROPOSITION 2: *A higher credit effect ε^C decreases the cost of public financing if and only if the interest rate on public debt r_d exceeds the social rate of time preference, i. In this case the initial burden from withdrawal of the credit sum (crowding out) is overcompensated by the stimulation of net tax transfer to credit suppliers*.

PROOF: see: (16). ■

$$\frac{\partial\mu}{\partial\varepsilon^C} = 1 - \frac{1-k}{1-k\gamma^N}, \qquad \text{hence: } \frac{\partial\mu}{\partial\varepsilon^C} < 0 \;\Leftrightarrow\; r_d > i \qquad (16)$$

ε^C is relevant for public revenues as well as public redemption and loan payments. For example, for a positive value of r_d and in the case of neutrality towards the future ($i = 0$) a higher burden of a monetary unit in relation to public credit caused by the withdrawal of the credit sum *decreases* the shadow price of public capital, because this elementary factor ε^C is dominantly used for payment to the private sector. The reason is that apart from the initial withdrawal of the credit sum out of the credit market future payments of redemptions and loans *all* have a stimulating effect because every money unit paid is financed by a share of k by credit suppliers (i.g., by $(1 - k)$ through tax payers) but in contrast fully expended to the credit suppliers. If and only if $r_d > i$ then this stimulation dominates the initial burden by the withdrawal of the credit sum, because γ is influences the substance factor f defined in(9).

4.2. Incentives and disincentives for public debt financing

Why do we stipulate so clearly the inferiority of public credit financing while simultaneously almost every nation has a growing GDP-share of public debt? In contrast, we have learned from (16) that the effect of economic stimulation by credit financing may dominate the initial crowding out if the rate of interest for public loans exceeds the social rate of time preference.

Let us extent the argument $\Delta_E \equiv \varepsilon^S (1 - g^S) x^S - \varepsilon^C (1 - g^S) x^S$.[15] First, the degree of globalization of capital is much higher than the effect from taxation of foreign markets (the global wedge: $g^C >> g^S$).

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[13] We presume $\varepsilon^{CR} = \varepsilon^{PR}$ ($\equiv \varepsilon^C$) because both public revenue and repayment with respect to the credit market should have similar effects but mostly, for the simplicity of discussion. This leads to $\varepsilon_D = (1 - k)\,\Delta$ with $\Delta \equiv (\varepsilon^S - \varepsilon^C)$ as the critical value of public debt financing (cf. below).

[14] Naturally, we have no reaction for complete financing ($\partial\mu / \partial\varepsilon^S = 0$ for $k = 1$) and, actually, a one-to-one reaction for credit financing ($\partial\mu / \partial\varepsilon^S = 1$ for $k = 0$).

[15] One can replace ε^y by $\varepsilon^y(1 - g^y)\, x^y$ for $y = S, C$ from the right beginning without principal differences.

Second, the marginal utility of tax payers may be lower as one of capital suppliers (the welfare wedge: $x^C < x^S$). In contrast one could argue that the marginal propensity to invest is higher for money of credit suppliers in comparison to tax payer leading to an argument for $\varepsilon^C > \varepsilon^S$ via $a^C > a^S$. If one follows our view that the first two influences are stronger then Δ_E is positive, or, say for simplicity, $\Delta = \varepsilon^S - \varepsilon^C$. The implication is that even for the reaching of the no-crowding out break even with a stimulation by tax transfer into the credit market this will support rich credit lenders abroad while the suffer from taxation more hurts poor or normal tax payers (or social transfer receptions) at home. This view is supported by the effect of the credit share k on the public burden of finance:

$$\frac{\partial \mu}{\partial k} = -\Delta \frac{1-\gamma^N}{(1-k\gamma^N)^2}, \qquad \frac{\partial \mu}{\partial^2 k} = -\Delta \frac{2\gamma^N(1-\gamma^N)}{(1-k\gamma^N)^3} \tag{17}$$

The second factors are negative if and only if $r_d > i$, e.g., for sufficient care for the future and a normal loan rate for public investors. If one accepts our arguments in favor for a positive Δ then society should minimize the credit share k. But, naturally, if our ignorance towards the future is high then we will increase k up to a level near by an explosive value of public debt.[16] Anyhow we have for values of k $(0 \leq k \leq 1)$:

$$\frac{\partial \mu}{\partial k} > 0 \quad \Leftrightarrow \quad \text{sign}(\Delta)\ \text{sign}(r_d - i) > 0 \tag{18}$$

What can we learn from this? The global wedge and the welfare wedge of public financing makes it attractive to *limit* debt financing. But for high pressure of tax payers i may be increase up to a level beyond r_d, and these wedges are overcompensated by ignorance for the future. But even is this case there is a mechanistic border which prevent full debt financing. Remember that we are in case $k\gamma^N < 1$ (or $k < \gamma^{-N}$) avoiding the divergence of the burden μ which society has to take per money unit taken in the hand for public investment. But be careful, the respective convergence does not necessarily prevent credit from diverging towards infinity. This is because i at least in part is only an expression of our weight for the future. The convergence of μ does not imply the convergence of the credit sum which per invested monetary unit after P series of refinancing is

$$c_{t+PN} = k^{P+1}(1 + r_d)^{PN} \tag{19}$$

This converges down to zero if and only if (else it diverges to infinity)

$$k < \left(\frac{1}{1+p_d}\right)^N \tag{20}$$

This is exactly the principle scenario of public debt which we observe today: Despite the fact that capital income is global the incentive for debt financing is high due to political ignorance towards the future (high δ) because taxation (or less social transfer) hurts more. This is limited by a more or less

[16] Naturally the local flip-flop character of the optimal k with respect to Δ is due to our simplistic model with constant rates without dynamic feedbacks (dynamic optimal programming or differential equations). But for the local purpose to illustrate the limits of mock capital values perhaps it is even preferable.

arbitrary imagination about the share of GDP which should not be exceeded. The high relevance of the critical factor $\Delta =_{def} \varepsilon^S - \varepsilon^C$ is additionally supported by:

$$\frac{\partial \mu}{\partial r_d} = \Delta \frac{(1-k)kN\gamma^N}{(1+r_d)(1-k\gamma^N)^2}, \qquad \text{hence, usually: } sign\left(\frac{\partial \mu}{\partial r_d}\right) = sign\,\Delta \tag{21}$$

To think that an increase of r_d increases the shadow price of public capital μ presumes that ε^S dominates ε^C because only in this case the credit time chain is a net burden. As already mentioned, the reason is that apart from the initial withdraw of the investment sum (which is independent of r_d) the influence of ε^C is positive because afterwards tax payers have to transfer money towards credit suppliers. This can go so far that $\Delta < 0$ signals a massive stimulus through this transfer. But we know that this is unrealistic due to the global and welfare wedge.

PROPOSITION 3: *The critical value Δ and the critical comparison of the rate of interest for public loans r_d and social rate of time preference i co-determine the attractiveness of credit financing. The global and the welfare wedge as well as rate of interest for public loans are disincentives for credit financing. The overall social rate of time preference is an incentive representing ignorance for the future. If the later dominates then the avoidance of credit explosion limits the credit financing share. Naturally, r_d normally hurts.*

PROOF: See(18): Remember: Δ_E (or, say Δ) > 0 for assumed $g^C >> g^S$ and $x^C < x^S$, dominating $a^C > a^S$. Case: $k\gamma^N \geq 1$. Then $\mu = \text{sign}(\Delta)\,\infty = +\infty$. Case: $k\gamma^N < 1$. Because of $\Delta > 0$ according to (17) or (18) we have $\partial\mu / \partial k > 0$ for all k ($0 \leq k \leq 1$) iff $r_d > i$. Subcase: $k < 1/(1 + p_d)^N$. Then due to (19) $\lim_{P\to\infty} c_{t+PN} = 0$. Subcase: $k \geq 1/(1 + p_d)^N$. Then due to (19) $\lim_{P\to\infty} c_{t+PN} = +\infty$. See(21). ■

PROPOSITION 4: *If we presume that the global and welfare wedge dominate and that private shadow prices are similar then the shadow price of public capital decreases ($\partial\mu/\partial i < 0$) for higher ignorance for the future.*

PROOF: We discuss the sign $\partial\mu/\partial i$. For all common concepts for shadow prices of private capital ν we can relay on $\partial\nu/\partial i < 0$ (e.g., cf. overview of Cline, 1992, *Annex 6A*, pp. 270–274). Case: $k = 1$. Then $\mu = \varepsilon^C =_{def} (1 - a^C) + a^C\, a\, v^C$. Hence, using $\partial\nu/\partial i < 0$, we have $\partial\mu/\partial i < 0$. Case: $k = 0$. Then $\mu = \varepsilon^S =_{def} (1 - a^S) + a^S\, a\, v^S$. Hence, again $\partial\mu/\partial i < 0$. Case: $0 < k < 1$: We differentiate(10):

$$\frac{\partial \mu}{\partial i} = (1-k)\frac{\partial \Delta}{\partial i} f + \Delta \frac{\partial f}{\partial i} + \frac{\partial \varepsilon_I}{\partial i} \tag{22}$$

We discuss the sign of the elements of $\partial\mu/\partial i$ using(22). For the last term is always negative ($\partial\varepsilon_I/\partial i < 0$), again, due to $\partial\nu/\partial i < 0$. Obviously, $\partial f/\partial i < 0$ as well as $f > 0$ (see f-def.(9)). Now it is useful the distinguish two subcases of a clear or ambiguous sign of $\partial\mu/\partial i$:

Subcase 1: $\partial\Delta/\partial i \leq 0$ and $\Delta \geq 0$ $\Rightarrow \partial\mu/\partial i < 0$

Subcase 2: $\partial\Delta/\partial i > 0$ and/or $\Delta < 0$ $\Rightarrow \partial\mu/\partial i$ unclear

Presuming that the global and the tax wedge dominate and that reaction of the shadow prices of private capital are similar one can argue that

$$\frac{\partial \Delta_E}{\partial i} = (1-g^S)x^S a^S \frac{\partial v^S}{\partial i} - (1-g^C)x^C a^C \frac{\partial v^C}{\partial i} \tag{23}$$

or for simplicity $\partial\Delta/\partial i$ is negative as well as $\Delta > 0$. ■

5. Conclusions

Instead of using a high social discount rate the opportunity costs of public capital have to be considered by shadow prices of public capital multiplied with initial public budged expenditure. This solves our first (*O*-) problem related to public cost–benefit analysis. Hence, in this regard the social rate of time preference is liberated from the pressure to count for opportunity costs. The Ramsey rule nevertheless states that this rate is equal to the private rate of return on investment and, hence, via market arguments equal to the market rate of interest. The dispute about the pure rate of social time preference between the descriptive and the prescriptive approach is solved by the acceptance of both positions. The first extrapolates BAU social self preferences (and self processes) into the future, the second regards the outcome of optimal, e.g., unbiased optimization of intertemporal welfare. The political choice of a value between both positions with the help of consumption equivalent public capital prices can freely be made and used in public cost–benefit analysis being aware that these prices take sufficiently care for opportunity costs of the public budget. The reaction of the consumption equivalent price of public capital with respect to central parameters demonstrates the robustness of our approach of public cost-benefit analysis, the consumption equivalent *public capital* method.

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