Spatial Discrimination, Nations' Size and Transportation Costs

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Abstract

In this paper we develop a spatial Cournot trade model with two unequally sized countries, using the geographical interpretation of the Hotelling line. We analyze the trade and welfare effects of international trade between these two countries. The welfare analysis indicates that in this framework the large country benefits from free trade and the small country may be hurt by opening to trade. This finding is contrary to the results of Shachmurove and Spiegel (1995) as well as Tharakan and Thisse (2002), who use related models to analyze size effects in international trade, where the small country usually gains from trade and the large country may lose.

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1 Introduction

The seminal paper of Shachmurove and Spiegel (1995) has been an important attempt to model the effect of nations' size differences in international trade flows using Hotelling's framework of spatial competition. Their paper studies the trade and welfare effects of opening a closed border between two neighboring nations of different size. The authors assume mill-pricing and fixed locations. First they analyze the price and welfare in autarky and after that they assume that the border opens. Finally they compare the price and welfare effects between autarky and free trade. In contrast to traditional trade theory or the new economic geography, the country size is not dimensionless and not solely represented by population size. Here country size is modelled in terms of geographical extension. The analysis of Shachmurove and Spiegel (1995) suggest that the small country always gains from free trade, while opening to free trade is not necessarily beneficial to the large country. The smaller country is able to gain from trade, because of its ability to expand its market share. Under certain assumptions on the locations of the firms the analyzed Nash equilibrium in Shachmurove and Spiegel (1995) does not exist, as Tharakan (2001) shows. The spatial setup of the Shachmurove-Spiegel-Model validates the stability-conditions in the Hotelling model with linear transportation costs imposed by d'Aspremont et al. (1979). To recover stability of equilibrium Tharakan and Thisse (2002) extend the model with quadratic transportation costs. The results show that, in the case of quadratic transportation costs the smaller country gains more from free trade than the larger country. This result is even stronger than in the case of linear transportation costs.[1] Another possibility to avoid the existence problem is to assume spatial discrimination. In Tharakan (2001), the author assumes that firms use spatial price discrimination as pricing policy. The analysis indicates that while the small country gains from opening up to free trade, the large country's welfare is unpersuaded. However, there is reasonable doubt about the use of Bertrand pricing policy. Greenhut (1981) shows in an empirical study that the delivered prices of real-world firms did not resemble Bertrand schedules. The predictions of the Cournot model in terms of delivered prices have been verified by Greenhut, Greenhut and Li (1980) in a representative Sample.

Theoretical support for the use of the Cournot model with spatial discrimination in Greenhut et al. (1991), as well as in Anderson and Neven (1991). The latter argue, that the Cournot assumption should be reasonable when quantity (or capacity) decision is inflexible, contrary Bertrand competition is relevant if price decisions are less flexible than quantity decisions. Another important issue, concerning the use of Cournot instead of Bertrand in space is the fact that markets overlap in Cournot and (all) market points are served by both firms. If we want to model markets where overlapping is unrealistic and a (sub)market is only served by a single firm, we should use Bertrand spatial discrimination.

[1] Using quadratic transportation costs instead of linear transportation costs is just a technical change. However, from an empirical point of view quadratic transportation costs are questionable.

The purpose of this paper is to extend Shachmurove and Spiegel's model to make the analysis possible and compare the results with the results of the above described existing corrections by Tharakan and Thisse (2002) and Tharakan (2001). We use a similar setup where firm's apply spatial discrimination in the sense of Cournot. Instead of an inelastic demand we use a more realistic elasitc demand function and linear transportation costs. We want to investigate the gains and losses associated with the opening to trade. Here it is of primary interest to verify the hypothesis of an advantage of being small. In particular, we find that with long-run firm locations the large country gains from free trade and the overall welfare in the small country is lower compared to the autarky equilibrium. This result is contrary to the results found by Shachmurove and Spiegel (1995), Tharakan (2001) and Tharakan and Thisse (2002), where the small country gains from free trade. The reason of this finding is the agglomeration of the Cournot firms at the center of the market. If nations' size differs, the firm located in the small country in autarky always relocates in the large country in the long run.[2] Clearly, this result is obtained in a very particular model, but we can show that the gains and losses of opening to free trade depend crucially on market conditions and firm's behaviour.

This paper is organized as follows. The following section describes the theoretical model. The comparison between autarky and free trade is done in the third chapter. The last chapter concludes.

2 The Model

Similar to Shachmurove and Spiegel (1995) we study first the autarky case. Therefore we assume two closed economies with a border between them. The countries differ only in size, size h for the small country and size *1-h* for the large country. There is just one firm in both countries. The second scenario analyses the effect of free trade. The border opens and the firms compete in the whole market. We use this model setup to analyze the trade and welfare effects of free trade compared to autarky. In the spirit of Hotelling (1929), we study a subgame perfect equilibrium with location choice as the first stage. In the second stage the firm sets its quantity at a particular location. The model is solved using backward induction in both scenarios.[3]

The firms sell a homogeneous product and face the same linear transport costs of t per unit to ship one unit of the product from its own location to a consumer. Production involves constant marginal costs (without loss of generality production costs are normalized to zero). Firms are able to discriminate between customers since they control transportation. The location of the firm in the small country is denoted as a and in the large country as $1-b$. Both

[2] For a detailed discussion on spatial agglomeration and Cournot competition, see Anderson and Neven (1991).

[3] It is possible to set new quantities and relocate for both firms after opening the border. The two-stage-game is played, independently, two times (one time in autarky and one time in competition). We assume, that the firm's cannot anticipate the opening of the border in the first game.

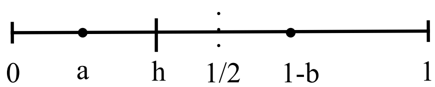

Fig. 1: The model.

countries have the same uniform distribution of consumers over the market space and all consumers have identical preferences. A consumer's location in the small country is indexed by $x\epsilon\,[0,h]$ and in the large country by $x\epsilon\,[h,1]$. Each point generates an inverse demand $p\,(x) = 1 - q(x)$, where p is the price of the homogeneous good and q is the total quantity offered to consumers at that point.[4] Obviously we have to assume $0 < h < \frac{1}{2}$ to makes sure that both countries differ in size. To ensure, that both firms serve the whole market in free trade and to avoid the case of local monopolies we follow Hamilton et al. (1988) and restrict the transport cost to $t \leqq \frac{1}{2}$. For simplification we assume $t = \frac{1}{2}$. We will call this condition the full market condition.[5]

2.1 Autarky

First autarky is assumed in both countries. The two firms are both a monopoly in their country. In case of monopoly it makes no difference between setting prices or quantities. We analyze quantity setting.

Small country: The profit earned by the firm in the small country is given by[6]

$$\Pi_m(x,a) = \int_0^h (1 - q_m - \frac{1}{2}|a-x|)q_m dx. \tag{1}$$

Under the assumptions the monopoly is able to maximize profit at every location separately. Standard calculation yield the following profit maximizing quantity at location x:

$$q_m^*(x,a) = \frac{1 - \frac{1}{2}|a-x|}{2}. \tag{2}$$

Furthermore, the resulting price schedule is

$$p_m^*(x,a) = \frac{1 + \frac{1}{2}|a-x|}{2}. \tag{3}$$

[4] It is necessary to work with the inverse demand function since Cournot competition assumes prices to be determined by market clearing. However, d'Aspremont et al. (1991) show that Cournot competition can be viewed as competition in prices and in quantities, where price competiton is coordinated by a pricing scheme. Therefore Cournot competition implies some form of price coordination.

[5] To ensure that the full market is served, we use the result that the monopoly price equals $\frac{1}{2}$. If the location of the firm is at the end point the inequality $\frac{1}{2} + t \leq 1$ must hold to ensure that the firm serves the whole market. Rearranging yields the full market condition.

[6] Notation: The index m is used to indicate the monopoly case. Throughout the paper we use capital letters for the large country and small letters for the small country.

These results indicate, that monopoly price and the quantity equal $\frac{1}{2}$ at location $a = x$.

We can find the profit maximizing location by substituting the equilibrium quantity in the profit function and differentiating w.r.t. a. The result is not surprisingly, since the monopoly chooses in equilibrium the transport cost minimizing location:

$$a_m^* = \frac{h}{2}. \tag{4}$$

Substitution of (2), (3) and (4) in (1) and by using standard calculation gives

$$\Pi_m^* = \frac{2}{96} h(12 - 3h + \frac{1}{4} h^2). \tag{5}$$

The aggregate consumer surplus by using linear demand yields

$$\begin{aligned} CS_m^* &= \int_0^h \frac{1}{2}(1 - p_m^*)^2 dx \\ &= \frac{1}{96} h(12 - 3h + \frac{1}{4} h^2). \end{aligned} \tag{6}$$

Overall welfare in the small country is the sum of aggregate consumer surplus and monopoly profit, which yields

$$\begin{aligned} W_m^* &= CS_m^* + \Pi_m^* \\ &= \frac{1}{32} h(12 - 3h + \frac{1}{4} h^2). \end{aligned} \tag{7}$$

Large country: The profit of the monopoly in the large country is given by

$$\Pi_M(x, b) = \int_h^1 (1 - q_M - \frac{1}{2}|1 - b - x|) q_M dx. \tag{8}$$

Profit maximizing quantity at location x is

$$q_M^*(x, b) = \frac{1 - \frac{1}{2}|x - 1 + b|}{2}. \tag{9}$$

The optimal price for monopoly appears to

$$p_M^*(x, b) = \frac{1 + \frac{1}{2}|x - 1 + b|}{2}. \tag{10}$$

Calculation of the optimal location yields, not surprisingly,

$$b_M^* = \frac{1}{2} - \frac{1}{2} h. \tag{11}$$

Again, the monopoly is located at the center of the market, here the transport costs are minimized.

Using (9), (10) and (11), the profit of the monopoly can be written as

$$\Pi_M^* = \frac{2}{96}(1-h)(12-3(1-h)+\frac{1}{4}(1-h)^2). \tag{12}$$

The aggregate consumer surplus by using linear demand, is given by

$$\begin{aligned} CS_M^* &= \int_h^1 \frac{1}{2}(1-p_M^*)^2 dx \\ &= \frac{1}{96}(1-h)(12-3(1-h)+\frac{1}{4}(1-h)^2). \end{aligned} \tag{13}$$

Overall welfare in the large country is the sum of aggregate consumer surplus and monopoly profit, which yields

$$\begin{aligned} W_M^* &= CS_M^* + \Pi_M^* \\ &= \frac{1}{32}(1-h)((12-3(1-h)+\frac{1}{4}(1-h)^2). \end{aligned} \tag{14}$$

Global welfare: Global welfare is simply the sum of the overall welfare in the small and in the large country, that is

$$W_{m+M}^* = \frac{1}{32}(12-3(h^2+(1-h)^2)+\frac{1}{4}(h^3+(1-h)^3)). \tag{15}$$

2.2 Free Trade with quantity competition

After getting the results of the autarky scenario in 2.1., we turn to the free trade case. The border, located at h, is opened and both firms compete in quantities at each point in the integrated market. Since production costs are constant and arbitrage is nonbinding the firms determine separate quantities for each location, which are strategically independent. In the second stage each firm chooses a profit maximizing quantity schedule given its rival schedule, for fixed locations a and b. In the first stage, given the equilibrium quantity schedules, each firm chooses a location to maximize profits given its rival's choice.

The quantity equilibrium: The profit of the firms are written as[7]

$$\Pi_f(x,a,b) = \int_0^1 (1-q_f(x)-q_F(x)-\frac{1}{2}|a-x|)q_f(x)dx \tag{16}$$

and

$$\Pi_F(x,a,b) = \int_0^1 (1-q_f(x)-q_F(x)-\frac{1}{2}|1-b-x|)q_F(x)dx. \tag{17}$$

Direct computation of the Cournot equilibrium yields

$$q_f^*(x,a,b) = \frac{1-|a-x|+\frac{1}{2}|x+b-1|}{3} \tag{18}$$

[7] We use the notation f for free trade. Again capital letters indicate the large country.

and

$$q_F^*(x,a,b) = \frac{1 - |x+b-1| + \frac{1}{2}|a-x|}{3}. \tag{19}$$

Using these results, the delivered price schedule is

$$\begin{aligned} p^*(x,a,b) &= 1 - (q_f^*(x,a,b) + q_F^*(x,a,b)) \\ &= \frac{1 + \frac{1}{2}|a-x| + \frac{1}{2}|x+b-1|}{3}. \end{aligned} \tag{20}$$

If the full market condition holds, each firm will supply all points in space. The distribution of output among firms at any point depends on their spatial locations: at any consumer point, the firm which is closer to that point will have a larger market share.

The location equilibrium: We use the equlibrium quantity schedules to find the first-stage location equilibrium. Each firm chooses a location to maximize profits given it's rival's location. The firm's total profit function for the location game are as follows:

$$\Pi_f^*(x,a,b) = \int_0^1 (p^*(x,a,b) - \frac{1}{2}|a-x|)q_f^*(x,a,b)dx \tag{21}$$

and

$$\Pi_F^*(x,a,b) = \int_0^1 (p^*(x,a,b) - \frac{1}{2}|1-b-x|)q_F^*(x,a,b)dx. \tag{22}$$

The first order condition of the maximization of firm f's profit is given by

$$\frac{\partial \Pi_f^*}{\partial a} = \frac{1}{18}(3 - 8a + 2a^2 - 2b + 2b^2 + 4ab) \equiv 0. \tag{23}$$

Solving (23) for a gives two solutions: $a_{1,2} = 2 - b \pm \frac{1}{2}\sqrt{10-12b}$. If $b = \frac{1}{2}$ then only one solution is inside the market and yields $a = \frac{1}{2}$.[8]

Therefore the first order condition is satisfied for both firms at the central location:

$$a^* = b^* = \frac{1}{2}. \tag{24}$$

The second order condition is written as

$$\frac{\partial^2 \Pi_f^*}{\partial a^2} = -\frac{4}{9} + \frac{2}{9}a + \frac{2}{9}b. \tag{25}$$

The second order condition is strictly negative. We can see that central agglomeration is an equilibrium with spatial discrimination and quantity competition. [9]

[8] The other solution gives $a = \frac{5}{2}$, which is outside the market.

[9] Gupta, Pal and Sarkar (1997) show that the agglomeration result depends on the assumed uniform distribution of consumers. They show that if population density is thin at the centre then central agglomeration never occurs in Cournot competition.

The Cournot equilibrium is described through (18), (19), (20) and (24). [10][11] Using these results it is easy to see that trade flows from the large to the small country. Here it is the country size that determines the direction of trade.

Welfare in the small country: As derived above, after opening the border, the former monopoly relocates at the center of the new market, which is given by $[0,1]$. Since $h < \frac{1}{2}$ the firm relocates in the large country. The remaining overall welfare of the small country is therefore only the consumer surplus in the small country

$$\begin{aligned} W_f^* &= CS_f^* = \int_0^h \frac{1}{2}(1-p^*)^2 dx \\ &= \frac{h}{54}(\frac{27}{4} + \frac{9}{2}h + h^2). \end{aligned} \tag{26}$$

Welfare in the large country: The overall welfare in the large country is the sum of consumer surplus in the large country and the profits of both firms, because they are both located in the large country. Consumer surplus in the large country is written as

$$\begin{aligned} CS_F^* &= \int_h^1 \frac{1}{2}(1-p^*)^2 dx \\ &= \frac{1}{54}(\frac{37}{4} - \frac{27}{4}h - \frac{9}{2}h^2 - h^3) \end{aligned} \tag{27}$$

Both firms earn the same profit in equilibrium, because of symmetry. The profits are calculated using (18), (19), (20) and (24)

$$\Pi_f^* = \Pi_F^* = \frac{37}{432}. \tag{28}$$

Furthermore, the overall Welfare in the large country is

$$\begin{aligned} W_F^* &= CS_F^* + \Pi_f^* + \Pi_F^* \\ &= \frac{1}{54}(\frac{37}{2} - \frac{27}{4}h - \frac{9}{2}h^2 - h^3). \end{aligned} \tag{29}$$

Global Welfare in free trade: As in the autarky case, the global welfare is simply the sum of the welfare in the large country and the welfare in the small country:

$$W_{f+F}^* = \frac{37}{108}. \tag{30}$$

[10] Existence and uniqueness of the equilibrium have been shown by Anderson and Neven (1986).

[11] If both firms are located at the same point and produce a homogeneous product, it is beneficial to them to merge and maximize joint profits. We assume that this merging is forbidden by law by an antitrust authority in the large country.

3 Results

The characterisation of the autarky and the free trade equilibrium enables us to analyze the welfare consequences of opening the border to international trade in both countries. We compare the changes in profits, consumer surpluses and welfare.

Changes in profits The opening of the border changes two important aspects for both firms: market size and competition. The first aspect is positive for the profit of a firm: if the full market condition holds, both firms will supply a larger market. The second aspect is negative, because the firm faces competitive pressure instead of a monopoly position.

Proposition 1: Assuming that the full market condition holds, under free trade, the profit change for the small country firm is always negative if $0.37701 < h < \frac{1}{2}$*. For country sizes* $0.37701 > h$ *the profit change is positve. The profit does not change between autarky and free trade for* $0.37701 = h$*.*

Proof. See the appendix.

Profit change for the firm is positive provided the small country is not too "large". If the small country is nearly as large as the large country, then the monopoly profit in the small market exceeds the free trade profit. If the small country is indeed small, than the profit change is always positive this is effect is due to the larger market.

Proposition 2: Assuming that the full market condition holds, under free trade, the profit change for the large country firm is always negative.

Proof. The change of the large country firm is negative, if $\Pi_F^* - \Pi_M^* = \frac{9h^3+81h^2+243h-185}{1728} < 0$, this is approximately satisfied for all $h<0.62299$. □

The profit change for the firm in the large country is always negative, because of the competetive pressure. It is easy to show, that the overall profit change is negative as well for all (allowed) values of h and t.

For consumer surplus, we have:

Proposition 3: The change in the consumer surplus for the small country as well as the change in the consumer surplus for the large country is positive.

Proof. The change is positive, because $CS_f^* - CS_m^* = \frac{11h^2(5h+36)}{3456}$, clearly this is positive, because $h>0$.

The change in the large country $CS_F^* - CS_M^* = \frac{1}{3456}(259 - 189h - 207h^2 - 55h^3)$ is always positive, if the full market condition holds and $h<0.71137$ (approximately).□

As it is often the case, free trade leads to a decrease in prices. The change of the equilibrium price influences the change of the consumer surplus.

Finally we can analyze the effect for the overall welfare. The result for the small country can be summarised in the following proposition.

Proposition 4: The overall welfare change in the small country is negative when it engages in free trade.

Proof. The overall welfare change is $W_f^* - W_m^* = \frac{1}{3456}(37h^3+612h^2-864h) < 0$, for any values $0 < h < \frac{1}{2}$.□

The explanation for this change is the fact, that the small country firm relocates in the large country and in the free trade case, the overall welfare in the small country depends only on consumer surplus. However, this result contradicts neo-classical trade theory of international trade. There a country either gains or is indifferent to international trade. Here this is not the case, because the small country has a higher overall welfare in autarky. The relocation argument might be a strong reason for policymakers to keep up barriers to trade.

For the large country we have, therefore:

Proposition 5: Overall welfare change is positive in the large country.

Proof. If the joint overall welfare change is positive (Proposition 6) and the welfare change in the small country is negative (Proposition 4), then the welfare change in the large country must necessarily be positive.$\square$

As argued above the welfare change must be positive, because the consumer surplus rises (see proposition 4) and the sum of the profits of both firms is larger than the monopoly profit for the firm in the large country alone. We can see that $W_F^* - W_M^* > 0$, holds for all values.

It remains the question of the change of the overall joint welfare.

Proposition 6: The joint overall welfare change is positive.

Proof. This result is derived by $W_{f+F}^* - W_{m+M}^* = \frac{1}{3456}(567h^2 - 567h + 185) > 0$ for any values $0 < h < \frac{1}{2}$.$\square$

The welfare gain in the large country over-compensates the welfare loss in the small country, because the resulting market structure is more efficient than the monopolistic structure in autarky.

However, the results show that opening to trade is globally efficient. Free trade leads to a higher welfare than autarky. But it is not the case that free trade is preferable for both countries, since the small country losses in this model setup.[12]

4 Conclusions

This paper constructs a model of international trade in the Hotelling model, using the geographical notation of the Hotelling line. We follow the approach of Shachmurove and Spiegel (1995) and change the model using a spatial discriminating Cournot duopoly. The results of Shachmurove and Spiegel (1995) as well as Tharakan and Thisse (2002) indicate, that the small country gains from trade. By a slight change in the model assumptions we can reverse the distributional results and find that the large country gains in our model in free trade, while the small country is worse off. We find that consumer surplus is higher in the free trade case than in autarky. Global welfare is higher with free trade compared to autarky. Furthermore, trade flows from the large to the small country.

[12] Results differ if we assume, that profits of the small country firm are transferred to the small country in free trade, because of (e.g.) ownership-reasons. In that case free trade would be beneficial to both countries.

However, our results are, as well as the results of Shachmurove and Spiegel (1995) and Tharakan and Thisse (2002), obtained in the context of a very particular model. We consider a single dimensional space and uniform linear demand. One assumption that is crucial for our results is the restriction on the transportation costs. If we allow for larger transportation costs, the market would support isolated sellers as local monopolies. Clearly there would be no agglomeration in this context and the results of the model would vanish.

The model has interesting consequences regarding the gains and losses of free trade. Especially if we compare our results with the results of Shachmurove and Spiegel (1995) and Tharakan and Thisse (2002), we see that a lot of work needs to be done to understand the impact of geographical size and industrial organization for trade and welfare effects across countries.

Appendix

Proof of Proposition 1

The change of the profit of the small country firm is $\Pi_f^* - \Pi_m^* = \frac{1}{432}(37 - 108h + \frac{108}{4}h^2 - \frac{9}{4}h^3)$. The change of the profit may be positive, negative or zero, depending on h. We set $\frac{1}{432}(37 - 108h + \frac{108}{4}h^2 - \frac{9}{4}h^3) = 0$ and solve for h to get the nations' size, where the profit change equals zero. We find three solutions, one real and two complex solutions, however, the real valued solution is of interest here.[13] The approximated real value solution is $h_{crit} = 0.37701$. For a nations' size below this critical value, the profit change for the small country firm is positive. If $0.37701 < h < 0.5$ the profit change for the small country firm is negative. $\square$

[13] Note that we only use the real value solution in later proofs as well.

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