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Matthias Kalkuhl
Max Franks
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Ottmar Edenhofer

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University of Potsdam
August-Bebel-Straße 89, 14482 Potsdam
Tel.: +49 331 977-3225
Fax: +49 331 977-3210
E-Mail: dp-cepa@uni-potsdam.de

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Pigou's Advice and Sisyphus' Warning: Carbon Pricing with Non-Permanent Carbon-Dioxide Removal*

Matthias Kalkuhl
MCC Berlin, University of Potsdam

Max Franks
PIK Potsdam, TU Berlin

Friedemann Gruner
MCC Berlin, University of Potsdam

Kai Lessmann
PIK Potsdam, MCC Berlin

Ottmar Edenhofer
PIK Potsdam, MCC Berlin, TU Berlin

ABSTRACT

Carbon dioxide removal from the atmosphere is becoming an important option to achieve net zero climate targets. This paper develops a welfare and public economics perspective on optimal policies for carbon removal and storage in non-permanent sinks like forests, soil, oceans, wood products or chemical products. We derive a new metric for the valuation of non-permanent carbon storage, the social cost of carbon removal (SCC-R), which embeds also the conventional social cost of carbon emissions. We show that the contribution of CDR is to create new carbon sinks that should be used to reduce transition costs, even if the stored carbon is released to the atmosphere eventually. Importantly, CDR does not raise the ambition of optimal temperature levels unless initial atmospheric carbon stocks are excessively high. For high initial atmospheric carbon stocks, CDR allows to reduce the optimal temperature below initial levels. Finally, we characterize three different policy regimes that ensure an optimal deployment of carbon removal: downstream carbon pricing, upstream carbon pricing, and carbon storage pricing. The policy regimes differ in their informational and institutional requirements regarding monitoring, liability and financing.

Keywords: Carbon Dioxide Removal, Carbon Capture, Social Cost of Carbon, Climate Policy, Impermanence

JEL Codes: D61, H23, Q54, Q58

Corresponding author:
Matthias Kalkuhl
Mercator Research Institute on Global Commons and Climate Change
Torgauer Str. 12-15
10829 Berlin, Germany
Email: kalkuhl@mcc-berlin.net

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> Aye, and I saw Sisyphus in violent torment, seeking to raise a monstrous stone with both his hands. Verily he would brace himself with hands and feet, and thrust the stone toward the crest of a hill, but as often as he was about to heave it over the top, the weight would turn it back, and then down again to the plain would come rolling the ruthless stone. But he would strain again and thrust it back, and the sweat flowed down from his limbs, and dust rose up from his head.
>
> — Homer, Odyssey

1. Introduction

By 2050, the European Union and the United States of America aim to become carbon neutral, and China plans to be so by 2060. However, carbon neutrality is not easily achieved on this timescale by reducing fossil fuel combustion alone. While low-carbon substitutes exist for electricity generation from fossil resources, other economic activities, such as aviation or the production of steel, cement, and chemicals, are difficult to decarbonize.

Scenarios with deep decarbonization and low temperature targets therefore rely on carbon dioxide removal (CDR) technologies. In some scenarios that limit global warming to 1.5°C, for example, the IPCC estimates that up to half of current carbon emissions have to be removed annually in the second half of the 21st century (Masson-Delmotte et al. 2018).

Although these scenarios project substantial use of CDR, research on the policies to govern CDR is still in its infancy.[1] In contrast, a vast literature has explored policies to reduce fossil emissions. Its foundations go back to the seminal work on taxation of pollution (Pigou 1920; Farzin 1996), recent work is surveyed in Edenhofer et al. (2021). However, the applicability of this literature to the case of carbon removal is limited.

A key difference is that, in general, removal of carbon is only temporary while emission abatement, that is, reducing fossil fuel combustion, has a permanent effect on carbon in the atmosphere.[2] Average times until removed carbon emissions are released from storage vary widely.

1. Franks et al. (2022), for example, analyze optimal pricing policies for CDR under inter-regional leakage.

2. Some authors consider also emission abatement to be only temporary since a unit of carbon left in the ground may be extracted at a later time. Then, reduced demand and therefore a lower price on carbon will even make such a postponed extraction more likely (Herzog et al. 2003; van Kooten 2009). Dynamic modeling as in this paper includes such demand

Removal and storage pathway	Storage duration *(half-life)*
Bioenergy with carbon capture and storage	millennia
Enhanced weathering	centuries
Forestry techniques & wood products	decades to centuries
Single family home	*100*
Furniture, residential upkeep and improvement	*30*
Paper	*2*
Soil carbon sequestration techniques	years to decades
Biochar	years to decades

TABLE 1. Storage time for different CO2 removal technologies. Based on Hepburn et al. (2019); Smith (2006); Hiraishi et al. (2014).

For instance, storage by sequestration to geological formations, which is used by biomass energy with carbon capture and storage or direct air capture and storage, is close to permanent. Land-based storage technologies, such as afforestation and reforestation, are less permanent. Depending on the management strategy, half-lives range from decades to centuries (Table 1). Further temporary carbon storage technologies are related to carbon capture and usage concepts that store the carbon in the productsphere. Typical examples include the production of bioplastics, harvested wood products but also chemicals or synthetic fuels. While some of these products are only short-lived, others can achieve storage durations comparable to other carbon removal technologies (Table 1). In particular, the use of wood in the construction of new buildings could store carbon for centuries and has the potential to store up to 20 Gt of carbon over the next thirty years (Churkina et al. 2020).

The non-permanent nature of carbon storage raises an important question: If the removed and stored carbon is returned to the atmosphere eventually, what (if anything) can CDR with non-permanent storage contribute to carbon neutrality? It stands to reason that non-permanence lowers the value of removing a ton of carbon from the atmosphere compared to abating the emission of that one ton of carbon once and forever. Hence, a holistic carbon pricing scheme must consider that the two activities should be priced differently. In this paper, we investigate such CDR policies in a dynamic model of carbon emissions and atmospheric carbon removal with storage in non-permanent sinks. Without knowledge of the optimal usage of CDR and tools to properly incentivize

responses. Equilibrium prices will adjust in response to reduced demand such that emission abatement is permanent. Thus, intertemporal leakage does not require explicit consideration.

CDR, regulators run the risk of allowing an over-utilization or fail to set incentives for an urgently needed set of technologies.

A number of studies have made an effort to differentiate between avoided and removed emissions, often suggesting differentiated carbon pricing. Groom and Venmans (2021) discuss the social value of avoided versus removed emissions based on carbon offsets. Carbon offsets are units of removed or avoided emissions that can compensate for emissions caused elsewhere. When offsets are generated by removal with non-permanent storage, their value is still positive but falls below the value of avoided or permanently removed emissions. Several studies have investigated how carbon pricing policies can take this difference into account. For example, Kim et al. (2008) compute price discounts for carbon offsets that are characterized by non-permanence. Compared to the price of an idealized perfect storage option, the price of these offsets takes into account the additional cost of released emissions. The price discount depends on the permanence of storage, the price path of carbon and the discount rate. van Kooten (2009) considers credits for sequestered carbon, and discusses the rate at which these credits should be discounted relative to avoided emissions. This rate depends on permanence (time until release), the growth rate of the carbon price and the discount rate on cash flows. As these characteristics are difficult to predict for a regulator, the author favors a market based approach to determine the discount rate. van Kooten follows Marland et al. (2001) in proposing that credits generated by sequestering emissions are not sold but rented for a limited time, such that the responsibility for premature release continues to lie with the originator of the credit. Market forces will then price the temporary credits.

Other economic studies on the governance of CDR address the problem of default risk rather than the permanence problem. Considering that default of the firm in control of the stored carbon may imply a premature release, their findings are relevant in this context. Evidence for the similarity of the problems is Groom and Venmans (2021), who consider default risk (risk of failure) in addition to permanence (as discussed above) and partial additionality. The authors identify discount factors in a similar way for all three problems.

Bednar et al. (2021) propose a different instrument to address default risk. In their proposal emissions that exceed a given emission budget translate into “carbon debt”, formalized as carbon removal obligations. The interest rate on carbon debt then becomes the central tool to differentiate and thus deter risky debt.

Lemoine (2020) follows the same intuitions as van Kooten (2009), namely to continue to hold firms responsible for their carbon but focuses on emitters of carbon into the atmosphere where van Kooten targets the originators of carbon offsets. Lemoine suggests a carbon stock tax, such that emitters pay a rental charge for storing carbon in the atmosphere until they remove it again. The incentive to reduce emissions is impaired when firms can forego the rental charge in case of bankruptcy. To address this moral hazard Lemoine introduces up-front payment via bonds and tradable carbon shares. However, the study does not consider where or how carbon is stored after removal, nor issues of non-permanence.

As a common theme, previous analyses have established that despite non-permanence of storage, carbon removal should be carried out but should be priced at a discount, which depends on the economy's interest rate or discount rate, the degree to which storage is temporary, and the development of the carbon price. This indicates a strong link between the appropriate pricing of non-permanence and the pathway of the economy. Consequently, assumptions about carbon prices, discount rates and growth rates of the economy are varied.

However, in most studies carbon prices and the time path of the social cost of carbon remain exogenous (but see Lemoine 2020). The question thus remains how CDR should be incentivized, in particular with non-permanent storage, when long-term climate stabilization is driven by optimal carbon pricing. Previous research has not yet provided a comprehensive analysis of optimal pricing policies that specifically includes both emissions abatement and CDR with non-permanent storage.

In the present paper, we address this question for the first time by introducing a welfare maximization approach in a dynamic partial equilibrium model to characterize optimal use of emissions abatement and carbon removal technologies and the carbon pricing policies for their implementation. In the model, economic activity requires energy inputs, which the economy obtains from fossil fuel combustion causing carbon emissions or from a carbon-free technology. We assume that the two inputs are perfect substitutes in production. Carbon emissions accumulate in the atmosphere, which leads to a global temperature increase that causes climate change damages. For removal, we introduce a set of CDR technologies, which may differ regarding their marginal cost and permanence.[3] We model the latter as release from storage sites at constant rates.

3. In the following, we always assume non-permanent CDR if not stated otherwise. CDR with permanent storage is a limiting case of our analysis that we discuss explicitly.

We use the model to conduct a welfare analysis of CDR use in the long term, that is, in the steady-state of the economy, and of CDR use as part of the transition dynamics. Our contribution to the literature arises from the use of the dynamic optimization framework, which allows us to study endogenous cost-benefit trade-offs and equilibrium effects, as well as optimal and second best policies.

We find that it is optimal to use CDR despite non-permanence of storage. During the transition, CDR with temporary carbon storage is employed to reduce near-time climate change damages. However, this comes at a cost: emissions released from storage need to be removed from the atmosphere in perpetuity. Thus, even though carbon emissions from fossil fuel are zero in the long-run steady state, utilization of CDR commits the economy to a continuous use of CDR in the steady state – to offset emissions that come from leaky storage sites of past removal activities. The commitment to perpetual removal of carbon is evocative of Sisyphus pushing the heavy stone to the top of the mountain only to watch it roll down again. However, without CDR and perpetual removal of the carbon that is released from non-permanent sites, discounted social welfare would be lower. The optimal amounts of carbon removal and the storage size in the steady state increase with the discount rate and fall with the half-life of storage.

The optimal long-term global temperature is, however, not affected by the availability or utilization of non-permanent CDR technologies when the initial stock of carbon is below the corresponding steady state level. In that case, CDR effectively creates an additional carbon sink that allows the economy to reach the same optimal temperature target with additional cumulative fossil resource use and lower cumulative climate damages. The optimal steady state atmospheric carbon concentration and temperature depend only on the balance of marginal costs of using carbon free energy, as well as marginal climate change damages. This finding emphasizes that CDR cannot substitute for emissions abatement in the long-run. If the initial stocks of carbon in the system are well above these optimal levels, it is not optimal to return to this steady state due to the cost of continuously returning released emissions to storage forever.

In our numerical simulations, we see that removal flows are substantial (ranging from 10 to almost 100 percent of current carbon emissions), even when the half-life of storage is only decades. We further reaffirm that the availability of CDR does not affect long-run carbon prices nor optimal temperature targets, but mainly reduces carbon prices in the near-term. Hence, CDR provides a way to dramatically reduce short-run mitigation costs by effectively increasing the available atmospheric carbon budget.

Regarding policy design, we find that non-permanent carbon removal introduces a new social cost of carbon metric, the social cost of carbon removal (SCC-R). Whereas the conventional social cost of carbon emissions (SCC-E) is a measure of the climate change damages from carbon emitted into the atmosphere, the SCC-R is a measure of climate change damages resulting from releasing emissions from storage. Hence, the SCC-R is a measure of the cost of a delayed carbon emission. The SCC-E and SCC-R metrics turn out to be central concepts for the design of tax and subsidy policies.

The government can incentivize the optimal use of CDR technologies by paying a Pigouvian subsidy on removed emissions or stored carbon. When the government can price emissions released from temporary storage, the optimal subsidy on removal is equal to the price on carbon emissions, the social cost of carbon emissions. We refer to such a regime as *downstream* carbon pricing, as all downstream emissions are covered. Alternatively, a subsidy can be applied to carbon in storage stocks, creating an incentive to remove carbon from the atmosphere and keep it stored with a single instrument (*storage subsidy*). Both instruments, however, require knowledge about the carbon stored and/or released. The regulator can forgo the effort of monitoring release and/or storage by reducing the subsidy on removal relative to the carbon tax on emissions to take future released emissions into account, that is, by *upstream* carbon pricing. The optimal discount factor that captures the ratio of removal subsidy and emissions tax is determined by the SCC-R and falls between 0 and 1. It is decreasing in the release rate and increasing in the discount rate. In our numerical model, we analyze a second-best policy that disregards the discount factor and applies the same carbon price to emissions and removals. We find that such a uniform carbon price can lead to significant excess emissions in the short and medium term and to worse welfare outcomes than in the absence of carbon removal technologies. However, we further show that applying a simple second-best discount factor to the carbon removal subsidy causes only minor welfare losses compared to first best. These findings suggest that the use CDR requires a careful design of the policy architecture to avoid adverse incentives to over- or under-use removal and carbon storage.

Our paper is structured as follows: In section 2, we discuss the economic value of removal and abatement. Therefore, we introduce the social planner model and characterize optimal removal flows and shadow prices in the transition and the steady state. In Section 3, we describe optimal

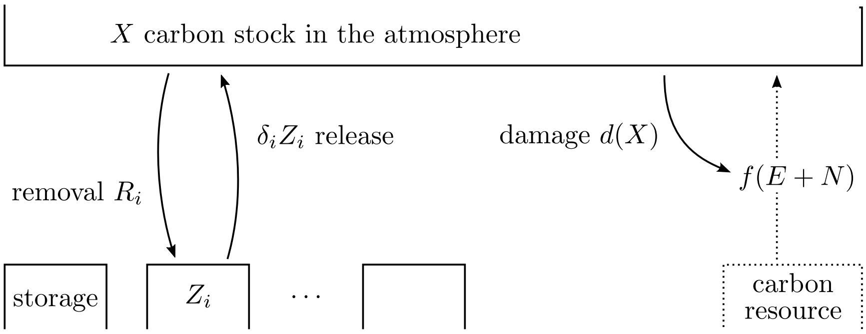

FIGURE 1. Model schematic of stocks (X, Z_i) and flows (E, R_i). Carbon resources (dotted) are not part of the model.

carbon taxes, and storage subsidies as optimal policy instruments for regulating carbon removal. Next, Section 4 discusses various modifications and extensions to the basic model. Finally, we develop a numerical model to illustrate transition dynamics and second-best policy scenarios in Section 5. Section 6 concludes.

2. The value of removal and abatement: Social planner model

We first develop a social planner model that serves as benchmark for an optimal allocation in the economy. The social planner model characterizes optimal removal quantities and stocks during the transition and in the long-run. The transition to a low-carbon economy is driven by increasing climate damages. The social planner solution also describes shadow prices associated to carbon flows; these shadow prices are fundamental to understand the value of CDR and emissions abatement. They constitute the foundation for calculating optimal taxes and subsidies in the decentralized economy (Section 3).

Before we discuss different policies in the decentralized economy, though, we first describe the model in Section 2.1. Then, we characterize its transition dynamics (Section 2.2) and its steady state (Section 2.3). We perform comparative statics in Section 2.4 to derive, among others, the result that CDR does not affect the optimal temperature target in the long-run.

2.1. *Model description*

Economic output $f(E+N)$ can be obtained by using fossil-based energy, which causes a flow of emissions $E(t)$, or by using non-fossil energy $N(t)$. The two energy sources are perfect substitutes and we assume an aggregate production technology with decreasing returns ($f' > 0$, $f'' < 0$). We further assume that fossil energy is supplied at zero costs, while non-fossil energy is characterized by convex costs $b(N)$ with $b'(N) > 0$, $b''(N) > 0$.[4] Fossil energy use causes emissions that accumulate in the atmosphere as carbon stock $X(t)$. The atmospheric carbon concentration leads to climate damages $d(X)$ that increase in X. Greenhouse gases can be removed from the atmosphere with different CDR technologies. Removing at rate $R_i(t)$ using technology $i \in \mathscr{I}$ comes at the cost $g_i(R_i)$ with g_i', $g_i'' > 0$. Removed carbon is stored in storage stocks $Z_i(t)$. Carbon storage is non-permanent, that is, the sink releases carbon back into the atmospheric stock $X(t)$ at rate $\delta_i > 0$. We relax this assumption in Section 4.1. We can understand these released emissions as a continuous flow of carbon emissions from stored carbon. The stock and flow dynamics are summarized in Figure 1. We suppress the time index for the sake of readability when possible.

The social planner maximizes intertemporal welfare

$$\max_{E,N,R} \int_0^\infty \left[f(E+N) - \sum_i g_i(R_i) - b(N) - d(X) \right] e^{-rt} \mathrm{d}t \tag{1}$$

$$\text{such that} \quad \dot{X} = E - \sum_i R_i + \sum_i \delta_i Z_i \qquad \perp \mu \tag{2}$$

$$\dot{Z}_i = R_i - \delta_i Z_i \qquad \perp \psi_i \tag{3}$$

$$E \geq 0 \qquad \perp m \tag{4}$$

4. This assumption implies that fossil fuel demand is only ’constrained’ by the climate damages, not by extraction costs or physical availability of fossil energy.

Initial values are given by $X(0) = X_0$ and $Z_i(0) = Z_{i0}$. The inequality constraint is considered by nesting the Hamiltonian $\mathscr{H}$ in a Lagrangian $\mathscr{L} = \mathscr{H} + mE$:

$$\mathscr{L} = f(E+N) - \sum_i g_i(R_i) - b(N) - d(X)$$

$$+ \mu(E - \sum_i (R_i - \delta_i Z_i)) + \sum_i \psi_i (R - \delta_i Z_i) + mE \tag{5}$$

The shadow prices associated with X and Z_i are μ and ψ_i. The Lagrangian multiplier of the non-negativity constraint on emissions is m. The optimal solution is, hence, characterized by the following first order conditions.

$$f'(E+N) = -\mu - m \tag{6}$$

$$f'(E+N) = b'(N) \tag{7}$$

$$g_i'(R_i) = \psi_i - \mu \tag{8}$$

$$\dot{\mu} = r\mu + d' \tag{9}$$

$$\dot{\psi}_i = r\psi_i + \delta_i(\psi_i - \mu) \tag{10}$$

$$0 = \lim_{t\to\infty} \mu(t)X(t)e^{-rt} \tag{11}$$

$$0 = \lim_{t\to\infty} \psi_i(t)Z_i(t)e^{-rt} \tag{12}$$

and the complementary slackness condition

$$mE = 0 \quad \text{with } m \geq 0,\ E \geq 0 \tag{13}$$

The optimal shadow prices for fossil energy use and for carbon removal grow at the rates

$$\hat{\mu} = r + \frac{d'(X)}{\mu} \tag{14}$$

$$\hat{\psi}_i = r + \delta_i(1 - \frac{\mu}{\psi_i}) \tag{15}$$

The shadow prices μ and ψ_i have an important economic intuition: μ measures the marginal damage of emitting one ton of carbon and ψ_i measures the marginal damage caused by one initially removed ton of carbon as it is released from storage over future periods.

PROPOSITION 1. *The social cost of a carbon emission (SCC-E) is*

$$SCC\text{-}E = -\mu = \left[\int_t^\infty d'\left(X(s)\right)e^{-rs}ds\right]e^{rt} \tag{16}$$

The social cost of a carbon removal (SCC-R) is

$$SCC\text{-}R_i = -\psi_i = \left[\delta_i \int_t^\infty SCC\text{-}E(s)e^{-\delta_i(s-t)}ds\right]e^{rt} \tag{17}$$

Proof. See Appendix A.1.1. □

The proposition indicates that considering non-permanent carbon removal creates two distinct social cost of carbon prices – one conventional for measuring the social damage of emitting one ton of carbon, and a new measure accounting for the social damage caused by one ton of carbon that is initially removed and released back into the atmosphere over time.

We can exploit the monotonicity of f and b to identify how the Lagrangian multiplier m of the non-negativity constraint for E evolves. Since f and b are monotonic, we can invert the functions to obtain an expression that gives us fossil energy use E as a function of the shadow price μ. By inverting (6) and (7) and taking their difference we have

$$(f')^{-1}(-\mu - m) - (b')^{-1}(-\mu - m) = E + N - N = E \tag{18}$$

Figure 2 illustrates the relationship between fossil energy demand, the SCC-E and the multiplier m by showing the use of emissions E, non-fossil energy N and total energy $E+N$ as inverse functions of the SCC-E, that is, of the shadow price $-\mu$. Since the inverse of strictly increasing functions are, again, strictly increasing, $b'' > 0$ implies that non-fossil energy use $N = (b')^{-1}$ is strictly increasing; similarly total energy $E+N = (f')^{-1}$ is strictly decreasing, and fossil energy demand $E = (E+N) - N$ likewise. As $E \geq 0$ there is a threshold $-\tilde{\mu}$ such that $E(\mu) = 0$ for all $-\mu \geq -\tilde{\mu}$. That is, for SCC-E below $-\tilde{\mu}$, fossil energy use remains positive (dashed-dotted line) and $m = 0$ (hatched). When the SCC-E exceeds $-\tilde{\mu}$ then negative fossil energy use would be optimal. In this case, E is at its lower bound (4) and $m > 0$. To see how m evolves, note that $E = 0$ only holds for $-\mu - m = -\tilde{\mu}$ in (18).[5] Thus, the multiplier m needs to adjust such that $m = \tilde{\mu} - \mu$. Consequently, m rises one-to-one with the SCC-E ($-\mu$).

5. As $f'' < 0$ and $b'' > 0$ there is a unique intersection of the two curves.

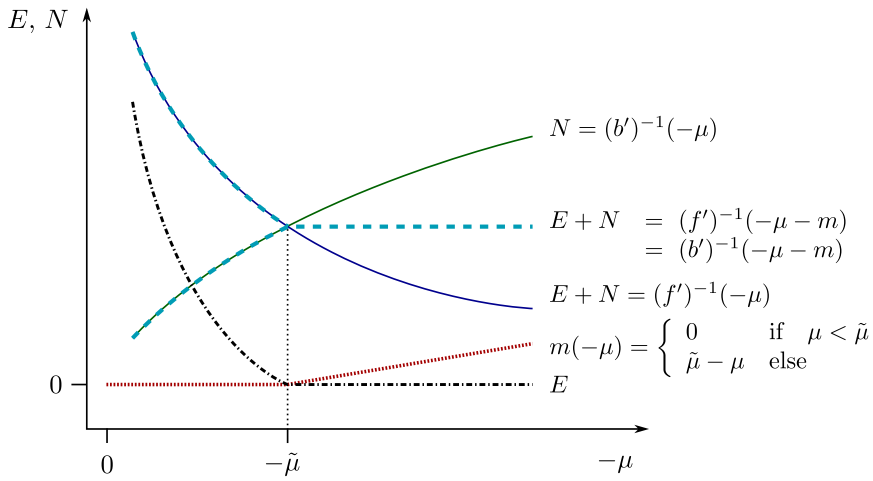

FIGURE 2. The social cost of carbon determine whether the non-negativity constraint on emissions binds. As long as SCC-E $< -\tilde{\mu}$, we have $E > 0$.

LEMMA 1 (Non-negativity of emissions). *Emissions E are declining in the SCC-E $(-\mu)$ until the SSC-E exceed a threshold value $\tilde{\mu}$. At $\mu = \tilde{\mu}$ the non-negativity constraint of emissions E (4) becomes binding, that is, emissions remain at zero. The associated Lagrangian multiplier m follows*

$$m(\mu) = \begin{cases} 0 & \textit{if } -\mu < -\tilde{\mu} \\ \tilde{\mu} - \mu & \textit{else} \end{cases} \tag{19}$$

2.2. Transition Phase

The evolution of the dynamic system crucially depends on the initial stocks of carbon in the atmosphere $X_0 = X(0)$ and in underground storage stocks $Z_{i,0} = Z_i(0)$. We have to consider two cases – one in which it is optimal to use the fossil resource for some time, that is, $E(t) > 0$ for some $t > 0$, and a second case, in which the total initial stock of carbon in the whole system $S_0 = X_0 + \sum_i Z_{i,0}$ is already so large that it is never optimal to burn more of the fossil resource, that is, $E(t) = 0\,\forall t$. We begin with the second case, in which the non-negativity constrain on emissions binds and we have excessive initial carbon stocks.

2.2.1. Transition phase with excessive initial stocks. We first consider a transition phase in which no fossil resources are used because initial atmospheric carbon and storage stocks exceed a certain threshold. It is characterized by the following proposition.

PROPOSITION 2. *If the social cost of carbon from time zero onward exceeds the threshold value $-\tilde{\mu}$, that is, if $-\mu(t) > -\tilde{\mu}$ for all $t \geq 0$, then no more carbon will be emitted and carbon will only be shifted between storage and atmosphere:*

$$X(t) + \sum_i Z_i(t) = X_0 + \sum_i Z_{i,0} \; \forall t \tag{20}$$

Proof. If $-\mu > -\tilde{\mu}$, then $m > 0$ due to (19) and, following (13), we have $E = 0$. Then, looking at the equations of motion reveals $\dot{X} = -\sum_i \dot{Z}_i$. Since no fossil resources are burned, the only movement in the system now consists of carbon moving between the atmospheric stock and the storage stock, but their sum remains constant. □

2.2.2. Unconstrained transition phase with fossil resource use. Now, we consider a transition phase with positive fossil fuel use $E > 0$ and thus $m = 0$. Because of strictly convex costs of non-fossil energy and carbon removal, we obtain an interior solution with $E > 0$, $N > 0$ and $R_i > 0$ that is characterized by

$$f' = b' = g_i' - \psi_i = -\mu \tag{21}$$

In this transition phase, marginal costs and benefits have to be equal to achieve the optimum.

We can characterize the time-profile of key variables in the transition phase as follows:

COROLLARY 1. *When the SCC-E increase over time, renewable energy use N increases and fossil energy use E decreases.*

Proof. In all cases, we assume that $-\mu > 0$ is growing over time. (i) From the first order conditions, we have $b'(N) = -\mu$. Taking the total derivative with respect to $-\mu$ implies that $dN/d(-\mu) > 0$ as $b'' > 0$. (ii) Taking the total derivative of (6) w.r.t. $-\mu$, together with the result of (i) implies that $dE/d(-\mu) < 0$ as $f'' < 0$. □

2.3. Steady state

We consider now properties of the *steady state*, that is, the case of a stationary economy where state variables do not change over time.

DEFINITION 1 (Steady state). *The economy is in a steady state at time t if* $\dot{X}(s) = 0 = \dot{Z}_i(s)$, $\forall i, s \geq t$.

Let us denote constant state variables in the steady state with superscript s. As a first result, we characterize the social cost of carbon in the steady state.

COROLLARY 2. *In the steady state, the social cost of a carbon emission (SCC-E) and the social cost of a carbon removal (SCC-R) are constant and given by:*

$$SCC\text{-}E = -\mu^s = \frac{d'(X^s)}{r} \tag{22}$$

$$SCC\text{-}R_i = -\psi_i^s = \frac{d'(X^s)\delta_i}{r(r+\delta_i)} = -\mu^s \frac{\delta_i}{\delta_i + r} \tag{23}$$

Proof. See Appendix A.1.2. □

Corollary 2 states that the SCC-R are determined by the SCC-E, corrected by a multiplicative factor $\delta_i(\delta_i + r)^{-1}$. When release rates are zero, the correction factor becomes zero and the social cost for removals are zero. When release rates are much higher than the discount rate r, the correction factor approaches one and the social cost for emissions and removals are equal. Similarly, as the discount rate approaches zero, the social cost of carbon removal converge to the social cost of carbon emissions. It follows that carbon removal with temporary storage has only very limited social benefits when discount rates are low.

Constant state variables imply that removal rates, fossil and non-fossil energy use are constant as well. From the equations of motion of stocks it follows directly that emissions in the steady state have to fall to zero, that is, $E^s = 0$ and that removals exactly offset released emissions, that is, $R_i = \delta_i Z_i$. Hence, in the steady state, emissions from fossil energy use are zero but CDR has to offset the leaking emissions from past removals perpetually. This result has some parallel to Sisyphus' task of pushing a heavy stone to the top of a mountain just to watch it roll down and having to start all over again.

With zero fossil resource use, non-fossil energy use is characterized by:

PROPOSITION 3. *The steady state level of non-fossil energy* N^s *depends only on the structure of costs and benefits, that is, the functional form of* b *and* f*. It is independent of the discount rate* r*, the release rate* δ_i *and removal costs* g*.*

Proof. Follows directly from $E = 0$ and (7). □

As a direct consequence of (8) we obtain with Corollary 2 that removal flows are determined by marginal damages

$$g_i'(R_i^s) = g_i'(\delta_i Z_i) = \frac{d'(X^s)}{r+\delta_i} \tag{24}$$

A remarkable implication is that CDR should be used even when storage is non-permanent. We summarize this in the next proposition.

PROPOSITION 4. *Even with positive release rates, the socially optimal solution includes CDR. The relation between the steady state levels of carbon in the atmosphere X^s and in storage sites Z^s is determined entirely by marginal damages, marginal removal costs, the release rate and the discount rate. X^s and Z^s are positively correlated.*

Proof. Follows from (24). □

Intuitively speaking, equation 24 says that, in the steady state, the marginal costs caused by the stock of carbon in storage sites should equal the marginal costs caused by the atmospheric carbon stock. The nature of the costs are different. The costs caused by stored carbon Z_i result from the impermanence of the storage technology and the constant need to replace released emissions with costly removal technologies. The costs caused by atmospheric carbon X are due to climate damages. Both types of costs, however, are flows caused by stocks. Efficiency dictates that they should be equal at the margin.

The results we obtain from our analytical model hold for general convex functional forms for removal costs, non-fossil energy costs and climate damages. For illustrative purposes, it is also useful to examine a specific parametrization. Therefore, we also consider the case of quadratic functions.

COROLLARY 3. *If we consider quadratic approximations $d(X) = \frac{d_0}{2}X^2$, and $g_i(R_i) = \frac{g_{i,0}}{2}R_i^2$, it holds that*

$$\frac{X^s}{Z_i^s} = \delta_i(r+\delta_i)\frac{g_{i,0}}{d_0} \tag{25}$$

Proof. In the steady state, $R_i = \delta Z_i$, hence from (24), we obtain

$$g_{i,0}\delta Z_i^s = \frac{d_0 X^s}{r+\delta_i}$$

Rearranging yields the results. □

The result on how atmospheric carbon stock and storage stock are related can help us to determine the steady state levels, too. Determining the levels, however, requires to differentiate the case of excessive initial stocks along with no fossil resource use and the unconstrained case with positive fossil resource use.

2.3.1. Steady state with excessive initial stocks. In the case of very high initial atmospheric carbon and carbon storage stocks, that is, $SCC\text{-}E(0) > -\tilde{\mu}$, we know from Proposition 2 that

$$X^s + \sum_i Z_i^s = X_0 + \sum_i Z_{i,0}. \tag{26}$$

Combining this with (24) in general, or (25) under the assumption of quadratic functional forms, yields the steady state levels of the stocks. We can also make a statement about the stability of the steady state for the case of excessive intital stocks:

PROPOSITION 5. *In the case of excessive intital stocks, the system is saddle-path stable. The stable arm is given by* (24) *in general, or* (25) *under the assumption of quadratic functional forms.*

Proof. See Appendix, Section A.1.3. □

Graphically, cases of excessive initial stocks are given by points that lie strictly above and to the right of the dashed diagonal line in Figure 3. In the left panel, we illustrate the direction of the flow over time. The right panel is obtained from numerical simulations for quadratic functional forms. It also includes the unconstrained case with fossil resource use, which is represented by points that lie below and to the left of the dashed line.

2.3.2. Steady state with prior fossil resource use. If the initial state allows for some fossil resource use to be optimal, then the optimal time path of the SCC-E determines the level of atmospheric and storage carbon stocks. It holds that

$$\dot{X} + \sum_i \dot{Z}_i = E = f'^{-1}(SCC\text{-}E(t)) - b'^{-1}(SCC\text{-}E(t))$$

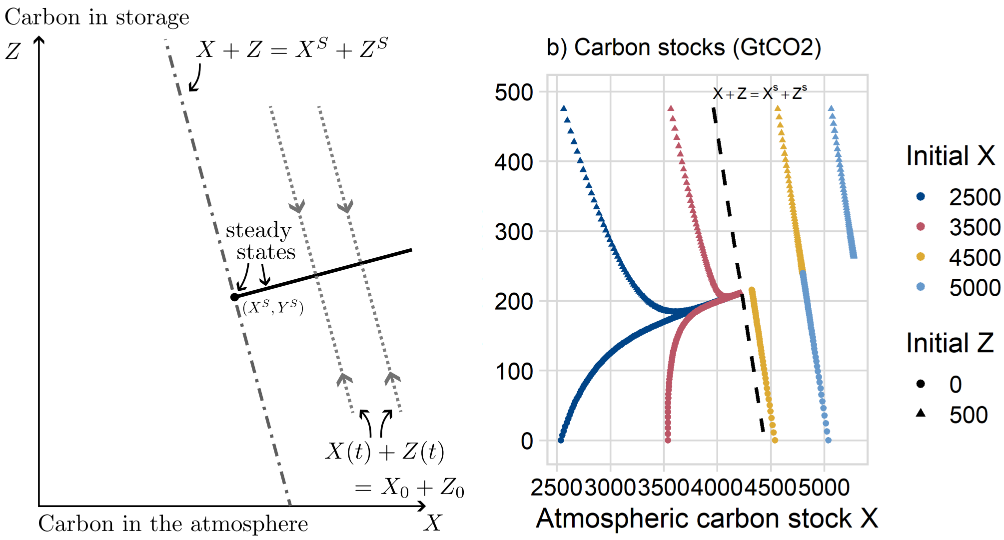

FIGURE 3. Steady states of the economy with one carbon removal technology.

From Corollary 2 we know that if the SCC-E are rising, then emissions fall. If emissions fall fast enough, the integral $\tilde{S} := \lim_{\tilde{t}\to\infty} X_0 + \sum_i Z_{i,0} + \int_0^{\tilde{t}} f'^{-1}(SCC\text{-}E(t)) - b'^{-1}(SCC\text{-}E(t))\mathrm{d}t$ converges. Then, combining $\tilde{S} = X^s + \sum_i Z_i^s$ with (24) or (25) determines the steady state levels X^s and Z_i^s.

We do not fully characterize the stability properties of the steady state when there is fossil resource use analytically due to the complexity of the system. For details, see Appendix A.1.3. However, in the right panel of Figure 3, we show results from numerical simulations with quadratic functional forms. The steady state to which the system converges if the initial atmospheric carbon and storage stocks are small enough is unique and independent of the starting point.

2.4. Comparative statics

In the following, we characterize how the steady state levels of the carbon stocks in the atmosphere and in the storage sites, as well as the removal rate depends on model parameters, that is, on the discount rate r, the release rate δ_i and on marginal removal costs g', marginal damages d' and marginal costs of fossil-free energy b'.

The comparative statics are quite different for the unconstrained case and the case of excessive initial stocks. We begin with the unconstrained case in which $m(t) = 0\ \forall t$.

2.4.1. Comparative statics for the unconstrained case. Recall that in the unconstrained case, initial stocks of atmospheric carbon and stored carbon are sufficiently low such that the economy exhibits an initial phase of non-zero fossil resource use.

PROPOSITION 6 (Comparative statics for the atmospheric carbon stock). *If $m(t) = 0$ for all t, the optimal atmospheric carbon stock in the steady state X^s*

- *increases when the discount rate r or marginal backstop costs increase,*
- *is independent of the release rates δ_i and marginal removal costs g_i'*
- *decreases if marginal damages d' increase.*

Proof. See Appendix A.1.4. □

The proposition confirms some well-established patterns regarding the usual structural parameters: optimal atmospheric carbon, and hence also temperatures, increase with the discount rate and the cost of non-fossil energy, and they decrease with climate damages.

With respect to carbon removal, however, the proposition emphasizes that the optimal temperature is independent from release rates and marginal removal costs. This finding is remarkable as it implies that availability of low-cost (or high-permanence) removal technologies should not lead to more ambitious climate policies. Instead, when removal technologies become more attractive, the optimal response is to increase carbon removals and storage stocks so that in the short run more fossil fuel can be combusted to generate economic benefits while keeping the long run temperature level constant.

COROLLARY 4. *The long-run optimal temperature level as well as the long-run social cost of carbon emission (SCC-E) are independent from costs and permanence properties of carbon removal technologies.*

Proof. The first part follows directly from Prop. 6. The second part (on SCC-E) follows from Corollary 2 and the first part. □

What is the economic intuition behind this result? To maximize welfare, the social planner has to find the right balance between marginal economic benefits of fossil-free energy use, their costs, marginal climate damages and marginal removal costs. That is, from (7), (22), (24) and $m = 0 = E$

we obtain $f'(N^s) = b'(N^s) = \frac{d'(X^s)}{r} = \frac{r+\delta_i}{r} g_i'(R_i^s)$. The derivatives f' and b' are independent of removal costs and release rates. Thus, changes in removal costs or release rates will not affect f' nor b' and thus, by equality, must also leave constant the other two expressions.

The availability of non-permanent CDR creates additional carbon sinks. While carbon in the atmosphere causes a flow of climate damages, carbon stored in the additional sinks does not. However, due to impermanence, the storage stocks of carbon cause a different typ of damage, namely the costs of removing released emissions. Following equation 24, efficiency dictates the equalization of marginal costs caused by both carbon sinks and, hence, the sinks' sizes. Over the course of the transition phase the sinks are filled. Thus, in the steady state, removal has no net effect. In particular, steady state removal does not allow to increase fossil resource use above zero. CDR is not a substitute for abating fossil resource use in the steady state. The optimal temperature level is therefore independent from availability and characteristics of CDR. Instead, the optimal temperature only depends on marginal benefits of energy use, the marginal costs of fossil free energy and marginal damages.

After characterizing the atmospheric carbon stock, we now turn to removal flows.

PROPOSITION 7 (Comparative statics for removal rates). *If $m(t) = 0\ \forall t$, optimal removal rates R_i in the steady state*

- *increase when the discount rate r or marginal costs of non-fossil energy b' increase,*
- *are independent of marginal damages d',*
- *decrease if the release rate δ_i or marginal removal costs g_i' increase.*

Proof. See Appendix A.1.4. □

The Proposition shows that removal flows increase in the discount rate and in the marginal costs of non-fossil energy b'. The intuition for the former is that since the costs of constantly removing carbon persist indefinitely, higher discounting effectively reduces removal costs. Also quite intuitively, higher marginal costs of fossil-free energy provide a comparative advantage for carbon removal compared to emission reduction. Similarly, higher marginal removal costs as well as higher release rates lead to lower removal rates as they increase the costs of effective carbon removals (compared to emission reduction).

Half time (years)	δ_i	Discount rate r 0.01	0.02	0.03	0.05	0.07
1	0.69	.	.	.	.	.
5	0.14	.	1.2	3.8	8.0	11.5
10	0.07	1.2	6.0	9.8	15.6	19.7
50	0.014	15.6	24.0	28.6	33.5	36.1
100	0.007	24.0	31.5	35.0	38.2	39.8
500	0.0014	38.2	41.1	42.1	42.9	43.3
1000	0.0007	41.1	42.6	43.2	43.6	43.8

TABLE 2. Steady state removal flows R_i in GtCO2, assuming SCC-E of 500\$/t and convex removal costs with marginal removal costs of 50\$/t for zero removal and 300\$/t for annual removal of 24.6 GtCO2.

Interestingly, removal rates are independent from the magnitude of marginal climate damages d'. If marginal damages were to increase, the optimal steady state carbon stock would decrease such that the left hand side of Equation (24) remains constant. Thus, the only advantage that the availability of carbon removal technologies offers is that it reduces overall mitigation costs as it allows us to emit more carbon than without its availability. Carbon removal does not affect the optimal level of global warming.

Using (24), we can calculate removal flows to illustrate the quantitative role of carbon removals in the long-run. As can be seen in Table (2), optimal steady state removals for half-lives above 50 years are in most cases above 50 percent of current carbon emissions from fossil fuels (43 GtCO2) and approach them for half-lives of 500 years or longer. Hence, despite storage being only temporary, removal flows are of similar order of magnitude as current carbon emissions.

Finally, we analyze how structural parameters affect carbon stocks held in managed storage sites.

COROLLARY 5 (Comparative statics for removal stocks). *If $m(t) = 0$ for all t, optimal removal stocks Z_i in the steady state*

- *increase when the discount rate r or marginal costs of non-fossil energy b' increase,*
- *are independent of marginal damages d',*
- *decrease if the release rate δ_i or marginal removal costs g_i' increase.*

Proof. See Appendix A.1.4. □

		Discount rate r				
Half time (years)	δ_i	0.01	0.02	0.03	0.05	0.07
1	0.69	.	.	.	.	.
5	0.14	.	9	27	57	82
10	0.07	18	86	140	223	281
50	0.014	1,113	1,716	2,045	2,394	2,577
100	0.007	3,432	4,506	4,996	5,463	5,687
500	0.0014	27,313	29,330	30,062	30,671	30,940
1000	0.0007	58,659	60,880	61,654	62,286	62,561

TABLE 3. Steady state carbon stocks Z_i in GtCO2, assuming SCC-E of 500$/t and convex removal costs with marginal removal costs of 50$/t for zero removal and 300$/t for annual removal of 24.6 GtCO2.

The Corollary shows that removal stocks are similarly affected as removal flows. Again, we can illustrate the implications for removal stocks numerically in Table (3).

For very low half-lives, no (positive) removal is used in the steady state. For longer half-lives such as fifty years – which is a typical order of magnitude for many land-based removal technologies related to forests or soil carbon – the steady state removal stock exceeds the current amount of CO_2 stored in global forest biomass (1,327 GtCO2, Mildrexler et al. 2020), at least when discount rates are larger than 1%. Similarly, for half-lives of 500 years and more the steady state removal stock would exceed the amount of carbon currently stored in global soils (9,167 GtCO2, Lal 2008). Thus, it seems likely that the steady state levels for high-permanence removal technologies in this scenario would not be reached due to physical constraints on storage capacity. While removed carbon stocks generally increase in the discount rate r, they become less sensitive to r when permanence is high. In general, optimal removal stocks are more sensitive to release rates than to discount rates.

2.4.2. Comparative statics for excessive initial stocks. When initial stocks of atmospheric and stored carbon are too high, the $SCC\text{-}E(0) > -\tilde{\mu}$, and the economy does not use fossil energy from the beginning on-wards. Rather, CDR is used to shift carbon from the atmosphere to non-permanent sinks, implying declining temperature levels (overshooting). While the results in this case stand in contrast to the unconstrained case, we discuss them here only very briefly due to their lower relevance.

LEMMA 2 (Comparative statics for $SCC\text{-}E(0) > -\tilde{\mu}$).

1. *Changes in the cost of non-fossil energy have no influence on the steady state levels of atmospheric carbon X^s, stored carbon Z_i^s and on removal R_i^s.*
2. *Changes in the discount rate r or the release rate δ_i give rise to changes as follows.*

$$\frac{\mathrm{d}X^s}{\mathrm{d}r} > 0 \qquad \frac{\mathrm{d}X^s}{\mathrm{d}\delta_i} > 0$$

$$\frac{\mathrm{d}Z_i^s}{\mathrm{d}r} < 0 \qquad \frac{\mathrm{d}Z_i^s}{\mathrm{d}\delta_i} < 0$$

$$sgn\left(\frac{\mathrm{d}R_i^s}{\mathrm{d}r}\right) = sgn\left(\delta g_i'' - (r+\delta_i)d''\right) \qquad sgn\left(\frac{\mathrm{d}R_i^s}{\mathrm{d}\delta_i}\right) = sgn\left(\frac{\delta_i^2 d' - (r+\delta_i)d''}{\delta_i(r+\delta_i)g'' - d''}\right)$$

3. *Increases in marginal removal costs g' lead to*
   - *increases in X^s*
   - *decreases in Z_i^s and R_i^s.*
4. *Increases in marginal damages d' lead to*
   - *decreases in X^s*
   - *increases in Z_i^s and R_i^s.*

Proof. To obtain 1., note that (24) is independent of b. We obtain 2. by using (24), $R_i = \delta_i Z_i$ and $X_0 + \sum_i Z_{i,0} = X^s + \sum_i X_i^s$ and taking partial derivatives with respect to r and δ_i. For changes in g' and d', we exploit the assumption that g and d are convex to obtain the results in 3. and 4. □

In case of excessive intitial stocks, the total amount of carbon in the economy $S = X + \sum_i Z_i$ remains constant over the entire time horizon and not only in the steady state. Emissions are always zero. Changes in structural parameters only affect how carbon is allocated between the different stocks. The direction of the change for atmospheric carbon X is always the opposite from carbon in storage sites Z_i. For the case of quadratic functional forms, changes in parameters can be visualized as rotations of the diagonal line of steady states in Figure 3.[6]

A higher discount rate or lower marginal damages lead to an increase in atmospheric carbon and to a reduction of carbon in storage stocks (a clockwise rotation of the line of steady states in Figure 3). In contrast to the unconstrained case with $E > 0$, changes in the cost of non-fossil energy

6. Note that changes in parameters may also affect the locus of the dashed diagonal line that separates the case of excessive initial stocks from the unconstrained case.

have no influence on the allocation of carbon because they cannot change fossil energy use, which always remains zero.

Most notably, when carbon storage becomes less attractive due to either increases in release rates or removal costs, the optimal atmospheric carbon stock – and thus, the optimal temperature – increases.

COROLLARY 6. *In steady states with excessive initial stocks, the long-run optimal temperature level as well as the long-run social cost of carbon emission (SCC-E) are not independent from costs and permanence properties of carbon removal technologies.*

3. Optimal carbon prices and removal subsidies

For the discussion of policy instruments, we focus on the production-side of the economy. Hence, we consider a representative firm that produces final goods $f(E+N)$ using carbon and non-carbon energy with the respective costs $p_E E$ and $b(N)$ and potentially engaging in carbon removal activities with costs $g_i(\cdot)$:

$$\pi = f(E+N) - p_E E - b(N) - \sum_i g_i(R_i)$$

As firms are owned by households that discount consumption flows at rate r, firms maximize net-present value of discounted profits at r as well. Contrary to the social planner model, households and firms ignore any external effects of carbon emissions and removal on climate change.

We consider three different policy regimes, where carbon prices are applied either *upstream* or *downstream* in the carbon management chain of emitting, removing and storing carbon, or as a subsidy on stock of stored carbon. First, under a downstream carbon price regime, a uniform carbon price is applied to E and to R_i and to the released emissions $\delta_i Z_i$. This, however, requires a perfect monitoring of released emissions. Second, we explore an upstream carbon price that does not cover released emissions. Instead, we allow the tax on the fossil emissions E and the subsidy on removal R_i to differ. Thus, the price incentive on storage R_i needs to consider the subsequent release of emissions. Third, we consider a subsidy on the stock of stored carbon instead of the flow of removal. Figure 4 shows an overview of the three pricing regimes.

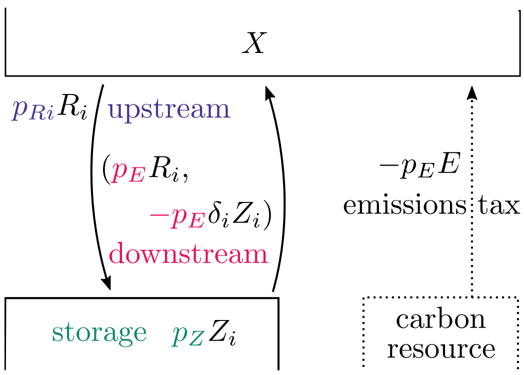

FIGURE 4. Overview of pricing instruments

3.1. Downstream carbon pricing

Under a downstream uniform carbon price, all emission fluxes are priced at the uniform carbon price p_E. Hence, firms maximize

$$\max_{E,R_i} \int_0^\infty \left[f(E+N) - p_E \left(E + \sum_i (\delta_i Z_i - R_i) \right) - \sum_i g_i(R_i) - b(N) \right] e^{-rt} \mathrm{d}t \tag{27}$$

The evolution of storage is a constraint to let firms anticipate future emission release:

$$\dot{Z}_i = R_i - \delta_i Z_i \tag{28}$$

PROPOSITION 8. *A uniform carbon price equaling the socially optimal social cost of carbon emissions, $p_E = -\mu^* = SCC\text{-}E > 0$, to all carbon fluxes in the economy constitutes an optimal policy.*

Proof. The result follows from by comparison of the first order conditions of the social planner problem (1-4) with those of the firm-owning household (27-28). See A.2.1 for details. □

Thus a comprehensive uniform carbon price (downstream carbon pricing) set to the level of the social cost of carbon emissions constitutes an optimal policy. Importantly, differentiation of removal technologies is not necessary as every ton of carbon removed is rewarded equally.

3.2. Upstream carbon pricing

Under the upstream carbon price regime, released emissions are not covered by a carbon price. Rather, firms pay a carbon price p_E on regular emissions E and receive a technology-specific subsidy $p_{R,i}$ on the removal of carbon. Hence in every period firms solve

$$\max_{E,R_i} f(E+N) - p_E E + \sum_i p_{R,i} R_i - \sum_i g_i(R_i) - b(N) \tag{29}$$

An optimal carbon pricing system is then characterized as follows.

PROPOSITION 9. *A carbon price $p_E > 0$ on emissions equal to the optimal social cost of carbon emissions $p_E = -\mu^* > 0$ and technology-specific removal subsidies $p_{R,i}$, equal to the optimal social cost of carbon emissions net of the social cost of carbon removal, $p_{R,i} = -\mu^* + \psi_i^* =$ SCC-E $-$ SCC-R, achieve an optimal allocation, without the necessity to price released emissions.*

Proof. See A.2.2. □

Proposition 9 gives a clear interpretation of optimal policies. Emitting carbon into the atmosphere is priced at the social cost of carbon while firms that remove carbon receive a subsidy that does not only take into account avoided damages through the SCC-E, but also induced damages due to emissions that are released from the carbon stocks at a later stage, valued by the SCC-R.

To facilitate the interpretation of the removal subsidy $p_{R,i}$, we define $\lambda_i := \frac{p_{R,i}}{p_E}$ as the share of the removal subsidy relative to the price on the emission of carbon into the atmosphere. We can thus interpret λ_i as an optimal 'discount factor' to the carbon price that must be applied in order to incentivize the optimal use of removal technology i. We can now characterize the discount factor as follows:

PROPOSITION 10. *(i) In general, the optimal discount factor for pricing carbon removal is $\lambda_i = 1 - \frac{SCC\text{-}R_i}{SCC\text{-}E}$. (ii) In the steady state, the discount factor is $\lambda_i^s = \frac{r}{r+\delta_i}$.*

Proof. (i) This follows directly from Proposition 9. In the steady state, we substitute μ^* and ψ^* from Corollary 2 into λ_i which gives the result. □

Intuitively, when $\delta_i = 0$ the full carbon price for removal is paid. When $\delta_i \approx r$, only half of the carbon price is paid to CDR. When $\delta_i = 1$, the CDR price is two orders of magnitude smaller than

Half time (years)	δ_i	Discount rate r 0.01	0.02	0.03	0.05	0.07
1	0.69	0.01	0.03	0.04	0.07	0.09
5	0.14	0.07	0.13	0.18	0.27	0.34
10	0.07	0.13	0.22	0.30	0.42	0.50
50	0.014	0.42	0.59	0.68	0.78	0.83
100	0.007	0.59	0.74	0.81	0.88	0.91
500	0.0014	0.88	0.94	0.96	0.97	0.98
1000	0.0007	0.94	0.97	0.98	0.99	0.99

TABLE 4. CDR subsidy as fraction of the carbon price in the steady state, λ_i^s.

the SCC-E for typical values of the discount rate r. Table 4 gives an overview over steady state discount factor for various combinations of r and δ_i (or, the respective half-lives). For example, removal technologies that store carbon with a half-life of 50 years, should be compensated with 42–83 percent of the carbon price. If storage is just 5 years, compensation should only be 7 percent (with low discount rates) up to 34 percent (with high discount rates). We explore the welfare implications of using λ_i^S to approximate $\lambda_i(t)$ as part of the numerical second-best analysis (Section 5.5).

3.3. Removal stock subsidy

Rather than subsidizing removal flows, the government subsidizes the removed stock Z_i. Firms would have to prove the size of the retained carbon in every period to obtain the subsidy p_Z:

$$\max_{E,R_i} \int_0^\infty \left[f(E+N) - \sum_i g_i(R_i) - p_E E - b(N) + \sum_i p_Z Z_i \right] e^{-rt} \mathrm{d}t \tag{30}$$

subject to (28). An optimal carbon pricing system is then characterized as follows.

PROPOSITION 11. *A carbon price $p_E > 0$ on emissions equal to the optimal social cost of carbon emissions $p_E = -\mu^* > 0$ and a technology-neutral subsidy p_Z on stored carbon Z_i, equal to the marginal damages $d'(X)$ achieve an optimal allocation, without a necessity to price released emissions.*

Proof. See A.2.3 □

3.4. Informational aspects of policy instruments

While upstream, downstream and removal stock carbon pricing all ensure a socially optimal outcome, they differ in their informational requirements (cf. Table 5 for an overview of the differences).

Downstream pricing. Under a downstream system, all carbon fluxes, including released emissions ($\delta_i Z_i$), need to be known in order to tax them. To know released emissions, the regulator either needs information on the storage specific release rates δ_i and the stocks Z_i – the latter by book-keeping of flows R_i or auditing Z_i. Alternatively, the regulator can monitor the release flow ($\delta_i Z_i$) directly. As monitoring of emission fluxes is associated with transaction costs, such an approach might involve high administrative costs.[7]

Furthermore, in the downstream pricing system, prices are applied to carbon flows *at that time*, that is, there are no upfront payments for future services such as maintenance of the sink. This matters, for example, when the release rate becomes a function of diligence. We explore this extension in Section 4.3. When low diligence, and thus a higher release rate, is cost-saving, pricing released emissions directly maintains the optimal incentives for storage management. Downstream pricing does, however, depend on the continued operation of the firm. If the firm goes out of business, for example in case of bankruptcy, diligence would drop to zero. The release rate would be maximal but no tax on released emissions could be levied from the insolvent firm. This problem of the firm being "judgment-proof", that is, being limited in its liability for release emissions is well-known (Shavell 1986; Shogren et al. 1993). Bonds posted by the firm as a collateral to be claimed by the regulator solve this problem in principle but come with their own transaction costs and constrain the firm liquidity (Gerard and Wilson 2009; White et al. 2012) unless backed up by further financial derivatives. Held and Edenhofer (2009), for example, suggest that storage of captured carbon should be ensured by demanding that storage operators must hold state-issued bonds ("CCS bonds"), which will earn an interest if continued storage is proven, thus turning bonds into a tradable asset. A similar judgement-proof problem arises for the stock tax (on carbon in the atmosphere) in Lemoine (2020) who also suggest that emitters (instead of paying a per-period charge on the stock) post a bond. Liquidity is created by providing tradable "carbon shares" in exchange for the bond, with a face value initially equal to the bond but reduced each period to

7. For example, for the wood products in Table 1 the life-cycle of houses, furniture and paper would need to be tracked.

TABLE 5. Pricing schemes: overview of differences

Informational requirements		
	pricing	quantities
Upstream	$p_{Ri} = \lambda_i p_E$	R_i
Downstream	—	$\delta_i Z_i$, R_i
Stock subsidy	$p_Z = d'(X)$	Z_i
Firm's financial flows		
	near-term	steady state
Upstream	profit	profit
Downstream	profit	deficit
Stock subsidy	deficit	profit
Government's payments		
	near-term	steady state
Upstream	$p_{Ri}R_i$	$p_{Ri}R_i^S$
Downstream	$p_E(R_i - \delta_i Z_i)$	0
Stock subsidy	$p_Z Z_i$	$p_Z Z_i^S > p_{Ri}R_i^S$

account for damages caused. The face value is paid back upon removal of the associated emissions from the atmosphere.[8]

Upstream pricing. By contrast, an upstream system requires the regulator to monitor only the removal quantity and to assess the release rate of a specific technology. Monitoring only the removal quantity might be cheaper than monitoring the release quantity and thereby lower the administrative costs.[9] If in addition release rates can be narrowed down considerably and, hence, uncertainty about release rates is not large for a specific removal technology, this could constitute a cost advantage for an upstream system.

The subsidy in upstream pricing is paid upfront and in anticipation of future release at the rate δ_i. Again, if realizing this release rate requires diligence, for example through costly operation and maintenance, firms have an incentive for minimum diligence unless they are committed to the

8. Beyond the judgement-proof problem, the full proposal in Lemoine 2020 also takes uncertainty about climate change damages into account: the initial face value is set to cover worst case climate change damages. Then, as climate change damage projections are revised towards lower values, the face value is reduced accordingly and the difference paid out as a dividend. To address the judegement-proof problem, this is not essential.

9. The reason for this cost asymmetry is twofold: (1) removal quantities are often spatially concentrated while later emission releases from stored carbon can be very dispersed, (2) removal can take place at a short period of time while emission release from storage might occur in small rates over large time horizons. Both aspects imply large monitoring costs relative to the size of the emission flows.

expected diligence. Similar to the downstream pricing discussed above, even when contracts can hold the firms accountable for the expected diligence, bankruptcy could limit firm liability, and its mere possibility will undermine the incentive for removal.

Stock subsidy. Finally, the removal stock subsidy only requires the regulator to observe the *contemporaneous* marginal climate damages $d'(X)$, which are a subset of the social cost of carbon that need to be estimated anyway for the emissions tax p_E. In addition, the regulator needs to observe the amount of carbon stored in a particular reservoir at any point in time. Notably, the regulator does not need to observe any carbon fluxes. Therefore, if monitoring stocks is cheaper than monitoring flows, the removal stock subsidy has the lowest informational requirement of the policy instruments considered.

The stock subsidy also has a favorable incentive structure. Since the subsidy is always paid for the stock at that time, there is no commitment problem, and neither can firms benefit from lack of diligence or strategic bankruptcy.

3.5. Financial flows

Another distinction of the different pricing schemes are the financial flows of the government and the removal firms. In particular, financial flows in the steady state, which are to be paid or received forever on, merit consideration as they are relevant for the insolvency risk of the firm. An overview of the financial flows is included in Table 5.

Upstream pricing. In the upstream pricing system, firms receive a revenue of $p_{Ri}R_i$ and do not need to pay for released emissions. Since R_i is produced with increasing marginal costs (remember g is convex), the firm earns a producer surplus – a profit if we assume zero fixed costs. This is true in every period such that storage is a continuous stream of income for the firm, paid by the government, that is, the tax payer via the subsidy on removal.

Downstream pricing. Under downstream pricing, firms receive a subsidy of p_E for the difference of removal and release emissions $(R_i - \delta_i Z_i)$. We know that these subsidy revenues vanish in the steady state, leaving the firm without income but removal costs of $g_i(R_i^S)$. In the near-term and for small $Z_i(0)$,[10] we have $R_i > \delta_i Z_i$ and hence the balance of subsidy on removal versus tax on release emissions is positive. In the aggregate, the net present value of profits can only be positive when the profits from operating removal and storage are maximized (since $R_t \equiv 0$ provides a lower bound of zero that the firm can improve upon). Hence, if the firm can move its early profits forward in time without loss of net present value, for example if it can invest profits and earn a return equal or greater than r, then all future removal costs can be financed from early profits.[11] Nevertheless, in a downstream pricing system, storage becomes a financial burden for the firm in the long-term.

Removal stock subsidy. When the regulator applies the removal stock subsidy, the firm earns $p_Z Z_i$. Initially removal costs will exceed the storage subsidy since $Z_i(0)$ is close to zero (unless the subsidy $p_Z = d'(X)$ is very large). In the long-run steady state however, the subsidy income will exceed removal costs:[12]

$$
\begin{aligned}
p_Z Z_i^S - g_i(R_i^S) &= d'(X^S) Z_i^S - g_i(R_i^S) \;= d'(X^S)\frac{R_i^S}{\delta_i} - g_i(R_i^S) \\
&= (r+\delta_i) g_i'(R_i^s)\frac{R_i^S}{\delta_i} - g_i(R_i^S) \;= \underbrace{\frac{(r+\delta_i)}{\delta_i}}_{>1} \underbrace{g_i'(R_i^s) R_i^S}_{>g_i(R_i^S)} - g_i(R_i^S) > 0
\end{aligned}
$$

In the aggregate the net present value of optimal removal can only be positive (as in the downstream pricing case). Thus if the firm can borrow at an interest rate less than or equal to r it can cover the early deficit from future profits.

In summary, firms may be hesitant to commit to removal under the removal stock subsidy, as they will initially run a deficit. Firms need to be patient, as a financial break-even point might be far in the future.

---

10. This seems to be the realistic case as there is not large scale CDR to date. In contrast, when $Z_i(0)$ is large such that $R_i < \delta_i Z_i$ the storage branch will run a deficit at all times. R_i will need to increase towards $\delta_i Z_i$ and zero revenues eventually – it is hard to imagine how positive revenues could be achieved along this transition.

11. A similar approach is taken in Germany's Nuclear Waste Disposal Fund which manages a share of the profits of nuclear power companies to finance long-term storage.

12. We know $p_Z = d'(X)$, and that in a steady state, $R_i = \delta_i Z_i$. Furthermore from Proposition 3 we have $g_i'(R_i^s) = \frac{d'(X^s)}{r+\delta_i}$ and hence $(r+\delta_i) g_i'(R_i^s) = d'(X^s)$.

The government. The financial flows of the government mirror the financial flows of the firm (except for removal costs). Table 5 summarizes the payments made by the government to the firms. For the government, downstream pricing has the advantage that no payments need to be made in the steady state, in contrast to upstream pricing and the removal stock subsidy. Of the latter two, the stock subsidy requires higher subsidy payments in the steady state.

In addition, the financial flows of the firm in the downstream pricing and the removal stock subsidy schemes may concern the government. For the downstream pricing scheme, financing the perpetual operation of the removal technology is challenging. Gerard and Wilson (2009) acknowledge that this kind of long-term responsibility may deter private actors and thus suggest, for the case of long-term maintenance of closed carbon storage sites, that the state ultimately assumes responsibility. A similar transfer of long-term responsibility to the public may be needed to incentivize private removal and storage in the downstream pricing scheme. In the case of the stock subsidy, capital markets may be unwilling to offer financing until break-even if they perceive the lengthy start-up time as too risky. In this case, financial constraints will reduce the ability of private firms to remove and store despite the attractive subsidy payments in the long-term. To overcome this barrier, governments could step in by de-risking capital market finance, for example, by providing guarantees, or complementing finance with special loan programs.

3.6. Upstream pricing with uniform carbon prices

So far, we have discussed *optimal* pricing regimes. In particular, the optimal upstream carbon pricing outlined in Proposition 9, which distinguishes removal technologies by release rates.

In practice, however, governments might disregard the impermanence aspect of different removal technologies. In an extreme case, a regulator may even ignore the social cost of carbon removal completely and grant the full carbon price to all removal technologies. This would, for example, also occur in an emissions trading scheme (ETS) where each ton removed would create an additional certificate that can be sold in the carbon market.[13]

13. To correctly account for impermanence of storage, the discount factor λ_i would also need to be applied here (or emission releases from storage should be covered under the ETS). Hence, each ton of carbon removed generates only $\lambda_i < 1$ carbon permits that can be sold in the carbon market.

Such a policy would be akin to an upstream carbon pricing system where all removal technologies receive the same carbon price as the tax that needs to be paid in the mitigation sector: $p_{R_i} = p_E$. When the carbon price is set to the SCC-E in the optimum, $p_{R_i} = p_E = -\mu^*$, removal rates always exceed the socially optimal removal rates. The latter are determined by $g_i'(R_i) = -\mu^* + \psi_i$ while removal rates under uniform pricing are determined by $g_i'(R_i) = -\mu^*$ with $\mu, \psi_i < 0$.

4. Model extensions

The basic model can be extended in various ways. In this section we discuss some relevant cases and how they would alter the results.

4.1. Permanent storage

With permanent storage, $\delta_i = 0$ and, thus, $\psi_i = 0$. Hence, the optimal price for emitting a ton of carbon is the same as the optimal subsidy for removing a ton, in both an upstream and a downstream pricing regime. Removal will be according to $g_i'(R_i) = -\mu$. In the steady state, energy use has to equal removal rates, $E = \sum_i R_i$, and optimal quantities are therefore

$$f'(\sum_i R_i + N) = b'(N) = g_i'(R_i)$$

Here, the balance of the marginal costs of non-fossil energy (b') and carbon removal (g_i') determines the relative shares of removal and non-fossil energy, whereas the marginal productivity of total energy (f') determines the overall quantity of energy.When CDR with permanent storage is used in the steady state, stored carbon grows to infinity. Hence, biophysical limits to storage (or availability of fossil carbon) would ultimately determine the long-run outcome of the economy. If biophysical limits to storage are binding, CDR will again only affect transition costs but not the long-run steady state in the unconstrained case in which some non-zero fossil resource use is optimal. While basically the same model dynamics apply, permanent storage does not require perpetual removal activities to address leaking emissions. It therefore does not impose perpetual removal costs to future generations (or firms). Also, the insights on informational requirements of different policy approaches apply: Permanent and non-permanent removals need to be distinguished and permanence may also depend on diligence.

4.2. Further environmental externalities

When a specific removal technology causes further positive or negative environmental externalities $h_i(R_i)$, this should be accounted for in the cost function, such that it becomes $\tilde{g}_i(R_i) = g_i(R_i) + h_i(R_i)$. While the social planner considers social costs $\tilde{g}_i(\cdot)$ of a removal technology, firms in a decentralized economy would only consider private costs $g_i(R_i)$. An additional Pigouvian tax $\pi_i = h_i'(R_i)$ would therefore be necessary to achieve the social optimum. The tax would be positive for a negative externality, that is,$h_i'(\cdot) > 0$ and negative – that is, a subsidy – for a positive externality, where $h_i'(\cdot) < 0$.

The consideration of additional external effects seems to be highly relevant for some removal technologies. For example, various technologies and management practices to increase soil carbon capture can increase soil fertility and biodiversity (Lal 2004), while afforestation can also have impacts on biodiversity, local climate and soil erosion (Fuss et al. 2018). Bioenergy with CCS (BECCS) – widely deployed in integrated assessment models – may increase demand for land (Lapola et al. 2010; Gawel and Ludwig 2011), increase the application of fertilizers and pesticides and reduce biodiversity of agricultural areas (Creutzig 2016).

4.3. Diligence and endogenous release rates

Rather than assuming a fixed release rate per technology δ_i, we could introduce diligence S_i that reduces release rates, $\delta_i'(S_i) < 0$ but causes additional costs $k_i(S_i)$ that are again assumed to be convex, $k_i'(S_i), k_i''(S_i) > 0$. From a social planner perspective, this would introduce an additional optimality condition that determines the optimal level of diligence S_i^*:

$$k_i'(S_i^*) = -(\psi_i^* - \mu^*)\delta_i'(S_i^*)Z_i = -g_i'(R_i)\delta_i'(S_i^*)Z_i \tag{31}$$

Hence, the optimal diligence level is determined by equating marginal costs of diligence $k_i'(S_i)$ with marginal benefits $-(\psi_i - \mu)\delta_i'(S_i)Z_i$ due to a marginal reduction of release rates $\delta_i'(S_i)$.

From a public policy perspective, a comprehensive downstream carbon pricing scheme as well as a removal stock subsidy would give sufficient incentives for employing an optimal amount of diligence. Under an upstream carbon pricing approach, diligence needs to be targeted by an additional policy instrument, for example, an additional subsidy on diligence. While this might be theoretically straight-forward, a subsidy requires that diligence levels are observable. This, however, is informationally demanding. If diligence is not observable and, thus, cannot be targeted with an instrument, private incentives vanish. Then, the level of diligence in a decentralized economy is determined by

$$k_i'(S_i^{\#}) = 0 \qquad \text{with } S_i^{\#} < S_i^* \tag{32}$$

The impossibility to target diligence has two major policy implications. First, as diligence is lower, release rates will be higher than optimal and a regulator should calculate the discount factor on the removal subsidy λ_i based on minimal diligence levels $S_i^{\#}$ and maximum release rates $\delta_i(S_i^{\#})$. Second, because release rates are higher than in the social optimum, less carbon removal should be used. While this does not affect the long-run optimal temperature level (see Proposition 6), it implies higher near-term costs of climate policy due to higher mitigation efforts and higher carbon prices along the transition.

5. Numerical analysis

Based on the analytical framework developed in Section 2, we use GAMS to analyze a simple numerical model of a production economy with fossil energy input E, non-fossil energy input N, carbon removals R and climate damages $d(X)$. We formulate a similar social planner problem as in Section 2.1 and solve it for a time horizon of $T = 600$ years.

$$\max_{E,N,R} \sum_{t=1}^{T} [f(E+N) - g(R) - b(N) - d(X)]\, \beta^{t-1} \tag{33}$$

subject to

$$X_t = X_{t-1} + E_t - R_t + \delta Z_t \tag{34}$$

$$Z_t = Z_{t-1} + R_t - \delta Z_t \tag{35}$$

$$E,N,R, \geq 0 \tag{36}$$

where $\beta = \frac{1}{1+r}$.

Description	Symbol	Value	Unit	Source
Initial values				
Carbon stock	X_0	2400	GtCO2	Canadell et al. (2021)
Removal stock	Z_0	0	GtCO2	calibrated
Global GDP	f_0	80	tril$	World Bank (2022)
Population size	L_0	7.90	billion	World Bank (2022)
Labor productivity	a_0	1.00		calibrated
Model parameters				
Discount rate	r	3.50	percent	Gollier (2021)
Annual release rate	δ	5.00	percent	calibrated
Income share of energy	$1-\alpha$	0.05		Melek and Orak (2021)
Transient climate response	$\tilde{\alpha}$	0.0004495	°C/GtCO2	Canadell et al. (2021)
Damage parameter	$\tilde{\beta}$	0.08		calibrated
Non-fossil energy cost (linear term)	b_1	0	tril$/GtCO2	calibrated
Non-fossil energy cost (quadratic term)	b_2	0.003	tril$/GtCO2	calibrated
Removal cost (linear term)	g_1	0	tril$/GtCO2	calibrated
Removal cost (quadratic term)	g_2	0.003	tril$/GtCO2	calibrated

TABLE 6. Model calibration

We assume no growth in the economy and impose a terminal condition that ensures the finiteness of the atmospheric carbon stock in the last period. We include only one single generic CDR technology for tractability. This is without loss of generality since there are no relevant interactions between different technologies with impermanent storage.

5.1. Functional forms and calibration

The model is calibrated to match current values for global population, economic output and fossil fuel emissions. An overview of initial values and parameter value choices is found in Table 6. For the results of an alternative calibration $(r = 1.5\%, \tilde{\beta} = 0.03)$ refer to Appendix A.3.1.

The production technology $f(\cdot)$ is given by a Cobb-Douglas function that takes effective labor (labor L times labor productivity a), fossil energy (E) and non-fossil energy (N) as inputs.

$$f(a_t L_t, E_t + N_t) = f_0 \left(\frac{a_t L_t}{a_0 L_0} \right)^{\alpha} \left(\frac{E_t + N_t}{E_0} \right)^{1-\alpha} \tag{37}$$

The elasticity of substitution α is calibrated to match an income share of energy of $1 - \alpha = 0.05$ in line with the estimates in Melek and Orak (2021). As we do not allow for economic growth the production function simplifies to

$$f(E_t + N_t) = f_0 \left(\frac{E_t + N_t}{E_0} \right)^{1-\alpha} \tag{38}$$

Removal cost are given by the convex function

$$g(R) = g_1 R + g_2 R^2 \tag{39}$$

with linear marginal removal cost of $g'(R) = g_1 + 2g_2 R$. We set $g_1 = 0$ and $g_2 = 0.003$, assuming that marginal removal cost range from 0$/tCO2 to 150$/tCO2 when a hypothetical global removal potential of 24.6 GtCO2 per year is reached.[14]

The cost of non-fossil energy are given by the convex function

$$b(N) = b_1 N + b_2 N^2 \tag{40}$$

with linear marginal cost of $b'(N) = b_1 + 2b_2 N$. By setting $b_1 = 0$ and $b_2 = 0.003$, we assume that marginal cost range from 0$/t to 250$/t when the quantity of 40 Gt per year is reached.[15]

We model the global mean temperature as a linear function of the atmospheric carbon stock

$$T(X) = \tilde{\alpha} X \tag{41}$$

and set $\tilde{\alpha} = 0.0004495°C/\text{GtCO2}$ as the transient climate response to cumulative emissions of carbon dioxide (Canadell et al. 2021).

Finally, we assume a convex climate damage function where damages increase quadratically in global mean temperatures.

$$D(X) = \frac{\widetilde{\beta}}{2} T(X)^2 f_0 \tag{42}$$

We calibrate the damage parameter $\widetilde{\beta}$ such that the optimal temperature in our numerical model lies between 1.5 and 2°C. In particular, by setting $\widetilde{\beta} = 0.08$ we assume that marginal damages are equal to 8% of global GDP for a $1°C$ temperature increase.

14. Assumptions for the removal cost and potential are based on Fuss et al. (2018). Estimates of removal cost and potentials remain highly uncertain and hence the calibration serves illustrative purposes only.

15. This calibration is similar to the marginal abatement cost in various versions of the DICE model (see Grubb et al. 2021).

5.2. *Illustrative model pathway*

Figure 5 summarizes the results from an illustrative model run with a release rate of 5%. Panel (a) shows the socially optimal pathways of fossil energy use, non-fossil energy use and removals in the economy. We observe two distinct phases in the model output.

The first phase is the transition phase. During this phase, fossil energy use E is positive and declining over time. The use of non-fossil energy N starts at a high level and increases over time. Similarly, the use of carbon dioxide removal R is positive and increases over time despite the non-permanence of storage.

In the second phase, the economy has converged to the steady state.[16] In the steady state, non-fossil energy N completely substitutes fossil energy in the production of global output. Since we do not assume a growing economy, the use of energy is constant. Importantly, the steady state removal rate equals exactly the amount of released emissions. Since the steady state flow of carbon emissions is zero, this equality implies constant atmospheric carbon and removal stocks. Consequently, the global mean temperature increases during the transition phase and stabilizes at about 1.8°C above pre-industrial levels (Panel (b)).

Panel (c) shows the socially optimal carbon prices for emissions and removals implied by the shadow prices of the atmospheric carbon stock X and the removal stock Z. Both the carbon price for emissions and the removal subsidy increase along the transition and remain constant in the steady state. Note that during the transition, the removal subsidy grows at a higher rate than the carbon price. Due to the non-permanence of storage, however, the subsidy is only a fraction of the carbon price. Therefore, the ratio of subsidy to carbon price increases over time until it reaches the optimal steady state discount factor given in Proposition 10.

5.3. *Comparative statics for the release rate*

Figure 6 summarizes the results of model runs with different release rate assumptions. There are four key insights.

First, panel (a) illustrates that carbon removal with low permanence is used less than carbon removal with high permanence. This confirms the analytical result in Proposition 7, which holds

16. Convergence to the steady state is only asymptotic, that is, the steady state is not reached in finite time. We say that the model has converged to the steady state when the change in stock variables is negligible.

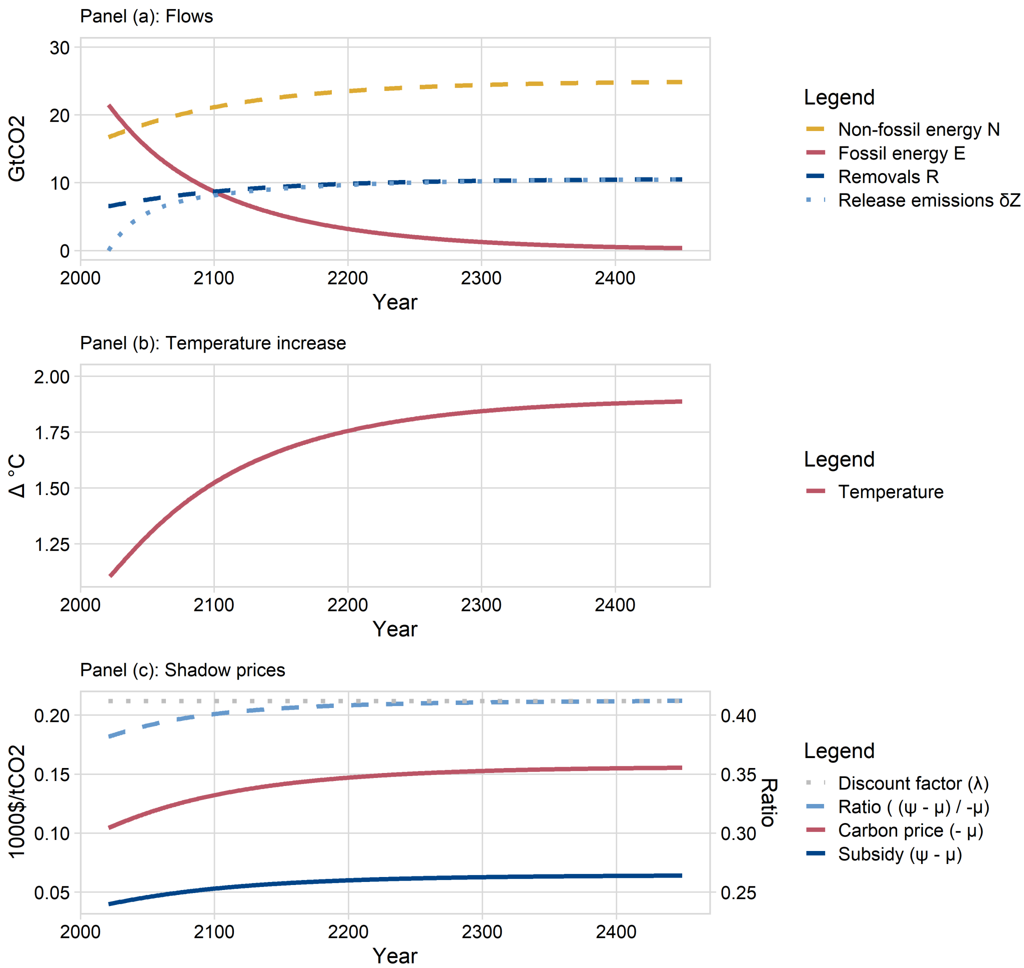

FIGURE 5. Illustrative model pathways for a release rate of $\delta = 5\%$ and a discount rate of $r = 3.5\%$.

that the optimal removal quantity decreases in the release rate. In addition, the figure suggests that released emissions reach their steady state level sooner if release rates are high. This observation implies that low-permanence carbon removal provides net-removals for a shorter period than carbon removal with high permanence. Thus, removal options with low permanence, such as storage in the biomass, exhaust their cost-saving potential sooner than high-permanence options, as, for example, storage in the geosphere.

Second, panel (b) shows that the optimal use of fossil energy decreases in the release rate. As a consequence, cumulative emissions along the transition are higher in scenarios with high storage permanence.

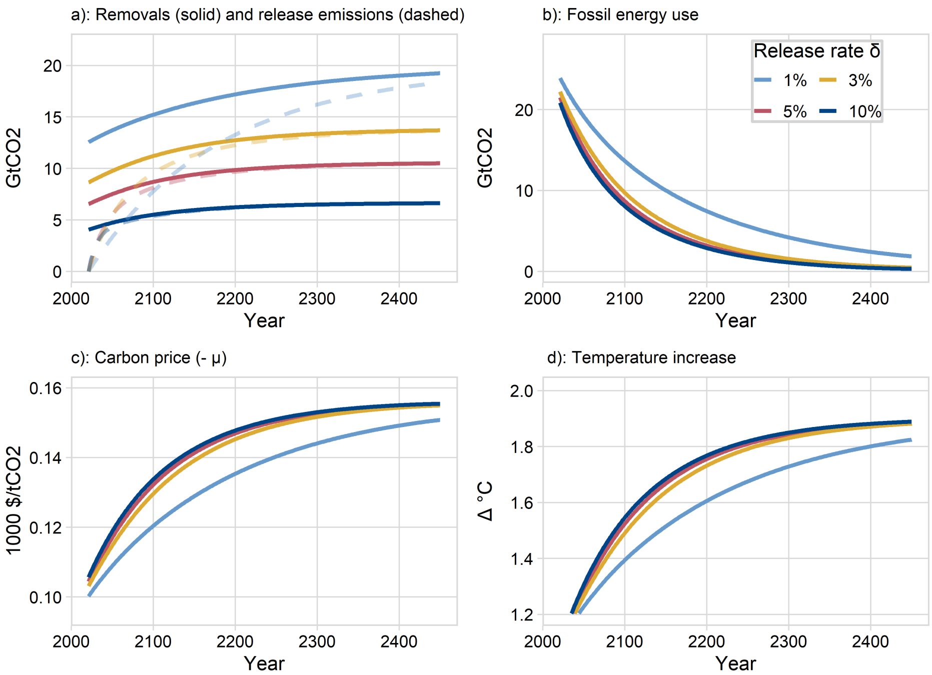

FIGURE 6. Comparative statics for removals, fossil energy use, carbon prices and temperature increase with respect to the release rate δ ($r = 3.5\%$).

Third, the social cost of carbon along the transition decrease in the permanence of carbon removal (panel (c)). This explains the so-called "mitigation deterrence" observed in panel (b): high-permanence carbon removal is used to remove larger quantities of carbon from the atmosphere, which in turn decreases the social cost of carbon and increases the use of fossil energy. In our model, "mitigation deterrence" is optimal because the social planner has perfect foresight and there is no uncertainty about the cost and availability of carbon removal options.

Fourth, panel (d) shows that the permanence of carbon removal has no impact on the optimal temperature level in the long-run. This illustrates the analytical result from Proposition 6. In addition, the figure shows that the speed at which the temperature converges to its long-run level depends on the release rate. In consequence, high-permanence carbon removal reduces the cumulative damage from climate change compared to low-permanence carbon removal.

In sum, Figure 6 illustrates the role of carbon removal with impermanent storage for climate policy. While CDR does not affect long-run temperature targets, it lowers the cost of the transition as indicated by the lower carbon price paths. In addition, using carbon removal slows the transition to the long-run temperature level, thereby reducing the cumulative damage from climate change.

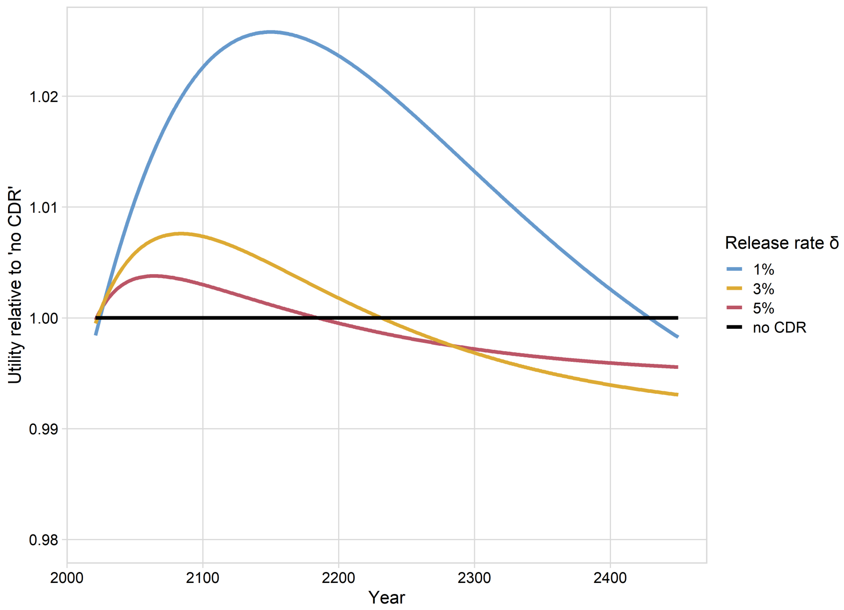

FIGURE 7. Utility over time relative to a scenario without CDR ($\delta = 5\%, r = 3.5\%$).

As a result, using carbon removal despite non-permanent storage is welfare-enhancing. However, Figure 7 shows that the welfare gains from the use of carbon removal technologies are unevenly distributed across generations. While generations in early periods benefit from the option to remove carbon from the atmosphere, later generations lose welfare relative to a scenario without carbon removal. Interestingly, both the benefits in early periods and the losses in later periods increase with the permanence of storage. This observation is explained by the positive relationship between permanence and removal quantities (see panel (a) of Figure 6).

5.4. Long-run temperature targets and overshooting

Figure 8 illustrates the comparative statics for the long-run temperature target and the removal stock with respect to the initial atmospheric carbon concentration X_0. A crucial distinction for the comparative statics is whether it is optimal to use fossil energy in the first period (Lemma 1).

At a low initial atmospheric carbon stock, the SCC-E are low such that the non-negativity constraint on E is not binding and it is still optimal to use some fossil energy. This case is represented by the solid lines in Figure 8. In line with Corollary 4, we observe that for low levels of the initial atmospheric carbon stock, the economy converges to the same long-run temperature level (panel a). Similarly, panel (b) shows that in these scenarios the initial carbon stock X has no

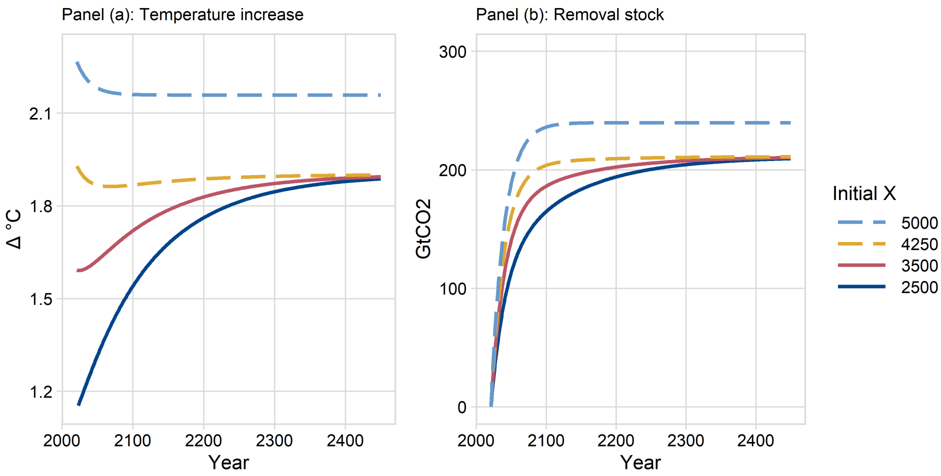

FIGURE 8. Temperature increase and removal stocks in scenarios with and without overshooting ($\delta = 5\%, r = 3.5\%$).

influence on the long-run removal stock. However, the closer the economy starts to the optimal long-run temperature level the faster carbon removal needs to be scaled up.

If, by contrast, the initial atmospheric carbon stock is high, we can observe cases in which the global temperature temporarily overshoots its long-run optimum. The upper one of the two dashed line in Figure 8 is an example of the nonnegativity constraint on emissions (4) being binding. In that case, it is never optimal to use fossil energy. In panel (a), we see that the temperature decreases and stabilizes at a carbon concentration that is lower than the initial level. However, both the long-run temperature level and removal stock depend on the initial atmospheric carbon concentration. Thus, the role of carbon removal in scenarios with excessive initial stocks is to limit the temperature increase and to enable the economy to reach a lower steady-state temperature level.

Note that overshooting can also occur if initial stocks are high but lie below the threshold specified in Proposition 2. An example for this case is when $X = 4250$, which is represented by the lower dashed line in Figure 8. Then, using fossil resources is still optimal and CDR allows the economy to return from an initial temperature overshooting to the lower long-run optimum.

5.5. *Second-best policies*

In Section 3, we have shown that an upstream or downstream carbon pricing scheme achieves the first best outcome in a decentralized economy. However, policy makers might perceive the informational requirements for these policies as prohibitively high. For this reason, they might choose to apply the same carbon price to emissions and removals while disregarding released emissions from storage. Alternatively, they might prefer to apply the steady state discount factor stated in Proposition (10) already during the transition because of its low informational requirement. In the following, we will discuss the outcome of such policy experiments in an exemplary run of the numerical model.

5.5.1. Uniform upstream carbon price. Figure 9 shows the results from applying a uniform upstream carbon price on fossil energy and carbon removal. As panel (a) illustrates, this policy leads to a steeper temperature increase and a higher long-run temperature level than under first-best. In addition, panel (b) and (c) show that the uniform carbon price leads to higher emissions along the transition and to higher removal levels than optimal. The reason is that a uniform carbon price addresses two policy objectives with only one instrument. The first objective is to incentivize the optimal amount of emissions, pinned down by the SCC-E. The second objective is to incentivize the optimal amount of carbon removal, pinned down by both the SCC-E and the SCC-R. The uniform carbon price balances these two objectives and thus lies between the first-best subsidy and the first-best emission tax (see Figure A.2 in the Appendix). In consequence, a lower than optimal emission tax induces higher emissions than first best, while a higher than optimal removal subsidy incentivizes higher removal quantities than first best.

The welfare effects of the uniform carbon price are ambiguous and depend on the considered time frame. In the short run, the removal quantity exceeds release emissions, thereby leading to a lower temperature level compared to a scenario without carbon removal. However, in the long run, released emissions increase while the distorted carbon price signal still induces higher fossil emissions than first-best. In consequence, the uniform carbon price leads to a higher temperature level in the long-run.

To evaluate the performance of the uniform carbon price it thus useful to compare the discounted welfare outcomes over a certain period to a scenario without carbon removal. This captures both the potentially lower short-run climate damages as well as the higher long-run climate damages that result from the uniform carbon price. Hence, Figure 10 shows the discounted welfare over a period of 400 years in the first- and second-best policy scenarios relative to the discounted welfare in a scenario without carbon removal.

In the first-best policy scenario, the availability of carbon removal increases welfare compared to the scenario without carbon removal. The extent depends on the permanence of storage and the

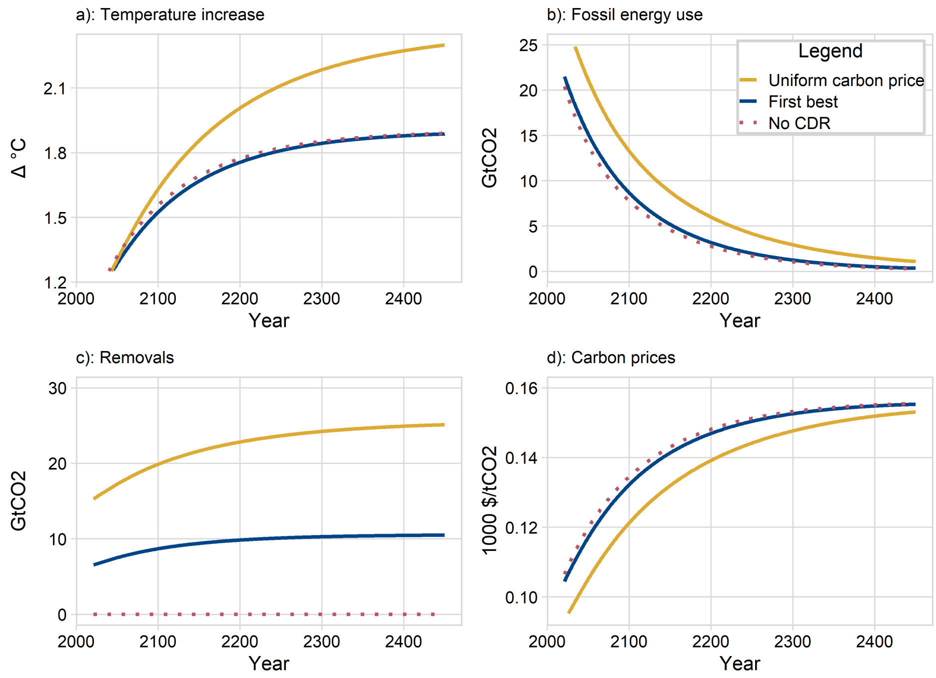

FIGURE 9. Temperature increase and emission and removal flows with first best and uniform carbon pricing

slope of the marginal removal cost curve. The reason is that removal quantities increase when storage becomes cheaper or more permanent. Higher removal quantities, in turn, imply lower cumulative damages at any point in time.

In the uniform carbon price scenario, the availability of carbon removal is only beneficial for low release rates. For higher release rates, the wedge between the optimal emission tax and the uniform carbon price is larger and thus leads to a stronger increase in emissions and removals. In turn, the economy suffers both higher climate damages as well as high total removal cost. The extent depends again also on the slope of the marginal removal cost curve. The steeper the marginal cost curve, the less strong do the removal quantity and the total removal cost react to a subsidy that is set above the first-best level. The uniform carbon price can thus be set closer to the first-best emission tax for higher marginal removal cost.

5.5.2. Steady state discount factor. Instead of applying a uniform carbon price, policy makers could also adjust the removal subsidy using the steady state discount factor $\lambda^S = \frac{r}{r+\delta}$ proposed in Proposition (10). This policy might be attractive to policy makers because it allows them to pay technology-specific subsidies and only requires to assess the release rate of a given removal option.

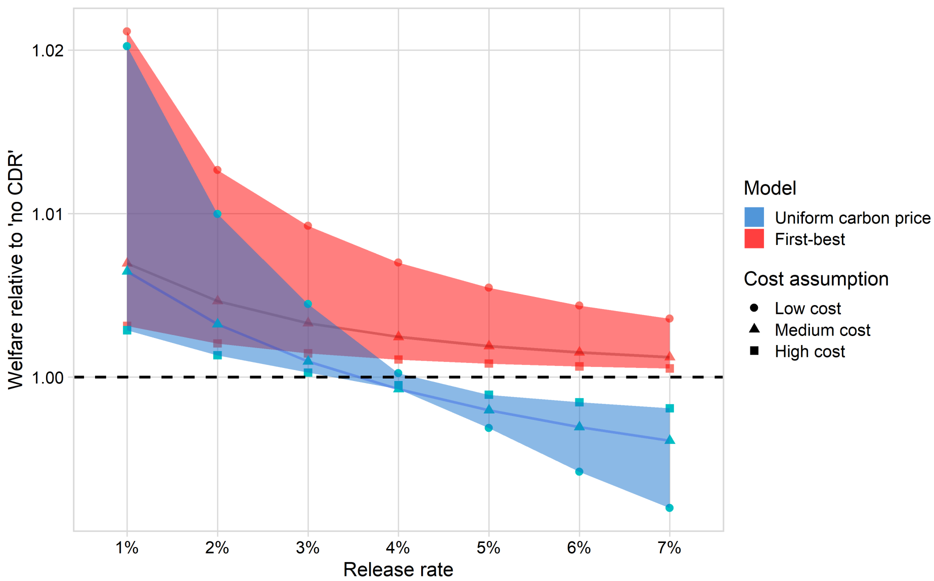

FIGURE 10. Discounted welfare over 400 years in the first-best and uniform carbon price scenarios, relative to a scenario without CDR ($\delta = 5\%, r = 3.5\%$). Low, medium and high cost scenarios refer to marginal removal cost of 50\$/t, 150\$/t and 350\$/t when a hypothetical potential of 24.6 GtCO2 per year is reached.

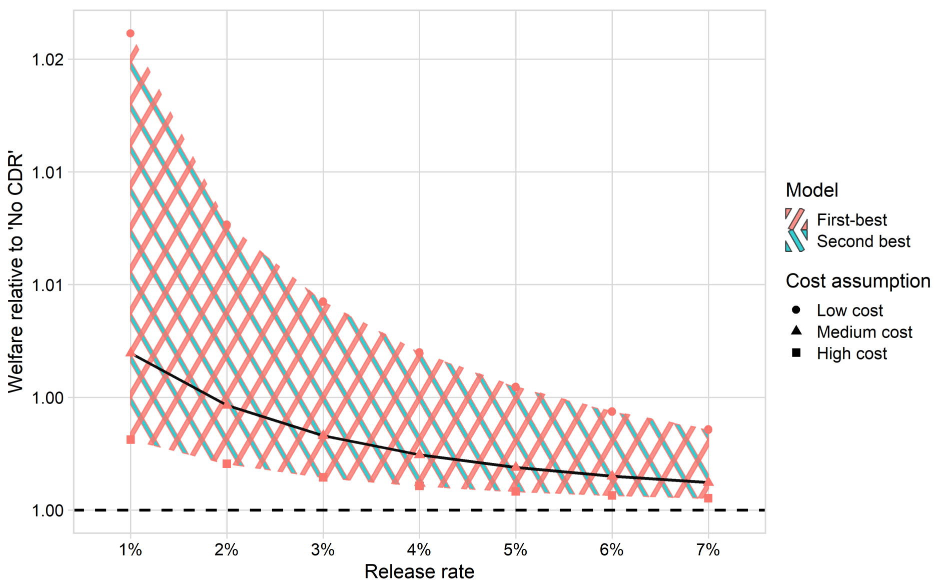

FIGURE 11. Discounted welfare over 400 years in the first-best and second-best scenarios, relative to a scenario without CDR ($\delta = 5\%, r = 3.5\%$). Low, medium and high cost scenarios refer to marginal removal cost of 50\$/t, 150\$/t and 350\$/t when a hypothetical potential of 24.6 GtCO2 per year is reached.

Remarkably, Figure 11 suggests that applying the steady state discount factor achieves welfare outcomes that are very close to first-best. The reason is illustrated by Figure A.3 in the Appendix. In this calibration of the numerical model, the first-best scenario implies flat carbon price trajectories. In turn, adjusting the removal subsidy by using the steady state discount factor results only in a modest over-incentivization of carbon removal in early periods.

6. Conclusions

Governments around the world have announced their plans to reach carbon neutrality by mid-century. Large scale deployment of carbon dioxide removal appears as a *conditio sine qua non* to achieve this goal, and has already become a part of private corporations' strategy to achieve carbon neutrality (Joppa et al. 2021). However, the role of technologies that cannot store carbon permanently is not self-evident. This applies in particular to various 'nature-based' solutions to increase carbon stocks in forests and soils, and to carbon capture and usage approaches, where carbon is stored in produced goods for the short-term (for example bioplastics, wooden furniture, chemicals) or medium-term (for example timber construction).

The present paper sheds light on whether and how carbon dioxide removal should be deployed when storage is non-permanent. Our findings lay out some fundamental principles upon which governments should design policy instruments to incentivize CDR. First, even if a specific technology cannot store carbon permanently, it is still a valuable mitigation option because it lowers the cost of mitigation along the transition. Lower release rates, higher cost of non-fossil energy or a higher discount rate imply higher levels of carbon storage in the long run. Second, if the sum of the initial carbon stock in the atmosphere and the removal sinks is not excessive, the optimal long-run temperature level is independent of the availability of storage technologies and the volume of stored carbon. In this case, high availability or low costs of CDR will not help to increase climate ambition levels. If initial carbon stocks are too high, CDR can help shifting some carbon from the atmosphere to (non-permanent) sinks. This is of particular relevance when climate policy has not been effective for some time and atmospheric carbon stocks became excessive. CDR can then reduce some atmospheric carbon. The optimal temperature level will, however, then depend on the initial carbon stocks in atmosphere and sinks. Without CDR, such a shifting between carbon stocks would not be possible.

Third, while Pigouvian carbon pricing can be extended to carbon removal, new instruments with additional informational requirements are necessary. Governments may not have enough information to directly tax emissions that are released from carbon storage. Nevertheless, they can still implement a first-best optimal pricing policy by adjusting the removal subsidy below the tax on carbon emissions such that the social cost of release emissions are taken into account. Differentiating emission pricing and removal pricing is important as second-best uniform pricing of emissions and removal might result in substantial welfare losses.

Our analysis shows that carbon removal technologies present policy makers with a double edged sword: non-permanent CDR facilitates short-term welfare gains during the transition but commits future generations to continuously return released emissions back to their reservoirs. Similar to Sisyphus's task, non-permanent CDR creates a perpetual "carbon debt" to future generations that consists of undertaking removal into leaky reservoirs. Furthermore, our findings highlight that pricing carbon removal via Pigouvian subsidies or taxes is not trivial. Without sufficient information or care, an over- or underutilization of CDR with substantial welfare costs may result.

Our analysis identifies ways to include carbon dioxide removal into a carbon pricing scheme, but many open questions remain. Our modeling, for example, abstracts from information asymmetries and uncertainty. Relaxing the assumption of perfect information could provide valuable insights for the design of policies when release rates are not observable by the government.[17] Similarly, future research could allow for uncertain storage duration or release rates that are stochastic. Our analysis has identified intertemporal financing as a prerequisite for two of the proposed policy instruments. If financial constraints were considered supplementary policies could be necessary to achieve efficient use of removal; we leave these consideration for future research. Our analysis furthermore suggests that the volume of carbon dioxide to be removed and stored may be large, raising questions about the physical limits of storage capacity as well as the scale of finance necessary to fund the removal subsidies. If revenues for subsidy payments need to

17. In a related paper, Held and Edenhofer (2009) propose bonds to incentivize removal firms to reveal the release rates of their storage sites.

be raised via distortionary taxation, the cost of public funds should be factored into the policy design. Finally, there are additional market failures related to the non-permanence of storage, such as the risk of accidental release or the default risk of firms in charge of maintaining the storage, that pose additional challenges for the design of optimal removal policies.[18] The analytical model developed in this paper provides an excellent framework to analyze these questions in future research.

18. See, Groom and Venmans (2021) for an overview.

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Appendix: Appendix

A.1. Cost-benefit analysis: Solution of the social planner's optimization problem

$$\max_{E,N,R} \int_0^\infty [f(E+N) - g(R) - b(N) - d(X)]\, e^{-rt} \mathrm{d}t \tag{A.1}$$

$$\text{s.t.} \quad \dot{X} = E - R + \delta Z \tag{A.2}$$

$$\dot{Z} = R - \delta Z \tag{A.3}$$

$$E \geq 0 \tag{A.4}$$

$$X(0) = X_0 \tag{A.5}$$

Due to the inequality constraints, we formulate a Lagrangian.

$$\mathscr{L} = f(E+N) - g(R) - b(N) - d(X) + \mu(E - R + \delta Z) + \psi(R - \delta Z) + mE \tag{A.6}$$

A.1.1. Transition Phase. In the transition phase, in which $E \neq 0 \neq N$, and thus $m = 0$ we have

$$f'(E+N) = -\mu \tag{A.7}$$

$$f'(E+N) = b'(N) \tag{A.8}$$

$$g'(R) = \psi - \mu \tag{A.9}$$

$$\dot{\mu} = r\mu + d' \tag{A.10}$$

$$\dot{\psi} = r\psi + \delta(\psi - \mu) \tag{A.11}$$

$$0 = \lim_{t\to\infty} \mu(t) X(t) e^{-rt} \tag{A.12}$$

$$0 = \lim_{t\to\infty} \psi(t) Z(t) e^{-rt} \tag{A.13}$$

In this transition phase, marginal costs and benefits have to be equal to achieve the optimum.

$$f' = b' = g' - \psi = -\mu \tag{A.14}$$

LEMMA A.A.1. *The shadow price $\mu(t)$ of the stock of carbon X is determined by the social cost of carbon.*

$$\mu(t) = -SCC(t)e^{rt}$$

Proof. The solution to (A.10) is

$$\mu(t) = \left(\mu_0 + \int_0^t d'(X(s))\, e^{-rs} ds\right) e^{rt} \tag{A.15}$$

Plugging (A.15) into the TVC (11) yields

$$\lim_{t\to\infty} \left(\mu_0 + \int_0^t d'(X(s))\, e^{-rs} ds\right) X(t) = 0 \tag{A.16}$$

Thus, either $X \to 0$, or

$$\lim_{t\to\infty} \int_0^t d'(X(s))\, e^{-rs} ds = -\mu_0 \tag{A.17}$$

Now, assume that $d'' = d_0 = \text{const.}$, then

$$\lim_{t\to\infty} \int_0^t \dot{X}(s) e^{-rs} ds = -\frac{\mu_0 r}{d_0} - X_0 \tag{A.18}$$

We can split up the integral in (A.15) as follows

$$\begin{aligned}
\mu(t) &= \left(\mu_0 + \int_0^t d'(X(s))\, e^{-rs} ds\right) e^{rt} \\
&= \mu_0 e^{rt} + e^{rt} \int_0^\infty d'(X(s))\, e^{-rs} ds - e^{rt} \underbrace{\int_t^\infty d'(X(s))\, e^{-rs} ds}_{SCC(t)}
\end{aligned}$$

$$\overset{(A.17)}{\Longrightarrow} \quad \mu(t) = -SCC(t)e^{rt}$$

□

LEMMA A.A.2. *The shadow price $\psi(t)$ of the stock of carbon stored Z is given by*

$$\psi(t) = \left[\psi_0 + \delta \int_0^t SCC(s) e^{-\delta s} \mathrm{d}s\right] e^{(r+\delta)t}$$

Proof. Analogous to the proof of Lemma A.A.1. The solution to (A.11) is

$$\psi(t) = \left[\psi_0 - \delta \int_0^t \left(\mu_0 + \int_0^s d'\left(X(k)\right) e^{-rk} dk\right) e^{-\delta s} ds\right] e^{(\delta+r)t} \tag{A.19}$$

Then, we can further calculate that

$$\begin{aligned}
\psi(t) &= \psi_0 e^{(r+\delta)t} - \delta e^{(\delta+r)t} \int_0^t \mu_0 e^{-\delta s} \mathrm{d}s - \delta e^{(\delta+r)t} \int_0^t e^{-\delta s} \int_0^s d'(X(k)) e^{-rk} \mathrm{d}k \mathrm{d}s \\
&= \psi_0 e^{(r+\delta)t} - \mu_0 \delta e^{(\delta+r)t} \left\{-\frac{1}{\delta}[e^{-\delta s}]_0^t\right\} \\
&\qquad - \delta e^{(\delta+r)t} \int_0^t e^{-\delta s} \left[\int_0^\infty d'(X(k)) e^{-rk} \mathrm{d}k - \int_s^\infty d'(X(k)) e^{-rk} \mathrm{d}k\right] \mathrm{d}s \\
&= \psi_0 e^{(r+\delta)t} + \mu_0 e^{(\delta+r)t}(e^{-\delta t} - 1) - \delta e^{(\delta+r)t} \int_0^t e^{-\delta s}[-\mu_0 - SCC(s)] \mathrm{d}s \\
&= (\psi_0 - \mu_0) e^{(r+\delta)t} + \mu_0 e^{(\delta+r)t} + \mu_0 \delta e^{(\delta+r)t} \int_0^t e^{-\delta s} \mathrm{d}s + \delta e^{(r+\delta)t} \int_0^t SCC(s) e^{-\delta s} \mathrm{d}s \\
&= \psi_0 e^{(r+\delta)t} + \delta e^{(r+\delta)t} \int_0^t SCC(s) e^{-\delta s} \mathrm{d}s
\end{aligned}$$

Plugging this expression into the transversality (12) condition yields

$$\lim_{t\to\infty} \left(\psi_0 + \delta \int_0^t SCC(s) e^{-\delta s} \mathrm{d}s\right) Z(t) e^{\delta t} = 0$$

Thus, either $Z \to 0$, or

$$-\psi_0 = \delta \int_0^\infty SCC(s) e^{-\delta s} \mathrm{d}s$$

Using again the expression above yields the expression for the SCC-R:

$$\begin{aligned}
-\psi(t) &= \left[\psi_0 + \delta \int_0^t SCC(s) e^{-\delta s} \mathrm{d}s\right] e^{(r+\delta)t} \\
&= \left[\delta \int_t^\infty SCC(s) e^{-\delta s} \mathrm{d}s\right] e^{(r+\delta)t}
\end{aligned}$$

□

A.1.2. Steady State. In the steady state, $\dot{X} = \dot{Z} = 0$. We use the superscript s to denote quantities in the steady state. Therefore, we obtain that

$$E = 0 < N \tag{A.20}$$

$$R_i^s = \delta Z_i^s \tag{A.21}$$

$$\dot{R}_i^s = 0 \tag{A.22}$$

$$f' = b' \tag{A.23}$$

$$g' = \psi - \mu \tag{A.24}$$

$$f' + \mu + m = 0 \qquad m \geq 0 \qquad E \geq 0 \tag{A.25}$$

$$\dot{\mu} = r\mu + d'(X^s) \tag{A.26}$$

We can further characterize the optimal solution in the steady state.

$$\overset{(A.22),(A.24)}{\Longrightarrow} \dot{\psi} = \dot{\mu} \tag{A.27}$$

$$\Longrightarrow (r+\delta)\psi = (r+\delta)\mu + d'(X^s) \tag{A.28}$$

We can easily solve the differential equation for the shadow price of the atmospheric carbon stock. That is, the solution to $\dot{\mu} = r\mu + \underbrace{d'(X^s)}_{\text{const.}}$ is given by

$$\mu(t) = \left(\mu_0 + \frac{d'(X^s)}{r}\right)e^{rt} - \frac{d'(X^s)}{r} \tag{A.29}$$

Inserting the latter in (A.28) gives us an explicit expression for ψ in the steady state, too.

$$\psi(t) = \left(\mu_0 + \frac{d'(X^s)}{r}\right)e^{rt} - \frac{d'(X^s)\delta}{r(r+\delta)} \tag{A.30}$$

Now, we plug (A.29) and (A.30) into (A.24) to obtain that

$$g'(R_i^s) = \frac{d'(X^s)}{r+\delta} \tag{A.31}$$

Finally, using the terminal condition for the stock of stored carbon (12) and (A.30), we find that

$$\mu_0 = -\frac{d'(X^s)}{r} \leq 0 \tag{A.32}$$

Thus, in the steady state we have

$$\Longrightarrow \mu(t) \equiv -\frac{d'(X^s)}{r} \quad \text{and} \quad \psi(t) \equiv -\frac{d'(X^s)\delta}{r(r+\delta)} \tag{A.33}$$

Plugging this explicit constant value for $\mu(t)$ in the steady state into (A.25), we obtain that

$$\dot{m} = 0 \quad \text{and} \quad m \in \left[0, \frac{d'(X^s)}{r}\right] \tag{A.34}$$

In summary, we obtain from (A.24), (A.25) and (A.33) that

$$\frac{d'(X^s)}{r+\delta} = g'(R_i^s) = f'(N^s) - \frac{d'(X^s)\delta}{r(r+\delta)} + m \tag{A.35}$$

$$\Longrightarrow \frac{d'(X^s)}{r} = f'(N^s) + m = b'(N^s) + m \tag{A.36}$$

Equations (A.35) and (A.36) summarize for the steady state how the social planner balances marginal benefits of renewables, their marginal costs, marginal removal costs and marginal climate damages to achieve the first best social optimum.

A.1.3. Stability analysis of steady state. We have a system of four ODEs in $v := (X, Z, \mu, \psi) \in \mathbb{R}^4$. Let $\dot{v} = F(v)$. From above and (19), we obtain

$$\dot{X} = (f')^{-1}(-\mu - m) - (b')^{-1}(-\mu - m) - (g')^{-1}(\psi - \mu) + \delta Z \tag{A.37}$$

$$\dot{Z} = (g')^{-1}(\psi - \mu) - \delta Z \tag{A.38}$$

$$\dot{\mu} = r\mu + d'(X) \tag{A.39}$$

$$\dot{\psi} = (\delta + r)\psi - \delta\mu \tag{A.40}$$

We linearize the system around the steady state v^s, for which $F(v^s) = 0$. We have to distinguish two cases.

Case 1: $m > 0$

When $m > 0$, then $E = 0$ and there is a small neighborhood around v such that $E = (f')^{-1}(\underbrace{-\mu - m}_{=-\tilde{\mu}}) - (b')^{-1}(\underbrace{-\mu - m}_{=-\tilde{\mu}}) = 0$. Then, the Jacobian is given by

$$J = \begin{pmatrix} 0 & \delta & (g')^{-1'} & -(g')^{-1'} \\ 0 & -\delta & -(g')^{-1'} & (g')^{-1'} \\ d''(X) & 0 & r & 0 \\ 0 & 0 & -\delta & \delta + r \end{pmatrix}$$

The Eigenvalues are

$$\lambda_1 = 0$$

$$\lambda_2 = r$$

$$\lambda_3, \lambda_4 = \frac{r}{2} \pm \sqrt{\delta^2 + d''(X^s)(g')^{-1'} + \delta r + r^2/4}$$

All Eigenvalues are real, two are positive, one is negative and one is zero. This implies saddle path stability along a one-dimensional path.

Case 2: $m = 0$

When $m = 0$, then we can only say that $(f')^{-1}(-\mu - m) - (b')^{-1}(-\mu - m) \geq 0$. Hence, the Jacobian is

$$J = \begin{pmatrix} 0 & \delta & (b')^{-1'} - (f')^{-1'} + (g')^{-1'} & -(g')^{-1'} \\ 0 & -\delta & -(g')^{-1'} & (g')^{-1'} \\ d''(X) & 0 & \rho & 0 \\ 0 & 0 & -\delta & \delta + \rho \end{pmatrix}$$

The Eigenvalues are

$$\lambda_j := \frac{\rho}{2} \pm \sqrt{2}\sqrt{\alpha \pm \sqrt{\beta}}, \qquad j \in \{1, 2, 3, 4\}$$

where we define

$$\alpha := \delta^2 + (b')^{-1'} d''(X) - d''(X)(f')^{-1'} + d''(X)(g')^{-1'} + \delta\rho + \rho^2$$

and

$$\beta := -4\delta d''(X)\left((b')^{-1'} - (f')^{-1'}\right)(\delta+\rho) + \left(\delta^2 + d''(X)((b')^{-1'} - (f')^{-1'} + (g')^{-1'}) + \delta\rho\right)^2$$

The general expression of the Eigenvalues is, thus,

$$\lambda_j = \frac{\rho}{2} \pm (a_j + ib_j)$$

Special case. Now, assume that $f(E+N) = f_0 \log(E+N)$, $b(N) = b_0 N^2 + b_1 N + b_2$, $d(X) = d_0 X^2 + d_1 X + d_2$ and $g(R) = g_{i,0} R^2 + g_1 R + g_2$. Then, the Eigenvalues simplify, since $(f')^{-1'} = f_0$, $(b')^{-1'} = \frac{1}{b_0}$ and $(g')^{-1'} = \frac{1}{g_{i,0}}$.

$$\alpha = \delta^2 + d_0\left[\frac{1}{b_0} + \frac{1}{g_{i,0}} - f_0\right] + \delta\rho + \rho^2$$

$$\beta = -4\delta d_0(\rho+\delta)\left(\frac{1}{b_0} - f_0\right) + \left[\delta^2 + \delta\rho + d_0\left(\frac{1}{b_0} + \frac{1}{g_{i,0}} - f_0\right)\right]^2$$

A.1.4. Comparative statics in the steady state.

PROPOSITION A.1. *The steady state level of non-fossil energy use N^s depends only on the structure of costs and benefits, that is, the functional form of b and f.*

Proof. Follows directly from (A.23). □

The impact of changes in r, δ, non-fossil energy costs and removal costs for the steady state level of the atmospheric carbon stock X^s, the removal rate R_i^s and the stock of stored carbon Z_i^s is different in the two cases $m(t) = 0$ for all t and $SCC - E(0) > -\tilde{\mu}$. If $m(t) = 0$ for all t, the impact of parameter changes is determined by equations (A.35) and (A.36).

PROPOSITION A.2 (Comparative statics for $m = 0$). *The steady state levels of*

1. *carbon in the atmosphere* X^s
   - *increase when the discount rate r or marginal backstop costs increase,*
   - *are independent of the release rates* δ_i *and marginal removal costs* g_i'
   - *decrease if marginal damages* d' *increase.*
2. *carbon removal* R_i^s *and carbon in storage stock* Z_i^s
   - *increase when the discount rate r or marginal costs of non-fossil energy* b' *increase,*
   - *are independent of marginal damages* d'*,*
   - *decrease if the release rate* δ_i *or marginal removal costs* g_i' *increase.*

Proof.

1. Differentiating (A.36) with respect to r yields.

$$0 = \frac{rd''(X^s)X_r^s - d'(X^s)}{r^2}$$

$$X_r^s = \frac{rd'(X^s)}{d''(X^s)} > 0$$

Further, differentiating (A.36) with respect to δ yields

$$\frac{d''(X^s)X_\delta^s}{r} = 0$$

$$\implies \quad X_\delta^s = 0$$

When marginal costs of non-fossil energy increase, N^s falls due to (A.23). Thus, f' increases and via (A.36) also X^s.

Marginal damages d' have no influence on N^s. Hence, due to (A.36), any increase in marginal damages must be accompanied by a decrease in X^s such that $d'(X^s)$ remains constant.

Finally, since marginal removal costs g' have no influence on N^s, (A.36) implies that X^s also remains constant when g' changes.

2. Differentiating the left equation in (A.35) with respect to r yields.

$$\frac{(r+\delta)d''(X^s)X_r^s - d'(X^s)}{(r+\delta)^2} = g''(R_i^s)R_{i\,r}^s$$

$$R_{i\,r}^s = \frac{(r+\delta)d''(X^s)X_r^s - d'(X^s)}{g''(R_i^s)(r+\delta)^2}$$

From above (1.), we have further $0 = rd''(X^s)X^s_r - d'(X^s)$, hence, adding $\delta d''(X^s)X^s_r$ implies $\delta d''(X^s)X^s_r = (r+\delta)d''(X^s)X^s_r - d'(X^s)$ which is positive (when δ is positive). Hence, $R^s_{i\,r} > 0$

Further, differentiating (A.35) with respect to δ yields

$$\frac{-d'(X^s)}{(r+\delta)^2} = g''(R^s_i)R^s_{i\,\delta}$$

$$R^s_{i\,\delta} = -\frac{d'(X^s)}{g''(R^s_i)(r+\delta)^2} < 0$$

When non-fossil energy costs increase and as above X^s increases, then via (A.35) so does R^s_i.

Since any increase in marginal damages results in a reduction of X^s such that $d'(X^s)$ remains constant, using (A.35) we conclude that R^s also has to remain constant.

Finally, recall that X^s is independent of g'. Then, from (A.35) it follows that increases in g' lead decreases in R^s_i such that $g'(R^s_i)$ remains constant.

3. Due to (A.21), changes in r, b', d' and g'_i have the same effect on Z^s_i as on R^s_i. To derive the effect of changes in δ_i, we differentiate (A.21) with respect to δ_i to obtain

$$R^s_{i\,\delta_i} = Z^s_i + \delta_i Z^s_{i\,\delta_i}$$

$$Z^s_{i\,\delta_i} = \frac{R^s_{i\,\delta_i} - Z^s_i}{\delta_i} < 0$$

□

A.2. Proofs in the decentralized economy

A.2.1. Proof of Proposition 8 (downstream pricing). We have to set-up a Lagrangian with $\tilde{\psi}_i$ as co-state variable for Z_i and m as the multiplier of the inequality constraint $E \geq 0$. The resulting first order conditions are:

$$f'(E+N) = p_E - m \tag{A.41}$$

$$f'(E+N) = b'(N) \tag{A.42}$$

$$g'_i(R_i) = \tilde{\psi}_i + p_E \tag{A.43}$$

$$\dot{\tilde{\psi}}_i = r\tilde{\psi}_i + \delta_i(\tilde{\psi}_i + p_E) \tag{A.44}$$

$$0 = \lim_{t\to\infty} \tilde{\psi}_i(t)Z(t)e^{-rt} \tag{A.45}$$

When we set $p_E = -\mu^*$ with μ^* the optimal carbon price from the social planner, the optimality conditions of the household are fully equivalent to those of the social planner (6–12) and, thus, imply the same allocation.

A.2.2. Proof of Proposition 9 (upstream pricing). The firms' first order conditions are:

$$f'(E+N) = p_E - m \tag{A.46}$$

$$f'(E+N) = b'(N) \tag{A.47}$$

$$g_i'(R_i) = p_{R,i} \tag{A.48}$$

With $p_E = -\mu^* = \text{SCC-E} > 0$ and $p_{R,i} = -\mu^* + \psi_i^* = \text{SCC-E} - \text{SCC-R}_i$, the optimality conditions equal those of the social planner (6–12).

A.2.3. Proof of Proposition 11 (stock subsidy). The firms' first order conditions are:

$$f'(E+N) = p_E - m \tag{A.49}$$

$$f'(E+N) = b'(N) \tag{A.50}$$

$$g_i'(R_i) = \varphi_i \tag{A.51}$$

$$\dot{\varphi}_i = (r+\delta_i)\varphi_i - p_Z \tag{A.52}$$

$$0 = \lim_{t\to\infty} \varphi_i(t) Z_i(t) e^{-rt} \tag{A.53}$$

Transition: For the transition, we need to show that $\varphi_i = \psi_i - \mu$ if $p_Z = d'(X)$. Solving the differential equation in (A.52) yields

$$\varphi(t) = e^{t(\delta+r)}\left(\varphi_0 - \int_0^t e^{-s(\delta+r)} p_Z(s) ds\right)$$

Inserting this into the transversality condition in (A.53) yields

$$\varphi_0 = \int_0^\infty p_Z(s) e^{-s(\delta+r)} ds$$

and thus we can conclude that

$$\varphi(t) = e^{t(\delta+r)} \int_t^\infty e^{-s(\delta+r)} p_Z(s) ds.$$

Expressing $\psi - \mu$ as a function of $d'(X)$ (using (A.29) and (A.30)):

$$\psi(t) - \mu(t) = \left[-\delta \int_t^\infty SCC(s) e^{-\delta s} \mathrm{d}s \right] e^{(r+\delta)t} + e^{rt} \int_t^\infty d'\left(X(s)\right) e^{-rs} \mathrm{d}s$$

$$= \left[-\delta \int_t^\infty e^{-\delta s} \int_s^\infty d'\left(X(k)\right) e^{-rk} \mathrm{d}k \mathrm{d}s \right] e^{(r+\delta)t} + e^{rt} \int_t^\infty d'\left(X(s)\right) e^{-rs} \mathrm{d}s$$

Using integration by parts, this simplifies to

$$\psi(t) - \mu(t) = e^{t(\delta + r)} \int_t^\infty e^{-s(\delta + r)} d'\left(X(s)\right) ds$$

which is equal to $\varphi(t)$ for $p_Z(s) = d'\left(X(s)\right)$.

Steady State: in the steady state, we have that $\dot{\varphi}_i = 0$. It follows from (A.51) and (A.52) that

$$\varphi_i = \frac{p_Z}{r + \delta}$$

which yields the same steady state removal quantities as the social planner for $p_Z = d'(X)$.

A.3. Numerical model: additional graphs

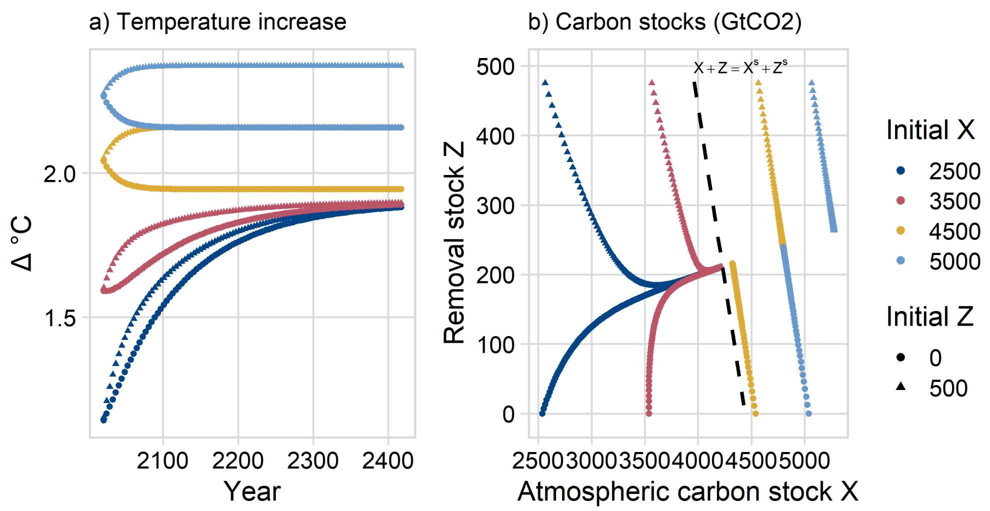

FIGURE A.1. Convergence of temperature and carbon stocks for different initial total carbon in the system.

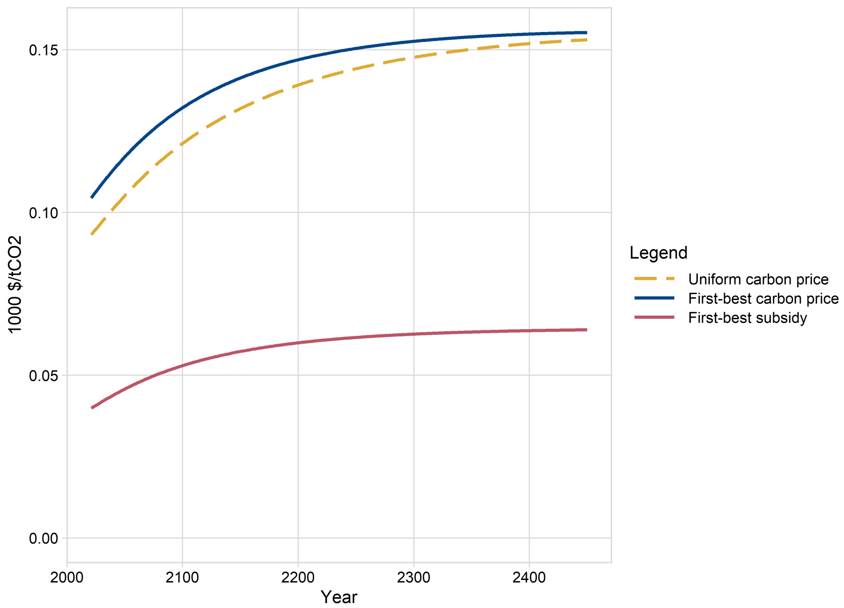

FIGURE A.2. The uniform carbon price lies between the optimal carbon price and the optimal subsidy.

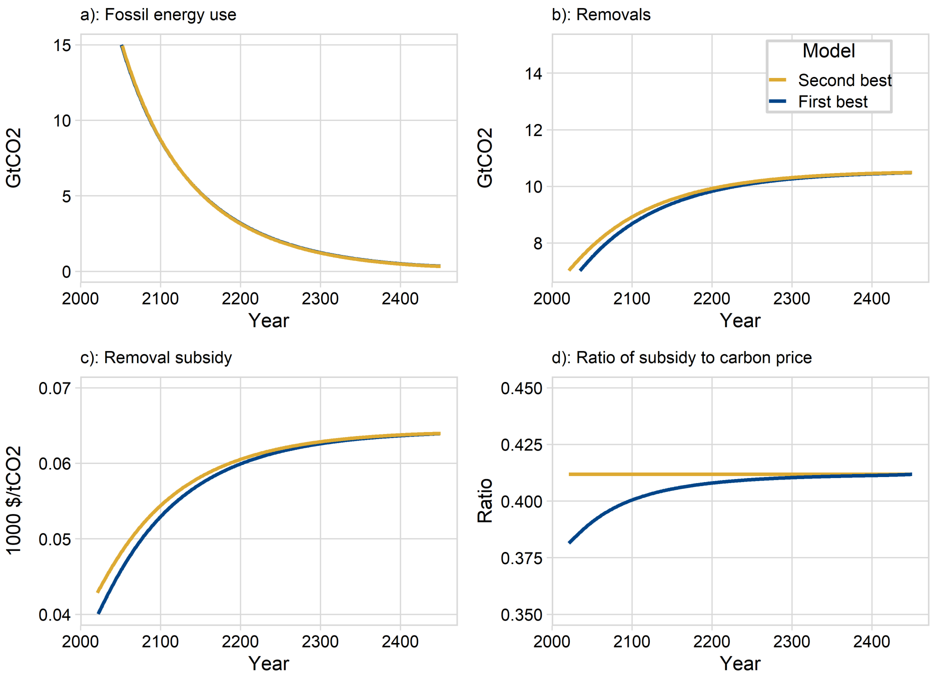

FIGURE A.3. Fossil energy use, carbon removal and removal subsidy with the second-best policy instrument $p_R = \lambda^S p_E = \frac{r}{r+\delta} p_E$.

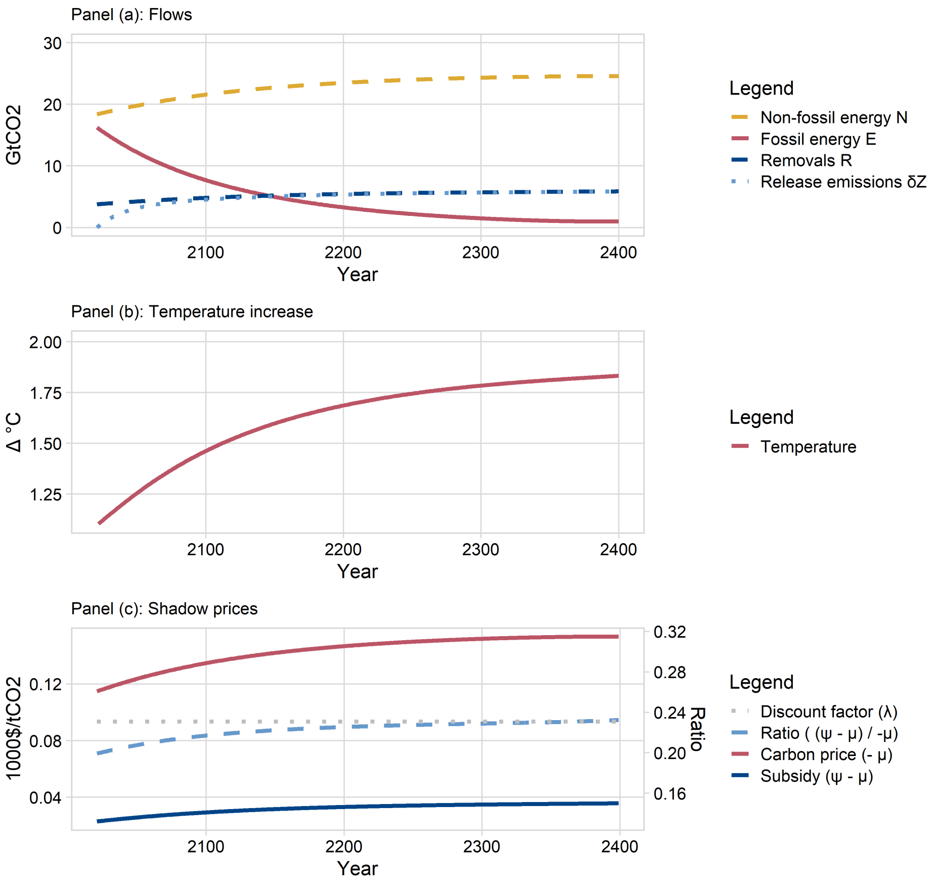

FIGURE A.4. Alternative calibration ($\delta = 5\%, r = 1.5\%, \tilde{\beta} = 0.035$): Illustrative model pathways for a release rate of $\delta = 5\%$.

A.3.1. Alternative calibration. This section shows the results of the numerical model for an alternative calibration. In particular, we choose a discount rate of 1.5%, the same as the social rate of pure time preference in the DICE-R model (Nordhaus 2014). We recalibrate the damage function by setting $\tilde{\beta} = 0.035$ to analyze a scenario with an optimal global warming between 1.5°C and 2°C.[19] All other model parameters remain as in Table 6. The recalibration does not affect the qualitative findings of the model.

19. Without recalibrating the damage function, the model starts in a scenario with excessive initial stocks of atmospheric carbon and carbon storage. The dynamics of such a scenario are illustrated in Figure 8.

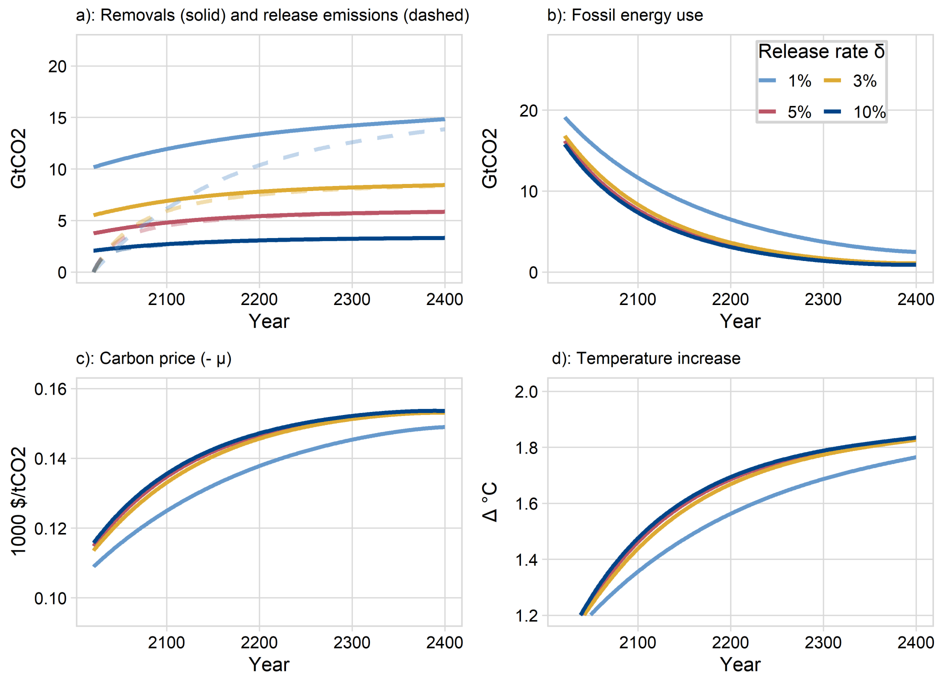

FIGURE A.5. Alternative calibration ($r = 1.5\%, \tilde{\beta} = 0.035$): Comparative statics for removals, fossil energy use, carbon prices and temperature increase with respect to the release rate δ.

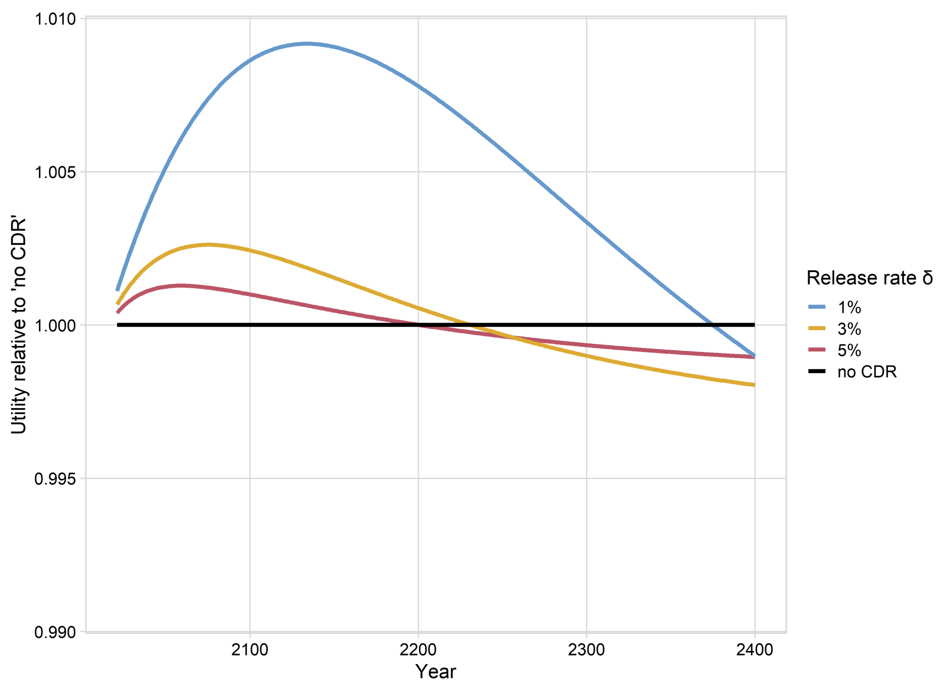

FIGURE A.6. Alternative calibration ($r = 1.5\%, \tilde{\beta} = 0.035$): Welfare trajectories relative to a scenario without CDR.

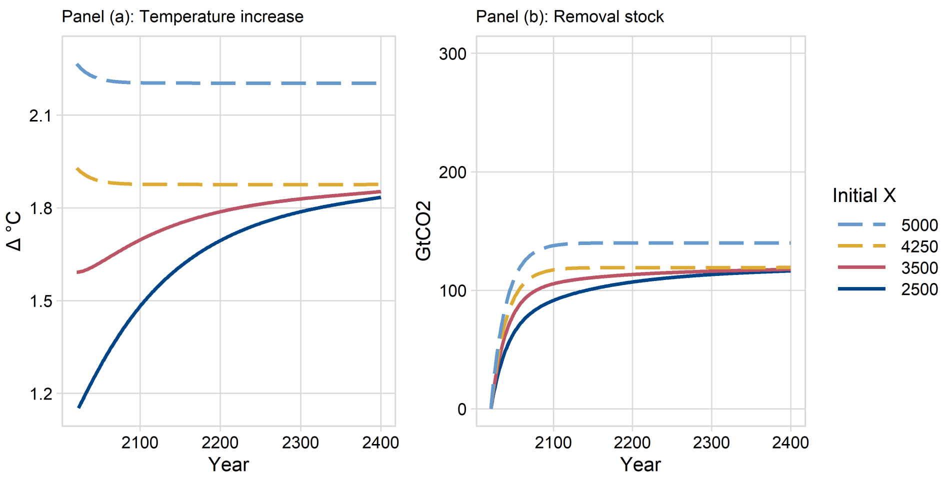

FIGURE A.7. Alternative calibration ($\delta = 5\%, r = 1.5\%, \tilde{\beta} = 0.035$): Temperature increase and removal stocks in scenarios with and without overshooting.

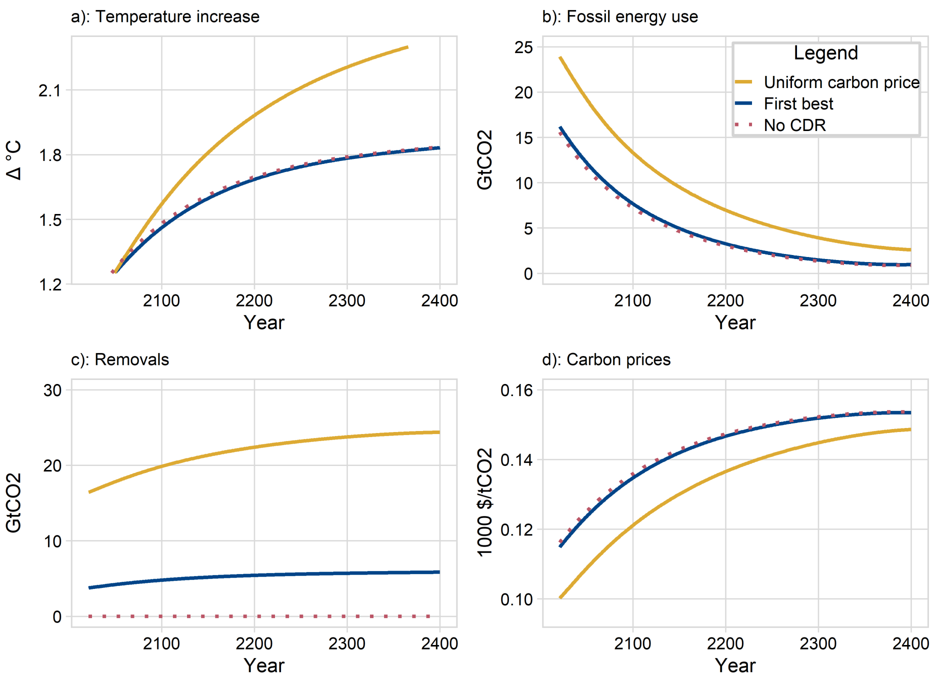

FIGURE A.8. Alternative calibration ($\delta = 5\%, r = 1.5\%, \tilde{\beta} = 0.035$): Temperature increase and emission and removal flows with first best and uniform carbon pricing.

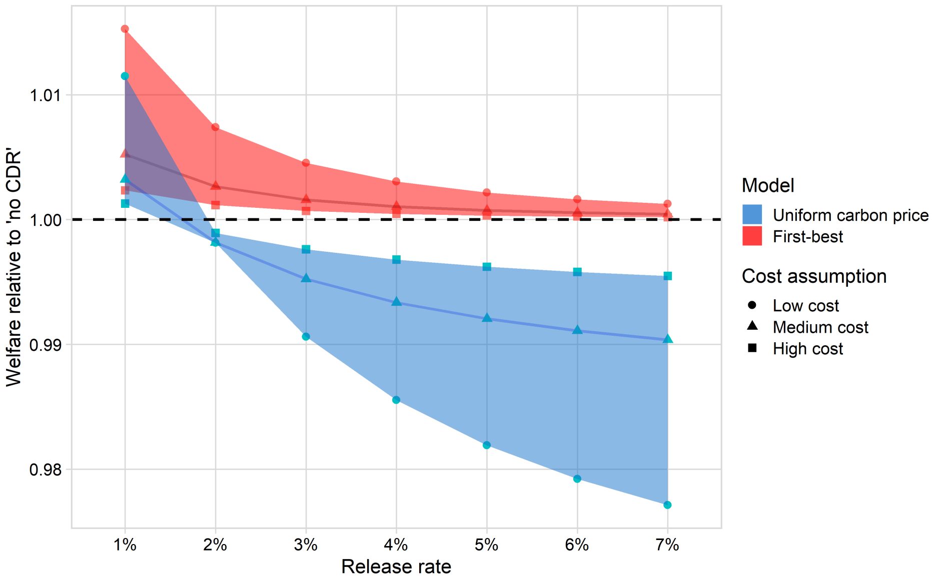

FIGURE A.9. Alternative calibration ($r = 1.5\%, \tilde{\beta} = 0.035$): Discounted welfare over 400 years in the firs-best and uniform carbon price scenarios, relative to a scenario without CDR. Low, medium, and high cost scenarios refer to marginal removal cost of 50$/t, 150$/t and 350$/t when a hypothetical potential of 24.6 GtCO2 per year is reached.