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A Vector Error Correction Model for the Relationship between Public Debt and Inflation in Germany

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Abstract

In the paper, the interaction between public debt and inflation including mutual impulse response will be analysed. The European sovereign debt crisis brought once again the focus on the consequences of public debt in combination with an expansive monetary policy for the development of consumer prices. Public deficits can lead to inflation if the money supply is expansive. The high level of national debt, not only in the Euro-crisis countries, and the strong increase in total assets of the European Central Bank, as a result of the unconventional monetary policy, caused fears on inflating national debt. The transmission from public debt to inflation through money supply and long-term interest rate will be shown in the paper. Based on these theoretical thoughts, the variables public debt, consumer price index, money supply m3 and long-term interest rate will be analysed within a vector error correction model estimated by Johansen approach. In the empirical part of the article, quarterly data for Germany from 1991 by 2010 are to be examined.

JEL-Classification: C32, E31, E51, H63

Keywords: Beveridge-Nelson Decomposition, Public Debt, Inflation, Money Supply, Vector Error Correction Model

1 Introduction

Governments in collaboration with the central banks stabilised the global financial and banking system during the Global Financial Crisis in 2008 and 2009. They started bank rescue programmes with an amount of billions of Dollars worldwide. Moreover, economic recovery plans slowed down the strong and fast economic fall in 2009. However, the economic stimulus packages led to an acceleration of getting in debt in many countries. Not only Germany has paid additional money for the bail-out of Greece and the two European bail-out funds (EFSF, ESM). The public debt in Germany has reached a record high of 2 trillion and 166 billion Euros (Maastricht criteria) at the end of the fourth quarter in 2012 (Eurostat, 2013). That is nearly 82 per cent of the gross domestic product. Furthermore, public debt is threatened by additional burdens from Target2-credits from the Deutsche Bundesbank against the Euro system.

The high level of national debt not only in the Euro crisis states could lead to an apprehension of a debt restructuring caused by a sovereign default. An alternative is to inflate the government debt away. Spending cuts and tax increases would be accompanied with the resistance of lobbies and large shares of the population. Consequently, the inflation could be an attractive solution for the government debt problem. Reinhart and Rogoff (2009) have shown, that governments around the world tried to devaluate their debts by inflation. This discussion will be expanded by the theory of financial repression. It describes how public regulation in combination with an expansionary monetary policy leads to a negative real interest rate and this may conduce a debt relief of states (Reinhart/Sbrancia, 2011). So the following question arises: Does an accelerated inflation development such as in the 1970s come back in the medium- or long-term? This fear will be intensified by the crises in Southern EMU countries. Economically stable countries such as Germany supported the crises countries and therefore they had to boost their own national debt. This can increase the incentive to depreciate the debt in real terms in the future.

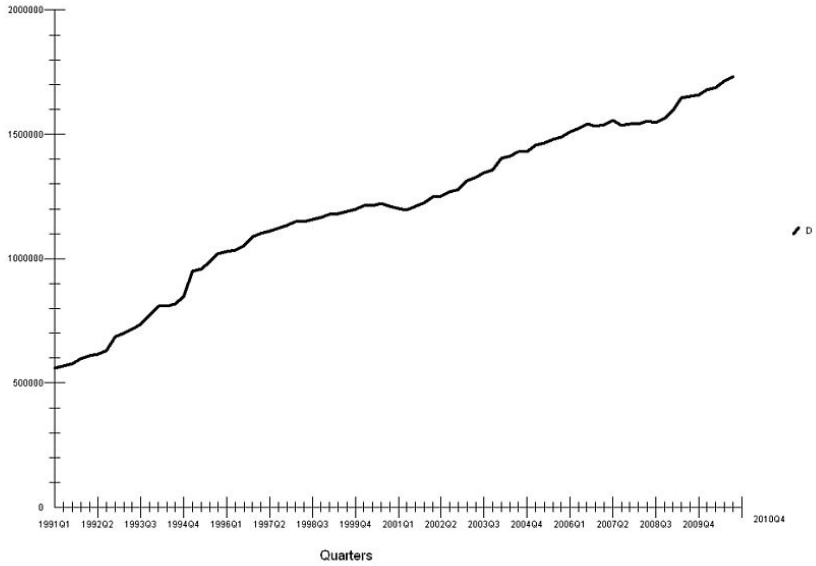

Figure 1: Public debt in Germany after the Reunification

Causes of public debt in Germany

The general reason for public debt is that the governmental expenditures were higher than the public revenues in most countries and periods. Special reasons for the strong increase in German debt are costs of the German Reunification, the financial support for the extensive social security system, several economic stimulus packages, bank rescue programmes and a lack of political intention for consolidation. That has been the main reasons for increasing public debts in Germany since the 1970s (Wagschal et al., 2009). The public debt problem got worse at the beginning of the latest financial crisis.

The concern about inflation will be strengthened by the strong expansion of money supply caused by the low interest rate policy and the unconventional monetary policy (quantitative easing) by the central banks in many countries. Moreover, the European Central Bank (ECB) has bought public bonds of the crisis countries and lowered their government yield payment. The first bond purchase programme (SMP) started at the peak of the European national debt crisis in May 2010 and had a volume of 219 billion Euros. A second bond purchase programme (OMT) was decided in September 2012 and covers an unlimited purchase of government bonds under certain conditions. The total assets of the Euro system expanded from 1.52 Trillion Euro in September 2008 to 3.02 Trillion Euro at the end of 2012. In May 2013 the ECB's main refinancing rate has reached a historical low rate (0.5 %). Furthermore, the lists of eligible safeties were continuously extended and so the criteria for deposit capable safeties were reduced significantly (qualitative easing).

Therefore some economists demanded the ECB to speed up the money supply. For example de Grauwe (2011) argues that in case of a disturbance of the bond market (e.g. liquidity crisis and domino effects) the central bank should purchase extensive government bonds. The monetary policy has to counteract against the crisis of confidence. So the central bank takes role as lender of last resort also for the governments. But that means the monetisation of a part of the public debt. The central banks should also take into account the possible moral hazard behaviour of the crises states after a purchase of public bonds. Lower yields on government bonds could reduce the market pressure and subsequently the governments could expand the fiscal deficits again. This development happened in Italy in the summer 2011. Beyond that, the chief economist of the International Monetary Fund (IMF) has already started a discussion about the level of the inflation rate target of the central banks (Blanchard et. al., 2010). According to that a higher average inflation rate enables stronger monetary policy impulses. However, those developments could strengthen the fear for inflation by economic subjects and unsettle their inflation expectations.

In this paper, the following questions will be examined:

- Does a high level of public debt cause an inflationary effect?
- What kind of effect has inflation on public debt?

A main point in this article will be the investigation of the role of the monetary policy by central banks in this interaction. Public deficits can lead to inflation if the money supply is expansive. We provide an overview of empirical studies about the relationship between the public debt and the inflation. Moreover, we show the main transmission mechanisms. The two questions above will be analysed empirically for Germany after the Reunification within a cointegration framework. We estimate a vector error correction model. That means the included variables are considered endogenous. The variables in the model will be tested for cointegrated relations and estimated with the Johansen-approach. Afterwards the generalised impulse response analysis is applied to the data of Germany. Moreover, a multivariate Beveridge-Nelson trend/cycle decomposition is be in progress to show the connections between the cyclical components.

2 Transmission from public debt to inflation

Following the classical-neoclassical theory a debt-financed government spending does not lead to an increasing output. There is only a displacement from private sector demand to the government (crowding out). Corresponding to the Ricardo-Barro-equivalence theorem debt-financed government expenditures with the consideration of a public budget restriction has no effect on output or on inflation (Brümmerhoff, 2011, p. 635f.). Considering the fact that private households act rational, the present public borrowing is equal to higher taxes in the future. In contrast, the Keynesian theory supposes that deficit spending has an expansive effect on the macroeconomic demand. Thus, the cyclical development in the short-term is determined by the macroeconomic demand. According to the monetarist view a with the help of the central bank debt-financed public spending leads to a higher money supply and corresponding to the quantity theory of money the price level will increase in the long-run (Mankiw, 2011, p. 480f.).

Public debt and the consumer price level are connected within a two-way relationship over different transmission ways. The analysis of the transmission mechanism from public debt to inflation has to consider the impact on money supply and aggregate demand and the role of the central bank:

- Debt-financed government spending stimulates macroeconomic demand in the short-term and inflation will rise in the long-term, i.e.
- directly through the purchase of public bonds by the central bank or
- indirectly through the demand of public bonds by the private sector with a simultaneous expansive monetary policy to stabilise the increasing interest rate,
- also indirectly through the demand of public bonds by the banking sector,
- and through the inflation expectation of the economic subjects.
- An increase in debt-financed government spending stimulates macroeconomics demand in the short-term too.

These five factors could lead to an increasing price level in the medium- or long-term. The relationship between public debt and inflation does not only exist through the money supply and the macroeconomic demand. The inflation expectations of the

economic subjects can influence the current price development. A high budget deficit or a high level of public debt could lead to increasing inflation expectations. As a result the nominal wages claims and later the de facto wages rise. The higher wage level affects the consumer prices and could start an inflation process. This channel describes a dangerous link between fiscal and monetary policy. If there are a high debt level and high nominal interest rates, the bonding of the inflation target within the inflation expectations of the economic subject would be more difficult. As a consequence central banks are interested in stabilising economic subjects' inflation expectations. A monetary police strategy, which is focussed on price stability, depends on the public debt level. Since the financial crisis a sound fiscal debt policy is also important for the financial stability. Banks and insurers are the most important creditors for the governments.

The transmission from inflation to public debt is given by

- the seigniorage as a part of the central bank profits, money creation is being an easy way to finance public debts,
- the short-term stimulation of economic growth caused by an expansive monetary policy, leading to rising public revenues and decreasing public spending,
- a progressive tax system because a high inflation rate leads to higher tax revenues - also without real economic growth,
- the devaluation of nominal public debt by inflation (inflation tax). If the nominal value is given, a high price increase leads to a decreasing real value of the nominal debt.

These four factors could reduce public debt in the short- or medium-term. Seigniorage and inflation tax are closely interconnected. Money creation leads to an increasing money supply and with a lag to inflation. The result of the higher inflation is through the inflation tax a declining of the public debt. If the inflation rate increases, the buyer of government bonds will require a higher nominal interest rate. They also will prefer short repayment terms. Is the debt process onward instable, the government (resp. the central bank) will extend the money supply again and therefore the inflation could strongly accelerate. The consequences are higher nominal interest requirements and that generates an additional push in the inflation necessary. The result of this development could be a hyperinflation.

Giannitsarou and Scott (2006) and Sill (2005) couldn't find a significant relationship between public debt and inflation in the industrial countries since the 1960s. Opposed to that, Sill (2005) and Catao and Terrones (2005) estimated a definite influence from public debt on inflation in the emerging economies for the period from 1960 to 2000. Independent central banks and a high reputation from the capital market players contribute a more stable financial position in the developing countries, so that public deficits predominantly do not lead to high inflation rates.

Taghavi (2000) examined the relation between inflation and the debt ratio for the period from 1970-1997 for Germany, Italy, France and United Kingdom. He couldn't find any cointegrated relations for these four countries. But the debt ratio was Granger-causal to inflation with a time lag of 3 or 5 years in all four states. The impulse-response functions showed that a shock in the debt ratio caused a significant reaction on inflation. However, the direction and the strength were very different. Bleaney (1996) applied a correlation analysis for the public debt level and inflation. The result was a positive mean correlation of 0.36 between the two variables for 15 OECD-countries in the period 1973-1982. In contrast, the correlation was negative (-0.19) during the period 1983-1989.

3 Econometric methods

The public debt/inflation function to be estimated later in this paper is considered as a kind of long-run equilibria or cointegrating relations. Cointegration might be characterised by two or more integrated variables indicating a common long-run development. However, transitory fluctuations are possible. This defines a statistical equilibrium which can be interpreted as a long-run economic relation. Equation (3.1) shows a vector error correction model (VECM) with the order (p – 1):

$$\Delta \mathbf{x}_t = \boldsymbol{\mu} + \mathbf{A}\mathbf{B}^t \mathbf{x}_{t-1} + \sum_{i=1}^{p-1} \boldsymbol{\Gamma}_i \Delta \mathbf{x}_{t-i} + \mathbf{u}_t \tag{3.1}$$

with a deterministic shift vector $\boldsymbol{\mu}$. $\boldsymbol{\Gamma}_i$ are ($k \times k$) parameter matrices of the lagged stationary differences, **B** being the ($k \times r$) matrix of k-dimensional cointegrating vectors and **A** the corresponding ($k \times r$) matrix of error correction coefficients. The matrix **Π=AB**' represents the long-run relation between the variables in $\mathbf{x}_t$.

This VECM is equivalent to a vector autoregression (VAR (p)) presentation of the levels $\mathbf{x}_t$. In a VAR-model each variable can be taken as endogenous. The changes in a selected target variable in period t depend on the deviations from that specific equilibrium in the previous period and the short-run dynamics. The VECM allows us to estimate the long-term effects and to analyse the short-term adjustment process within one model. Actually, the variable vector $\mathbf{x}_t$ is assumed to be vector integrated of order 1 (I(1)), i.e. $\Delta\mathbf{x}_t$ is vector stationary. But for the purpose of this paper, it will be sufficient to test each individual variable independently for integration and stationarity by augmented Dickey-Fuller-test (ADF).

An intercept can be included in cointegrating relations alternatively, as well as a deterministic time trend. The maximum lag p can be easily found by Schwarz-Bayes or Akaike information criterion. The number r of cointegrating vectors (lines in B‘) can be determined as the rank of the matrix **Π=AB‘** by several tests, such as the test of maximum eigenvalue of **Π** (Nastansky/Strohe, 2011, p. 12-13). Under rather general conditions, the coefficient matrices **A, B** and $\boldsymbol{\Gamma}_i$ can be estimated by least squares (LS), generalised least squares (GLS) and maximum likelihood (ML). In the following sections of this paper, the ML method known as Johansen procedure as presented in the software MICROFIT (Pesaran/Pesaran, 2009, p. 496-511) will be used.

Generalised Impulse Response

Following the paper by Koop et al. (1996) concerning impulse response functions in nonlinear econometric models, Pesaran and Shin (1998) developed generalised impulse response functions because of the lack of unambiguity of the orthogonal shock analysis. Orthogonal impulse response functions depend on the sequence of the elements within the vector of jointly dependent variables. There are $n!$ different sequences of n economic variables in the VAR model. There are no unambiguous criteria for the choice of an optimal sequence (Pesaran/Shin, 1998, p. 20). In contrast, the generalised impulse response functions for the individual variables are unambiguous, i.e. invariant towards the chosen order of variables within vector $\mathbf{x}_t$.

Under the assumptian that the variables in $\mathbf{x}_t$ are I(1), i.e. have stationary first differences, the latter can be written in the infinite version of a moving average presentation (Pesaran/Shin, 1998, p. 5) with certain coefficient matrices $\mathbf{C}_k$ as long-term multiplicators after k periods:

$$\Delta \mathbf{x}_t = \sum_{k=0}^{\infty} \mathbf{C}_k \mathbf{u}_{t-k} \tag{3.2}$$

Then the generalised impulse response function in a cointegrated VAR measures the effect after k periods onto vector $\Delta\mathbf{x}_t$ of a single impulse in the j-th equation. The effect for the i-th equation is given by:

$$\psi^{G}_{\Delta x,ij}(k) = \frac{\mathbf{e}_i^T \mathbf{C}_k \boldsymbol{\Sigma}_{\mathbf{u}} \mathbf{e}_j}{\sqrt{\sigma_{jj}}}, \tag{3.3}$$

where the $\mathbf{C}_k$ come from the infinite MA representation (3.2), $\mathbf{e}_j = (0,0,...,1,0,...,0)'$ is the j-th unit vector for the purpose of selecting the j-th element, and σ_{jj} is a diagonal element of the variance-covariance matrix $\boldsymbol{\Sigma}_{\mathbf{u}}$ of the shock variables $\mathbf{u}_t$, i.e. the variance σ_j^2 of shocks u_j in x_j.

The cumulative effect of a one-standard-deviation shock on $\mathbf{x}_{t+k}$ results to:

$$\psi^{G}_{x,j}(k) = \frac{\mathbf{D}_k \boldsymbol{\Sigma}_{\mathbf{u}} \mathbf{e}_j}{\sqrt{\sigma_{jj}}} \quad \text{with} \quad \mathbf{D}_k = \sum_{j=0}^{k} \mathbf{C}_j \quad \text{and} \quad \mathbf{D}_0 = \mathbf{C}_0 = \mathbf{I}_n \tag{3.4}$$

Beveridge-Nelson Decomposition

The Beveridge-Nelson decomposition procedure (BN) for vector-autoregressive processes allows extracting cyclical components of cointegrated time series and estimating the degree of co-movement between these transitory components. The VAR representation of an integrated multivariate process offers an interesting decomposition into a nonstationary (permanent) component, that consists itself of the deterministic and the stochastic trend, and a stationary transitory component easily recognisable as cyclical component.

Beveridge and Nelson (1981) have shown that a ARIMA process x_t with stationary first differences representable corresponding to Wold's decomposition theorem as infinite MA process (Beveridge/Nelson, 1981, p. 155f.),

$$\Delta x_t = \mu + \varepsilon_t + \varphi_1\varepsilon_{t-1} + \varphi_2\varepsilon_{t-2} + \ldots, \qquad \sum_{i=1}^{\infty}\varphi_{i_t} < \infty, \tag{3.5}$$

with a long-term average increase μ and white noise ε_t, can be split to

$$x_t = x_t^P + x_t^C, \tag{3.6}$$

where x_t^P is a random walk, i.e. a stochastic trend, with drift μ:

$$x_t^P = \mu + x_{t-1}^P + (\sum_{i=0}^{\infty}\varphi_i)\varepsilon_t\ . \tag{3.7}$$

Beveridge and Nelson show, that the difference between process x_t and its nonstationary component x_t^P, *i.e.*

$$x_t^C = x_t - x_t^P = (\sum_{i=1}^{\infty}\varphi_i)\varepsilon_t + (\sum_{i=2}^{\infty}\varphi_i)\varepsilon_{t-1} + (\sum_{i=3}^{\infty}\varphi_i)\varepsilon_{t-2} + ... \tag{3.8}$$

is stationary. It is referred to as cyclical component. It is easily to show that an analogous split is possible also in presence of an additional deterministic trend. This is then classed as belonging to the permanent component.

The Beveridge-Nelson decomposition can be generalised to VAR processes and particularly integrated in the analysis of vector error correction models (Pesaran/Pesaran, 2009, p. 518-521). In the empirical part of this paper, a modification is applied that was developed by Pesaran and Pesaran (2009).

The determination of the multivariate cyclical component firstly requires an adequate specification of the vector error correction model, particularly concerning its restrictions on the existence of constants or linear trends in the error correction model or the cointegrating relations. For this purpose, the Johansen procedure is most suitable.

Relationships between the extracted cyclical components of the individual variables x_{it} of a vector process $\mathbf{x}_t$ can be found e.g. by correlation or regression analysis. This way there is a third aspect of the interactions between cointegrated variables: If the cointegrating relations or the cointegrating vectors in $\mathbf{B}$ explain the long-term relationships and the coefficients of the lagged differences in $\mathbf{\Gamma}$ explain short-term relation within the error correction model, regression coefficients between components of different variables can be interpreted as medium-term relationships. Using terms of spectral analysis can be noticed, that they allow the view through a medium frequency window on the complex fabric of relationships between the individual variables while the permanent component is the result of a low-frequency window and the relationships between first differences would correspond to a high pass window.

4 Statistical dataset

In the following section we present the data sources and describe the executed transformation procedures.

Public debt

The public debt series is taken from the data base of Deutsche Bundesbank. The variable includes all sorts of public debts corresponding to the Maastricht criteria except for the debt of state owned hospitals with a commercial accountancy. The series also includes the debt of the 2008 established Special Financial Market Stabilisation Funds (SoFFin). Since March 2009 the debt has been including the debt of the Investment and Repayment Fund. The dataset are quarterly volumes measured in million Euros.

Consumer price index

The inflation will be represented by the consumer price index. The index includes all goods and services in the economic region, if they are part of the consumer spending of the private households. The monthly dataset is seasonally adjusted by Census-X-12-ARIMA. The monthly data were transformed into a quarterly period by applying the chronological mean. The series is standardised to 100 for the year 2000.

Money supply

The third variable in the analysis is the money supply used in the m3 classification. The monthly dataset is available from Deutsche Bundesbank. By the chronological mean, the monthly series is transformed to a quarterly periodicity. The money supply is also seasonally adjusted with the Census-X-12-ARIMA and will be measured in million Euros. Since January 2002 currency in circulation has been embodied from the money supply m3.

Long-run interest rate

The long-term interest rate will be measured by the yield on debt securities outstanding of domestic public bonds in per cent. The series has a monthly periodicity. The chronological mean was applied to transform the monthly data into a quarterly series.

5 Empirical results

In the following section, we present empirical research, which is based on aggregate time series for unified Germany for the period 1991, quarter 1, to 2010, quarter 4. This period partially includes the effects from the latest financial and economic crisis. The data will be analysed within the framework of a vector error correction model and the variables will be tested for cointegrating relations. The econometric model is estimated with the Johanson approach. Afterwards, the results of the generalised impulse response analysis and the multivariate Beveridge-Nelson trend/cycle decomposition are shown.

5.1 The econometric model

The analysis of the relationship between public debt and inflation is using the following variables vector

$$\mathbf{x}_t = \begin{bmatrix} \ln D_t \\ \ln P_t \\ \ln M_t \\ I_t \end{bmatrix}$$

with $\ln$ as the natural logarithm.

The estimation of the determinants of public debt in Germany is based on the following log-linear macroeconomic debt function:

$$(5.1) \quad \ln D_t = \alpha + \beta \ln P_t + \gamma \ln M_t + \delta I_t + u_t$$

Where D_t is the public debt in quarter t, P_t is the consumer price index, M_t is the money supply m3 and I_t is the long-term interest rate. Equation (5.1) implies a linear relationship between the four variables in $\mathbf{x}_t$. Because of applying the natural logarithm, the coefficients α, β and γ are to be interpreted as elasticity's. However, the coefficient δ is a semi-elasticity of the interest rate.

Equivalent to the public debt function a consumer price equation can be estimated:

$$(5.2) \quad \ln P_t = \alpha + \beta \ln D_t + \gamma \ln M_t + \delta I_t + u_t .$$

Before estimating the VECM, we need to ensure that the variables are in fact integrated, I(1).

5.2 Results of test on integration

The variables will be tested with the Dickey-Fuller test or the augmented Dickey-Fuller test. The results are shown in table 1.

Table 1: Results of Augmented Dickey-Fuller Tests

Variables	Regression	Lags	Test-statistic	95% critical value	
ln*D*	C,T	4	-3.2532	-3.4696	ns
ln*P*	C,T	4	-3.3085	-3.4696	ns
ln*M*	C,T	1	-2.9366	-3.4673	ns
I	C,T	1	-3.2241	-3.4673	ns
Δln*D*	C	2	-3.7076	-2.9001	s
Δln*P*	C	3	-3.8518	-2.9006	s
Δln*M*	C	0	-5.4645	-2.8986	s
Δ*I*	C	1	-6.3789	-2.8991	s

Note: The Akaike criterion suggests the order of ADF regression be selected.
The Dickey-Fuller regressions include an intercept (C) or a linear trend (T).
ns - non-stationary, s - stationary

Unit root tests confirm, at a level of 5 % significance, that money supply, consumer price index and long-run interest rate are integrated of order one. They have significantly stationary growth rates. The variable public debt is also integrated of order one, if we loosen the strength of the Akaike criterion and look at ADF test with lag order 2. For the ADF(2) test rejects the null hypothesis of non-stationarity on the 5-%-level. Hence the first differences of log public debt can be considered as stationary. Nevertheless, as a consequence of easing the Akaike criterion, we have to interpret the empirical results critically.

5.3 Results of test on cointegration

Because we have more than two I(1) variables, the cointegration rank must be estimated. First, the order of the cointegrating VAR needs to be selected. The Akaike criterion (ML version) suggests a VAR of order two, which equals a VECM of order one. We include a restricted intercept but not a trend in the cointegrating relations. The maximum eigenvalue test and the trace test are applied to test the rank *r*.

Table 2: Results of Johansen Cointegration Tests

H_0	max-eigenvalue test statistic	critical value (95 %)	trace-test statistic	critical value (95 %)
$r = 0$	64.94	28.27	100.55	53.48
$\mathbf{r \leq 1}$	19.73	22.04	35.60	34.87
$r \leq 2$	13.10	15.87	15.88	20.18
$r \leq 3$	2.78	9.16	2.78	9.16

Note: r denotes the number of cointegrating vectors.

The maximum eigenvalue tests suggest that there is $r = 1$, i.e. one single cointegrating relation between public debt, consumer price index, money supply and the long-term interest rate (table 2). The trace test suggests that there are $r = 2$ cointegrating vectors. Akaike and Schwarz information criteria also prefer $r = 1$. Therefore, in the following we suggest a single cointegrating relation between the four variables in **x**.

To identify the VECM, we normalise the coefficient of public debt to -1. As a result, we can interpret equation (5.1) as a public debt function. Alternatively to that we can also normalise the other coefficients to one. Instead of debt function, we get equations for money supply, consumer price level or the long-run interest rate. Estimating the vector error correction model yields the following long-term relationship for public debt:

$$(5.3) \quad \hat{d}_t = \underset{(1.58)}{5.06}\, p_t - \underset{(0.55)}{1.01}\, m_t + \underset{(0.03)}{0.02}\, I_t + \underset{(2.01)}{5.5}$$

The asymptotic standard errors of the coefficients can be found in the brackets. The lower case notation denotes natural logarithms. As we can see in equation (5.3), the consumer prices have a significant (5%) positive effect on public debt in Germany. If the consumer price index increases by 1 %, public debt rises by 5.1 % on an average c.p. In the long-run a higher consumer price level doesn't lead to lower public debt. Especially in the period after the Reunification in the early 1990s, a strong increase in public debt was linked to comparatively high inflation rates in Germany. The money supply and the long-term interest rate have no significant (5 %) influence on public debt.

The corresponding estimated equation for the consumer price level is:

$$(5.4) \quad \hat{p}_t = \underset{(0.06)}{0.2}\, d_t + \underset{(0.05)}{0.2}\, m_t - \underset{(0.006)}{0.004}\, I_t - \underset{(0.47)}{1.09}$$

In the sample period the effect of public debt on consumer prices is significantly positive. A 1 % increase in public debt causes a 0.2 % raise in the consumer price level. As described in section 2, a higher debt level could lead to increasing consumer prices. However, the quantity of the debt effect is quite low. Moreover, we estimated a rectified relationship between money supply and consumer prices. We proved that inflation in Germany after the reunification was as well a monetary phenomenon in the long-run.
In the long-run, trends in public debt are closely related to trends in inflation and money supply. Public debt deviates from this long-run equilibrium in the short term, but will tend to gradually revert to equilibrium over time. This process is modelled as an error correction mechanism.

The estimated vector error correction model is the following:

$$
\begin{aligned}
\Delta\hat{d}_t &= \underset{(0.097)}{0.03}\Delta d_{t-1} - \underset{(0.392)}{1.99}\Delta p_{t-1} - \underset{(0.134)}{0.02}\Delta m_{t-1} + \underset{(0.005)}{0.006}\Delta I_{t-1} - \underset{(0.014)}{0.11}\, ecm_{t-1} \\
\Delta\hat{p}_t &= \underset{(0.032)}{0.05}\,\Delta d_{t-1} + \underset{(0.129)}{0.01}\Delta p_{t-1} + \underset{(0.044)}{0.05}\Delta m_{t-1} - \underset{(0.0016)}{0.002}\Delta I_{t-1} - \underset{(0.0046)}{0.01}\, ecm_{t-1} \\
\Delta\hat{m}_t &= -\underset{(0.077)}{0.07}\Delta d_{t-1} + \underset{(0.308)}{0.4}\Delta p_{t-1} + \underset{(0.106)}{0.34}\Delta m_{t-1} - \underset{(0.004)}{0.003}\Delta I_{t-1} - \underset{(0.011)}{0.028}\, ecm_{t-1} \\
\Delta\hat{l}_t &= -\underset{(2.21)}{0.89}\Delta d_{t-1} + \underset{(8.878)}{17.49}\Delta p_{t-1} + \underset{(3.041)}{5.41}\Delta m_{t-1} + \underset{(0.114)}{0.3}\Delta I_{t-1} + \underset{(0.317)}{0.89}\, ecm_{t-1} \\
ecm_t &= 0.94 d_t - 4.74 p_t + 0.95 m_t - 0.02 I_t - 5.15
\end{aligned}
\tag{5.5}
$$

The dynamic specification exhibits a significant error correction coefficient for public debt, with the expected minus sign. The value of this coefficient, i.e. the speed of return to the equilibrium debt level appears to be relatively moderate. The money supply and the long-term interest rate also contribute to lowering the deviations from the long-run relationship. The magnitude of the error correction coefficient in the interest rate equation is quite large. In contrast, the estimated error correction coefficient in the consumer price growth equation doesn't contribute to the error correction of a shock, because the coefficient has the same sign as in the equilibrium relationship. Changes of the consumer prices cause a restraining effect on public debt growth in the short-term. The government profits from higher inflation only in the short-term, whereas in the long-term a mutual relationship was observed. Money supply and interest rates are significantly positive influenced by their own lagged first differences.

5.4 Results of the impulse response analysis

In the following the results of the cumulative effects of the variable- (or equation-) specific shocks for the variables in $\mathbf{x}_t$ are shown.

Public debt shock

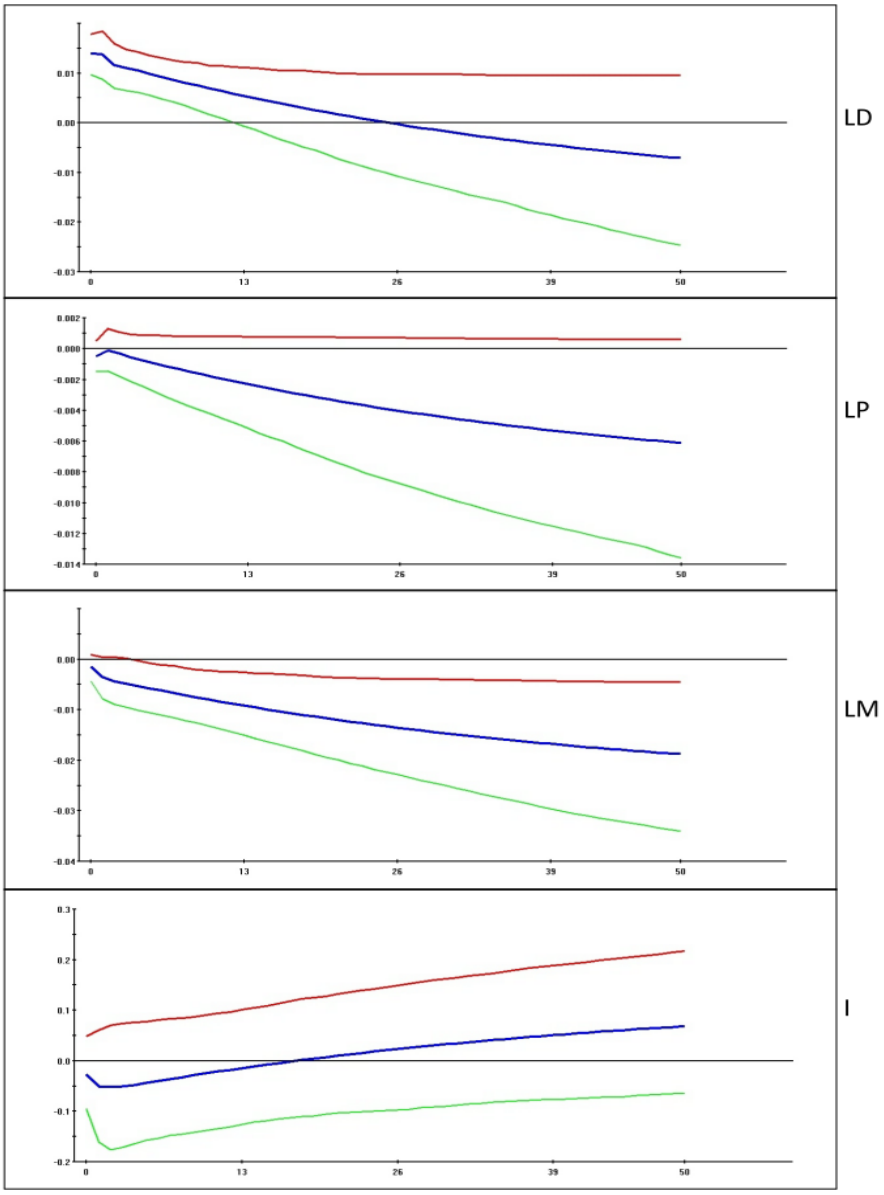

Figure 2: Generalised impulse responses to a one S.E. shock in the equation for public debt

An impulse in the equation for public debt induces a significant negative effect on money supply in Germany with a lag of one year. However, the debt shock doesn't cause a significant reaction on consumer prices and the long-run interest rate.

Consumer price shock

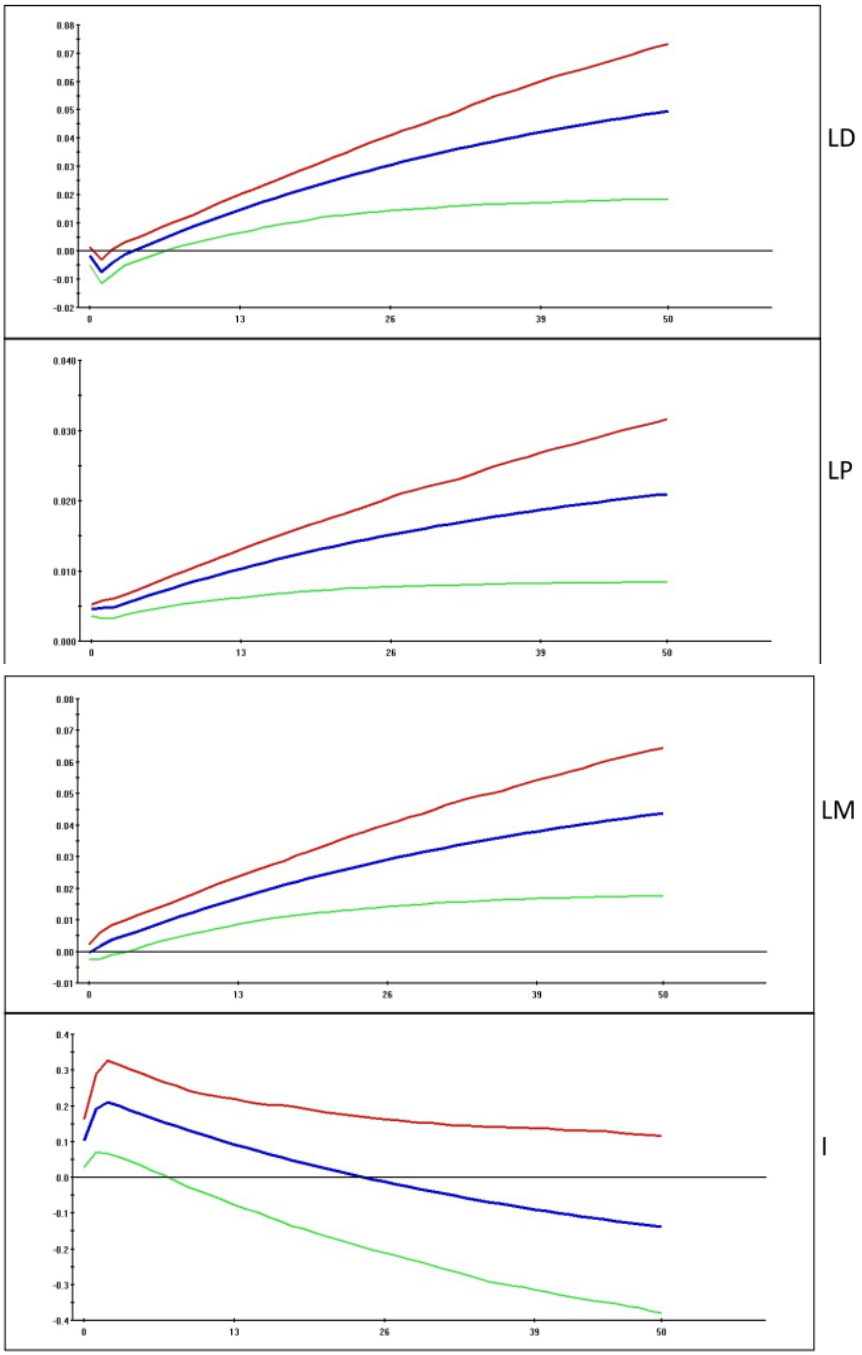

Figure 3: Generalised impulse responses to a one S.E. shock in the equation for consumer prices

A consumer price shock generates directly a negative reaction on public debt and in the medium- and long-term a positive effect on government debt. This supports the VECM results in the previous section. The response of money supply is significantly positive after one year. Moreover, the price shock determines an increasing effect on the interest rate only in the short-term.

Money supply shock

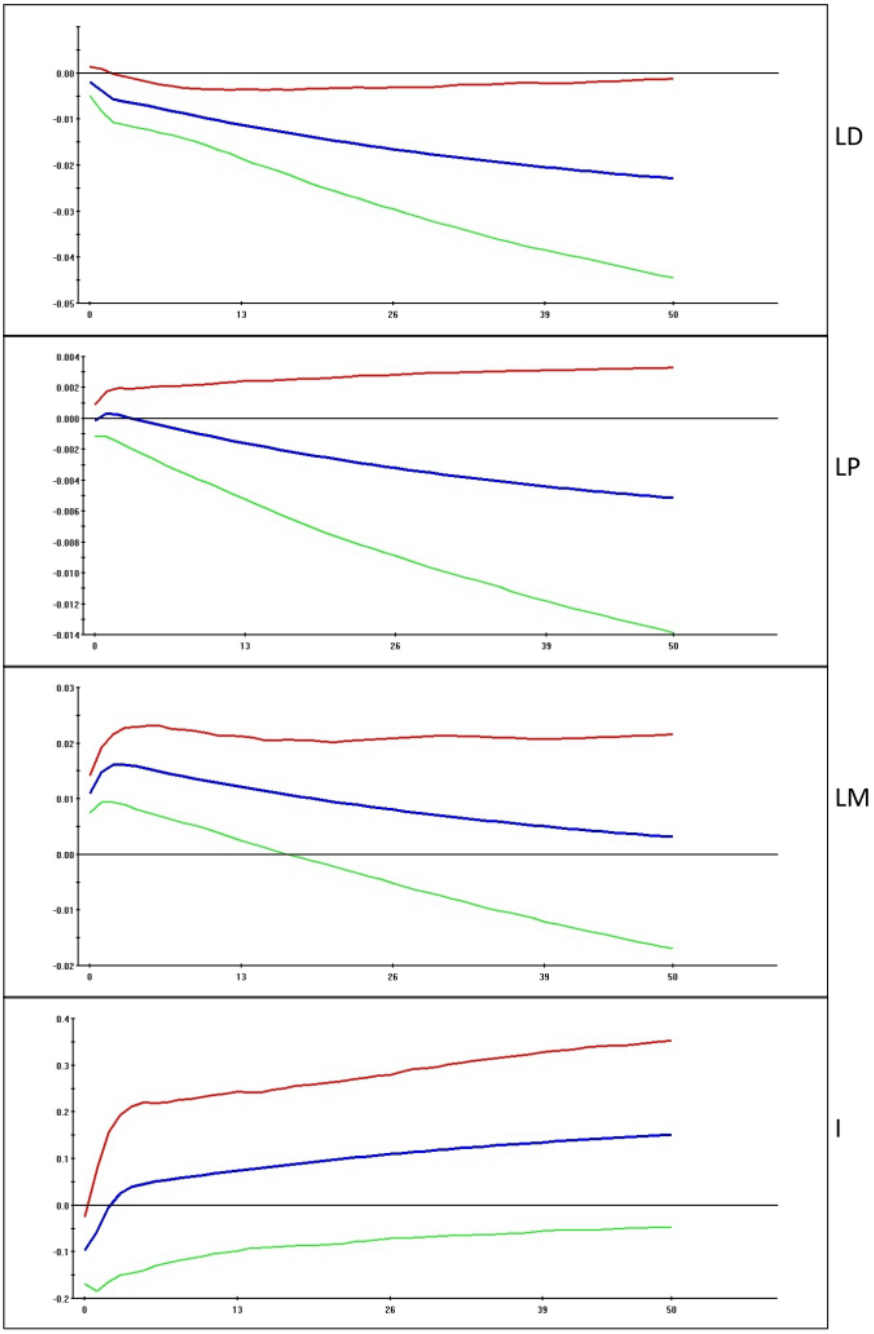

Figure 4: Generalised impulse responses to a one S.E. shock in the equation for money supply

A money supply shock causes a slightly contractive effect on public debt after the third quarter. After two years, the estimated impulse-response function tends against its long-term multiplier. The impulse response for the interest rate is only contemporarily significant. The money shock doesn't cause a significant effect on consumer prices during the observed period.

Interest rate shock

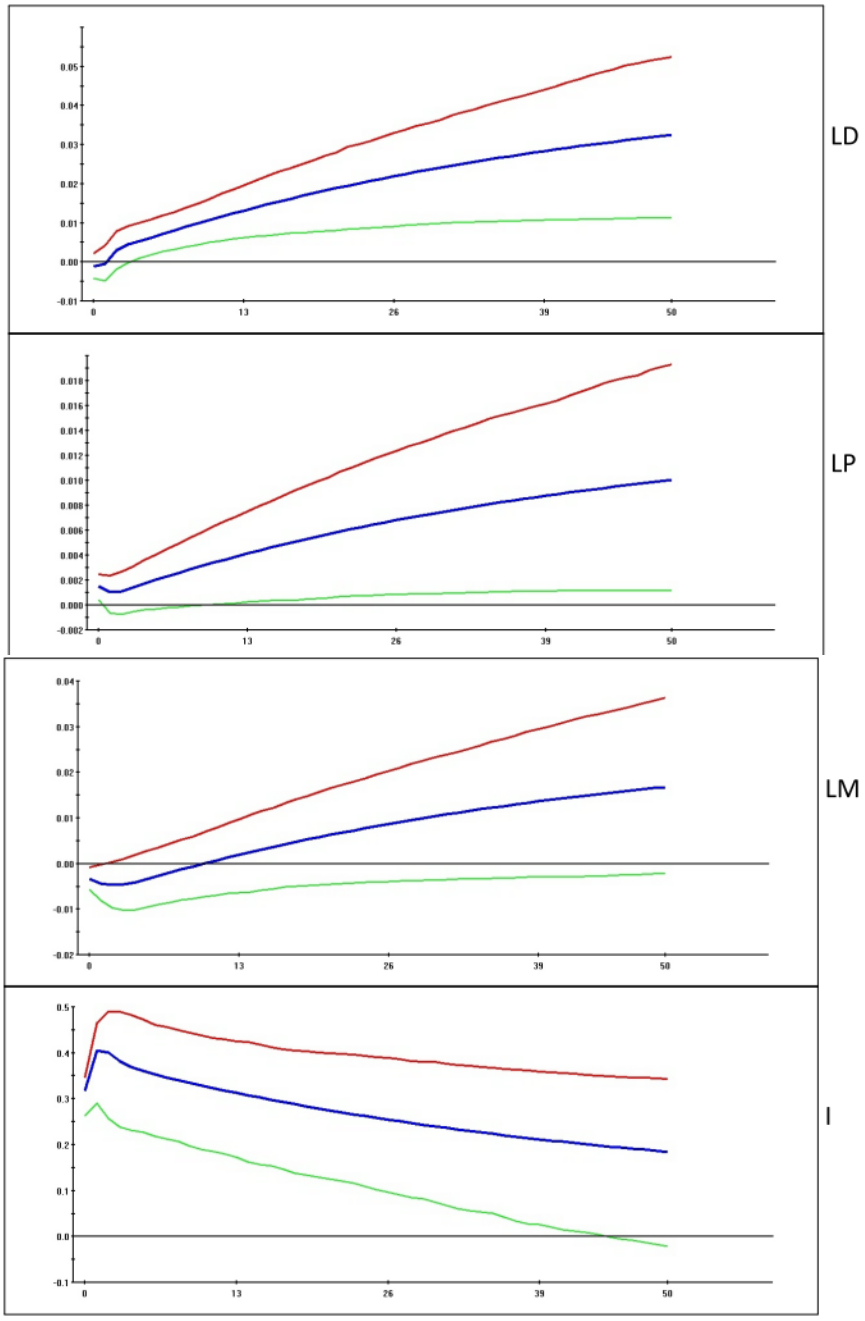

Figure 5: Generalised impulse responses to a one S.E. shock in the equation for long-run interest rate

An impulse to the long-run interest rate generates a slightly stimulating effect on public debt after two quarters. It seems that higher interest rates don't restrict the growth of government debt. Consumer prices show an increasing reaction to the shock. The response of money supply is significant only during the first quarter.

5.5 Results of the multivariate Beveridge-Nelson decomposition

The cyclical components of the four variables are determined by applying the multivariate Beveridge-Nelson decomposition on the estimated vector error correction model.

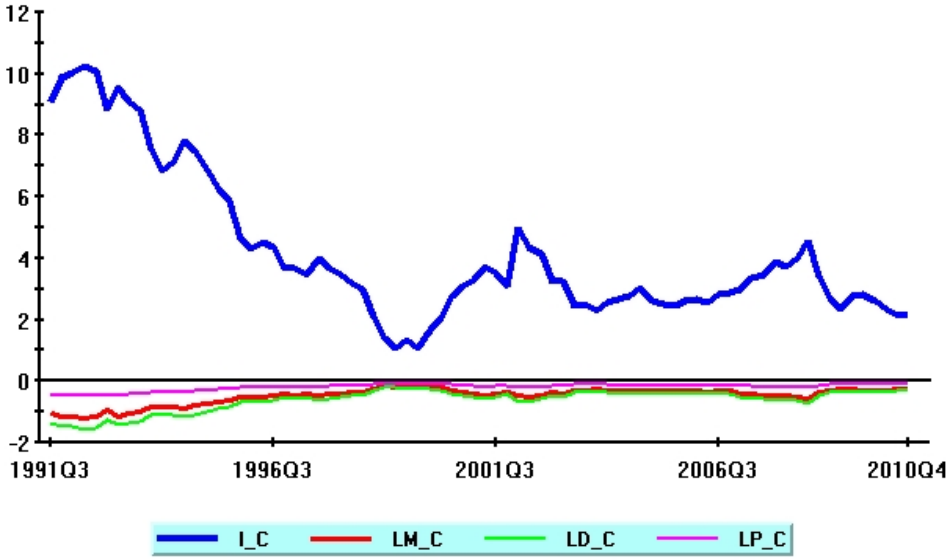

Figure 6: Plot of the transitory (cyclical) components

Figure 6 represents the cyclical components of the variables in **x**. The long-term interest rate is featured by an intense cyclical behaviour. Whereas the variables public debt, money supply and consumer price index have low cyclical variations during the period from the 1st Quarter 1991 to the 4th Quarter 2010.

To determine the dependency in the cyclical behaviour between the variables in $\mathbf{x}_t$, given the multivariate approach, we conduct four OLS estimations between the cyclical or stationary components marked by "…^{c}":

$$(5.6) \quad \hat{d}_t^c = \underset{(0.75)}{5.66}\, p_t^c - \underset{(0.24)}{0.86}\, m_t^c + \underset{(0.008)}{0.009}\, I_t^c .$$

The estimated standard errors of the coefficients can be found in the brackets. The transitory component of public debt is significantly positive influenced by the consumer prices and significantly negative by the money supply. An increase in inflation leads to an expansion of government debt, considering the cyclical behaviour.

$$(5.7) \quad \hat{p}_t^c = \underset{(0.01)}{0.08}\, d_t^c + \underset{(0.01)}{0.25}\, m_t^c - \underset{(0.0006)}{0.006}\, I_t^c .$$

The cyclical component of the consumer price index in Germany is positively influenced by the corresponding components of public debt and money supply. However, the impact of "cyclical" public debt on inflation is quite low. Monetary conditions play an important role for the cyclical behaviour of the consumer prices.

$$(5.8) \quad \hat{m}_t^c = \underset{(0.05)}{-0.17}\, d_t^c + \underset{(0.15)}{3.57}\, p_t^c + \underset{(0.002)}{0.02}\, I_t^c .$$

The cyclical component of money supply is significantly positive determined by the long-run interest rate and the consumer prices and negatively influenced by public debt.

$$(5.9) \quad \hat{I}_t^c = \underset{(1.74)}{2.08}\, d_t^c - \underset{(10.12)}{96.44}\, p_t^c + \underset{(2.46)}{26.89}\, m_t^c .$$

The influence of the cyclical component of the consumer prices on the interest rate is highly restraining, while the impact of the CPI growth rates was positive. Furthermore, the money supply is contemporary increasing the interest rate in a medium-term frequency window.

By a spectral analysis reflection, the relationship between public debt and consumer prices in Germany since the reunification depend on the used frequency window. There is an aligned development between both economic variables in the medium- and long-term. In contrast, consumer prices cause a restraining effect on public debt growth in the short-term.

6 Conclusion

Public debt and inflation are mutually connected through several channels. Public deficits can lead to higher inflation if the money supply is expansive. The liquidity condition of the banking sector and the institutional framework such as the independency of the central bank determine the relationship between national debt and inflation: The lower the level of independence the higher the potential of debt-caused inflationary processes. In this context it is important where (domestic or abroad) and from whom (private or institutional investors) the government lends money and how the investors evaluate the public bonds. An essential factor represents the government's solvency, which means on the one hand the ability to repay and on the other hand the attendance to repay. This is shown currently by the examples of Greek or Cyprus. A not as sustainable evaluated public debt level could lead to a loss in financial standing and cut the access to the credit markets.

The results of the empirical analysis for Germany show that consumer prices had a positive impact on public debt in the long-run after the German reunification. We estimated a significant long-run relationship. On the contrary, the short-term changes of the consumer price index have a restraining effect on public debt growth. The government profits from higher inflation only in the short-term, whereas in the medium- and long-term a mutual relationship was observed. Moreover, consumer prices were significantly positive affected by the public debt level in the long-run. That means public debt statistically causes inflation vice versa. Furthermore, a stable connection between the level of money supply and consumer prices was detected in Germany.

The central banks of some highly indebted countries (i.e. FED, Bank of England) have purchased a huge amount of government bonds since 2008 to lower the public refinancing costs. The returns on public bonds were successfully kept under the inflation rate in countries such as the United Kingdom and the United States. The uncertainty on bonds markets as a result of the European sovereign debt crisis also has led to negative real interest rates in Germany. The real interest rates are remaining in a negative range and contribute to a short-term debt relief of states with a high solvency. The Japanese central bank (Bank of Japan) has also decided to start a massive government bond purchase programme. The ECB keeps the option of further secondary market purchases with the OMT open.

Considering the high level of public debts and the fragile economic situation some central banks announced to fix the interest on a low level for a considerable time (forward guidance). Thereby the asset of the bond owner and depositors will be devaluated in real terms. Furthermore, a long-lasting low level of interest rates could cause or encourage the development of speculative bubbles on the bond, real estate or the commodity market. As a consequence, the capital accumulation of the economic subjects could be distorted. The crisis of banks and the new regulatory capital requirements have detained the financial sector to multiply the increasing central bank money base and thus to start an inflationary process so far. Independent central banks with the focus on price stability are an important factor to limit the inflation potential caused by high public debt.

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