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The Social Power of Spillover Effects: Educating Against Environmental Externalities*

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ABSTRACT

Economists are worried that the lack of property rights to natural capital goods jeopardizes the sustainability of the economic growth miracle that has existed since industrialization. This article questions their position. A vertical innovation model with a portfolio of technologies for abatement, adaptation, and general (Harrod-neutral) technology reveals that environmental damage spillovers have a comparable effect on research profits as technology spillovers so that the social costs of depleting public natural capital are internalized. As long as there is free access to information and technology, growth is sustainable and the allocation of research efforts among alternative technologies is socially optimal. While there still is a need to address externalities from monopolistic research markets, no environmental policy is necessary. These results suggest that environmental externalities may originate in restricted access to information and technology, demonstrating that (i) information has a similar effect as an environmental tax and (ii) knowledge and technology transfers have an impact comparable to that of subsidies for research in green technology.

Keywords: endogenous growth, horizontal innovation, sustainability
JEL Codes: O30, O44, Q55, Q56

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Whenever a theory appears to you as the only possible one, take this as a sign that you have neither understood the theory nor the problem which it was intended to solve.

Popper (1979)

1 Introduction

It was the power of innovations like the steam engine and electricity which brought our generation to a standard of living that previous generations had only dreamed of. However, the price of this development is fundamental burden on the environment. Visible evidence can be found in the pollution that marks urban and rural landscapes, the smog that obscures the view in the global metropolises, and the rise in sea levels around the proverbial pristine island due to climate change.

Economists have not been tired raising concerns about the sustainability of this development for decades, see e.g. Boulding and Jarrett (1966), Meadows et al. (1972), Stern, Common, and Barbier (1996), Arrow et al. (2004), or a recent overview in Drupp et al. (2020). The quality of the environment is a public good, a lack of property rights can lead to overuse and thus to an environmental externality that must be internalized, especially with a view to sustainable development. While economists typically call for a rethinking of production processes, some have hope in the potential of technology to compensate for the exhaustion of natural capital.

Technologies can reduce both actual pollution and the use of natural resources, or at least make the economy more resilient to environmental damage. To achieve such a 'green' technology transition, the literature emphasizes the importance of environmental policy, see e.g. Kemp and Never (2017). Yet, this article argues that this view does not pay enough attention to decentralized incentives for green production and innovation. These incentives are essential. In recent decades, consumers have become much more sensitive to their individual ecological footprints, and producers are aware of their environmental responsibility, see e.g. Fransen (2015), Fransen (2018) and Lambin and Thorlakson (2018).

A simple vertical innovation model reveals that a change in perspective from the demand side to the supply side of technologies can have a deep impact on the discussion of sustainable growth. Agents who have complete access to information and can combine alternative technologies choose a socially optimal environmental research strategy. Environmental externalities only occur if either a limited access to technology leads to lock-in effects or if the information about the environmental impact of individual actions is incomplete. In the former case, one-time research subsidies can shift innovation in the preferred direction if they do not have to augment different technologies. In the latter case, information policies (e.g. education) and Pigouvian environmental taxation are qualitatively comparable. These results go hand in hand with a highly stylized theory and are hence not intended to provide unequivocal evidence. Rather, they point out that literature may underestimate the potential of an ‘educated’ (well-informed, well-trained) society to solve environmental challenges. Improving access to information and technologies can thus have a similar effect as market-based environmental policy.

To demonstrate this, the paper is organized as follows: Section (2) gives an overview of how sustainable growth and depletion of natural capital are assessed in the literature. Section (3) presents the model, Section (4) evaluates possible equilibria and balanced growth. Section (5) discusses the sustainability and social optimality of balanced growth, whereas Section (6) assesses decentralized economy results and evaluates measures to improve welfare. Finally, Section (7) critically discusses the model and its implications, while Section (8) concludes.

2 Sustainable Growth and Natural Capital

The assessment of the sustainability of growth has a long tradition in economic theory (consider e.g. Pearce, 2002, for a retrospective). From a historical point of view, the foundation stone for the discussion was laid by the Aristotelian ethics which regard a good life as the greatest good for human beings. It is therefore important to practice virtues such as justice (Crisp, 2014). Sustainability is all about justice as it examines how a society should allocate its limited resources over time.

With the development of environmental economics in the 1950s, such considerations began to occupy economists on a broader scope, see Sandmo (2011) for a review. While early work by Barnett and Morse (1963) provided an optimistic analysis of the potential of technology to compensate for the exhaustion of non-renewable resources, Boulding and Jarrett (1966) and Meadows et al. (1972) only a little later raised concerns that resource scarcity threatens growth, see also Daly and Daly (1973), Georgescu-Roegen (1975), Georgescu-Roegen (1977), Holdren John et al. (1971), Novak (1973) to mention a few. This ‘limits to growth’ debate is still not solved today, see Drupp et al. (2020) for an update.

Theory usually assesses sustainability based on the definition by the World Commission on Environment and Development (1987) as a ‘*development that meets the needs of the present without compromising the ability of future generations to meet their own needs*’ (WCED, 1987). At the heart, this definition is about intertemporal consumption options. An intuitive path for their evaluation is the concept of genuine investments (GI), requiring that the value of the investments in the productive base must not decrease in order to achieve non-decreasing genuine wealth, see e.g. Hanley, Dupuy, and McLaughlin (2015), Arrow et al. (2004) and Ferreira and Vincent (2005). The productive base combines manufactured capital, human capital, natural capital, and technology. Since capital and technology can compensate for natural capital exhaustion, the concept may be interpreted as a weak sustainability criterion. However, as the value of genuine wealth ’shall’ not decrease, complete depletion of natural capital will likely lead to a deterioration of the value of the productive base (depending on the type of natural resource considered), and hence has to be avoided. Anyhow, strong sustainability would require that all individual assets at least remain constant, see e.g. Dedeurwaerdere (2014).

Yet, natural capital is a heterogeneous conglomerate that essentially contains all goods that do not have anthropogenic origins, see Barbier (2019) for an overview. The natural capacity to absorb pollution and emissions can thereby also be understood as natural capital. Such capacities are usually discussed as common-pool resources (Gardner, Ostrom, and Walker, 1990) or environmental sinks (Andersen, 2007) and are characterized by non-excludability and non-rivalrous consumption. They are thus typical public goods.

According to the Samuelson condition (Samuelson, 1954), a socially optimal allocation of these goods is only achieved if the marginal social benefit and the marginal costs of their provision (or maintenance) coincide. However, the costs and benefits of their use are not reflected in market prices which in most cases leads to the externality of their socially suboptimal provision (or maintenance), see e.g. Laffont (1989). This externality can become a serious problem as nature's ability to absorb pollution and emissions is limited. Yet, the economy can shift to less emission-intensive production processes to prevent an exploitation of the sinks.

The literature traditionally assumes that such a green technology shift requires environmental policy. For example, Bovenberg and S. Smulders (1995) and Bovenberg and S. A. Smulders (1996) demonstrated that ambitious environmental policy can promote long-run growth if technological change[1] enables less pollution-intensive production. Tahvonen (1997) discussed the optimal emissions tax to reduce the consumption of fossil fuels and to increase the use of a backstop technology (green energy). Hart (2004) showed that an environmental tax (sales tax) has the potential to improve environmental research, can shift vintage technology in a new (cleaner) direction and can increase the pace of innovations. Acemoglu, Philippe Aghion, et al. (2012) and Acemoglu, Akcigit, et al. (2016) elaborated that environmental taxes and research subsidies can achieve (weakly) sustainable growth if dirty and clean input technologies are sufficiently substitutable.

Rare literature indicating that socially optimal environmentally-friendly research efforts do not require environmental policy[2] is Schou (2000) and Schou (2002). One reason for this finding is that, similarly to this work, the two papers focus on market goods and therefore ignore the direct effects of the environment on utility. As will be discussed in Section (7), there are good arguments for this omission. What distinguishes this work is that it evaluates the use of a technology portfolio rather than just one technology and, for this purpose, refers to two special cases in the literature

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[1] This is labeled pollution-augmenting technological change. While *technological* change describes the entire process of invention, innovation and technology diffusion, see e.g. Jaffe, Newell, and Stavins (2002), *technical* change describes the shift from one technological focus to another.

[2] Acemoglu, Philippe Aghion, et al. (2012) and Acemoglu, Akcigit, et al. (2016) also indicate that such a path is possible but only if non-renewable resources scarce. This can evidently lead to a shift from dirty to clean production even without environmental policies.

on directed technological change that cannot be treated with just one technology. One in which the elasticity of substitution of alternative technologies in the final production function is unity, another in which this elasticity goes to infinity. This connection is discussed in the appendix, so the model is introduced next.

3 The Model

This article refers to the vertical innovation literature introduced by Grossman and Helpman (1991) and Philippe Aghion and P. Howitt (1990) and follows a methodology by Grimaud and Ricci (1999), which offers a simple possibility to aggregate between disaggregated innovation sectors. To not to be distracted by the coordination of global political efforts, the economy is closed. For parsimony, time indices are ignored if possible and the focus is on a per capita representation.

3.1 Final Production

There is a perfectly competitive final production sector that produces a numeraire, y, that can be consumed or saved. The production thus requires a labor share, $1-n \in (0,1)$, and a continuum of intermediate goods, x_i. Production is described with

$$y = (1-n)^{1-\alpha} N \int_0^1 \mathcal{T}_j x_j^{\alpha} dj \tag{1}$$

where $\alpha \in (0,1)$ denotes an elasticity and $\mathcal{T}_j$ refers to an intermediate gross production technology with

$$\mathcal{T}_j = A_j R_j \tag{2}$$

where a general technology, A_j, scales an environmental robustness technology, R_j. Further

$$N = \frac{1}{\bar{E}^{\phi}} \leq 1, \quad with \ \phi = \iota\omega \geq 0$$

represents a natural capital stock which reduces if an environmental effect, $\bar{E}$, increases. As a standardization, $\bar{E} \geq 1$ so that if there is no environmental effect

$N = 1$. From a technical point of view, $\bar{E}$ can describe any deterioration in natural capital. The damage sensitivity is then subject to the environmental damage elasticity, ϕ, accounting for that the consequences of the net degeneration elasticity, ω, on production is subject to the impact channel, ι. The environmental damage is neutral if $\iota = 1$, labor-biased if $\iota = 1 - \alpha$ (e.g. health effects) and capital-biased if $\iota = \alpha$ (e.g. physical damage to the manufactured capital stock). The environmental effect is proportional to the production activity while an abatement technology, G_j, can reduce the environmental footprint of intermediate products so that

$$\bar{E} = (1-n)^{1-\alpha}\Big[\int_0^1 \frac{\mathcal{T}_j}{G_j} x_j^{\alpha} dj\Big] \geq 1. \tag{3}$$

Note that even without abatement, it is required that $\left[\frac{R_j}{\bar{E}^{\phi}}\right] \leq 1$ because adaptation efforts can eliminate at most all damage caused by natural capital degeneration, but productivity will not be further increased. Final producers then solve

$$\max_{n,x_j} (1-n)^{1-\alpha} \int_0^1 \left[\frac{\mathcal{T}_j}{\bar{E}^{\phi}}\right] x_j^{\alpha} dj - \int_0^1 (1-T_p) p_j x_j dj - w\big(1-n\big)$$

with p_j as the intermediate price, T_p as a price subsidy that will enable to correct markup pricing and w as a wage. This gives the two factor demand equations

$$w = (1-\alpha)(1-n)^{-\alpha} \int_0^1 \left[\frac{\mathcal{T}_j}{\bar{E}^{\phi}}\right] x_j^{\alpha} dj \tag{4}$$

$$(1-T_p)p_j = \alpha(1-n)^{1-\alpha}\Big[\frac{\mathcal{T}_j}{\bar{E}^{\phi}}\Big] x_j^{\alpha-1} dj. \tag{5}$$

3.2 Intermediate Production

Each intermediate producer provides an intermediate that can be equipped with up to three of the above introduced types of technology or knowledge (both here interpreted as the same): A general technology, A_j, that summarizes all technology that is not used to directly reduce environmental damage and two technologies explicitly addressing environmental damage. On the one hand, there is an environmental robustness technology, R_j including any knowledge that protects against damage without reducing pollution (indoor filters, medicine, different materials, etc.) which is called adaptation. On the other hand, there is knowledge about how to reduce pollution which is called abatement and is denoted by, G_j. Examples include emission filters or measures to improve energy efficiency.

It is assumed that the respective knowledge scales the *technology intensity* of an intermediate according to $\mathcal{I}_j = A_j R_j F_j$ so that F_j measures the proportional abatement efforts. The greater the effort, F_j, to reduce the environmental footprint of the production process associated with a certain gross productivity, $\mathcal{T}_j = A_j R_j$, the higher the technological intensity of production, $\mathcal{I}_j$. Similarly, the higher the gross productivity, $\mathcal{T}_j = A_j R_j$, the higher the technological intensity of production for a certain intensity of the abatement effort, F_j. For the sake of simplicity, there is a directly proportional relationship between the abatement efforts and the actual abatement intensity, described by $F_j = G_j$ so that $\mathcal{I}_j = A_j R_j G_j$. Following literature standards, intermediate production is capital-intensive and proportional to the technology intensity, here described with $\mathcal{I}_i$, thus

$$x_j = \frac{k_j}{\mathcal{I}_i} \tag{6}$$

so that $K(t) = \int_0^1 \mathcal{I}_i x_j dj$. An intermediate producer who has a patent for the portfolio of technologies included in $\mathcal{I}_j$ is hence faced with a profit function that follows

$$\pi_{j,i} = \left[p_j - r(1 + T_{E_j})\mathcal{I}_j\right] x_j$$

where $T_{E,j}$ represents an environmental tax that affects the operational costs. These costs are proportional to the interest rate. The intention to maximize monopoly rents results in[3]

$$p_j = \frac{r(1 + T_{E_j})\mathcal{I}_j}{\alpha} \tag{7}$$

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[3] With (5), this is based on $\max_{p_j} \left[p_j - r(1 + T_{E_j})\mathcal{I}_j\right] \frac{(1-n)\alpha^{\frac{1}{1-\alpha}} (A_j R_j^{\iota})^{\frac{1}{1-\alpha}}}{((1+T_p)p_j)^{\frac{1}{1-\alpha}} \bar{E}^{\frac{\iota\omega}{1-\alpha}}}$.

$$x_j = (1-n)\Big[\frac{\alpha^2}{r\bar{E}^{\iota\omega}}\Big]^{\frac{1}{1-\alpha}}\Big[\frac{\mathcal{T}_j}{\varrho_j\mathcal{I}_j}\Big]^{\frac{1}{1-\alpha}}, \quad \textit{with} \quad \varrho_j := (1-T_p)(1+T_{E_j}) \tag{8}$$

so that

$$\pi_j = \frac{\Lambda(1-n)}{r^{\frac{\alpha}{1-\alpha}}\bar{E}^{\frac{\iota\omega}{1-\alpha}}}\Big[\frac{\mathcal{T}_j}{(\varrho_j\mathcal{I}_j)^{\alpha}}\Big]^{\frac{1}{1-\alpha}} \quad \textit{with} \quad \Lambda := (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}. \tag{9}$$

3.3 Research and Development

With regard to access and use of alternative technology, this theory examines how two characteristics affect the innovation decisions: (1.) access to information on the environmental impacts of innovations, (2.) access to technologies.

3.3.1 Access to Information

Intuitively, if individuals do not receive all of the information they need, there is a high risk of biased decisions. Stiglitz (1985) emphasizes that imperfect information usually introduces a moral hazard problem as people increase their exposure to risks if they do not anticipate that they bear the full costs of that risk. Such challenges are often observed in the context of climate change and environmental pollution and have many faces. For example, in recent years, heterogeneous groups have emerged that doubt the existence of anthropogenic climate change for different, mostly unscientific reasons. The literature discusses these groups as *climate deniers*, see e.g. Corry and Jørgensen (2015), Ploeg and Rezai (2019) or Krishna (2021). The question is how to address such individuals with neoclassical theory.

One possibility is to suggest that their information set is restricted, so they do not have the information necessary to be aware of the complete environmental consequences of their actions. To address such agents, this theory distinguishes between agents that lack information about their environmental footprint, indicated with $\mathcal{I} = 0$, and agents that know their innovation will exhaust natural capital, indicated with $\mathcal{I} = 1$. While both types are exposed to a certain current environmental damage when determining their research strategy, only the latter group anticipates that their innovation will contribute to the depletion of natural capital if it is not accompanied by abatement. The reason for a lack of information is not explained further. It is possible that it is simply too costly to take into account the environmental impact of innovations, or that socio-cultural and political forces are hindering access to the relevant information. Yet, $\mathcal{I} = 1$ will remain the benchmark.

3.3.2 Access to Technologies

Another important distinction that characterizes research is whether and how technologies can be combined and how the research sector can provide these technologies. For example, a car manufacturer has not the skills to produce solar panels and a civil engineer cannot produce lithium batteries, however, a car manufacturer may be able to switch from using gasoline to using batteries. Likewise, a civil engineer can possibly move from using concrete (which is relatively robust to environmental damage but emission-intensive) to using wood or other environmentally-friendly materials (which are less robust to environmental damage but less emission-intensive). So branches can possibly combine alternative technologies in their products and shift to greener innovations.

The question is thus whether innovators use all technologies and benefit from combined spillover effects or whether they have to specialize so that the spillover effects become path-specific. While specialization is excessively discussed with regard to comparative advantages in trade (see e.g. Laursen, 2015), this theory investigates its role on innovation in a closed economy. Thereby, specialization not only needs to be related to technical complexities, but can also be caused by very restrictive patent laws (so it can be used to assess various innovation constraints). To investigate this feature in more depth, this theory distinguishes two regimes:

(i) A **Generalized Knowledge (GK) regime** refers to a setting where the *R&D* sector has access to a shared pool of knowledge to improve alternative technologies. This e.g. represents the civil engineer who can switch from using concrete to using wood, or combine both as a hybrid.

(ii) A **Specialized Knowledge (SK) regime** refers to a setting where the *R&D* sector can improve the general productivity based on shared knowledge, but further needs to specialize for either improving adaptation or abatement. A combination of both technologies is thus not possible here. This e.g. represents the civil engineer who can either use concrete or wood, but no hybrid.

3.3.3 Research Labor

While the direction of technical change depends on how researchers use their time to improve existing technologies, the intensity of research depends on the fraction of researchers, $n \in (0,1)$. This fraction is determined by a standard no-arbitrage condition equating the wage earned in production with the expected value of an innovation according to

$$w = \lambda(1 + T_V)V \tag{10}$$

where $\lambda > 0$ presents a Poisson parameter for the likelihood to innovate, T_V denotes a price instrument (tax or subsidy) to influence the profitability of R&D, and

$$V(s) = \int_0^\infty e^{-\int_0^t r(s,t)+\lambda n(s,t)dt}\pi(s,t)dt \tag{11}$$

represents the value of all innovations in the market where r represents the interest rate and $\pi(s,t)$ denotes the profits of period s innovations in $t \geq s$ that are capitalized until a new drastic innovation takes over, for what λn measures how many vintage technologies are replaced by new innovations.

3.3.4 Research Efforts

Each researcher considers how to improve the available technology package, $\mathcal{I}_i$. It is thus necessary to assess how to use a fraction $\eta_i \in (0,1)$ of the research time to improve general productivity, A_j, $\kappa_i \in (0,1)$ of the time to improve abatement technologies R_j, while the remaining $1 - \kappa_i - \eta_i$ is used for abatement activities, G_j. Since it was simplified that $G_j = F_j$, the innovation function is directly related to G_j. In total, this leads to the following three path specific innovation difference equations

$$\dot{A}_i = \lambda n_i \varsigma_A \eta_i^\theta A_i \tag{12}$$

$$\dot{R}_i = \lambda n_i \varsigma_R \kappa_i^{\theta} R_i \tag{13}$$

$$\dot{G}_i = \lambda n_i \varsigma_G \big(1 - \kappa_i - \eta_i\big)^{\theta} G_i \tag{14}$$

where $\theta \in [0,1]$ is responsible for decreasing returns in efforts and $\varsigma_j > 0$, $j = A, R, G$ denotes a path-specific research efficiency.

3.3.5 Research in a GK Regime

Every researcher knows that if an innovation is successful, its research efforts will be immediately visible in the next period. After an innovation, a researcher becomes an entrepreneur, until a new drastic innovation replaces the invention. Researchers, therefore, allocate efforts to maximize the potential next period profit increase, $\dot{\pi}$, so that if there is access to all technologies, the research effort allocation principles are described with

$$\kappa_i = \arg \max_{\kappa_i \in [0;1]} \dot{\pi}(t)(\kappa_i, \eta_i, n) \tag{15}$$

$$\eta_i = \arg \max_{\eta \in [0;1]} \dot{\pi}(t)(\kappa_i, \eta_i, n_i). \tag{16}$$

In aggregate $\int_0^1 \kappa_i di = \kappa$, $\int_0^1 \eta_i di = \eta$ and $\int_0^1 n_i di = n$, whereby n follows with (10), which is restated to emphasize the difference among a GK regime and a SK regime discussed next

$$w = \lambda n (1 + T_V) V(\kappa, \eta, n). \tag{17}$$

3.3.6 Research in an SK Regime

With specialization, researchers must determine the direction of research at an early stage of their career. As a result, a fraction $\gamma \in (0,1)$ of n choose adaptation. They face $\eta + \kappa = 1$ and decide upon η and κ. The remaining $1 - \gamma$ researchers then choose abatement so that they face $\kappa = 0$ and select the profit maximizing η. The research effort allocation principles thus follow

$$\kappa_i = \begin{cases} \arg\max_{\kappa_i \in [0;1]} \dot{\pi}_{i,R}(t) & \textit{if } i = R \ \ (\textit{adaptation}), \textit{relevant for } \gamma \\ 0 & \textit{if } i = G \ \ (\textit{abatement}), \textit{relevant for } 1-\gamma \end{cases} \tag{18}$$

$$\eta_i = \arg \max_{\eta_i \in [0;1]} \dot{\pi}_i(t). \tag{19}$$

The allocation of γ follows with the nested no-arbitrage condition

$$(1+T_V)V_R\big(\kappa_R, \eta_R, (1-\gamma)n\big) = \lambda(1+T_V)\big(1-T_{V,R})V_G(\eta_G, \gamma n\big) \tag{20}$$

with $T_{V,R}$ as an path specific research tax or subsidy (described in detail later). Finally, the allocation of n follows with the no-arbitrage condition

$$w = \lambda n \max\Big\{(1+T_V)V_R\big(\kappa_R, \eta_R, (1-\gamma)n\big), \lambda(1+T_V)(1-T_{V,R})V_G\big(\eta_G, \gamma n\big)\Big\}. \tag{21}$$

3.4 Households

There is an infinitely living representative household that offers its labor inelastically, owns all capital, and obtains utility by using the unsaved output for consumption. Its intertemporal consumption preferences are described with a standard CRRA function. Capital is the only source for savings ($\dot{K}$). With (10), the per capita consumption saving decision then follows with the solution to

$$\max_{\{c(t)\}_{t=0}^{\infty}} \int_{t=0}^{\infty} e^{-\rho t} \frac{c(t)^{1-\epsilon}}{1-\epsilon} dt \quad s.t. \quad \dot{K}_t = w(t) + r(t)K(t) - c(t) - P(t) \tag{22}$$

where $\epsilon > 1$ scales the degree of risk aversion[4], $\rho > 0$ denotes time preferences, and $P(t)$ is a tax expenditure or income channel which finances the governmental budget

$$P(t) = (1+T_V)V + T_p \int_0^1 x_j dj - r \int_0^1 T_{E_j} \mathcal{I}_j x_j dj.$$

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[4] The $\epsilon > 1$ satisfies the necessary conditions for balanced growth in the baseline Hewitt and Aghion (1998) specification this theory relates to and is set as a precondition for simplicity.

For simplicity, population growth is ignored. This leads to[5] the standard Euler equation

$$g_c(t) = \frac{r(t) - \rho}{\epsilon}. \tag{23}$$

With this, the next section will address a balanced growth path. Following the reference literature on vertical innovation, the focus is on the existence and characteristics of a balanced growth path (BGP) while the transition paths are not addressed[6].

4 Balanced Growth and Sustainability

To start, the market equilibrium is defined as follows:

Definition 1. *A competitive equilibrium is a sequence of labor and effort allocations* $\{n(t), \eta(t), \kappa(t)\}_0^\infty$*, endowments* $\{x(t), K(t), c(t), P(t), A(t), D(t), G(t)\}_0^\infty$*, policy instruments* $\{T_p, T_E, T_V, T_{V,R}\}_0^\infty$ *and prices* $\{r(t), w(t), p(t)\}_0^\infty$ *that arise with the clearing of the markets for production factors, labor, and goods and services. Thereby, final producers hire labor and capital to maximize profits, taking* $w(t)$ *and* $p(t)$ *as given, researchers choose their efforts and labor* $\eta(t)$*,* $\kappa(t)$*,* $\gamma(t)$*,* $n(t)$ *to maximize expected innovation values taking* T_p*,* T_E*,* T_V *and* $T_{V,R}$ *as given. If researchers become entrepreneurs they rent capital* $K(t)$ *to maximize profits, taking* $r(t)$ *and* T_p *and* T_E *as given. Household optimize their utility subject to their budgets whereas saving market clearing equates the interest on consumption with the investment rate.*

A balanced growth path is then defined as:

Definition 2. *A balanced growth path (BGP) is a trajectory along which* g_y*,* g_k*,* g_c*, and technologies grow at a constant positive (not necessarily the same) rate.*

Before examining whether a balanced growth path exists, some of its properties must be elaborated first. This is addressed with Lemma (1):

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[5] Assuming $lim_{s \to \infty} e^{(-\int_0^t r(t)dt)} K(t) \geq 0$ (no Ponzi scheme) and $K(0) = K_0 > 0$ (initial assets).

[6] As detailed in the Appendix, addressing transition paths would overload the discussion without making a significant theoretical contribution. Note that the theory is based on a Ramsey-Cass-Koopmans model so that if BGP exists, there is also a stable trajectory leading to that path.

Lemma 1. *If $1 > \phi$, a BGP can exist in two alternative scenarios of resource use*

(a) $g_E > 0$ (exhaustion scenario), characterized by

$$y(t) = (1-n)^{(1-\alpha)(1-\phi)} K(t)^{\alpha(1-\phi)} \bar{b}(t) \quad \textit{with} \quad \bar{b}(t) := \frac{B(t)}{1+\mathcal{B}} \tag{24}$$

$$\mathcal{B} := (1-\phi)(1-\alpha)[\varsigma_A(\kappa\eta)^\theta + \varsigma_R((1-\kappa)\eta)^\theta] + [\phi - \alpha(1-\phi)]\varsigma_G(1-\eta)^\theta \tag{25}$$

$$B(t) := R(t)^{(1-\alpha)(1-\phi)} A(t)^{(1-\alpha)(1-\phi)} G(t)^{\phi-\alpha(1-\phi)} \tag{26}$$

$$g_y = \lambda n \frac{\mathcal{B}}{1-\alpha(1-\phi)} \tag{27}$$

with $\eta \in (0,1]$, $\kappa \in (0,1]$, and $n \in (0,1)$ all constant.

(b) $g_E = 0$ (non-exhaustion scenario), characterized by $g_G \geq g_A$ and

$$y(t) = (1-n)^{(1-\alpha)} K(t)^\alpha \hat{\bar{b}}(t) \quad \textit{with} \quad \hat{\bar{b}}(t) = \frac{\hat{B(t)}}{1+\hat{\mathcal{B}}} \tag{28}$$

$$\hat{\mathcal{B}} := \frac{(1-\alpha)\varsigma_A\varsigma_G}{\left(((1-\alpha)\varsigma_A)^{\frac{1}{\theta}} + \varsigma_G^{\frac{1}{\theta}}\right)^\theta}. \tag{29}$$

$$\hat{B}(t) := \frac{A(t)^{(1-\alpha)}}{G(t)^\alpha} \tag{30}$$

$$g_y = \lambda n \hat{\mathcal{B}} \tag{31}$$

with $n \in (0,1)$ constant.

If $\phi > 1$ only $g_E = 0$ (non-exhaustion scenario) as described with (b) is possible.

Proof. See Appendix.

The lemma differentiates two alternative scenarios: either the environmental effect increases, $g_{\bar{E}} > 0$, or remains constant, $g_{\bar{E}} = 0$. An increasing environmental effect states that innovations exhaust the natural capital stock. The intensity of this exhaustion is scaled with the environmental damage elasticity, ϕ.

Within both scenarios, we learn that along a balanced growth path disaggregated innovation activity can be interpreted as a sequential aggregated innovation process.

Hence, technologies can be described via the relation among leading technologies, bundled with $B(t)$, respectively $\hat{B}(t)$, and average technologies, bundled with $\bar{b}(t)$, respectively $\hat{\bar{b}}(t)$. The proportionality among both is sensitive to the net research efforts, $\mathcal{B}$, respectively $\hat{\mathcal{B}}$, which describe how the combination of research efforts along alternative innovation pathways affects net productivity growth rates. Higher net efforts have thus two effects: on the one hand, they widen the gap between average and leading technology stocks[7], on the other hand, they increase the production growth rate, g_y. Note here that the net research efforts are the higher the higher the research efficiencies, ς_A, ς_G, and ς_R if $g_{\bar{E}} > 0$, and the higher the technology weight in production, $1-\alpha$. Yet, if $g_{\bar{E}} > 0$, the elasticity of the environmental damage relativizes the efficiency of the research effort combination in $\mathcal{B}$. So, the higher ϕ, the smaller the net effects of the aggregate research efforts, but the more effective the abatement efforts.

With the lemma we further learn that the output growth rate, g_y, is proportional to the number of new inventions which are measured through the product of the probability of an innovation, λ, and the fraction of researchers, n. Thereby, $\frac{1}{1-\alpha(1-\phi)}$ implicitly adjusts for the amount of production that is reinvested in manufactured capital and technology to compensate for the natural capital use. The higher α, the less this adjustment while the lower ϕ, the higher the output growth rate[8].

As soon as $g_E = 0$, the production elasticities are no longer weighed with ϕ as there is no increasing environmental effect. Consequently, there is no need for adaptation so that $\kappa = 1$. Yet, $0 < \eta < 1$ is required to achieve $g_E = 0$, which will be explained in more detail after having discussed the determinants of a BGP next.

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[7] As addressed in the Appendix in depth, an increase in $\mathcal{B}$, respectively $\hat{\mathcal{B}}$, has a positive effect on $\bar{b}(t)$, respectively $\hat{\bar{b}}(t)$, since the technology distribution is biased towards newer technologies.

[8] This is since a higher ϕ leads to greater environmental damage and reduces the elasticity of the manufactured capital used in production so that a larger part of the numeraire output must be used for compensation.

Proposition 1. *If a BGP exists, it is unique and characterized by*

$$\hat{g} = g_y = g_\pi = g_V = g_w = g_{\bar{\iota}} + g_x = g_k = \begin{cases} \lambda n \frac{\mathcal{B}}{1-\alpha(1-\phi)} & if \;\; 1 > \phi \\ \lambda n \frac{\hat{\mathcal{B}}}{1-\alpha} & if \;\; \phi \geq 1. \end{cases}$$

If $1 > \phi$*, a BGP exists if* $n \in (0,1)$*. If* $\phi \geq 1$*, a BGP exists if* $\in (0,1)$ *and* $g_G \geq g_A$.

Proof. See Appendix.

The proposition summarizes the necessary (and sufficient) conditions for a balanced growth path, highlights the corresponding growth characteristics, and emphasizes the uniqueness of the path. Specifically, a balanced growth path requires that $0 < n < 1$ and is also sensitive to whether $\phi \geq 1$ or $1 > \phi$. If $\phi \geq 1$, an increase of the environmental effect cannot be sustained, so that balanced positive growth is only possible if the natural capital stock remains constant[9]. This requires that $g_G \geq g_A$. Further, along a BGP all key aggregates grow at the same rate as production so that the Kaldor facts hold[10].

Importantly, the sustainability of balanced growth depends on the environmental damage elasticity and the technical potential to compensate for the exhaustion of natural capital. Weak sustainability requires that the value of the productive base is non-decreasing. The consequence is that as long as $1 > \phi$, both technology, and manufactured capital can compensate for the depletion of natural resources. Hence, the economy continues to grow irrespective of whether production exhausts the environment. When natural capital is an essential factor of production, a complete exhaustion of the resource is critical. However, in the simplified framework of this theory, this state of extinction is only approached but never achieved. As a result, this theory can discuss a weak sustainability path that continuously reduces the natural capital good [11]. Therefore, $g_{\bar{E}} > 0$ is feasible. A strong sustainability criterion

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[9] While it would be possible to further differentiate the case with $\phi = 1$, as this special case would allow the evaluation of both a non-growth and a positive growth scenario, the focus is placed on the latter so that it is discussed along with the case that $\phi > 1$.

[10] I.e. constant expenditure shares among capital and labor, constant growth rates for capital and output per worker, constant capital-output ratio, a constant interest rate.

[11] Note that in this theory, the shadow value of a asset is not affected by amenity effects, a feature

applies whenever $\phi \geq 1$. In this case, the environmental damage is so severe that balanced growth requires avoiding any depletion of the natural capital stock, so $g_{\bar{E}} = 0$ is necessary. If this is reached, ϕ will no longer affect production growth rates.

Which of the two sustainability criteria to use is a question of the application of this theory. For example, when being interested in the health effects of air pollution, a narrow interpretation of the model would suggest that a strong sustainability criterion must be met since agents can hardly survive ever-increasing pollution without damage. Practically, this would relate to $\iota = 1 - \alpha$ and $\omega > \frac{1}{1-\alpha}$. However, it is also possible to make this discussion less narrow by abstractly assuming that technologies enable a life with low pollution growth, so $\omega < \frac{1}{1-\alpha}$, what would describe a weak sustainability scenario.

Against this background, two questions emerge: First, whether a decentralized economy can achieve sustainable growth; second, whether this growth path (if achieved) is socially optimal. The latter question is especially interesting if alternative technologies are available and is addressed with the planner solution next.

5 Social Planner

This theory understands a social planner as a benevolent force that maximizes the infinite utility of a representative household based on the technology available in the decentralized economy. The intriguing question is how to organize research.

Proposition 2. *Suppose a social planner can select κ, η, and n to maximize the utility of a representative household.*

(i) If $1 > \phi$, then $j^{**} = j^* \in \{brown, gray, green\} \mid g_{j^{**}} \geq g_{j^*}$, with

(a) $j^* = green$, characterized by $\kappa^* = 0$, $\eta^* = \frac{1}{1+\left[\frac{\varsigma_A}{\varsigma_G}\right]^{\frac{1}{\theta}}}$, and $g^*_{\bar{E}} = 0$.

(b) $j^* = gray$, characterized by $\kappa^* = \frac{\varsigma_R^{\frac{1}{1-\theta}}}{\varsigma_A^{\frac{1}{1-\theta}}+\varsigma_R^{\frac{1}{1-\theta}}+(\Gamma\varsigma_G)^{\frac{1}{1-\theta}}}$, $\eta^* = \frac{\varsigma_A^{\frac{1}{1-\theta}}}{\varsigma_A^{\frac{1}{1-\theta}}+\varsigma_R^{\frac{1}{1-\theta}}+(\Gamma\varsigma_G)^{\frac{1}{1-\theta}}}$,

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which would otherwise increase the shadow price of natural capital.

and $g^*_{\bar{E}} > 0$, *only an alternative if* $\Gamma = \frac{\phi}{(1-\phi)(1-\alpha)} - \frac{\alpha}{1-\alpha} > 0$ *and* $\frac{\varsigma_A^{\frac{1}{1-\theta}} + \varsigma_R^{\frac{1}{1-\theta}}}{\varsigma_G^{\frac{1}{1-\theta}}} > \Gamma^{\frac{\theta}{1-\theta}} \big(\frac{\alpha}{(1-\alpha)} + (1-\phi)(1-\alpha) \big)$.

(c) $j^* = brown$, *characterized by* $\kappa^* = \frac{1}{1+\left[\frac{\varsigma_R}{\varsigma_A}\right]^{\frac{1}{1-\theta}}}$, $\eta^* = 1 - \kappa^*$, *and* $g^*_{\bar{E}} > 0$.

Thereby, $g^*_{y,j} = \lambda n^*_j \mathcal{B}^*_j$ *with*

$$\mathcal{B}^*_j = \begin{cases} \frac{(1-\phi)(1-\alpha)}{1-\alpha(1-\phi)} (\varsigma_A^{\frac{1}{1-\theta}} + \varsigma_R^{\frac{1}{1-\theta}})^{1-\theta} & \text{if } j^* = brown \\ \frac{(1-\phi)(1-\alpha)}{1-\alpha(1-\phi)} \left(\varsigma_A^{\frac{1}{1-\theta}} + \varsigma_R^{\frac{1}{1-\theta}} + \Gamma \varsigma_G^{\frac{1}{1-\theta}} \right)^{1-\theta} & \text{if } j^* = gray \\ \frac{(1-\alpha)\varsigma_A \varsigma_G}{\left(((1-\alpha)\varsigma_A)^{\frac{1}{\theta}} + \varsigma_G^{\frac{1}{\theta}} \right)^{\theta}} & \text{if } j^* = green \end{cases}$$

$$n^*_j = \begin{cases} \left(\frac{1 + \mathcal{B}^*_j - \frac{\rho}{\lambda \mathcal{B}^*_j}(1-\alpha)(1-\phi)}{\frac{\phi + \epsilon(1-\alpha)(1-\phi)}{1-\alpha(1-\phi)} + \mathcal{B}^*_j(\phi + \alpha(1-\phi))} \right) & \text{if } j^* \in \{brown, gray\} \\ \left(\frac{(1-\alpha)}{\epsilon(1-\alpha)+\alpha} \right) \left(\frac{(1+\mathcal{B}^*(1-\alpha))}{(1-\alpha)} - \frac{\rho}{\lambda \mathcal{B}^*(1-\alpha)} \right) & \text{if } j^* = green. \end{cases}$$

(ii) If $\phi \geq 1$, $j^{**} = green$ *following (i)(a).*

In $j^* \in \{brown, gray\}$, $n^* \in (0,1)$ *if* $\frac{\left(\frac{(1-\epsilon)^2}{1-\alpha(1-\phi)^2} + 4\frac{\rho}{\lambda} \right)^{\frac{1}{2}} - \frac{(1-\epsilon)}{1-\alpha(1-\phi)}}{2(1-\alpha(1-\phi))} > \mathcal{B}^*_j > \frac{(1 + \frac{4\rho(1-\alpha)(1-\phi)}{\lambda})^{\frac{1}{2}} - 1}{2(1-\alpha(1-\phi))}$.

In $j^* = green$, $n^* \in (0,1)$ *if* $\frac{\left(\frac{(1-\epsilon)^2}{(1-\alpha)^2} + 4\frac{\rho}{\lambda} \right)^{\frac{1}{2}} - \frac{(1-\epsilon)}{(1-\alpha)}}{2} > \mathcal{B}^*_j > \frac{(1 + \frac{4\rho(1-\alpha)(1-\phi)}{\lambda})^{\frac{1}{2}} - 1}{2}$.

Proof: See Appendix.

The proposition presents the socially optimal research strategy among a weak and a strong sustainability criterion. This strategy combines research efforts with research labor and needs to completely abate any environmental effect whenever strong sustainability is required. For a weak sustainability scenario, however, alternative innovations strategies are available as technically any combination among abatement and adaptation leads to sustainable positive growth if some efforts are used to improve general technology, thus $\eta > 0$. The proposition structures the innovation pathways in three characteristic directions of technical change, depending on whether the planner combines general technology with abatement, with adaptation or with both. Each direction is then described by the specific socially optimal effort allocation which are consequently all indicated with ‘*’.

A *brown* direction is free of any abatement ($\eta^* + \kappa^* = 1$) so that the environmental effect increases steadily. The planner concentrates on general innovations and adaptation and selects κ^* and η^* independently of ϕ because improvements in both technologies compensate for the damage. Hence, the only parameters to consider are the research efficiencies ς_A and ς_R.

A *gray* direction describes a strategy that combines research in all directions. Since there is no compete abatement, an increasing environmental effect remains. The crucial parameter for this path is $\Gamma = \frac{\phi}{(1-\phi)(1-\alpha)} - \frac{\alpha}{1-\alpha}$ representing an efficiency indicator for abatement. Thereby, $\frac{\phi}{(1-\phi)(1-\alpha)}$ measures the net benefits of abatement whereby ϕ measures direct benefits which are rescaled by $(1-\phi)$ accounting for that environmental damage reduces production and thus the depletion of the natural capital stock while $(1-\alpha)$ scales this effect by the amount of abatement-focused research efforts which cannot be used for other productivity enhancing activities. The costs of abatement are measured with $\frac{\alpha}{1-\alpha}$ with α describing that abatement-focused research efforts cannot be used for general productivity improvements while $1-\alpha$ rescales this by the benefits due to damage reduction. Consequently, there is only abatement if $\Gamma > 0$ while the efforts face an upper limit if $\frac{\varsigma_A^{\frac{1}{1-\theta}} + \varsigma_R^{\frac{1}{1-\theta}}}{\varsigma_G^{\frac{1}{1-\theta}}} < \Gamma^{\frac{\theta}{1-\theta}} \left(\frac{\alpha}{(1-\alpha)} + (1-\phi)(1-\alpha) \right)$ as at this intensity, the abatement is so effective that $g_{\bar{E}} = 0$, so the direction of technical change is not longer gray but green.

This *green* direction describes a scenario where the planner abates any increase in the environmental effect, so $g_E = 0$. There is thus no need for adaptation, what leads to $\kappa^* = 1$. For the innovation and growth equations, this has the consequence that $\phi = 0$ so Γ is no longer relevant for η^*.

The choice of the socially optimal direction of technical change is subject to various parameter weights and thus best assessed with a calibration later. However, without identifying parameter, two features are known. First, whenever the gray direction is feasible, it dominates the brown direction, as the combination of general innovations, adaptation, and abatement is more efficient due to decreasing returns in research efforts. This is also the reason why alternative strategies such as only using general innovations or focusing on general innovations and abatement without fully eliminating an increase in the environmental effect are never an alternative. Second, if strong sustainability is required, the green direction is evidently the only option.

To conclude, the allocation of research labor n_j^* is subject to the direction of technical change, while the planner weighs the advantages of using labor to either generate direct consumption by allocating labor to production or to improve consumption growth rates by allocating labor to the research sector. Note that $\frac{\partial n_j^*}{\partial \mathcal{B}_j^*} > 0 \forall j$ since the larger the (marginal) productivity of research, the stronger the social value of research. Further, $\frac{\partial n_j^*}{\partial \rho} < 0$ since greater impatience increases the benefits of immediate consumption and thus the value of labor in production. In addition, $\frac{\partial n_j^*}{\partial \lambda} > 0$ since a higher likelihood to innovate increases the consumption growth rate and thus the social value of research labor. To determine the welfare maximizing direction of technical change in a weak sustainability scenario, the model is calibrated next.

Calibrated Social Planner Results

Figure (1) reveals the social planner's net research efforts, $\mathcal{B}^*$ (top left), research labor n^* (top right), the depletion rate of natural capital $g_{\bar{E}.}^*$ (bottom left), and the corresponding production growth rates g_y^* (bottom right) for all three directions of technical change, for different intensities of $\phi \in (0, 1)$ in a calibrated version of the model. While the underlying parameter selection is described in the appendix, the focus is set on a benchmark scenario where research is equally efficient ($\varsigma_A = \varsigma_R = \varsigma_G$). In general, since the green direction of technical change eliminates environmental damage, the marginal product of labor in production is relatively high along this path. The social planner therefore allocates more labor to production than in a brown or gray direction of technical change. As a result, the latter two directions have a higher proportion of research labor and thus a higher rate of growth until the environmental damage becomes so severe that the green direction has the highest growth rates. For relatively low damage elasticities of up to 25%, $\Gamma < 0$, so the gray direction is not efficient while the brown direction generates the highest growth

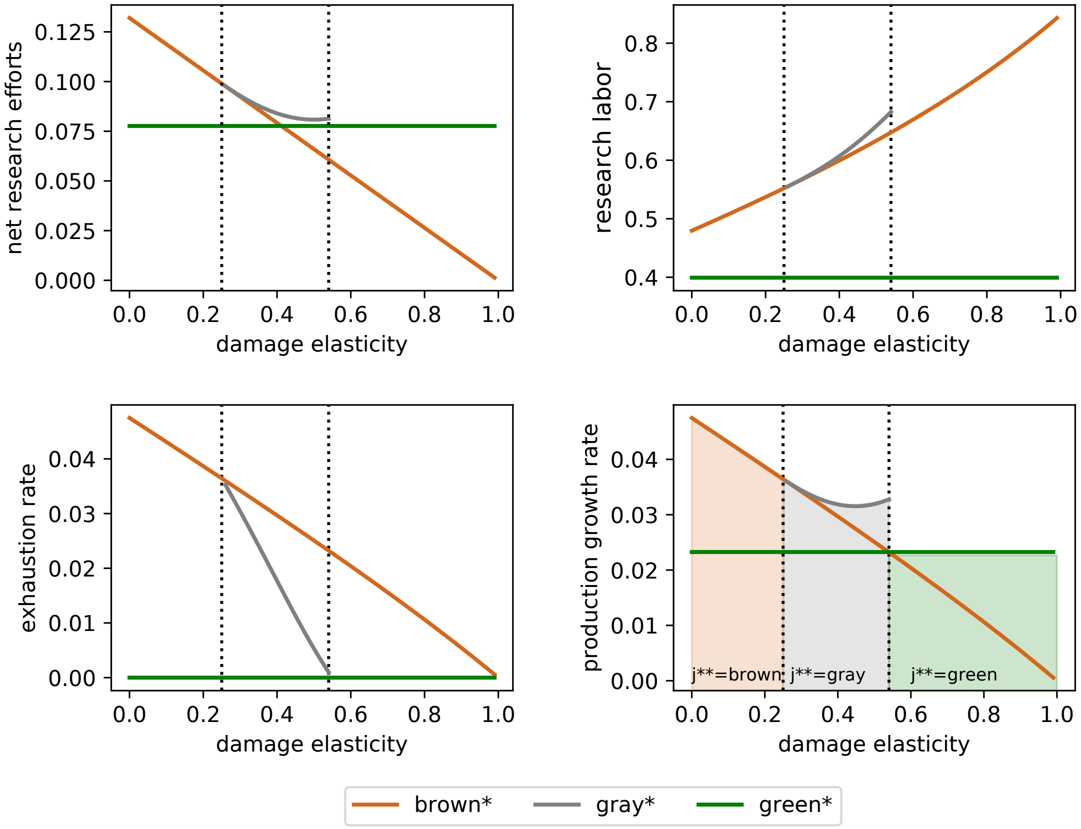

Figure 1: Social planner responses of net efforts $\mathcal{B}^*$ (top left), research labor n^* (topright), natural capital depletion rate $g_{\tilde{E}}^*$ (bottom left) and production growth rates g_y^* (bottom right) to damage elasticities ϕ from 0 to 1 (abscissa).

rates and is, therefore, the choice of the planner. For higher damage elasticities, $\Gamma > 0$, a gray direction becomes efficient and is the social planner's choice. Yet, at a critical damage elasticity around $\phi \approx 0.55$, the marginal effect of using research for abatement is high enough to incentivize a complete abatement of an increase in the environmental effect. Ergo, the gray direction turns green. At this level, a brown direction experiences such a high environmental damage that the green direction is the social planner's choice[12].

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[12] Note that in a gray direction, the social planner's net research efforts, $\mathcal{B}^*$, initially decrease with ϕ, but then increase. This is because marginal abatement effects increase with larger damages,

The appendix additionally presents a scenario where adaptation and abatement-focused research efforts are only 50% as efficient as research in general technologies ($0.5\varsigma_A = \varsigma_R = \varsigma_G$). This does not affect the qualitative results, only the critical level at which the gray direction turns green. This level is close to 75% and thus higher since general innovations are (relatively) more efficient. Hence, path-specific research efficiencies play a subordinate role for the fact that the brown direction is socially optimal for low environmental elasticities until the gray direction becomes efficient, which is then socially optimal until its abatement efforts turn so intense that the direction becomes green, which is socially optimal thereafter.

6 Decentralized Economy

For the decentralized economy, it is first essential to understand the research efforts and labor allocation in a benchmark with full access to information and technology. The focus is on whether this allocation leads to sustainable growth, and if so, whether this path is socially optimal or could be improved with adequate policies. With this orientation, the section then investigates the role of both, access to information and access to technology. For this discussion, note that while this theory refers to a Ramsey-Cass-Koopmans growth model, it adds natural capital and a monopolistic research sector to the framework. Both are potential sources of externalities which are not observed in the original model.

When improvements in gross productivity, $\mathcal{T}$, are not neutralized by a sufficient increase in abatement, G, the natural capital stock, N, exhausts. However, the previous section has shown that on a weakly sustainable growth path a continuous depletion of natural capital can be socially optimal. Environmental externalities only occur when decentralized research efforts do not consider the social costs of natural capital depletion. In that case, there is a static inefficiency as the inventory

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making this research direction more effective. Also note that there is a sharp decrease in the production growth rates at $\phi \approx 0.55$, because the green direction eliminates all environmental damages thereby increasing the marginal product of labor in numeraire production what shifts labor from research to production. Therefore, this is simply a tipping point.

of available technologies is not socially optimal (so immediate production is below its potential) and a dynamic inefficiency since the innovation rate is not socially optimal.

At the same time, the research sector is a potential source of externalities. A new innovation creates a positive externality in expanding the stock of publicly accessible knowledge used for subsequent innovations. However, there is only innovation if the opportunity costs of research are sufficiently compensated. Patent protection serves this appropriability requirement[13]. Yet, patents privatize the returns to public knowledge. As a result, a monopolistic competitive research sector emerges which charges price markups. One the one hand, these markups lead to static inefficiencies since they reduce intermediate demand, on the other hand they cause a dynamic inefficiency since they inflate the value of innovations which incentivizes too intense research. The second cause of bias is that research laboratories base their innovation decisions on private rather than social research returns. Thereby innovation values are discounted above a socially optimal rate because the R&D sector ignores the benefits of technology spillovers for future production, resulting in insufficient research[14]. Yet, researchers also ignore the business stealing effect that occurs when their innovation replaces a vintage technology, which leads to too excessive research.

Which of these distortions dominates is widely debated, parameter sensitive, and in some cases subject to whether the economy has access to information and technology, as detailed in the following step by step.

6.1 Research with Open Access to Technology (GK Regime)

As a first step, it is important to assess the decentralized research effort and research labor allocation in the benchmark economy.

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[13] See e.g. Philippe Aghion, Akcigit, and P. Howitt (2015) for a short overview and Decker (2014) and Akcigit and Kerr (2010) for a discussion.

[14] The focus on individual profits can affect the evaluation of labor productivity which can be profound in case that labor uses distinct technologies in different sectors, see Aghion and Hewitt (1998) for details. However this does not directly affect the presented theory.

Proposition 3. *Suppose an economy is characterized by a GK regime and* $\mathcal{I} = 1$*, then along a BGP* $\mathcal{B} = \mathcal{B}^{**}$ *while*

$$n = \frac{\lambda(1+T_V)(1-\alpha) - \rho}{(\lambda \mathcal{W}_j + (1+T_V)(1-\alpha))} \gtreqless n^{**},$$

$$\mathcal{W}_j := \begin{cases} \frac{\mathcal{B}_j^{**}\left(\epsilon + \frac{\phi}{(1-\phi)(1-\alpha)}\right)}{1-\alpha(1-\phi)} + 1 - \frac{\phi(\phi - \alpha(1-\phi)\varsigma_G(1-\eta_j-\kappa_j)^{\theta}))}{(1-\phi)(1-\alpha)} & \text{if } j^{**} \in \{brown, gray\} \\ \epsilon \frac{\mathcal{B}_j^{**}}{1-\alpha} + 1 & \text{if } j^{**} = green. \end{cases}$$

Proof: See Appendix.

The proposition reveals that access to information and technology are crucial for the social optimality of decentralized research decisions. If there is complete information about the environmental impact of innovations and full access to different technologies, there are no environmental externalities. Hence, decentralized economies not only find a sustainable growth path, but a socially optimal direction of technical change, $\mathcal{B} = \mathcal{B}^{**}$. The reasons is that the damage to the natural capital stock is proportional to the available technology stock, so an agent that anticipates the effects of research spillovers also anticipates the innovation effects on the depletion of natural capital. Patent protection therefore not only privatizes the technology spillovers but also the costs of the natural capital exhaustion. The sale of the intermediates (and hence the value of its innovation) is influenced by both what leads to socially optimal research efforts and internalizes potential environmental externalities.

Yet, the research labor allocation is biased (with the exception of an arbitrary case) because decentralized economies do not weigh the benefits of knowledge spillover effects against the business stealing effect. While similarly to the social planner, $\frac{\partial n}{\partial \rho} < 0$ since a larger impatience reduces the value of an innovation, distinct to the planner $\frac{\partial n}{\partial \lambda} < 0$ because a stronger replacement rate no longer stimulates research and $\frac{\partial n}{\partial \mathcal{B}} < 0$ (via $\mathcal{W}$) because the no-arbitrage condition in the labor market draws labor from the increasingly productive innovation sector to production[15].

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[15] Note that a higher α reduces the marginal product of production labor but increases $\mathcal{B}$ and thus the pace of innovation. Because technologies have a minor role in production, the aggregate effects of α are ambiguous. Also note that whenever $1 > \eta > 0$, $\mathcal{W}$ is scaled by $\varsigma_G(1-\eta)^{\theta}$. An increase in

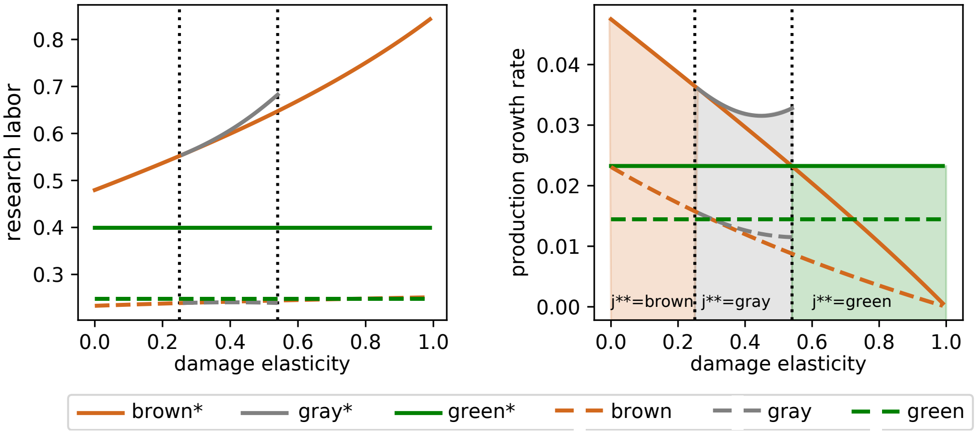

Figure 2: Comparison of the decentralized and social planner growth rates and research labor allocation under all three directions of technical change in the benchmark specification.

Figure (2.3) illustrates the differences between the social planner and the decentralized economy by sketching the respective research labor allocation (left figure) and the corresponding production growth rate (right figure) for all directions of technical change given different environmental damage elasticities ϕ. Since the decentralized net research efforts of the benchmark economy, $\mathcal{B}$, are socially optimal, the only difference between the decentralized economy and the social planner is the allocation of research labor, n. There, we learn that the decentralized economy consistently provides too little labor for research and therefore suffers from lower production growth rates, regardless of the direction of technical change[16].

As discussed with the robustness test in the appendix, these results are qualitatively identical when adaptation and abatement are only 50% as effective as general research so the discussion will proceed with policy paths for welfare improvements.

Corollary 1. *Suppose an economy is characterized by a GK regime and* $\mathcal{I} = 1$, *then*

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future intermediate demand elevates the environmental effect. Abatement-focused research hence reduces the environmental footprint of a vintage intermediate good what reduces $\mathcal{W}$ and thus partially offsets the positive impact of the effort effect $\mathcal{B}$.

[16] Note also that when a gray direction turns green at $\phi \gtrapprox 0.55$, the proportion of research labor increases as a greater abatement decreases the efficiency of research efforts. This is since the no-arbitrage condition in the labor market compensates for this development by shifting labor from production to research.

$n = n^{**}$ is achieved with

$$T_V = \begin{cases} \left(\frac{(1+\mathcal{B}_j)\lambda\mathcal{B}_j-\rho(1-\alpha)}{\rho(1-\alpha)-(1-\epsilon+(1-\alpha)\mathcal{B}_j)\lambda\mathcal{B}_j}\right)\frac{\lambda\mathcal{W}+\rho}{(1-\alpha)} - 1 & \textit{if } j^{**} \in \{brown, gray\} \\ \frac{\left(1+\frac{\mathcal{B}_j}{1-\alpha}-\frac{\rho(1-\alpha)^2}{\lambda\mathcal{B}_{green}}\right)\left(\lambda\mathcal{W}+\rho\right)}{(1-\alpha)\left(\frac{\rho(1-\alpha)^2}{\lambda\mathcal{B}_{green}}+\epsilon-1-\mathcal{B}_{green}\right)} - 1 & \textit{if } j^{**} = green, \end{cases}$$

while $T_p = \frac{1-\alpha}{\alpha}$ eliminates intermediate demand distortions due to markup pricing. In combination, these policies are first best.

Proof See Appendix.

The corollary details the design of the price subsidy, T_p, and the research subsidy, T_v, required to achieve static and dynamic efficiency. The latter is the higher, the larger $\mathcal{B}$ and λ because of the labor market distortions discussed above. Since there are no environmental externalities and since eliminating all monopolistic market externalities, these policies are first best.

Environmental Externalities in a GK Regime

With free access to technology, an explanation of this model for environmental externalities is a lack of information, as detailed in Proposition (4).

Proposition 4. *Suppose an economy is characterized by a GK regime and $\mathcal{I} = 0$, then along a BGP $j = brown$, which is*

- *(i) not sustainable if $\phi \geq 1$.*
- *(ii) socially not optimal if $1 > \phi$ and $j^{**} \in \{green, gray\}$.*
- *(iii) socially optimal if $1 > \phi$ and $j^{**} = brown$.*

Further, $n \gtreqless n^{**}$ (ambiguous) with

$$n = \frac{(1+T_V)(1-\alpha)-\rho}{(\lambda\mathcal{W}_{anticipated}+(1+T_V)(1-\alpha))},$$

$\mathcal{W}_{anticipated} := \epsilon\frac{\mathcal{B}_{anticipated}}{(1-\alpha)} + 1$, $\mathcal{B}_{anticipated} := (1-\alpha)(1-\alpha)(\varsigma_A^{\frac{1}{1-\theta}} + \varsigma_R^{\frac{1}{1-\theta}})^{1-\theta}$.

Proof See Appendix.

The proposition demonstrates that agents lacking information on the environmental impact of innovations cause an environmental externality whenever the socially optimal direction of technical change is not brown. The proposition therefore details that such agents ignore the environmental damage induced rescaling of the innovation $(1-\phi)$ so that they base their labor allocation decision on an overestimated efficiency, described with $\mathcal{B}_{anticipated}$. A lack of information hence shifts labor from research to production which intensifies the distortion of the labor allocation discussed above. While for a weak sustainability requirement, this bias only influences the social optimality of the decentralized growth path, it brings the economy to a collapse when a strong sustainability criteria is relevant. This requires immediate political action. Yet, market-based policies can help.

Corollary 2. *Suppose an economy is characterized by a GK regime and* $\mathcal{I} = 0$. *then*

(i) $\mathcal{B} = \mathcal{B}^{**}$ *is achieved with*

$$T_E(t) := \alpha(1+T_p)\bar{E}_0 e^{(g_{\Gamma_E}+\Omega g_E)t} - 1,$$

whereby

(a) $T_E(t) = 0$ *if* $j^{**} = brown$.

(b) $\Omega := \frac{1}{1+\alpha} + \alpha$ *and* $g_{\Gamma_E} := \frac{\Omega\phi}{(1-\alpha)}\big(\frac{1}{1-\alpha(1-\phi)}g_G - (g_A+g_R)\big)$ *if* $j^{**} = gray$.

(c) $\Omega := \frac{1}{\alpha+\left(\frac{1-\phi}{\phi-\alpha(1-\phi)}\right)}$ *and* $g_{\Gamma_E} := 0$ *if* $j^{**} = green$.

(ii) $n^{**} = n$ *is achieved with*

$$T_V = \begin{cases} \left(\frac{(1+\mathcal{B}_j)\lambda\mathcal{B}_j - \rho(1-\alpha)}{\rho(1-\alpha)-(1-\epsilon+(1-\alpha)\mathcal{B}_j)\lambda\mathcal{B}_j}\right)\frac{\lambda\mathcal{W}_{anticipated}+\rho}{(1-\alpha)} - 1 & \text{if } j^{**} \in \{brown, gray\} \\ \frac{\left(1+\frac{\mathcal{B}_{green}}{1-\alpha}-\frac{\rho(1-\alpha)^2}{\lambda\mathcal{B}_{green}}\right)\left(\lambda\mathcal{W}_{anticipated}+\rho\right)}{(1-\alpha)\left(\frac{\rho(1-\alpha)^2}{\lambda\mathcal{B}_{green}}+\epsilon-1-\mathcal{B}_{green}\right)} - 1 & \text{if } j^{**} = green \end{cases}.$$

(iii) *markup pricing is eliminated with*

(a) $T_p = \frac{\Gamma_0\bar{E}_0^{\Omega}}{\alpha} - 1$ *if* $j^{**} \in \{gray, green\}$.

(b) $T_p = \frac{1-\alpha}{\alpha}$ *if* $j^{**} = brown$.

Proof See Appendix.

With the corollary we learn that irrespective of the sustainability criteria policies can still be first best if agents lack information about the environmental impact of their innovations. Thereby, it is not only necessary to correct monopolistic market externalities with research subsidies, T_v, and intermediate price subsidies, T_p, but also to impose an environmental tax that eliminates the environmental externality whenever $j^{**} \in \{gray, green\}$. A fixed tax on the environmental effect would not be sufficient since it is necessary to specifically affect research efforts. This tax addresses the social costs of innovations and can thus be understood as a Piguvian tax.

If $j^{**} = gray$, the environmental tax must increase proportionally to $\bar{E}(t)$ but must be accompanied by cost adjustments for vintage providers, denoted by g_{Γ_E}. These adjustments are required to reduce the operating costs of older providers, who would otherwise be crowded out as they would be disproportionately affected by a constantly increasing tax burden[17]. Distinctively, if $j^{**} = green$, the natural capital stock remains constant, so that no dynamic tax adjustments are necessary[18], as expressed with $g_{\Gamma_E} = 0$.

Since the research labor allocation principles are the same as those for agents who have complete information on the environmental impact of innovations, the instrument T_V to achieve $n = n^{**}$ follows the same structure as the one outlined in Corollary (1). The actual tax is then larger if $j^{**} = brown$, since in this case $\mathcal{B}_{anticipated}$ is above $\mathcal{B}_{brown}$ (see the discussion above). In addition, as before, a price subsidy, T_p, is required to eliminate price markups. However, this time this subsidy needs to be adjusted to the environmental taxif $j^{**} \in \{brown, green\}$.

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[17] To give an example, if $j^{**} = gray$, it is socially optimal to improve abatement, but not at an intensity that stops the depletion of natural capital.With new innovations, however, each unit of emissions becomes more expensive, although older providers have not added any further emissions. This increases their operating costs so they are crowded out. Therefore, the environmental tax rate of vintage suppliers needs to be adjusted to stabilize the technology distribution. While it would also be possible to apply a discriminating tax that charges distinct prices for new innovators, such a tax would be subject to strong vertical equity concerns and is, therefore, not further discussed.

[18] Note further that if $j^{**} \in \{gray, green\}$, then $\Gamma_{E,0} = \alpha \frac{1+T_p}{\bar{E}_0^{\Omega}}$, so $\Gamma_{E,0}$ and T_p need to be set in a fixed proportion but are not further identified.

6.2 Research in an SK Regime

When innovation requires specialization, research spillover effects become path-specific. The consequences of such a scenario are addressed next.

Proposition 5. *Suppose an economy is characterized by an SK regime and* $\mathcal{I} = 1$*, then*

$$\gamma = \begin{cases} 1 & \textit{if } \; R(t)^{(1-\alpha)(1-\phi)} > (1 + T_{V,R})G(t)^{\phi-\alpha(1-\phi)} \\ 0 & \textit{if } \; R(t)^{(1-\alpha)(1-\phi)} < (1 + T_{V,R})G(t)^{\phi-\alpha(1-\phi)} \\ \gamma \in [0,1] & \textit{if } \; R(t)^{(1-\alpha)(1-\phi)} = (1 + T_{V,R})G(t)^{\phi-\alpha(1-\phi)} \;\; \textit{and} \;\; \varsigma_R = \varsigma_G = \phi. \end{cases}$$

Along a BGP $\mathcal{B} = \mathcal{B}^{**}$ *if* $\phi \geq 1$*, while if* $1 > \phi$*,* $\mathcal{B} = \mathcal{B}^{**}$ *only if:*

(i) $\varsigma_R = \varsigma_G = \phi$.

(ii) $j^* =$ *brown and* $R(t)^{(1-\alpha)(1-\phi)} > (1 + T_{V,R})G(t)^{\phi-\alpha(1-\phi)}$.

(iii) $j^* =$ *green. and* $R(t)^{(1-\alpha)(1-\phi)} < (1 + T_{V,R})G(t)^{\phi-\alpha(1-\phi)}$.

Further, $n \gtreqless n^{**}$*, whereby n follows the principles established in Proposition (3).*

Proof: See Appendix.

The proposition sheds light on the fundamental role of the access to technology for the sustainability and social optimality of decentralized research. If a strong sustainability is required, agents specialize in abating all pollution growth, which is sustainable and socially optimal. The picture may differ in a weak sustainability scenario. Thereby first note that as there is no gray direction of technical change, it is questionable to consider this direction as social optimal. In the sense of the analysis, however, it is appealing to clarify the costs of restricted access to technologies. Therefore, the gray direction will remain a reference scenario.

Against this background, in a weakly sustainable growth scenario specialization evidently leads to socially suboptimal research if a gray direction of technical change is socially optimal. Yet, it is also possible to observe socially suboptimal research if alternative paths are welfare superior because of technology-related lock-in effects.

With specialization, researchers compare how path-specific spillover effects impact the value of an innovation in the next period. This results in a comparison of the net technology effects of adaptation, $R(t)^{(1-\alpha)(1-\phi)}$, with the tax-weighted net technology effects of abatement, $(1+T_{V,R})G(t)^{\phi-\alpha(1-\phi)}$, as all other determinants of the value of an innovation are identical. The higher the weighted technology inventory, the higher the returns on research in a specific direction. In the arbitrary case where these two components are equivalent, research typically chooses the more efficient direction, so γ is indefinite only in the stylized instance where $\varsigma_R = \varsigma_G = \phi$. In any other case, the economy is tied to the direction in which the weighted technology stock dominates.

The allocation of researchers follows the same principles as discussed for GK regimes (see Proposition (3)) because research efforts allocate independently of labor. However, $\mathcal{B}$ determines n so that the fraction of research labor may differ in both regimes. In any case, Corollary (3) reveals when and how policies can help to make decentralized research strategies socially optimal.

Corollary 3. *Suppose an economy is characterized by an SK regime with $\mathcal{I} = 1$ while lock-in effects prevent a socially optimal direction of technical change, then*

(i) $j^{**} = gray$ cannot be achieved but $T_{V,R} = \frac{R(t)^{(1-\alpha)(1-\phi)}}{G(t)^{\phi-\alpha(1-\phi)}} - 1 < (>) 0$ improves welfare if $R(t)^{1-\alpha} > (<) G(t)^{\frac{1}{1-\alpha}}$ and $g_{green} > (<)\ g_{brown}$.

(ii) if $j^{**} = \{brown, green\}$, $\mathcal{B} = \mathcal{B}^{**}$ can be achieved with

$$T_{V,R} = \begin{cases} \frac{R(t)^{(1-\alpha)(1-\phi)}}{G(t)^{\phi-\alpha(1-\phi)}} - 1 < 0 & \textit{if } j^{**} = green \\ \frac{R(t)^{(1-\alpha)(1-\phi)}}{G(t)^{\phi-\alpha(1-\phi)}} - 1 < 0 & \textit{if } j^{**} = brown. \end{cases}$$

In addition, T_v and T_p that follow the principles discussed in Corollary (1)

(i) are first best, leading to $n = n^{**}$ if $j^{**} \in \{brown, green\}$.

(ii) can improve welfare if $j^{**} = gray$.

Proof: See Appendix.

The corollary demonstrates that whenever technology lock-in effects hold a decentralized specialized economy on a socially inefficient path, a path-specific research subsidy, $T_{V,R}$, can shift research in the preferred direction. Evidently, if $j^{**} = gray$, such a shift is not possible. In this case, policies can still improve welfare if lock-in effects keep the economy from the faster-growing research direction. The corollary also reveals that the policy recommendations to reach a socially optimal fraction of researchers with T_V and to eliminate markup pricing with T_p follow the principles outlined in Corollary (1) so that these policies can internalize monopolistic market externalities. Therefore, when $j^{**}\{brown, green\}$, policies can be first best.

Environmental Externalities in an SK Regime

Proposition (5) revealed that if in an SK regime, lock-in effects prevent an economy from achieving a socially optimal direction of technical change, the depletion of the natural capital stock is socially suboptimal so an environmental externality occurs. Environmental externalities can further be caused by a lack of information on the environmental impacts of innovations, as addressed next.

Proposition 6. *If $\mathcal{I} = 0$, research efforts and labor allocation principles are not affected by whether the economy is characterized by an SK regime or a GK regime.*

Proof See Appendix.

The proposition emphasizes that agents that are not aware about the environmental impact of their innovation choose a brown innovation strategy 'by default' and hence specialize in abatement even if they could combine technologies. The labor allocation principle is thus identical for both innovation regimes, following the description in Proposition (3). However, policy measures to improve the efficiency of research need to account for a lack of information, as explained next.

Corollary 4. *Suppose an economy is characterized by an SK regime and $\mathcal{I} = 0$. If lock-in effects prevent a BGP from being socially optimal, then* *(i) combining* T_E *introduced in Corollary (2) with* $T_{V,R} = \max\{0, \frac{R(t)^{1-\alpha\Omega}}{G(t)^{\frac{\Omega}{1-\alpha}}} - 1\}$ *(a) achieves* $\mathcal{B} = \mathcal{B}^{**}$ *if* $j^{**} = green$*, (b) can improve welfare if* $j^{**} = gray$*.*

(ii) T_v *and* T_p *following the principles outlined in Corollary (1) (a) lead to* $n = n^{**}$ *if* $j^{**} \in (brown, green)$ *and (b) can improve welfare if* $j^{**} = gray$*.*

Proof See Appendix.

The corollary underlines that the knowledge regime affect the design of any policy aimed at improving decentralized research decisions. While there is no policy to reach $j^{**} = gray$, policy actions intending to reach $j^{**} = green$ not only require the use of an environmental tax that follows the principles outlined in Corollary (1) but an additional subsidy for green innovation, $T_{v,R}$, in case that the preexisting stock of adaptation knowledge is so large that the economy is locked-in in $j = brown$. In addition, all scenarios require the R&D policy introduced in Corollary (1), i.e. T_v for socially optimal research labor and T_p to eliminate price markups, both according to the patterns discussed. So if $j^{**} = gray$, policies can only lead to a second best scenario, while if $j^{**} = \{brown, green\}$ policies are first best.

6.3 A Chance for Education Policy

Table (1) summarizes the policy related key results of this theory. The table reveals that while research sector policies (i.e. intermediate price subsidy, research subsidy) are necessary to internalize extrernalities of the monopolistic competitive R&D sector, environmental policy (i.e. environmental tax) is unnecessary as long as access to information and technology is not restricted.

What is interesting with these results is their implicit policy implication. Any measures to facilitate access to information and technology have a similar effect as tax-based environmental policy. This puts non-market-based policies such as education policy into the spotlight. Examples include investments in education infrastructure (schools, universities, and public research laboratories, etc.) and investments in knowledge exchange (simplified patent and cooperation agreements, etc.). Their promising feature is that they can reduce income inequality, see e.g. Biggs and Dutta

	GK Regime		**SK Regime**	
	$\mathcal{I} = 1$	$\mathcal{I} = 0$	$\mathcal{I} = 1$	$\mathcal{I} = 0$
no env. externality	√	√ if j^{**}=brown ×j^{**}=gray, green	√ if j^{**}=brown, green & if no lock-in × if j^{**}=gray	√ if j^{**}=brown × if j^{**}=gray, green
no R&D externalities	$\times^i$ i=except in an arbitrary case	$\times^i$	$\times^i$	$\times^i$
policies first best	√	√	√ if j^{**}=brown, green × if j^{**}=gray	√ if j^{**}=brown, green × if j^{**}=gray

Table 1: Decentralized market characteristics and the potential of policy.

(1999), Sylwester (2002), or Abdullah, Doucouliagos, and Manning (2015) what is questionable for market-based environmental policy. The latter is well-known for affecting lower-income groups disproportionally, see e.g. Büchs, Bardsley, and Duwe (2011), Fullerton (2017), or Edenhofer, Franks, and Kalkuhl (2021), and raises the ethical question of a fair tax burden on higher-income groups, a dispute that is crucial but deflects from the core purpose of the directive. In that regard, this work can be read as an encouragement to shift the focus of environmental policy from taxation to education. Accordingly, the next section discusses the results.

7 Discussion

This article demonstrates that a decentralized economy can find socially optimal environmental research efforts, although its production exhausts public natural capital that lacks property rights. These results are based on strong simplifying assumptions, some of which are critically discussed in this section.

This theory differs from other models in the depletion of natural capital only affecting production. Hence, the environmental quality has no impact on household utility (which is often discussed as environmental amenity effects, see e.g. Shechter, 1991). However, the discussion of such effects is only useful if households have the opportunity to choose between products with different ecological footprints. In most theoretical assessments, they are not given such a choice. Hence, there is an environmental externality by default. This theory shifts the discussion from the demand side for goods and services to the supply side of alternative technologies and evaluates the ability of innovators to influence the environmental quality of production. The environmental amenity effect is thus modeled on behalf of the production damage[19]. Innovators face this damage and can improve a portfolio of alternative technologies what reveals that there are indeed endogenous incentives for socially optimal environmental research; similar insights could be obtained if consumers were allowed to choose between products with alternative environmental footprints. Importantly, these results are not intended to call into question the validity of other literature findings but raise concerns of whether the supply side of technology is sufficiently represented in standard environmental economic theory.

A feature of this theory is that the environmental effect can in principle be interpreted as a stock or a flow effect. While environmental pollution is often represented via flow effects, climate change is usually modeled as a stock challenge, see e.g. Perman et al. (2003) for an orientation[20]. Technically, this theory states that there is an environmental effect which is proportional to production what relates to a flow representation. Yet, the net damages are proportional to the technology stock. While an accumulation of flow effects over time would refer to a different channel of environmental impact than looking at technology inventory, the theory could interpret its inventory variables slightly differently to discuss a standard stock challenge.

The developed model represents a new way to evaluate a portfolio of technologies, the question arises as to whether it can be used for other discussions. A promising area is patent protection. For this purpose, R could be understood as an attempt to improve the patent protection of individual products (e.g. branding or a technical

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[19] Note that many topics that are commonly associated with utility effects can also be analyzed via production effects. Examples are health effects or indirect consumption impacts (the environmental effect 'rescales' consumption-based utility).

[20] Yet, there is early environmental growth theory indicating that environmental growth effects are qualitatively not strongly affected by whether the environmental effect describes a stock or a flow variable, see e.g. Keeler, Spence, and Zeckhauser (1971), Maler (1974), or Brock (1977).

identity). The higher R the more difficult it is to combine products. So while the increased individuality of products increases production, it also reduces the ability of the economy to innovate, as fewer technologies can be developed that resemble one another. The corresponding damage would then be related to $\bar{E}$. Ultimately, there will be efforts to circumvent the strictness of patent protection through clever technological adaptations that are not hindered by patent law. They could be represented with G. The greater G, the less damage caused by patent laws. Such an application of theory enables a nuanced discussion about the role of patent design that can e.g. be related to Schovsbo, Riis, and Petersen (2015) who assess the design of the Unified Patent Court (UPC) as the new patent judiciary for enforcement of European patents. For a more general discussion see e.g. also Lanjouw and Mody (1996) analyzing the role of patent protection for environmentally friendly technology and Haber (2015) for a general assessment on how patents affect innovation. Leaving this promising debate to future research, it remains to conclude.

8 Conclusion

There were various fundamental discoveries in environmental economic growth theory that have greatly improved our understanding of the determinants of sustainable and socially optimal growth. Yet, the main focus of the debate seems on how innovations should be used to address environmental externalities. This view can be misleading as it disregards decentralized R&D incentives for initiating green research.

With a theory shift on the supply-side of innovations, this chapter shows that a decentralized economy which (1.) has extensive information on the environmental impact of its actions and (2.) can access and improve a technology portfolio that combines general technologies with adaptation and abatement knowledge, finds socially optimal environmental research efforts so that decentralized growth is sustainable. The only remaining externalities come with a monopolistic R&D sector. A calibration suggests that they lead to too little research and too little innovation. These distortions do neither affect the sustainability of decentralized growth nor the social optimality of the environmental innovation strategy and can be internalized if (1.)

price subsidies reduce monopolistic intermediate price markups and (2.) research subsidies attract sufficient research labor.

The socially optimal research strategy depends on the elasticity of environmental damage, which describes the sensitivity of production to a production-related depletion of natural capital. As long as this elasticity is below unity, a weak sustainability criterion must be fulfilled so that the economy can compensate for the exhaustion of natural capital through general innovations, adaptation, and abatement. However, if the elasticity is above unity, strong sustainability is required so that the representative natural capital stock of this theory must remain constant.

A calibration reveals that for environmental damage elasticities below 25%, a brown direction of technical change combining general technological improvements and adaptation is socially optimal. For damage elasticities between approximately 25% and 55%, a gray direction of technical change relating to a combination of general technological improvements with adaptation and abatement (in which the exhaustion of natural capital continues) is socially optimal. For higher environmental damage elasticities, it is socially optimal to stop the exhaustion of natural capital with sufficient abatement, a path referred to as a green direction of technical change.

Decentralized markets apply these socially optimal research strategies, so no environmental externalities emerge. However, such externalities can arise when agents do not have complete information on the environmental impact of their actions or when R&D needs to specialize in either adaptation or abatement. In the former case, a Pigouvian-type environmental tax can internalize the environmental externalities. In the latter case, research subsidies can help overcome technology-related locked-in effects. Nevertheless, they cannot lead to parallel innovations in adaptation and abatement. So without a specialization, policies can achieve a first best result while there is no need for environmental policy if agents have complete information. With specialization, policies can achieve a first best result for all but a gray direction of technical change.

Summarized, this article should be interpreted as a theory that shifts the focus from environmental policy, which deals with a lack of property rights on natural capital, to environmental policy that aims to make environmental information and technology widely available. The article reveals that perfect information on the environmental footprint of production can make environmental taxes unnecessary, while free access and the ability to combine technologies have a comparable effect on green innovation as research subsidies. While the reasons for a lack of information are diverse and can be related to psychological factors, complexity, political will, or information costs, the difficulty of accessing alternative technologies can arise when these require path-specific expertise, or when patent-related restrictions prevent sectors from combining different technologies. This refines the already rich discussion on the role of knowledge for economic growth and development (see e.g. Van den Berg, 2016 for an overview) with an environmental perspective and increases the focus on the role of structural conditions for socially optimal technical change.

These findings beg the question of how policies can improve environmental awareness and how they can make environmental and general knowledge widely accessible. Practical recommendations are educational programs, less restrictive patent laws, better information networks, and open-access platforms for technology, just to name a few. Accordingly, a promising agenda for future research is to assess the cost-effectiveness of such measures compared to price and subsidy programs. Also interesting is the role and potential of digitization. The pandemic health crisis has enhanced digitization in public and private sectors abruptly. It remains exciting to recognize the extent in which this can facilitate access to information and flexibility in the application of new technologies. Such new forces can support achieving a sustainable growth path and strongly improve welfare.

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10 Appendix

CES function correspondence

The theory provides a special representation of a typical CES function employed in the literature of directed technological change. In a standard CES function production is described by $p = [ax^p + by^p + cz^p]^{\frac{1}{p}}$ with p as the production output, a, b, c as input weights, and x, y, z as alternative inputs. These inputs are produced with their production functions, usually combining technology with some production factors. The elasticity of substitution between inputs in the production of the final good is constant and described by $\sigma = 1/(1-p)$. The combination of no-arbitrage conditions in the factor markets and decreasing returns in factor inputs then leads to the sector with the lower technology growth rates attracting all production factors if the inputs are complements, as described with $0 \leq \sigma < 1$, while the sector with the higher technology growth rates attracts all production factors if the inputs are substitutes, as described with $1 < \sigma$.

The direction of technical change is thus determined by σ, which is the parameter of interest for the literature. While any σ below or above unity will lead to an either-or decision between technologies, this theory is interested in the intermediate case where $p \to 0$, $\sigma \to 1$ which leads to $I = x^a y^b z^c$. Therefore, a kind of hybrid model is presented that can be compared well with the standard literature on endogenous growth. The contribution of this model is that it takes a technology portfolio into account, provided that there is free access to technology. This is then compared to a perfect substitute scenario which is discussed as specialization and characterized by $p \to 1$ so that $\sigma \to \infty$. So essentially, only strong complements are not discussed. While they are dealt with in detail in Acemoglu, Philippe Aghion, et al. (2012), for this theory they would mainly influence the discussion if the decentralized technology portfolio were to differ from the social planner portfolio. This may only be interesting when discussing environmental amenity effects which are not addressed here, for the reasons explained in the discussion section.

Lemma (1) Following the appendix of Grimaud and Ricci (1999) but applying their methodology to three rather than one innovation direction, consider the cumulative distribution function (CDF) for a productivity parameter A_j within the $j \in [0,1]$ dimension along a BGP. Denote $\bar{A}$ as the leading edge technology at an arbitrary date $t = s$, then trivially $F(\bar{A}, s) = 1$, whereas given the replacement rate, it holds that $\frac{dF(\bar{A},t)}{dt} = -\lambda n F(\bar{A}, t)$ so that for $t \geq s$, $F(\bar{A}, t) = e^{-\lambda n(t-s)}$. Now since (12) also has to hold for $A(t) = A(s) = \bar{A}$, the leading edge technology improves according to $A(t) = \bar{A} e^{\lambda n \varsigma_A (\eta)^\theta (t-s)}$. The relation among the leading edge technology in $t = s$ and in $t > s$ thus follows $\frac{\bar{A}}{A(t)} = e^{-\lambda n \varsigma_A (t)(\eta)^\theta (t-s)}$. This gives the CBD: $F(\bar{A}, t) = \left[\frac{\bar{A}}{A(t)}\right]^{\frac{1}{\varsigma_A \eta)^\theta}}$. Now use $a_j(t) := \frac{A_j}{\bar{A}(t)} \in (0,1]$ to relate a sector technology to the leading edge productivity in t, then along a BGP, the CBD needs to exhibit $F(a) = a^{\frac{1}{\varsigma_A \eta^\theta}}$. The probability density function (PDF) thus follows $f(a) = \frac{a^{\frac{1}{\varsigma_A \eta^\theta} - 1}}{\varsigma_A (\eta)^\theta}$. Denoting the average technology with $\bar{a}(t)$, then $\bar{a}(t) := \int_0^1 A_j(t) dj = A(t) \int_0^1 a_j(t) dj = A(t) \int_0^1 a f(a) da = A(t) \int_0^1 \frac{a^{\frac{1}{\varsigma_A \eta^\theta}}}{\varsigma_A \eta^\theta} da = \frac{A(t)}{1 + \varsigma_A \eta^\theta}$. Thus

$$\bar{a}(t) = \frac{A(t)}{1 + \varsigma_A \eta^\theta}.$$

In this theory, a BGP will follow basic Ramsey properties, i.e. $i(t) = \dot{K}(t) = y(t) - c(t) - P(t)$, so that $g_k(t) = g_k = g_c = g_y = g_P$ if $P(t) > 0$, respectively $g_k(t) = g_k = g_c = g_y$ if $P(t) = 0$ (see later). Now consider $\mathcal{I}(t) := A(t)R(t)G(t)$, which grows with $g_{\mathcal{I}}(t) = g_A(t) + g_R(t) + g_G(t)$, thus $g_{\mathcal{I}}(t) = g_A(t) + g_R(t) + g_G(t) = \lambda n(t) \mathcal{J}(t)$ with $\mathcal{J}(t) := \varsigma_A \eta(t)^\theta + \varsigma_R \kappa(t)^\theta + \varsigma_G (1 - \kappa(t) - \eta(t))^\theta$. Given the proportional technology relation, along a BGP the average representation of this technology bundle must satisfy

$$\bar{\imath}(t) = \frac{\mathcal{I}(t)}{1 + \mathcal{J}(t)}$$

where $\bar{\imath}(t)$ denotes the average technological intensity. Since $\int_0^1 \mathcal{I}_j(t) x_j(t) dj = K(t)$, a BGP requires that $\bar{\imath}(t)$ grows at a constant rate, so $g_{\mathcal{I}} = g_{\bar{\imath}}$, while κ and η need to be constant. Including $x_j(t) = \frac{K(t)}{\mathcal{I}_j(t)}$ in (1) leads to $y(t) = (1-n)^{1-\alpha} K(t)^\alpha \int_0^1 \frac{A_j(t)^{1-\alpha} R_j(t)^{1-\alpha}}{\bar{E}(t)^{\iota\omega} G_j(t)^\alpha} dj$.

Similarly,

$$\bar{E}(t) = (1-n)^{1-\alpha} K(t)^{\alpha} \Big[\int_0^1 \frac{A_j(t)^{1-\alpha} R_j(t)^{1-\alpha}}{G_j(t)^{1+\alpha}} dj \Big].$$

Next, distinguish $g_{\bar{E}}(t) > 0$ from $g_{\bar{E}}(t) = 0$. If $g_{\bar{E}}(t) > 0$, including $\bar{E}(t)$ in $y(t)$ gives

$$y(t) = (1-n)^{(1-\alpha)(1-\phi)} K(t)^{\alpha(1-\phi)} \frac{\int_0^1 \frac{(A_j(t)R_j(t))^{1-\alpha}}{G_j(t)^{\alpha}} dj}{\Big[\int_0^1 \frac{(A_j(t)R_j(t))^{1-\alpha}}{G_j(t)^{1+\alpha}} dj \Big]^{\phi}}.$$

Defining $H_j(t) := \frac{A_j^{1-\alpha} R_j^{1-\alpha}}{G_j^{\alpha}}$, along a BGP, the average technology bundle must follow $\bar{h}(t) = \frac{H(t)}{1+\mathcal{H}}$, whereby $\mathcal{H} := (1-\alpha)\varsigma_A \eta^{\theta} + (1-\alpha)\varsigma_R \kappa^{\theta} - \alpha \varsigma_G (1-\kappa-\eta)^{\theta}$. Similarly, defining $P_i := \int_0^1 \frac{A_j^{1-\alpha} R_j^{1-\alpha}}{G_j^{1+\alpha}} dj$, then along a BGP $\bar{p}(t) = \frac{P(t)}{1+\mathcal{P}}$ with $\mathcal{P} := (1-\alpha)\varsigma_A \eta^{\theta} + (1-\alpha)\varsigma_R \kappa^{\theta} - (1+\alpha)\varsigma_G (1-\kappa-\eta)^{\theta}$. In combination, $y(t) = (1-n)^{(1-\alpha)(1-\phi)} K(t)^{\alpha(1-\phi)} \frac{\bar{h}(t)}{\bar{p}(t)^{\phi}}$. Along a BGP, $\bar{h}(t)$ and $\bar{p}(t)$ both grow with the same sequence of innovations. So using standardization, it must be possible to express the above results with $y(t) = (1-n)^{(1-\alpha)(1-\phi)} K(t)^{\alpha(1-\phi)} \bar{b}(t)$ with $\bar{b}(t) = \frac{B(t)}{1+\mathcal{B}}$, where

$$\bar{b}(t) := \bar{r}(t)^{(1-\alpha)(1-\phi)} \bar{a}(t)^{(1-\alpha)(1-\phi)} \bar{g}(t)^{\phi-\alpha(1-\phi)} = \frac{R(t)^{(1-\alpha)(1-\phi)} A(t)^{(1-\alpha)(1-\phi)} G(t)^{\phi-\alpha(1-\phi)}}{1+\mathcal{B}}$$

with $\bar{r}(t), \bar{a}(t), \bar{g}(t)$ as average adaptation knowledge, average general knowledge, and average abatement knowledge respectively. Thus

$$B(t) := R(t)^{(1-\alpha)(1-\phi)} A(t)^{(1-\alpha)(1-\phi)} G(t)^{\phi-\alpha(1-\phi)}$$

$$\mathcal{B} := (1-\phi)(1-\alpha)[\varsigma_A \eta^{\theta} + \varsigma_R \kappa^{\theta}] + [\phi - \alpha(1-\phi)]\varsigma_G (1-\kappa-\eta)^{\theta}.$$

Also note that $g_B = \lambda n \mathcal{B}$. Yet, along a BGP, $g_y = g_K$, thus

$$g_y = \frac{(1-\alpha)(1-\phi)\big(g_R + g_A\big) + (\phi - \alpha(1-\phi))g_G}{1-\alpha(1-\phi)} = \frac{\lambda n \mathcal{B}}{1-\alpha(1-\phi)}.$$

As detailed with Proposition (1) there is then only positive growth if $1 > \phi$. Note that $g_{\bar{E}} \leq 0$ ultimately leads to $\phi = 0$, thus $\kappa = 0$, so that with standardization

$\bar{E}(t) = \bar{E}_0 = R_0 = 1$. In this case, $x_j = \frac{k_j}{A_j G_j}$ so that expression (1) reads

$$y(t) = (1-n)^{1-\alpha} K(t)^{\alpha} \int_0^1 \frac{A_j(t)^{1-\alpha}}{G_j(t)^{\alpha}} dj$$

which accounts for that investments in G do not have a direct productivity effect, but an indirect productivity effect, as they make the production of intermediates more capital intense. Thereby, $g_y = \alpha g_K + (1-\alpha) g_A - \alpha g_G$. Since $g_y = g_K$, this gives $g_{\bar{E}} = g_y - g_G$, thus $g_y = g_A - \frac{\alpha}{1-\alpha} g_G = \lambda n \big(\varsigma_A \eta^{\theta} - \frac{\alpha}{1-\alpha} \varsigma_G (1-\eta)^{\theta} \big)$. Since $g_{\bar{E}} = g_y - g_G$, $g_{\bar{E}} = 0$ requires that $g_A = \frac{1}{1-\alpha} g_G$, $\bar{\eta} = \frac{1}{1+\left(\frac{(1-\alpha)\varsigma_A}{\varsigma_G}\right)^{\frac{1}{\theta}}}$. Therefore $\hat{\mathcal{B}} := \frac{(1-\alpha)\varsigma_A \varsigma_G}{\left(((1-\alpha)\varsigma_A)^{\frac{1}{\theta}} + \varsigma_G^{\frac{1}{\theta}}\right)^{\theta}}$ and $g_y = \lambda n \hat{\mathcal{B}}$, whilst $\hat{\bar{b}}$ and $\hat{B}$ follow with the above principles. □

Proposition (1) First, for (ii), Lemma (1) has derived that along a BGP $g_K = g_c = g_y$. Further, $g_K = g_x + g_{\bar{\iota}}$, with $g_{\bar{\iota}} = \lambda n \mathcal{J}$, where $\mathcal{J} := \varsigma_A \eta^{\theta} + \varsigma_R \kappa^{\theta} + \varsigma_G (1-\kappa-\eta)^{\theta}$. With (9), a BGP requires $g_p = g_{\varrho} + g_{\bar{\iota}}$. If this is the case, then along a BGP: $g_p = g_A + g_R + g_G + g_{\varrho} = g_{\bar{\iota}} + g_{\varrho}$ so that with (6), $g_x + g_{\bar{\iota}} = g_K$, where g_x refers to the (average) intermediate goods growth rate and g_K to the average capital growth rate. Along a BGP, we thus find that $g_x + g_{\bar{\iota}} = g_x + g_p - g_{\varrho} = g_K = g_y$. Further (9) states that $g_{\pi} = g_p + g_x$. Therefore, $g_{\pi} = g_p + g_x = g_{\varrho} + g_K$. However, for a BGP, (11) must follow

$$V(t) = \Big(\frac{1-\alpha}{\alpha}\Big) \frac{\pi(t)}{r + \lambda n - g_{\bar{\pi}}}$$

with $g_{\bar{\pi}}$ denoting the profit growth rate after innovation. This rate must remain constant because a constant n in the no arbitrage conditions (17) and (21) requires that $V(t)$ grows at a constant rate. Since g_{π} is constant, $g_{\bar{\pi}}$ needs to be constant as well. Now with (10) and (4), a constant n requires that $g_w = g_V$ so that $g_y = g_w = g_V = g_{\pi}$. However, with (9) and (6), along a BGP, it is then necessary that $g_{\pi} = g_p - g_{\bar{\iota}} + g_K$. Noting that $g_{\pi} = g_y = g_K$, this states that $g_p = g_{\bar{\iota}}$. Yet since $g_p = g_{\varrho} + g_{\bar{\iota}}$, balanced growth can only be reached with $g_{\varrho} = 0$. As vintage providers cannot adjust their technologies, any continuous change in ϱ will skew the profit distribution among vintage providers. As discussed with (10) and (4), this is not feasible along a BGP. As a consequence, ϱ needs to remain constant. Now with (9)

$$g_{\bar{\pi}} = \frac{\iota\omega}{1-\alpha} g_{\bar{E}}.$$

Then, with Lemma (1), the marginal product of capital in final production follows

$$r = \alpha(1-\phi)\frac{y}{K}.$$

Note that $g_x = \frac{g_B}{1-\alpha(1-\phi)} - g_{\bar{\iota}}$ yields $g_x = -\frac{(1-\phi)g_G+\phi(g_R+g_A)}{1-\alpha(1-\phi)}$, showing that the physical dimension of intermediate production steadily decreases but is proportionally replaced by technologies. This can be interpreted in response to the fact that environmental impacts are constantly causing damage that motivates individuals to increasingly rely on technology. The environmental effect thus results in an implicit capital depreciation, which is compensated for by technology. The net effect is that capital and output still grow at the same rate. Note therefore that $\Pi(t) = \frac{\Lambda(1-n)}{r^{\frac{\alpha}{1-\alpha}}\bar{E}^{\frac{\iota\omega}{1-\alpha}}} \int_0^j \frac{A_j R_j}{\varrho_j^{\frac{\alpha}{1-\alpha}} G_j^{\frac{\alpha}{1-\alpha}}} dj$. Similarly, (1) and (8) give $y = \frac{\alpha^{\frac{2\alpha}{1-\alpha}}(1-n)}{r^{\frac{\alpha}{1-\alpha}}\bar{E}^{\frac{\iota\omega}{1-\alpha}}} \int_0^j \frac{A_j R_j}{\varrho_j^{\frac{\alpha}{1-\alpha}} G_j^{\frac{\alpha}{1-\alpha}}} dj$ so that $\Pi(t) = (1-\alpha)\alpha(1-n)^{(1-\alpha)(1-\phi)} K(t)^{\alpha(1-\phi)} \frac{B(t)}{1+\mathcal{B}}$. Therefore, $\Pi(t) = \frac{\pi(t)}{1+\mathcal{B}}$, so that $\pi(t) = (1-\alpha)\alpha(1-n)^{(1-\alpha)(1-\phi)} K(t)^{\alpha(1-\phi)} B(t) = (1-\alpha)\alpha y(t)$ yielding $V(t) = \frac{(1-\alpha)^2 y(t)}{(r+\lambda n - g_{\bar{\pi}})}$. If $1 > \phi$, a BGP is therefore described with

$$\hat{g} = g_y = g_\pi = g_V = g_w = g_{\bar{\iota}} + g_x = g_k = \frac{\mathcal{B}}{1-\alpha(1-\phi)}$$

with $\mathcal{B}$ introduced in Lemma (1). The corresponding BGP is then characterized by $g_{\bar{E}} = \alpha g_K + (1-\alpha)(g_A+g_R) - (1+\alpha)g_G$. Thus using $g_y = \frac{1}{1-\alpha(1-\phi)}\big[(1-\alpha)(1-\phi)\big(g_R + g_A\big) + (\phi - \alpha(1-\phi))g_G\big]$ and $g_y = g_K$ leads to $g_{\bar{E}} = \frac{(1-\alpha)(g_R+g_A)-(\alpha+(1-\phi)(1-\alpha^2))g_G}{(1-\alpha(1-\phi))}$, respectively $g_y = (1-\phi)g_E - (\phi - \alpha(1-\phi)g_G)$.

If $\phi \geq 1$, then a BGP is only possible if there is no exhaustion of the natural capital stock what requires sufficient abatement, following the conditions defined with Lemma (1), what completes (ii). For (i), proofing the existence of a BGP follows standard Ramsey theory. Denoting variables in efficiency units with a hat, the two differential equations $\dot{\hat{c}} = \frac{(r-\rho)}{\epsilon}\hat{c}$ and $\dot{\hat{k}} = \hat{y} - \hat{c} - \hat{g}\hat{k}$ describe the entire dynamic framework so that the Jacobian matrix evaluated at a steady state reads

$$J(\hat{k}, \hat{c}) = \begin{bmatrix} \partial\dot{\hat{k}}/\partial\hat{k} & \partial\dot{\hat{k}}/\partial\hat{c} \\ \partial\dot{\hat{c}}/\partial\hat{k} & \partial\dot{\hat{c}}/\partial\hat{c} \end{bmatrix} = \begin{bmatrix} r - \hat{g} & -1 \\ 0 & \frac{r-\rho}{\epsilon} \end{bmatrix}$$

where the determinant of the Jacobian matrix proofs local saddle point stability. As $r = \alpha(1-\phi)(1-n)^{(1-\alpha)(1-\phi)} K_0^{\alpha(1-\phi)-1} \frac{(A_0 R_0)^{(1-\alpha)(1-\phi)} G_0^{\phi-\alpha(1-\phi)}}{1+\mathcal{B}}$, there is always a bundle $\{A_0, R_0, G_0, K_0\}$ that clears the savings investment market with

$$\epsilon\frac{\lambda n\mathcal{B}}{1-\alpha(1-\phi)} + \rho = \alpha(1-\phi)(1-n)^{(1-\alpha)(1-\phi)} K_0^{\alpha(1-\phi)-1} \frac{(A_0 R_0)^{(1-\alpha)(1-\phi)} G_0^{\phi-\alpha(1-\phi)}}{1+\mathcal{B}}$$

which satisfies $r > \rho$ so that local saddle point stability exists. The Hamiltonian is jointly concave in control and states, hence Mangasarian's sufficiency theorem applies such that given initial state and transversality conditions, the Maximum Principle yields first order conditions which complete the set of necessary conditions that are sufficient for local stability. It can also be shown that the saddle point is globally stable (for a proof Philippe Aghion, P. Howitt, et al., 1998). Finally, (iii) directly follows since the innovation and production equations are monotonous in their arguments. $\square$

Proposition (2) First, for $g_{\bar{E}} > 0$ and $1 > \phi$, the social planner intends to find the κ, η, n combination that maximizes $c = c(t)$ based on the Hamiltonian

$$H : \frac{c^{1-\epsilon}}{1-\epsilon} + \psi_K(y-c) + \psi_b(\lambda n\mathcal{B}\bar{b}(1+\mathcal{B}))$$

s.t. $\lim_{t\to\infty} e^{-\rho t}\psi_k k(t) = 0$ and $\lim_{t\to\infty} e^{-\rho t}\psi_g \bar{b}(t) = 0$ with $\bar{b}_0, K_0 > 0$ as predetermined and $\bar{b} = \bar{b}(t) = \frac{B(t)}{1+\mathcal{B}}$ with $\dot{B}(t) = \lambda n\mathcal{B}B(t)$. Given this, if $1 > \phi$,

$$H_c = c^{-\epsilon} = \psi_k \tag{i}$$

thus $-g_{\psi_k} = \epsilon g$. Then, $\dot{\psi}_k = -H_k + \rho\psi_k = -\frac{(\alpha-\phi))y}{K}\psi_k + \rho\psi_k$, so that

$$-\frac{\dot{\psi}_K}{\psi_k} = -g_{\psi_k} = \frac{\alpha(1-\phi)y}{K} - \rho \tag{ii}$$

what with (i) results in the social planner Euler equation

$$\frac{\alpha(1-\phi)y}{K} = \epsilon g_c + \rho = \epsilon g + \rho. \tag{iii}$$

Next, $\dot{\psi}_{\bar{b}} = -H_{\bar{b}} + \rho\psi_{\bar{b}}$ yields

$$g_{\psi_{\bar{b}}} = -\frac{\psi_k}{\psi_{\bar{b}}}\frac{y}{\bar{b}} - \lambda n\mathcal{B}(1+\mathcal{B}) + \rho. \tag{iv}$$

Further, H_n gives

$$\psi_k(1-\alpha)(1-\phi)\frac{y}{1-n} = \psi_{\bar{b}}\lambda\mathcal{B}\bar{b}(1+\mathcal{B}). \tag{v}$$

Considering the dynamics of (v), along a BGP $g_{\psi_k} + g_y = g_{\psi_{\bar{b}}} + g_B$. Further, (i) gives $-g_{\psi_k} = \epsilon g_y$, thus $(1-\epsilon)g_y - g_B = g_{\psi_{\bar{b}}}$. Along a BGP, $g_y = \frac{\lambda n\mathcal{B}}{1-\alpha(1-\phi)}$ and $g_B = \lambda n\mathcal{B}$. This gives $\big(\frac{(1-\epsilon)}{1-\alpha(1-\phi)} - 1\big)\lambda n\mathcal{B} = g_{\psi_{\bar{b}}}$. Including this in (iv) gives $\big(\frac{(1-\epsilon)}{1-\alpha(1-\phi)} + \mathcal{B}\big)\lambda n\mathcal{B} - \rho = -\frac{\psi_k}{\psi_{\bar{b}}}\frac{y}{\bar{b}}$, so with (v)

$$n = \frac{\big(1 + \mathcal{B} - \frac{\rho}{\lambda\mathcal{B}}(1-\alpha)(1-\phi)\big)}{\frac{\phi+\epsilon(1-\alpha)(1-\phi)}{1-\alpha(1-\phi)} + \mathcal{B}(\phi + \alpha(1-\phi))}.$$

For $n > 0$, it is necessary that $1 + \mathcal{B} > \frac{\rho}{\lambda\mathcal{B}}$ thus $(1+\mathcal{B})\frac{\lambda\mathcal{B}}{(1-\alpha)(1-\phi)} > \rho$, so that there is a critical $\underline{\mathcal{B}}$ where $(1+\underline{\mathcal{B}})\frac{\lambda\underline{\mathcal{B}}}{(1-\alpha)(1-\phi)} = \rho$. Since the corresponding quadratic equation has a unique (reasonable) solution, this states that $\mathcal{B} > \frac{(1+\frac{4\rho(1-\alpha)(1-\phi)}{\lambda})^{\frac{1}{2}}-1}{2}$. Further, $1 > n$ yields $\rho > \lambda\mathcal{B}\big(\mathcal{B} + \frac{(1-\epsilon)}{1-\alpha(1-\phi)}\big)$, requiring that $\mathcal{B}$ is below a critical $\overline{\mathcal{B}}$ where $\rho = \lambda\overline{\mathcal{B}}\big(\overline{\mathcal{B}} + \frac{(1-\epsilon)}{1-\alpha(1-\phi)}\big)$, thus $\frac{\big(\frac{(1-\epsilon)^2}{1-\alpha(1-\phi)^2}+4\frac{\rho}{\lambda}\big)^{\frac{1}{2}} - \frac{(1-\epsilon)}{1-\alpha(1-\phi)}}{2} > \mathcal{B}$. Combining both critical intensities gives $\frac{\big(\frac{(1-\epsilon)^2}{1-\alpha(1-\phi)^2}+4\frac{\rho}{\lambda}\big)^{\frac{1}{2}} - \frac{(1-\epsilon)}{1-\alpha(1-\phi)}}{2} > \mathcal{B} > \frac{(1+\frac{4\rho(1-\alpha)(1-\phi)}{\lambda})^{\frac{1}{2}}-1}{2}$.

Direction of technical change H_η gives $\psi_b \frac{\partial \mathcal{B}}{\partial \eta}(1+2\mathcal{B}) = 0$, what simplifies to $\psi_b \frac{\partial \mathcal{B}}{\partial \eta} = 0$ and yields $\left(\frac{\varsigma_A}{\Gamma \varsigma_G}\right)^{\frac{1}{1-\theta}} = \frac{\eta}{1-\kappa-\eta}$ so that $\frac{1}{1+\left(\frac{\Gamma\varsigma_G}{\varsigma_A}\right)^{\frac{1}{1-\theta}}}(1-\kappa) = \eta$ with $\Gamma := \left(\frac{\phi}{(1-\phi)(1-\alpha)} - \frac{\alpha}{1-\alpha}\right)$, while H_κ gives $\psi_b \mathcal{B}_\kappa (1+2\mathcal{B})) = 0$, thus $\psi_b \mathcal{B}_\kappa = 0$ so that $\left(\frac{\varsigma_R}{\Gamma\varsigma_G}\right)^{\frac{1}{1-\theta}} = \frac{\kappa}{1-\kappa-\eta}$ thus $1-\kappa\left(1+\left(\frac{\Gamma\varsigma_G}{\varsigma_R}\right)^{\frac{1}{1-\theta}}\right) = \eta$. Therefore, $\kappa^* = \frac{\varsigma_R^{\frac{1}{1-\theta}}}{\left(\varsigma_A^{\frac{1}{1-\theta}} + \varsigma_R^{\frac{1}{1-\theta}} + \left(\Gamma\varsigma_G\right)^{\frac{1}{1-\theta}}\right)}$, $\eta^* = \frac{\varsigma_A^{\frac{1}{1-\theta}}}{\left(\varsigma_A^{\frac{1}{1-\theta}} + \varsigma_R^{\frac{1}{1-\theta}} + \left(\Gamma\varsigma_G\right)^{\frac{1}{1-\theta}}\right)}$ and $1-\kappa^*-\eta^* = \frac{\left(\Gamma\varsigma_G\right)^{\frac{1}{1-\theta}}}{\left(\varsigma_A^{\frac{1}{1-\theta}} + \varsigma_R^{\frac{1}{1-\theta}} + \left(\Gamma\varsigma_G\right)^{\frac{1}{1-\theta}}\right)}$. Yet, for $1 > \phi$ with $g_{\bar{E}} > 0$, there is only abatement if $\Gamma > 0$. As there are improvements in abatement and adaptation, this is called a *gray* direction of technical change. Reformulations yield

$$g_{gray} = \frac{\lambda n}{1-\alpha(1-\phi)} \mathcal{B}_{gray} \;\; \textit{with} \;\; \mathcal{B}_{gray} := (1-\phi)(1-\alpha)\left(\varsigma_A^{\frac{1}{1-\theta}} + \varsigma_R^{\frac{1}{1-\theta}} + \left(\Gamma\varsigma_G\right)^{\frac{1}{1-\theta}}\right)^{1-\theta}.$$

Yet, other alternatives are possible. Consider a complete concentration on general technologies without abatement or adaptation, then $\eta = 1$. Referring to this scenario as a *black* direction of technical change, this leads to

$$g_{black} = \frac{\lambda n}{1-\alpha(1-\phi)} \mathcal{B}_{black} \;\; \textit{with} \;\; \mathcal{B}_{black} := (1-\phi)(1-\alpha)\varsigma_A.$$

Further, consider a *brown* direction of technical change in which there is no abatement but both adaptation and general innovation. Therefore, $\eta^* + \kappa^* = 1$, leading to $\eta^* = \frac{1}{1+\left[\frac{\varsigma_R}{\varsigma_A}\right]^{\frac{1}{1-\theta}}}$ and $\kappa^* = \frac{1}{1+\left[\frac{\varsigma_A}{\varsigma_R}\right]^{\frac{1}{1-\theta}}}$, thus

$$g_{brown} = \frac{\lambda n}{1-\alpha(1-\phi)} \mathcal{B}_{brown} \;\; \textit{with} \;\; \mathcal{B}_{brown} := (1-\phi)(1-\alpha)(\varsigma_A^{\frac{1}{1-\theta}} + \varsigma_R^{\frac{1}{1-\theta}})^{1-\theta}.$$

In addition, consider a no adaptation case, $\kappa = 0$, with abatement, but $g_{\bar{E}} > 0$, called a *yellow* direction of technical change. In this case, efficient research leads to

$\eta^* = \frac{1}{1+\left(\frac{\Gamma\varsigma_G}{\varsigma_A}\right)^{\frac{1}{1-\theta}}}$ again with $\Gamma := \left(\frac{\phi}{(1-\phi)(1-\alpha)} - \frac{\alpha}{1-\alpha}\right)$, so that

$$g_{yellow} = \frac{\lambda n}{1-\alpha(1-\phi)}\mathcal{B}_{yellow} \;\; \textit{with} \;\; \mathcal{B}_{yellow} := (1-\phi)(1-\alpha)\left(\varsigma_A^{\frac{1}{1-\theta}} + (\Gamma\varsigma_G)^{\frac{1}{1-\theta}}\right)^{1-\theta}.$$

Since $N = \frac{1}{\bar{E}^{\iota\omega}} \leq 1$, it is not possible that $g_{\bar{E}} < 0$. Since $g_{\bar{E}} = \frac{(1-\alpha)(g_R+g_A)-(\alpha+(1-\phi)(1-\alpha^2))g_G}{(1-\alpha(1-\phi))}$, see Proposition (1), $(1-\alpha)(g_R+g_A) < (\alpha+(1-\phi)(1-\alpha^2))g_G$. Therefore, $(\varsigma_A\eta^\theta + \varsigma_R\kappa^\theta) > (\frac{\alpha}{(1-\alpha)} + (1-\phi)(1-\alpha))\varsigma_G(1-\kappa-\eta)^\theta$ which translates into $\eta > \frac{1}{1+\left(\frac{\varsigma_A}{\Lambda\varsigma_G}\right)^{\frac{1}{\theta}}}$ with $\Lambda := \left(\frac{\alpha}{(1-\alpha)} + (1-\phi)(1-\alpha)\right)$. Thus $\left(\frac{\varsigma_A}{\varsigma_G}\right)^{\frac{1}{1-\theta}} > \Gamma^{\frac{\theta}{1-\theta}}\Lambda$. Whenever this is not the case, then $g_{\bar{E}} = 0$. For a yellow direction, this condition reads $\frac{\varsigma_A^{\frac{1}{1-\theta}}}{\varsigma_G^{\frac{1}{1-\theta}}} > \Gamma^{\frac{\theta}{1-\theta}}\Lambda$, for a gray direction, this condition reads $\frac{\varsigma_A^{\frac{1}{1-\theta}}+\varsigma_R^{\frac{1}{1-\theta}}}{\varsigma_G^{\frac{1}{1-\theta}}} > \Gamma^{\frac{\theta}{1-\theta}}\Lambda$.

Further, $g_{\bar{E}} = 0$ relates to a *green* direction of technical change, characterized by $\kappa = 0$ (thus no adaptation). As derived with Lemma (1), then $\eta^* = \bar{\eta} = \frac{1}{1+\left[\frac{\varsigma_A}{\varsigma_G}\right]^{\frac{1}{\theta}}}$,

$$g_{green} = \lambda n \mathcal{B}_{green} \;\; \textit{with} \;\; \mathcal{B}^*_{green} = \hat{\mathcal{B}} := \frac{(1-\alpha)\varsigma_A\varsigma_G}{\left(((1-\alpha)\varsigma_A)^{\frac{1}{\theta}} + \varsigma_G^{\frac{1}{\theta}}\right)^\theta}.$$

Importantly, the allocation principle for n is distinct here. Thereby, $g_{\psi_{\bar{b}}} = -\frac{\psi_k}{\psi_{\bar{b}}}\frac{y}{\bar{b}} - \lambda n\mathcal{B}(1+\mathcal{B}) + \rho$, $\psi_k(1-\alpha)\frac{y}{1-n} = \psi_{\bar{b}}\lambda\mathcal{B}\bar{b}(1+\mathcal{B})$, so $-g_{\psi_{\bar{b}}} = \lambda n\mathcal{B}(1+\mathcal{B})\left(\frac{1-n\alpha}{n(1-\alpha)}\right) - \rho$. Further, (i) gives $-g_{\psi_k} = \epsilon g_y$, thus $(1-\epsilon)g_y - g_B = g_{\psi_{\bar{b}}}$. However, along a green direction $g_y = g_B = \lambda n\mathcal{B}$ so $-\epsilon\lambda n\mathcal{B} = g_{\psi_{\bar{b}}}$. Therefore,

$$n_{green} = \left(\frac{(1-\alpha)}{\epsilon(1-\alpha)+\alpha}\right)\left(\frac{(1+\mathcal{B}(1-\alpha))}{(1-\alpha)} - \frac{\rho}{\lambda\mathcal{B}(1-\alpha)}\right).$$

So for $1 > n$, it is necessary that $\frac{\left(\frac{(1-\epsilon)^2}{(1-\alpha)^2}+4\frac{\rho}{\lambda}\right)^{\frac{1}{2}} - \frac{(1-\epsilon)}{(1-\alpha)}}{2} > \mathcal{B}_{green}$, so $\frac{\left(\frac{(1-\epsilon)^2}{(1-\alpha)^2}+4\frac{\rho}{\lambda}\right)^{\frac{1}{2}} - \frac{(1-\epsilon)}{(1-\alpha)}}{2} > \mathcal{B}_{green} > \frac{(1+\frac{4\rho(1-\alpha)(1-\phi)}{\lambda})^{\frac{1}{2}}-1}{2}$.

Now, for $1 > \phi$, any combination of research efforts is sustainable. The planner chooses the path with the highest consumption growth rate as utility is purely consumption based while growth effects dominate level effects. Since n and $\mathcal{B}$ are positively correlated, a planner will never choose a black and a yellow direction of technical change since $\mathcal{B}_{brown} > \mathcal{B}_{black}$, what reduces to $(\varsigma_A^{\frac{1}{1-\theta}} + \varsigma_R^{\frac{1}{1-\theta}})^{1-\theta} > \varsigma_A$ and is always satisfied when $\varsigma_R > 0$. The same argument holds for $\mathcal{B}_{gray} > \mathcal{B}_{yellow}$, as long as $\varsigma_R > 0$, and for $\mathcal{B}_{gray} > \mathcal{B}_{brown}$ as long as $\Gamma > 0$. $\square$

Proposition (2) With Proposition (1), $\pi(t) = (1-\alpha)\alpha(1-n)^{(1-\alpha)(1-\phi)}K(t)^{\alpha(1-\phi)}B(t) = (1-\alpha)\alpha y(t)$, so that if $1 > \phi$, (15) and (16) result in

$$\max_{\kappa,\eta} \mathcal{B} = \max_{\kappa,\eta}(1-\phi)(1-\alpha)[\varsigma_A(\kappa\eta)^\theta + \varsigma_R((1-\kappa)\eta)^\theta] + [\phi - \alpha(1-\phi)]\varsigma_G(1-\eta)^\theta$$

yielding $\eta = \eta^{**}$ and $\kappa = \kappa^{**}$. Further, whenever $\phi \geq 1$, a rational agent will anticipate that there are only profits if there is no exhaustion of the natural capital stock. Therefore, $\kappa = 0$, $\eta = \eta^{**} = \bar{\eta}$, see Proposition (1) and (2). Next, for the labor allocation, Proposition (1) has shown that $(1-\alpha)\frac{y}{(1-n)} = w$ and $V(t) = \frac{(1-\alpha)^2 y(t)}{(r+\lambda n-g_{\bar{\pi}})}$. With this, (23) can be reformulated to $\frac{1}{1-n} = (1+T_V)\frac{(1-\alpha)}{(r+\lambda n-g_{\bar{\pi}})}$. If $1 > \phi$, $\frac{(g_y-(\phi-\alpha(1-\phi))g_G)}{(1-\phi)} = g_{\bar{E}}$, thus with $g_{\bar{\pi}} = \frac{\phi}{1-\alpha}g_{\bar{E}}$, we find $g_{\bar{\pi}} = \frac{\phi(g_y-(\phi-\alpha(1-\phi)g_G)}{(1-\phi)(1-\alpha)}$. Now with (23), it also holds that $\epsilon\lambda n\frac{\mathcal{B}}{1-\alpha(1-\phi)} + \rho = r$ so that

$$\frac{\lambda n\mathcal{W} + \rho}{1-n} = \lambda(1+T_V)(1-\alpha)$$

with $\mathcal{W} = \frac{\mathcal{B}}{1-\alpha(1-\phi)}\left(\epsilon + \frac{\phi}{(1-\phi)(1-\alpha)}\right) + 1 - \frac{\phi(\phi-\alpha(1-\phi)\varsigma_G(1-\eta)^\theta))}{(1-\phi)(1-\alpha)}$. Therefore, reformulating results gives $n = \frac{\lambda(1+T_V)(1-\alpha)-\rho}{(\lambda\mathcal{W}+(1+T_V)(1-\alpha))}$, so that since $\partial n/\partial T_V > 0$ (as $\partial n/\partial T_V = \frac{(1-\alpha)(1-n)}{(\lambda\mathcal{W}+(1+T_V)(1-\alpha))}$), a research subsidy $T_V > 0$ increases n. Whenever $j^{**} = green$, then decentralized agents choose to fully abate any increase in the environmental effect. Therefore, $g_{\bar{\pi}} = 0$ so that $\frac{1}{1-n} = (1+T_V)\frac{(1-\alpha)}{(r+\lambda n)}$ what then with $\epsilon\lambda n\hat{\mathcal{B}} + \rho = r$ again yields $n = \frac{(1+T_V)(1-\alpha)-\rho}{(\lambda\mathcal{W}+(1+T_V)(1-\alpha))}$, this time with $\mathcal{W} = \epsilon\hat{\mathcal{B}} + 1$. $\square$

Calibration The calibration is based on the reference literature in combination with current data. It is thus best related to an early stage of environmental degradation where the elasticity of environmental damage is relatively low, so research is following a brown direction of technical change per assumption. The economy is in the process of updating its information on the environmental impact and is starting to consider a gray, or green direction of technical change. Current literature usually sets α between 0.3 and $\frac{1}{3}$. Therefore, $\alpha = \frac{1}{3}$ and $\alpha(1-\phi) = 0.3$ what leads $\phi = 0.1$. Proposition (3) then gives $\lambda n \mathcal{W}_{brown} = \frac{2}{3}(\lambda - n) - \rho$ and $\mathcal{W}_{brown} = \frac{\mathcal{B}\left(\epsilon + \frac{\phi}{(1-\phi)(1-\alpha)}\right)}{1-\alpha(1-\phi)} + 1 \approx 2.96\mathcal{B} + 1$, so that $\frac{\mathcal{W}_{brown}-1}{2.96} = \mathcal{B}$.

With this $g_y = \lambda n \frac{\mathcal{B}}{1-\alpha(1-\phi)}$, so $0.0432 \approx \lambda n(\mathcal{W}_{brown} - 1)$ thus combining results and using $\rho = 0.015$ (Nordhaus, 2007) leads to $0.0873 \approx \lambda - n - \frac{3}{2}\lambda n$ which with $\lambda = 0.5$ (Ricci, 2007) gives $n \approx 0.236$. Note that this fraction of researchers can be interpreted quite broadly as it simply describes individuals engaged in innovative activities. In any case, $g_y = \lambda n \frac{\mathcal{B}}{1-\alpha(1-\phi)}$ gives $0.124 \approx \mathcal{B}$.

Now since $\mathcal{B}_{brown} = (1-\phi)(1-\alpha)(\varsigma_A^{\frac{1}{1-\theta}} + \varsigma_R^{\frac{1}{1-\theta}})^{1-\theta}$, $0.2 \approx (\varsigma_A^{\frac{1}{1-\theta}} + \varsigma_R^{\frac{1}{1-\theta}})^{1-\theta}$, while with $x\varsigma_A = \varsigma_R$, $\frac{0.2}{(1+x^{\frac{1}{1-\theta}})^{1-\theta}} \approx \varsigma_A$. It is then worth to consider 2 scenarios. Scenario (1) relating to a case where adaptation is 100% as efficient as general research, so $x = 1$ which serves as a benchmark.

Scenario (2) (serving as a robustness check) considers $x = 0.5$, stating that adaptation research is only 50% as effective as general research. For direct comparability of adaptation and abatement, $\varsigma_R = \varsigma_G$. With this, Scenario (1) leads to $\varsigma_A \approx 0.14$ so that $\varsigma_A := 0.14$ and $\varsigma_R := 0.14$. Scenario (2) leads to $\varsigma_A \approx 0.18$ so that $\varsigma_A := 0.18$ and $\varsigma_R := 0.09$. Scenario (2) is presented with Figure (3) here, while the benchmark results are presented and discussed in the text.

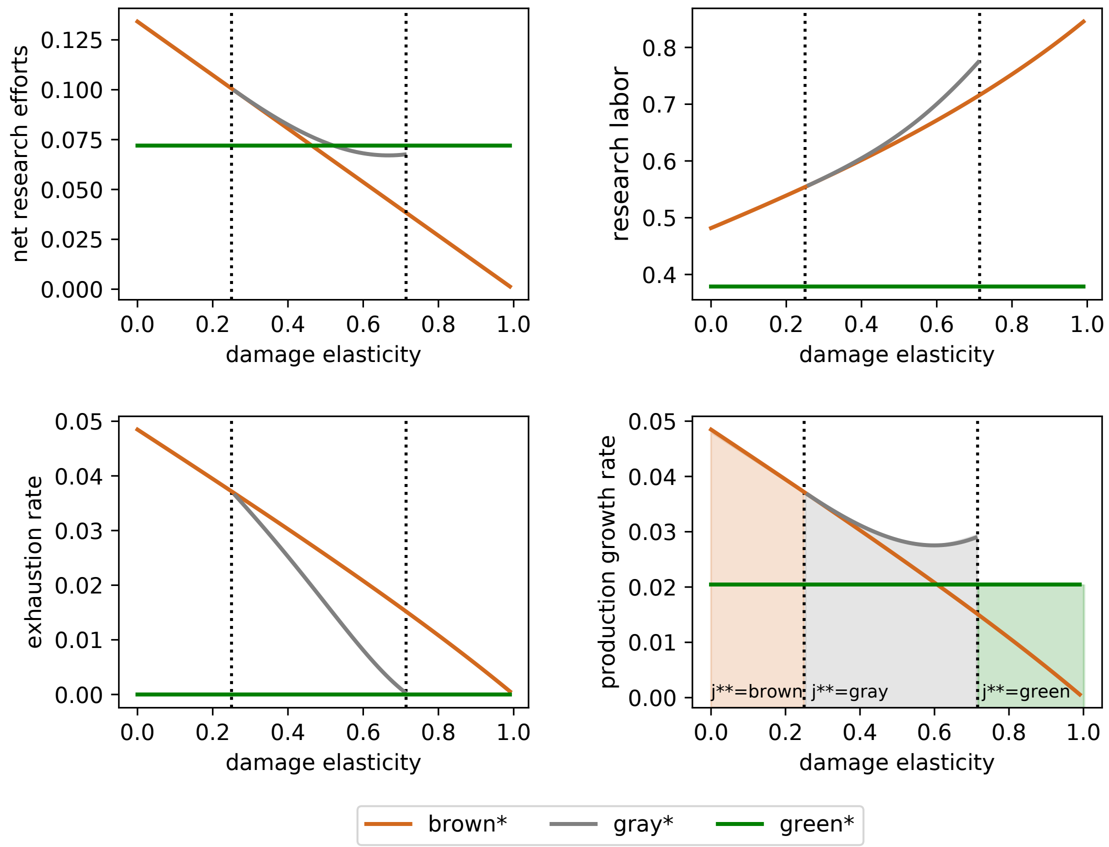

Figure 3: Social planner responses of net efforts $\mathcal{B}^*$ (upper left), research labor n^* (upper right), natural capital depletion rate $g^*_{\bar{E}}$ (lower left) and production growth rates g^*_y (lower right) when adaptation and abatement focused research is only 50% as efficient as general research.

Corollary (1) Reformulating the research labor allocation principle for T_V and setting $n = n^{**}$ leads to $\frac{\lambda n^{**}\mathcal{W}+\rho}{(1-n^{**})(1-\alpha)} - 1 = T_V$. Since the markup is determined by $\frac{1}{\alpha}$, it is necessary to choose $1 - T_p = \frac{1}{\alpha}$, so $T_p = \frac{1-\alpha}{\alpha}$. $\square$

Proposition (4) Agents that have no information about the environmental implications of an innovation consider a production and innovation technology based on $\phi = 0$. Therefore, they face $y = \frac{(1-n)^{1-\alpha}}{\bar{E}^{\iota\omega}} K^{\alpha} \underline{b}$ with $\underline{b} = \frac{\underline{B}}{1+\underline{\mathcal{B}}}$ with $\underline{B} := A(t)^{1-\alpha} D(t)^{1-\alpha}$, whereas the intermediate profits in period t described with (9) now follow

$$\pi(t) = \frac{\Lambda(1+T_p)^{\frac{\alpha}{1-\alpha}} A(t)R(t)(1-n)}{(r(1+T_E))^{\frac{\alpha}{1-\alpha}} G(t)^{\frac{\alpha}{1-\alpha}} \bar{E}^{\frac{\iota\omega}{1-\alpha}}}.$$

The agents choose $\kappa + \eta = 1$. Agents therefore address current damages but do not anticipate how they will evolve in near and distant future. Evidently, if $\phi \geq 1$, this innovation strategy is not sustainable, whereas if $1 > \phi$ it is.

For the labor allocation first note that if an environmental effect is ignored, then (4) gives $w = (1-\alpha)\frac{(1-n)^{-\alpha}}{\bar{E}^{\iota\omega}} K^\alpha \underline{b}$ so that with $V(t) = \frac{(1-\alpha)^2}{\lambda n (r+\lambda n - g_{\bar{\pi}})} \frac{(1-n)^{1-\alpha}}{\bar{E}^{\iota\omega}} K^\alpha \underline{b}$, (17) yields $\frac{1}{1-n} = (1+T_V)\frac{(1-\alpha)}{(r+\lambda n - g_{\bar{\pi}})}$. However, the agents do not anticipate environmental effects, so $g_{\bar{\pi}} = 0$ whilst (23) gives $\epsilon \lambda n \frac{\mathcal{B}_{anticipated}}{1-\alpha} + \rho = r_{anticipated}$ as an anticipated discount rate with $\mathcal{B}_{anticipated} := \frac{\mathcal{B}_{brown}}{1-\phi} = (1-\alpha)[\varsigma_A(\kappa)^\theta + \varsigma_R((1-\kappa))^\theta]$. Hence

$$n = \frac{(1+T_V)(1-\alpha) - \rho}{(\lambda \mathcal{W}_{anticipated} + (1+T_V)(1-\alpha))}$$

with $\mathcal{W}_{anticipated} = \epsilon \frac{\mathcal{B}_{anticipated}}{(1-\alpha)(1-\phi)} + 1$. There is thus a twofold bias. Firstly, $\mathcal{B}_{brown} < \mathcal{B}_{anticipated}$. Secondly, $\epsilon \lambda n \frac{\mathcal{B}_{brown}}{1-\alpha(1-\phi)} + \rho = r_{brown} < \epsilon \lambda n \frac{\mathcal{B}_{anticipated}}{1-\alpha} + \rho = r_{anticipated}$ (which is beyond the effort effect). The agents therefore apply the same labor allocation rule as agents with complete information, however, with an explicit focus on $j = brown$.
$\square$

Corollary (2) Expression (9) gives $\pi(t) = \frac{\Lambda(1+T_p)^{\frac{\alpha}{1-\alpha}} A(t)R(t)(1-n)}{(r(1+T_E))^{\frac{\alpha}{1-\alpha}} G(t)^{\frac{\alpha}{1-\alpha}} \bar{E}^{\frac{\iota\omega}{1-\alpha}}}$. Lemma (1) has derived that $\bar{E}(t) = (1-n)^{1-\alpha} K(t)^\alpha \Big[\int_0^1 \frac{A_j(t)^{1-\alpha} R_j(t)^{1-\alpha}}{G_j(t)^{1+\alpha}} dj \Big]$ so that the contribution of the newest innovator to the environmental effect is

$$E(t) = (1-n)^{1-\alpha} K(t)^\alpha \Big[\frac{A(t)^{1-\alpha} R(t)^{1-\alpha}}{G(t)^{1+\alpha}} \Big].$$

For the sake of the argument first assume that the government simply taxes the intensity of the environmental effect with $T_E := \Gamma_E(t)(1-n)^{1-\alpha} K(t)^\alpha \Big[\frac{A(t)^{1-\alpha} R(t)^{1-\alpha}}{G(t)^{1+\alpha}} \Big] - 1$, then the intermediate profit function follows $\pi(t) = \big[A(t)R(t)\big]^{1-\alpha} G(t)^{\frac{1}{1-\alpha}} \mathcal{C}(t)$

with $\mathcal{C}(t) = \frac{\Lambda(1+T_p)^{\frac{\alpha}{1-\alpha}}}{\Gamma_E(t)^{\frac{\alpha}{1-\alpha}}} \frac{(1-n)^{1-\alpha} K(t)^{\alpha}}{(r)^{\frac{\alpha}{1-\alpha}} \bar{E}^{\frac{\iota\omega}{1-\alpha}}}$. Note that along a BGP, $K(t)$ is proportional to $\pi(t)$ so that the marginal contribution of efforts on capital is proportional to $\big(A(t)R(t)\big)^{1-\alpha} G(t)^{\frac{1}{1-\alpha}}$. Therefore, for the following argument, the capital related effects (i.e. repercussions of a tax on the capital endowment) can be ignored since they do not affect the relative weight among technologies. Given this, an innovator evaluates $\max_{\kappa,\eta} \underline{\hat{\mathcal{B}}} = (1-\alpha)\big(\varsigma_A \eta^\theta + \varsigma_R (\kappa)^\theta\big)\eta^\theta + \frac{\varsigma_G (1-\kappa-\eta)^\theta}{(1-\alpha)}$ what results in $\kappa = \frac{1}{1+\left(\frac{\varsigma_A}{\varsigma_R}\right)^{\frac{1}{1-\theta}}}$ and $\eta = \frac{1}{1+\left(\frac{\varsigma_G}{(1-\alpha)^2 \left[\varsigma_A \kappa^\theta + \varsigma_R (1-\kappa)^\theta\right]}\right)^{\frac{1}{1-\theta}}}$. This example reveals that an unweighted environmental tax simply incentivizes to reduce the tax burden that affects profits.

In order to affect the abatement intensity to the socially optimal level, the tax has to weight the environmental effect according to its social costs. This results in a dynamic Pigouvian tax which considers the externality of innovations on damages in the production growth rate. Hence

$$(1+T_E) = \Gamma_E(t)\Big[(1-n)^{1-\alpha} K(t)^{\alpha} \Big[\frac{A(t)^{1-\alpha} R(t)^{1-\alpha}}{G(t)^{1+\alpha}}\Big]\Big]^{\Omega}.$$

Therefore, $\pi(t) = \big[A(t)R(t)\big]^{1-\alpha\Omega} G(t)^{\frac{\Omega}{1-\alpha}} \mathcal{C}(t)$ with $\mathcal{C}(t) = \frac{\Lambda(1+T_p)^{\frac{\Omega\alpha}{1-\alpha}}}{\Gamma_E(t)^{\frac{\alpha}{1-\alpha}}} \frac{(1-n)^{1-\Omega\alpha} K(t)^{\Omega\alpha}}{(r)^{\frac{\alpha}{1-\alpha}} \bar{E}^{\frac{\iota\omega}{1-\alpha}}}$, so an innovator evaluates $\max_{\kappa,\eta} \underline{\hat{\mathcal{B}}} = (1-\alpha\Omega)\big[\varsigma_A \eta^\theta + \varsigma_R \kappa^\theta\big] + \big(\frac{\Omega}{1-\alpha}\big)\varsigma_G (1-\kappa-\eta)^\theta$ what results in $\kappa = \frac{1}{1+\left(\frac{\varsigma_A}{\varsigma_R}\right)^{\frac{1}{1-\theta}}}$ and $\eta = \frac{1}{1+\left(\frac{\left(\frac{\Omega}{1-\alpha}\right)\varsigma_G}{\left(1-\Omega\alpha\right)\left[\varsigma_A \kappa^\theta + \varsigma_R (1-\kappa)^\theta\right]}\right)^{\frac{1}{1-\theta}}}$.

A *gray* direction of technical change is then characterized by

$$\eta = \eta^* = \frac{1}{1 + \Big(\frac{\left(\frac{\phi}{1-\phi} - \alpha\right)\varsigma_G}{(1-\alpha)\left[\varsigma_R (1-\kappa)^\theta + \varsigma_A \kappa^\theta\right]}\Big)^{\frac{1}{1-\theta}}}$$

what is achieved with $\frac{\frac{\Omega}{1-\alpha}}{\left(1-\Omega\alpha\right)} = \frac{\left(\frac{\phi}{1-\phi} - \alpha\right)}{(1-\alpha)}$, which simplifies to $\Omega = \frac{1}{\alpha + \left(\frac{1-\phi}{\phi - \alpha(1-\phi)}\right)}$.

Finally, the *green* direction requires

$$\eta^* = \bar{\eta} = \frac{1}{1 + \left[\frac{\varsigma_A}{\varsigma_G}\right]^{\frac{1}{\theta}}},$$

so $\bar{\eta} = \frac{1}{1+\left[\frac{\varsigma_G}{\varsigma_A}\right]^{\frac{1}{\theta}}}$, thus $\Omega := \frac{1}{1+\alpha} + \alpha$. Further, since $(1 + T_E)$ needs to be constant in order to not endanger the stability of the vintage technology distribution (see Proposition (1)), it is necessary that

$$g_{\Gamma_E} := \Omega\big((1+\alpha)g_G - \alpha g_k - (1-\alpha)(g_A + g_R)\big).$$

Since along a BGP $g_k = g_y$, we thus find that $g_{\Gamma_E} = \frac{\Omega\phi}{(1-\alpha)}\big(\frac{1}{1-\alpha(1-\phi)}g_G - (g_A + g_R)\big)$. This, however, is only required for a gray direction since it is associated with an increasing environmental effect and thus a potentially increasing environmental tax burden that skews the vintage cost distribution. In a green direction, the natural capital stock remains constant so that there is no increasing tax burden on its exhaustion. Further, since $\Gamma_E(t) = \Gamma_{E,0}e^{g_{\Gamma_E}t}$, it holds that $\Gamma_{E,0} = \frac{(1+T_E)}{\bar{E}_0^{\Omega}}$. Setting $n = \frac{(1+T_V)(1-\alpha)-\rho}{(\lambda\mathcal{W}_{anticipated}+(1+T_V)(1-\alpha))}$ equal to n^{**} and reformulating results yields $T_V = \frac{n^*\lambda\mathcal{W}_{anticipated}+\rho}{(1-n^*)(1-\alpha)} - 1$, so including the n^{**} findings of Proposition (2) leads to the stated results. For the markup prices, it is necessary to adjust for the environmental tax so $T_p = 1 - \frac{1+T_E}{\alpha}$. Now since $\Gamma_{E,0}\bar{E}_0^{\Omega} = (1+T_E)$, it thus holds that $T_p = \frac{\Gamma_0\bar{E}_0^{\Omega}}{\alpha} - 1$, so $\Gamma_{E,0} = \alpha\frac{1+T_p}{\bar{E}_0^{\Omega}}$. $\square$

Proposition (5) If $\phi \geq 1$, an agent with information on the environmental effect will correctly anticipate the required research strategy and thus select socially optimal research efforts. With specialization, research spillover effects are path specific so that an innovator compares alternative innovation values. Now, aggregate technologies would evolve according to

$$B(t) = \gamma\big(A(t)R(t)\big)^{(1-\alpha)(1-\phi)} + (1-\gamma)A(t)^{(1-\alpha)(1-\phi)}G(t)^{\phi-\alpha(1-\phi)}.$$

In the long run, three growth paths for this bundle are possible

(1.) (indifference) $(1-\alpha)(1-\phi)g_R = \phi - \alpha(1-\phi)g_G$, so that $0 < \gamma < 1$ and $g_B = (1-\alpha)(1-\phi)(g_A + g_R) = (1-\alpha)(1-\phi)(g_A + g_G)$,

(2.) (brown growth) $(1-\alpha)(1-\phi)g_R > \phi - \alpha(1-\phi)g_G$, so that $\gamma = 1$ and $g_B =$

$(1-\alpha)(1-\phi)(g_A+g_R)$,

(3.) (green growth) $(1-\alpha)(1-\phi)g_R < \phi-\alpha(1-\phi)g_G$, so that $\gamma=0$ and $g_B = (1-\alpha)(1-\phi)g_A + \big(\phi-\alpha(1-\phi)\big)g_G$.

However, for γ, an innovator compares $V_{brown}(t) \lesseqgtr V_{green}(t)$. With Proposition (1) $V(t) = \left(\frac{1-\alpha}{\alpha}\right)\frac{(1-\alpha)\alpha(1-n)^{(1-\alpha)(1-\phi)}K(t)^{\alpha(1-\phi)}B(t)}{r+\lambda n-g_{\bar{\pi}}}$. Hence, an innovator is indifferent if

$$\frac{K_{brown}(t)^{\alpha(1-\phi)}R(t)^{(1-\alpha)(1-\phi)}}{r+\lambda n-g_{\bar{\pi}_{brown}}} = \frac{K_{green}(t)^{\alpha(1-\phi)}(1+T_{V,R})G(t)^{\phi-\alpha(1-\phi)}}{r+\lambda n-g_{\bar{\pi}_{green}}}.$$

In this case, we are in Scenario (1.) were both paths face the same $g_{\bar{\pi}}$, hence $K_{brown}(t)^{\alpha(1-\phi)}R(t)^{(1-\alpha)(1-\phi)} = K_{green}(t)^{\alpha(1-\phi)}(1+T_{V,R})G(t)^{\phi-\alpha(1-\phi)}$. Yet, since capital is priced with r, a standard marginal product consideration says that the path specific capital intensity is proportional to technology. It is consequently not possible to have $K_{brown} > K_{green}$ and $R(t)^{(1-\alpha)(1-\phi)} < (1+T_{V,R})G(t)^{\phi-\alpha(1-\phi)}$. Hence, there is only indifference among the research direction if $R(t)^{(1-\alpha)(1-\phi)} = (1+T_{V,R})G(t)^{\phi-\alpha(1-\phi)}$. If $R(t)^{(1-\alpha)(1-\phi)} > (1+T_{V,R})G(t)^{\phi-\alpha(1-\phi)}$, $\gamma=1$, thus the economy enters brown research, if $R(t)^{(1-\alpha)(1-\phi)} < (1+T_{V,R})G(t)^{\phi-\alpha(1-\phi)}$, $\gamma=0$, thus the economy enters green research. Yet, in Scenario (1.)

$$g_B = \lambda n\Big((1-\alpha)(1-\phi)(\varsigma_A\eta^\theta+\varsigma_R\kappa^\theta\Big) = \lambda n\Big(1-\alpha)(1-\phi)(\varsigma_A+\varsigma_G(1-\kappa-\eta)^\theta)\Big).$$

Stability requires that both innovators spend the same amount of research on general innovations, so that it is necessary that $\kappa=\eta$. Scenario (1.) can only occur if $\varsigma_R=\varsigma_G$ which requires that $(1-\alpha)(1-\phi)g_R = \phi-\alpha(1-\phi)g_G$, thus $(1-\alpha)(1-\phi)\varsigma_R(1-\kappa)^\theta = \phi-\alpha(1-\phi)\varsigma_G(1-\eta)^\theta$, so $\kappa=\eta$ is necessary and hence $\varsigma_G=\phi$. Therefore, there is only indifference among the research direction if $\varsigma_R=\varsigma_G=\phi$ and $R(t)^{(1-\alpha)(1-\phi)} = (1+T_{V,R})G(t)^{\phi-\alpha(1-\phi)}$ so that $\gamma\in(0,1)$. In any other case, $\gamma=0$ or $\gamma=1$. □

Corollary (3) For agents with complete information, no environmental tax is required since the innovators correctly anticipate the social costs that come along with the alternative innovation directions. Yet, Proposition (5) has emphasized that it is possible to face environmental lock-in effects. Reformulating the equality condition for research indifference leads to $\frac{R(t)^{(1-\alpha)(1-\phi)}}{G(t)^{\phi-\alpha(1-\phi)}} - 1 = T_{V,R}$. □

Proposition (6) Since agents without the information on the environmental effect do not anticipate any profits of green innovations, they set $\eta + \kappa = 1$ and maximize the innovation related profit growth rate for η. The profit growth rate is proportional to $\mathcal{B}$ and in their view given with $\mathcal{B} = \varsigma_A \eta^\theta + (1-\eta)^\theta \varsigma_R$ so that they choose $\gamma = 1$ and allocate the research efforts as in a brown direction. As with general access to technologies, the general no arbitrage condition for labor described with (21) follows the above described logic, resulting in $n = \gamma n + (1-\gamma) n = \frac{1}{1+\frac{\lambda \mathcal{W}+\rho}{(1+T_V)(1-\alpha)}}$. □

Corollary (4) Any tax that intends to incentivize green research needs to consider both η and γ. Without information about the environmental impact of innovations, $\gamma = 1$, $\eta = 1-\kappa$, and $\eta = \eta^{**}$. To achieve green innovations, it is necessary that $\kappa = 0$ and $\eta = \frac{1}{1+\left(\frac{[\frac{\phi}{(1-\phi)}-\alpha]\varsigma_G}{(1-\alpha)\varsigma_A}\right)^{\frac{1}{1-\theta}}}$ what requires the environmental tax introduced in Corollary (3). Since a gray direction is not possible here, agents have to decide among adaptation and abatement. Therefore, $B(t) = \gamma\big(A(t)R(t)\big)^{(1-\alpha)(1-\phi)} + A(t)^{(1-\alpha)(1-\phi)} G(t)^{\phi-\alpha(1-\phi)}$ with $g_A = \gamma \varsigma_A + (1-\gamma)\varsigma_A \left(1 - \frac{1}{1+\left(\frac{\left(\frac{\Omega}{1-\alpha}\right)\varsigma_G}{\left(1-\Omega\alpha\right)\left[\varsigma_A \kappa^\theta + \varsigma_R (1-\kappa)^\theta\right]}\right)^{\frac{1}{1-\theta}}}\right)$.

□