

# Essays in Macroeconomics and Financial Market Imperfections

#### INAUGURAL-DISSERTATION

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#### Overview

#### Co-authorship

This dissertation consists of four papers where three of them are in collaboration. The first paper is written in collaboration with Philipp Engler and the second and the fourth paper are written in collaboration with Maik Heinemann. In total, the four papers are given as follows:

- 1. "Opposition to Capital Market Opening" (with Philipp Engler)
- 2. "Idiosyncratic Risk, Borrowing Constraints and Financial Integration" (with Maik Heinemann)
- 3. "Incomplete Information, Financial Market Imperfections, and Aggregate Saving"
- 4. "Financing of Government Spending in an Incomplete-Markets Model: The Role of Public Debt" (with Maik Heinemann)

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#### Presentations of Papers

- 2018: Colloquium TU Berlin;
- 2017: Annual Conference of the Society for Computational Economics (New York);
  Annual Congress of the European Economic Association (Lisbon);
  Annual Conference of the Verein für Socialpolitik (Vienna);
  Potsdam Research Seminar in Economics;
- 2016: Annual Conference of the Society for Computational Economics (Bordeaux);
  Brown Bag Seminar at HU Berlin;
  Potsdam Research Seminar in Economics;
- 2015: Annual Conference of the Royal Economic Society (Manchester);
  Annual Conference of the Verein für Socialpolitik (Münster);
  Brown Bag Seminar at HU Berlin;
- 2014: Conference of the *Irish Society of New Economists* (Galway);

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## List of Abbreviations

DSGE Dynamic Stochastic General Equilibrium

SIM Standard Incomplete Markets

PPP Purchasing Power ParityGDP Gross Domestic ProductFDI Foreign Direct Investment

OECD Organization for Economic Cooperation and Development

EIS Elasticity of Intertemporal Substitution

CARA Constant Absolute Risk Aversion

NDL Natural Debt Limit

MA(1) Moving Average Process of Order 1

OLG Overlapping Generation RBC Real Business Cycle VAR Vector Autoregression

NIPA National Income and Product Account

## Chapter 1

## Introduction

This dissertation consists of four self-contained papers that deal with the implications of financial market imperfections and heterogeneity primarily within the class of incomplete-markets models. The following review briefly relates this class of models to the evolution of macroeconomic modeling and summarizes its main ideas. Subsequently, the contributions of the four papers are described in more detail.

#### A brief history of macroeconomic modeling

The evolution of macroeconomic modeling is characterized by different waves of major and minor changes of the predominant paradigm, reflecting the different views of competing macroeconomic schools of thought. Up to the late 70s, macroeconomic modeling mainly finds its expression in large-scale macroeconometric models, following the Cowles commission approach (cf. Heathcote et al. 2009). Macroeconometric models allow for a very detailed description of different economic sectors and a large number of parameters to estimate. The high flexibility of this approach has been exploited to build large-scale models that have been used for simulation and forecasting purposes. Macroeconometric models still play a role in today's forecasting industry.

However, the high degree of freedom these models provide has not hidden the fact that the underlying modeling strategy was mainly data-driven rather than based on sound economic theory. The dissatisfaction with this approach amplified in the late 70s and found its climax in the famous Lucas critique and the rational expectations revolution. Based on the work by Lucas, Sargent, Kydland and Prescott, the research agenda of macroeconomics was then put forward to dynamic general equilibrium models in which aggregate behavior is derived from optimal individual decision making (cf. Heathcote et al. 2009). Since then, DSGE models have become one of the dominant classes of models in modern macroeconomics. The possible applications of DSGE models are broad and range from classical real business cycle applications, asset pricing, fiscal

<sup>&</sup>lt;sup>1</sup>The following brief review can neither be seen as completely comprehensive nor as capturing all strands of literature. Instead, it serves to provide a general overview of selected main developments in macroeconomic modeling. A more detailed introduction can be found in Heathcote et al. (2009), Guvenen (2011), and Quadrini (2011).

policy to long-run economic growth. Furthermore, the incorporation of nominal frictions bridges the gap towards traditional Keynesian theory and new Keynesian DSGE models are widely used to analyze monetary policy. The ascendancy of DSGE models also finds its roots in the development of numerical toolboxes, in particular Dynare, that allow to handle even large-scale models. Nowadays, DSGE models are also a standard tool for central banks and finance ministries. However, despite the widespread applications and the introduction of various frictions, a certain aspect has initially attracted slightly less attention within the standard framework: the role of financial markets and the consequences of financial market imperfections. Apart from early work by Bernanke and Gertler (1989), the common approach was to assume that financial markets are complete so that financial market imperfections or financial frictions do not play a crucial role (see, Quadrini 2011). The neglection of the financial channel seems less surprising as it constitutes a natural implication of the representative-agent approach that characterizes standard DSGE models. As emphasized by Quadrini (2011), there is no reason to trade state-contingent financial claims if all agents are homogeneous. Furthermore, the fact that some markets are missing may be irrelevant if agents choose voluntarily not to trade in these markets as equilibrium prices perfectly cancel out aggregate demand. Hence, the assumption of complete markets and the representative-agent approach are closely tied and form two sides of the same coin. However, the interest in understanding the implications of incomplete markets and heterogeneity found its way along mainstream macroeconomic modeling. As Lucas (1992) puts it.<sup>2</sup> "If the children of Noah had been able and willing to pool risks, Arrow-Debreu style, among themselves and their descendants, then the vast inequality we see today, within and across societies, would not exist (...)". Hence, the assumption of complete markets was more and more regarded as an unrealistic benchmark, mainly imposed to avoid model complexity rather than reflecting fundamental economic principles. With increasing interest and among different approaches, a new class of models therefore emerged that nowadays is called 'standard incomplete-markets models (SIM)'. The work on incomplete-markets models at least dates back to Bewley (1983) and gained greater attention based on the work by Huggett (1993) and Aiyagari (1994). Incomplete-markets models share a lot of features with traditional macroeconomic models: i.) aggregate behavior is derived from optimal individual decision making, ii.) agents respond to exogenous shocks by adjusting consumption, saving, and possibly labor supply, iii.) prices adjust to equate aggregate demand and supply. However, the core characteristics of incomplete-markets models lie in the emphasis on individual specific shocks, i.e. idiosyncratic risk, and the relaxation of the complete markets assumption. The presence of idiosyncratic shocks, for example to individual labor productivity or more generally to income, induces heterogeneity among agents and is closely linked to market incompleteness. If financial markets were

<sup>&</sup>lt;sup>2</sup>This quote also appears in Heathcote et al. (2009).

<sup>&</sup>lt;sup>3</sup>Ljungqvist and Sargent (2012) also give the name 'Bewley-type' models. A different strand of research, for example, focuses on overlapping generation models (see, Diamond 1965).

complete, providing full insurance against idiosyncratic risk, individual choices would be the same across all agents and the incomplete-market model essentially would boil down to the standard representative-agent approach. Hence, incomplete-markets models possess a close connection between heterogeneity and market incompleteness. Of course, various types of frictions have also been introduced in other types of models, leading to different strands of research besides the standard incomplete-markets approach (cf. Brunnermeier et al. 2012). However, incomplete-markets models explicitly emphasize the importance of heterogeneity and thereby, offer a natural framework for analyzing the implications of financial market incompleteness.

To gain deeper insight into the specific characteristics of the standard incompletemarkets model, it is useful to separate between the individual and the aggregate level. On the individual level, the main building block is the income fluctuation problem (e.g., Heathcote et al. 2009). Each agent chooses an optimal consumption and saving plan subject to idiosyncratic income shocks. Financial markets are incomplete so that full insurance against idiosyncratic risk is not obtainable. Agents can only partially self-insure against idiosyncratic risk by accumulating a stock of assets that pay out a risk-free return. Furthermore, agents also face a borrowing constraint on assets, which further restricts them in their ability to allocate resources over time. The assumption of incomplete insurance markets and the presence of borrowing constraints are supported by a number of reasons. First, the volume of state contingent contracts that is traded in the real economy is limited and a large volume of financing is in the form of standard debt contracts (e.g., Quadrini 2011). Furthermore, differences in the amount of idiosyncratic risk also reflect differences in insurance opportunities provided by the financial sector, which allows taking account of differences in financial development (e.g., Angeletos and Calvet 2006). Finally, and most importantly, a number of reasons such as moral hazard, limited commitment and limited enforcement, prevent an unlimited supply of financing and the provision of a full set of individual state contingent claims (e.g., Zhang 1997, Quadrini 2011). Hence, though more emphasis is put on understanding the consequences of financial market imperfections rather than on why markets are incomplete, a number of reasons support the standard incomplete-markets approach.

Turning from the individual level to the aggregate level, the next step is to describe the behavior of a continuum of agents facing idiosyncratic risk. In representative-agent models, the aggregation step is rather simple, based on the 'Big-K-little-k' trick, individual variables are simply replaced by aggregate variables (see, Ljungqvist and Sargent 2012). In incomplete-markets models, however, computing aggregate variables and the stationary equilibrium of the economy are more complex tasks, requiring to determine the entire wealth distribution of agents. The fact that the wealth distribution is non-degenerated in incomplete-markets models follows from the presence of idiosyncratic risk and market incompleteness. An agent's current position in the wealth distribution reflects the individual history of shocks and the agent's adjustment in consumption and saving. Though the wealth distribution is time-invariant in stationary equilibrium, the

individual agent steadily travels through the state space, reflecting the recurrent appearance of idiosyncratic shocks. The fact that solving incomplete-markets models is often more demanding than solving representative-agent models also contributes to explaining the dominance of the representative-agent approach. Increasing the number of state variables in incomplete-markets models quickly leads to the curse of dimensionality and solving large models becomes a time-consuming task. Furthermore, though progress has been made, no standard numerical toolbox exists for solving incomplete-markets models.<sup>4</sup> Standard solution algorithms for incomplete-markets models therefore use global solution methods for the individual problem and rely on repeated updating of equilibrium prices to determine the general equilibrium. Heer and Maussner (2009), for example, provide a detailed description of commonly used numerical methods and algorithms.

The goal of the incomplete-markets literature is not to just add complexity to macroeconomic models. Instead, a large and fast growing literature shows the importance of incomplete markets and borrowing constraints for individual and aggregate behavior. An important motive that is commonly analyzed within the incomplete-markets framework is the precautionary saving motive that refers to the change in individual saving in response to the presence of uninsurable risk. In standard incomplete-markets models with idiosyncratic income shocks and borrowing constraints on the household side, agents engage in positive precautionary saving under mild conditions on preferences.<sup>5</sup> That means, agents build up a buffer-stock of savings to protect themselves against future income drops (e.g., Carroll 1997). Higher savings on the individual level then translate into higher savings on the aggregate level so that the interest rate is lower in stationary equilibrium than in the complete markets case (Huggett 1993). However, it is important to note that the agent's response to financial market imperfections can vary with the underlying source of risk and the specification of borrowing constraints. For example, capital risk or financing constraints on the production side may lead to different implications for saving and investment than the standard incomplete-markets approach (e.g., Gertler and Rogoff 1990, Angeletos 2007). This clearly increases the flexibility of incomplete-markets models to explain various patterns, but also means that understanding the overall implications of financial market imperfections becomes a more complex task.<sup>6</sup>

A quite natural application of incomplete-markets models is to explain the evolution and current state of income and wealth distributions. This topic has recently gained

<sup>&</sup>lt;sup>4</sup>Winberry et al. (2016) developed a toolbox for solving incomplete-markets models in continuous time. However, up to now, this toolbox has not reached a similar standing as Dynare for DSGE models. For more recent progress to use standard toolboxes see Le Grand and Ragot (2017) and Boppart et al. (2017)

<sup>&</sup>lt;sup>5</sup>In case of stochastic fluctuations in income, the early literature shows that agents engage in positive precautionary saving if the third derivative of the period utility function is positive (e.g. Sandmo 1970, Kimball 1990). This assumption can be relaxed even further if agents face occasionally binding borrowing constraints.

<sup>&</sup>lt;sup>6</sup>The effects of capital risk and financing constraints on the production side will play a crucial role in Chapter 3 which focuses on financial integration.

special interest based on the book by Piketty and Goldhammer (2014), but was also a leading theme in earlier work (e.g., Cagetti and Nardi 2006). Current research especially focuses on matching the upper tail of the wealth distribution to explain the observed increase in top income dispersion (e.g., Nirei and Aoki 2016). Furthermore, incomplete-markets models also contribute to understanding the effects of monetary and fiscal policy, providing deeper insights beyond the complete-markets representative-agent approach. For example, in the presence of borrowing constraints and uninsurable risk, Ricardian equivalence fails to hold and fiscal policies that are neutral under complete markets may have substantial quantitative effects (e.g., Aiyagari and McGrattan 1998, Heathcote 2005). The topic of fiscal policy and incomplete markets has gained special interest after the recent financial crisis and has led to a renewed debate about the role of public debt and debt policy.

Finally, understanding the individual specific welfare implications also plays an important role in heterogeneous-agents economies. In his influential work, Lucas (1987) showed that for standard preferences, fluctuations in consumption have very little impact on the welfare of a representative consumer. However, in heterogeneous-agents economies, welfare effects may be substantially larger for some agents depending on their position in the wealth distribution (e.g., Storesletten et al. 2001). Furthermore, agents may also be affected very differently by certain types of policies. For example, the process of financial integration may lead to very different effects for agents depending on whether they derive income predominately from wages or interest payments. Hence, the support for certain policies will differ between agents, which emphasizes the importance of the distributional effects for policy considerations.

In summary, incomplete-markets models with heterogeneous agents nowadays play an important role in macroeconomic modeling. However, though the literature in this field grew rapidly, especially after the recent financial crisis, it is still in an early phase and various topics, ranging from classical puzzles and paradoxes to recent patterns, are yet to be explored. The aim of this dissertation is to contribute to this strand of research. Though different topics are covered by the four papers, they are strongly linked by the following underlying aims: understanding the implications of financial market imperfections and heterogeneity for individual and aggregate behavior, explaining patterns that cannot be fully explained by standard theory, and analyzing the corresponding welfare and policy implications.

#### Contributions of the dissertation

The first paper (Chapter 2) and second paper (Chapter 3) both deal with the topic of financial integration. Specifically, these papers emphasize the importance of distributional effects and financial market imperfections for explaining the pattern of international capital flows. The model considered in the first paper is not yet an incomplete-markets model, but bridges the gap by introducing heterogeneity in an otherwise neoclassical growth model. The second paper then considers a comprehensive incomplete-markets model with uninsurable risks and different types of financing constraints. In the following, I describe the two papers in greater detail.

The first paper, Opposition to Capital Market Opening, which is joint work with Philipp Engler, deals with the topic of financial integration. The integration of financial markets has long been regarded as a panacea for economic development by mainstream economists: Free capital flows would allow an efficient global allocation of savings, thereby amplifying economic growth and increasing welfare. These benefits should be particularly large for developing countries and standard theory predicts huge 'downhill' flows of capital, i.e. from rich to poor countries to close the gap in domestic returns. However, the literature of the past years has shown that these predictions did not materialize in a significant and robust order of magnitude. In particular, Lucas (1990) showed that capital flows to developing countries were much too low to fit into the picture of standard theory. Furthermore, as shown by Schularick (2006), the puzzle of low capital flows has not vanished over time as capital flows to developing countries in the recent period of financial integration have been even lower than in the period before World War I.

In this paper we add to the theoretical debate about potential causes of low capital flows to developing countries. We focus on the distributional effects of financial market integration and show that it might not be surprising at all that poor countries do not opt for large inflows of foreign capital because owners of the domestic capital stock take a massive hit to their income. In an otherwise standard neoclassical growth model of a small and developing country, we implement a heterogeneous-agent approach and look at the polar case of concentration of the domestic capital stock in the hands of capital owners. They derive income from the returns to their stock of capital only, while workers derive only wage income. We show in a benchmark calibration that capital owners suffer a permanent decrease in consumption of 42 percent under financial integration due to lower returns while only workers gain 8 percent of consumption due to higher wages. These huge gross impacts contrast with the small positive net effect found in a neoclassical representative agent model by Gourinchas and Jeanne (2006). Our estimates thus suggest that transfers from winners to losers of financial integration have to be huge and may be hard to achieve in any political process. But if capital owners are not compensated, they have a strong incentive to oppose an opening of capital markets and their losses provide a measure of their willingness to pay to reach that aim. Therefore, our quantitative results, which predict large losses, indicate a strong political relevance of the distributional effects which cannot be identified by just looking at the small net effects.

The results of the first paper show that the size of distributional effects may contribute to explain why the magnitude of capital flows to poor countries does not fit into the picture of standard theory. However, the counter-intuitive behavior of international capital flows has become even more puzzling in recent years. Prasad et al. (2006, 2007) show that since the end of the 20th century, the average income of countries running current account surpluses has fallen below the average income of countries running current account deficits, implying that capital nowadays even flows from poor to rich countries. In other words, we observe a 'second generation' of the Lucas paradox. Furthermore,

domestic economic development in the light of capital outflows also challenges conventional wisdom. Prasad et al. (2006, 2007) find periods of capital outflows from highgrowth nonindustrial countries and Sandri (2010) notes improvements in the current accounts of developing countries during periods of high per capita income growth. To explain these patterns, the literature has moved beyond the predictions of the frictionless neoclassical model and emphasizes the importance of financial market imperfections in the form of borrowing constraints and uninsurable idiosyncratic risks. As pointed out by Mendoza et al. (2009a), countries still differ significantly in their level of financial development despite the persistent increase in the volume of cross-border capital flows. However, the questions whether and under what conditions the presence of uninsurable risks and borrowing constraints contributes to explain the empirical findings have not been completely resolved yet. In fact, the literature shows that the results are extremely diverse and strongly vary with the underlying source of risk, the exact specification of borrowing constraints and further model assumptions. Hence, while progress has been made to understand some of the individual effects, we are still lacking a clear understanding of the rich interactional effects and therefore, of the joint overall effect of financial market imperfections on the process of financial integration. The paper presented in Chapter 3, Idiosyncratic Risk, Borrowing Constraints and Financial Integration, which is joined work with Maik Heinemann, aims at filling this remaining gap. Motivated by the mixed results from the literature, we employ an incomplete-markets model in which entrepreneurs face capital risk, receive riskless wage income and also face income risk due to risky profits. Moreover, borrowing constraints simultaneously impede consumption smoothing and limit the access to external funds for scaling up production. To keep track of the individual effects, we consider different scenarios, increasing the model complexity step by step. We find that in the presence of uninsurable risk only and for plausible parameter restrictions, it is in fact the initially poor country that builds up a positive net foreign asset position under financial integration and that an increase in the interest rate leads to higher levels of capital and output in the integrated steady state. However, we also find that tight borrowing constraints and high persistence of shocks strongly affect the model predictions and lead to significantly tighter parameter restrictions to explain the empirical facts. In particular, we find that in times of strong turmoil in financial markets, with an almost collapsing lending channel, the model predictions drastically change and indicate that financial integration may easily become an impediment for domestic economic development. Hence, as an overall result, we can conclude that a very careful consideration of the country specific characteristics is needed in order to fully understand the implications of financial integration. Even if two countries are characterized by a similar level of financial development, the implications of financial integration may be very different, depending on the dominant source of risk, the exact specification of borrowing constraints and the persistence of risk. This paper provides a basis for understanding these specific differences in financial market performance. Furthermore, we compare the welfare implications under the different model predictions and identify those members of society who are most likely to either win or lose from financial integration.

Taking stock, the first two papers show how the evolution of macroeconomic modeling contributes to explaining the puzzling behavior of international capital flows and domestic economic development. In the first paper, we emphasized the importance of the distributional effects for explaining low capital flows to developing countries as domestic capital owners take a massive hit to their income. However, introducing heterogeneity alone does not explain the reverse of capital flows, with poor countries building up a positive net foreign asset position. Therefore, the literature has considered the implications of financial market imperfections where heterogeneity endogenously arises from market incompleteness. In the second paper, we discussed the conditions under which financial market imperfections contribute to explaining the empirical pattern in a comprehensive incomplete-markets model. With these insights, we leave the topic of financial integration.

The third paper (Chapter 4) and the fourth paper (Chapter 5) both remain within the incomplete-markets framework but consider two different topics. The third paper deals with the implications of incomplete information and the fourth paper considers the financing of government spending. In the following, I describe the two papers in greater detail.

The third paper, Incomplete Information, Financial Market Imperfections, and Aqgregate Saving, investigates the importance of incomplete information about idiosyncratic income shocks within the incomplete-markets framework. Thereby, the paper links the existence of financial frictions to the existence of informational frictions and contributes to the strand of literature casting doubt on the optimistic view that agents are able to observe and to distinguish each type of income shock. As Goodfriend (1992) and Pischke (1995) point out, not all information is instantaneously available and collecting information is costly. Therefore, agents may not be able or willing to attribute each change in income to the respective underlying income shock. The literature in this field shows that understanding the agent's information set is not only important for explaining the individual behavior, but also takes a big step towards understanding the observed pattern of aggregate variables. Pischke (1995) and Ludvigson and Michaelides (2001) for instance show that the agent's inability to distinguish between individual and aggregate income shocks contributes to explaining the excess smoothness and sensitivity of aggregate consumption. In this paper, I too find a strong impact of incomplete information; however, the focus is more directed towards the interaction between incomplete information and financial market imperfections and the joint impact on aggregate saving.

The model I consider is an Aiyagari-type heterogeneous-agents, incomplete-markets model (see, Aiyagari 1994). Agents are subject to two types of productivity shocks, which expose each of them to an uninsurable income risk. Under the common assumption of complete information, agents can distinguish between shocks, whereas under

incomplete information, agents cannot distinguish the two components. In the latter case, agents have to use a simpler, single-shock model to predict future productivity growth. I find that in this setting, incomplete information does not only have a strong quantitative effect on aggregate saving, but also a qualitatively ambiguous effect. Dependent on the interest rate level and, thereby, dependent on the strength of the financial frictions, aggregate savings may be around 25 percent lower but also up to 40 percent higher under incomplete information than in the complete information case. An immediate implication of these high numbers is that estimating the agents' saving behavior becomes a more complex task: Depending on whether the interest rate is rather high or low, aggregate savings may be considerably under- but also considerably overestimated under the common assumption of complete information.

I show that the ambiguous behavior of aggregate saving can be traced back to the interplay of incomplete information and the financial market imperfections. The no-borrowing constraint breaks the symmetry in the agent's saving behavior based on a good or bad shock, and agents save more based on a good shock under complete information. Furthermore, the observation of larger aggregate asset holdings under incomplete information can be explained by the interplay of incomplete information and the income risk. I isolate the income-risk channel and show that agents have a higher precautionary saving demand under incomplete information. All in all, the results show that incomplete information plays a complex role within the incomplete-markets model framework and is more than just another source of risk.

In the fourth paper, Financing of Government Spending in an Incomplete-Markets Model: The Role of Public Debt, which is joint work with Maik Heinemann, we consider the implications of debt policy in an incomplete-markets model. The role of public debt and the design of debt policy have regained a lot of interest after the recent financial crisis and special emphasis has been placed on the interplay of debt policy and financial market imperfections. Since the work by Woodford (1990), Aiyagari and McGrattan (1998) and Flodén (2001), it has been known that public debt plays a special role in the presence of borrowing constraints and uninsurable idiosyncratic risk. However, the recent literature shows a remarkable debate on whether the positive or the negative effects of a high level of public debt finally prevail. This applies to both the long-run and the transitional dynamics of public debt (e.g., Dyrda and Pedroni 2016, Röhrs and Winter 2016, 2017).

In this paper, we contribute to the ongoing debate by studying the role of debt policy regarding the financing of government spending in an incomplete-markets model. In particular, we analyze the influence of different types of fiscal rules on the response of key macroeconomic variables to a government spending shock. The fiscal rule is called debt-intensive if public debt strongly increases in the first periods following the increase in government spending and the fiscal rule is called non-debt-intensive if it keeps the primary deficit small. Note that in the complete-markets case, the economy's response to the government spending shock does not depend on the underlying fiscal

rule. However, in the presence of borrowing constraints and uninsurable idiosyncratic risk, we observe significant differences in the responses of consumption, leisure, capital and output between the two fiscal rules. In particular, we find that the debt-intensive fiscal rule contributes to stabilizing consumption and leisure in the first periods following the change in government spending, whereas the non-debt-intensive fiscal rule leads to a faster recovery of consumption, leisure, capital and output in later periods. Hence, the possible benefits of raising the level of public debt in the first periods come along with lower consumption and leisure in later periods.

To understand the observed impact of public debt, we have to consider the interplay of public debt and financial market imperfections. In the presence of borrowing constraints and uninsurable risk, Ricardian equivalence fails to hold and agents with low financial wealth respond differently to a change in either the lump-sum tax or in public debt. Compared to a tax increase that leads to an unavoidable negative wealth effect, an increase in public debt allows borrowing constrained agents to mitigate the negative effect on consumption. This explains the more moderate drop in aggregate consumption and leisure under the debt-intensive fiscal rule in the first periods after the change in government spending. However, in later periods, taxes have to also increase under the debt-intensive fiscal rule. Furthermore, the sluggish increase in aggregate saving leads to a higher interest rate and, thereby, to a lower capital stock along the adjustment path. Both effects contribute to explaining why the recovery of consumption, leisure, capital and output is slower under the debt-intensive fiscal rule.

The observed differences in the adjustment paths of aggregate variables do not offer a simple answer to the question of which type of fiscal rule maximizes aggregate welfare. On the one hand, agents may prefer the debt-intensive fiscal rule due to its stabilizing effect on consumption and leisure in the first periods of transition. On the other hand, agents may also prefer the non-debt-intensive fiscal rule due to the faster recovery of consumption, leisure, capital and output in later periods of transition. Comparing the aggregate welfare effects between the two fiscal rules, we find that aggregate welfare is higher under the debt-intensive fiscal rule than under the non-debt-intensive fiscal rule. Although the differences in aggregate welfare remain quantitatively rather small, this result indicates that a temporary strong increase in public debt in response to the increase in government spending may be justified on the grounds of optimal debt policy. Hence, from a political economy perspective, our results provide some rationale for the observed pattern of public debt. Furthermore, we find that the individual welfare gain is particularly high for wealth-poor agents with low productivity who suffer most from the financial market imperfections. This result highlights the importance of the individual effects for understanding the role of debt policy.

## Chapter 2

# Opposition to Capital Market Opening<sup>1</sup>

#### Abstract

We employ a neoclassical growth model of a developing country to assess the impact of financial liberalization on capital owners' and workers' consumption and welfare. We find in a baseline calibration for an average non-OECD country that capital owners suffer a 42 percent reduction in permanent consumption because financial integration leads to lower returns while workers gain 8 percent of permanent consumption due to higher wages. These huge gross impacts contrast with the small positive net effect found in a neoclassical representative agent model by Gourinchas and Jeanne (2006). We further show that the result for capital owners is insensitive to enhanced productivity catch-up processes induced by capital inflows. Our findings can help to explain why poorer countries tend to be less financially open as capital owners' losses are largest for countries with the lowest capital stocks, generating strong opposition to capital market opening.

<sup>&</sup>lt;sup>1</sup>This paper was written in collaboration with Philipp Engler. Comments by Moritz Schularick, Thomas Steger, Dominic Quint, Christoph Große Steffen and Wolfgang Strehl are gratefully acknowledged.

#### 2.1 Introduction

The international integration of financial markets has long been regarded as a panacea for economic development by mainstream economists. In an often-cited paper, Fischer et al. (1998) summarized one of the main hopes associated with cross-border capital flows between economically developed and economically less developed countries: Free capital flows would allow an efficient global allocation of savings, thereby amplifying economic growth and increasing welfare. Another potential benefit is a reduction of consumption volatility because of better diversified portfolio holdings that reduce exposure to country or industry specific shocks (see, Eichengreen et al. 1998). These benefits should be particularly large for developing countries and standard theory predicts huge 'downhill' flows of capital, i.e. from rich to poor countries, in order to close the gap between domestic returns.

However, the fast growing literature of the past few years has shown that these predictions did not materialize in a significant and robust order of magnitude. Kose et al. (2003), for example, show that the ratio of consumption growth volatility to income growth volatility remained flat for low income developing countries. Even more striking is the finding that the size of capital flows from rich to poor countries dramatically fails to fit into the picture of standard theory: In his famous article, Lucas (1990) showed that capital flows to developing countries were much too low; a puzzle that has not vanished over time. As shown by Schularick (2006), capital flows to developing countries in the recent period of financial integration were even lower than in the period before World War I.<sup>2</sup>

In this paper we add to the theoretical debate about potential causes of the last of these stylized facts, the low inflow of capital to developing countries. We focus on the distributional effects of financial market integration and show that it might not be surprising at all that poor countries do not opt for large inflows of foreign capital because owners of the domestic capital stock take a massive hit to their income. In an otherwise standard neoclassical growth model of a small and developing country, we implement a heterogeneous-agent approach and look at the polar case of concentration of the domestic capital stock in the hands of capital owners. They derive income only from the returns to their stock of capital while workers only derive wage income. We show in a benchmark calibration that capital owners suffer a permanent decrease in consumption of 42 percent under financial integration due to lower returns while only workers gain 8 percent of consumption due to higher wages. These massive losses for capital owners imply that consequences for policies with respect to capital flows are to be expected. Even though Pareto improvements are possible if re-distributional institutions are in place, the huge amount of transfers that are necessary to leave no one worse off seems simply too big to achieve for most countries. Therefore, opposition to capital inflows is

<sup>&</sup>lt;sup>2</sup>Prasad et al. (2006) show that since the end of the twentieth century, capital even tends to flow 'uphill', i.e. from poor to rich countries.

likely to be pervasive on the side of capital owners and this opposition can be expected to be particularly strong in the poorest countries since capital owners' losses are the bigger the poorer the country is.

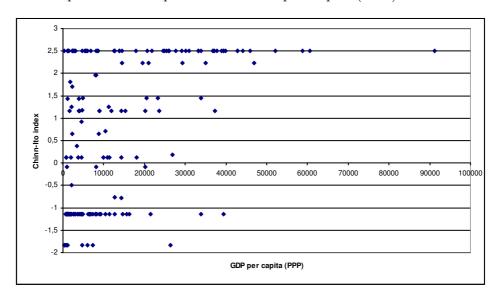


Figure 2.1: Capital Market Openness and GDP per Capita (PPP)

An indication that such concerns are indeed relevant is that the strongest restrictions to capital flows can be found among the poorest countries. Figure 2.1 shows the Chinn-Ito index of capital market openness<sup>3</sup> and the respective GDP per capita (PPP) for 170 countries in 2008.<sup>4</sup> This index increases with the degree of openness and one can see that the severest restrictions can be found among the countries with less than US-\$ 16000 per capita GDP. Above this threshold there are hardly any countries in the lower part of this graph. Consequently, even though there is an overall tendency towards financial integration, we still observe a number of poor countries with tight capital controls.<sup>5</sup>

Our analysis of the distributional effects of financial integration closely relates to the literature on the political issues of capital flows and capital controls. Alfaro (2004) shows in an overlapping generation model that in a capital-importing country, old people who only receive capital income would vote against capital account liberalization while young people would prefer to remove capital flow barriers. Pinto (2005) finds similar results by looking at the influence of FDI: Labor favors foreign direct investment while capital suffers from the downward pressure on the interest rate.<sup>6</sup> Furthermore, Aizenman (2005)

<sup>&</sup>lt;sup>3</sup>The Chinn-Ito index measures the extensity of capital controls and is based on information from the IMF's *Annual Report on Exchange Arrangements and Exchange Restrictions* (see Chinn and Ito 2008 for details).

 $<sup>^4</sup>$ These are all countries for which both variables are available. For GDP data, we use the World Bank's World Development Indicators & Global Development Finance database.

<sup>&</sup>lt;sup>5</sup>Our results also contribute to explain the findings of Reinhardt et al. (2010) that in countries with a closed capital account, there is no systematic relationship between net capital flows and the level of development.

 $<sup>^6\</sup>mathrm{See}$  also Pinto and Pinto (2008) for a sectoral analysis.

shows that FDI in terms of 'green-field' investments may also have a negative impact on the capitalists' welfare by reducing the return on domestic capital. However, this literature did not calculate the size of the distributional effects of capital market integration, which we believe is crucial for political economy considerations. This becomes most clear from comparing our results to the standard representative agent approach analyzed by Gourinchas and Jeanne (2006) (G&J in what follows). G&J show that in the neoclassical growth model with a single representative agent, the net welfare gain of a capital scarce country is only 1.7% of permanent consumption even though the domestic capital stock more than doubles under financial integration. This is in sharp contrast to the large gross effects we observe for the two groups of agents, indicating a strong political relevance of the distributional effects of financial integration that cannot be identified by just looking at the small net effects. Simply, if capital owners are not compensated, they have a strong incentive to prevent an opening of capital markets and the size of their losses provide a measure of their willingness to pay to reach that aim.

The remainder of this paper is organized in the following way. Section 2.2 presents the model with two types of agents, capital owners and workers. Section 2.3 describes the impact of financial openness on consumption and welfare and discusses the impact of a productivity catch-up process, which may be induced by financial inflows. Section 2.4 concludes.

#### 2.2 The model

We consider a small economy that is populated by two types of agents, capital owners and workers. Without loss of generality, we normalize the population of capital owners to one and assume a number of L identical workers. Therefore, the ratio of capital owners to workers is given by  $\eta = 1/L$  which, by assumption, is constant over time. The capital owner does not work but receives income from the return to his stock of capital. Workers supply their labor to a domestic firm and receive wage income only.

In order to analyze the effects of foreign capital inflows, we compare two cases, financial autarky and financial integration. Under financial autarky, there are no international capital flows and the development of the domestic capital stock is completely determined by the capital owner's savings. Under financial integration, the capital owner is allowed to freely trade a riskless bond on the international capital market given the world interest rate  $R^*$ .

#### 2.2.1 Financial autarky

#### **Production**

In the economy, a single final good is produced by a representative firm, employing the input factors capital and labor. Technology is given by a Cobb-Douglas production function of the form

$$Y_t = K_t^{\alpha} (A_t L_t)^{1-\alpha}, \tag{2.1}$$

where  $Y_t$  is the output of the single good,  $A_t$  is a labor-augmenting measure of productivity,  $K_t$  is the capital stock, and  $L_t$  denotes labor. Following G&J, we assume that labor productivity grows at constant rate, g > 0, which is the same for all countries.<sup>7</sup>

In order to ensure stationarity in the presence of productivity growth, we normalize variables by labor productivity. Output per effective labor input is thus given by

$$\widetilde{y}_t \equiv \frac{Y_t}{A_t L_t} = \widetilde{k}_t^{\alpha}, \tag{2.2}$$

where the tilde on top of the variables indicates the normalization. Factor markets are perfectly competitive so that labor and capital are paid according to their marginal products

$$r_t = \alpha \tilde{k_t}^{\alpha - 1} - \delta \tag{2.3}$$

$$w_t = A_t (1 - \alpha) \tilde{k_t}^{\alpha}, \tag{2.4}$$

where  $\delta$  is the constant depreciation rate.

#### Workers

Workers have preferences over consumption and maximize discounted lifetime utility. Preferences regarding momentary consumption are described by the log-function, so that the representative worker's objective function is given by

$$U_w = \sum_{t=0}^{\infty} \beta^t \ln(c_t^w). \tag{2.5}$$

 $c_t^w$  denotes the worker's consumption and  $\beta$  is the constant discount factor. Workers face the budget constraint

$$c_t^w = w_t, (2.6)$$

where  $w_t$  is the wage rate. The worker's optimization problem is the same under financial autarky and financial integration and simply means that the level of consumption in each period is equal to the level of wage income.

#### Capital Owners

Capital owners have the same preferences regarding momentary consumption as workers. The representative capital owner's objective function is given by

$$U_c = \sum_{t=0}^{\infty} \beta^t \ln(c_t^c), \tag{2.7}$$

where  $c_t^c$  denotes the capital owner's consumption. Under financial autarky, capital is

 $<sup>^{7}</sup>$ In the next section, we also consider the case of changes in productivity induced by financial integration.

the only asset available in the economy so that the capital owner's budget constraint is given by

$$c_t^c + k_{t+1}^c = R_t k_t^c, (2.8)$$

where  $R_t = (1 + r_t)$  is the domestic gross interest rate. The capital owner maximizes (2.7) subject to the constraint in (2.8). From the first-order conditions we get the Euler equation

$$\widetilde{c}_{t+1}^c = \frac{1}{1+q} \beta R_{t+1} \widetilde{c}_t^c, \tag{2.9}$$

where the tilde indicates normalization with productivity, i.e.  $\tilde{c}_t^c \equiv c_t^c/A_t$ . Given logutility, we can solve analytically for the optimal policy functions for normalized consumption and capital<sup>8</sup>

$$\widetilde{k}_{t+1}^c = \frac{1}{1+q} \beta R_t \widetilde{k}_t^c \tag{2.10}$$

$$\widetilde{c}_t^c = (1 - \beta) R_t \widetilde{k}_t^c. \tag{2.11}$$

Equation (2.11) shows that the capital owner consumes a constant fraction of income in each period.

#### Stationary equilibrium

As it is well-known from the standard neoclassical growth model, the economy under financial autarky converges toward a steady state where the interest rate satisfies the following condition

$$R^* = \frac{1+g}{\beta}.\tag{2.12}$$

The steady state interest rate then pins down the corresponding capital stock per efficient unit of  $labor^9$ 

$$\widetilde{k}^* = \left(\frac{R^* - 1 + \delta}{\alpha}\right)^{\frac{1}{\alpha - 1}}.$$
(2.13)

#### 2.2.2 Financial integration

We now consider the case of financial integration where cross-border capital flows are unrestricted. In contrast to the case of financial autarky, the domestic capital owner is now allowed to trade a bond given the world interest rate  $R^*$ . Following G&J, we assume that the world interest rate is equal to the steady state interest rate under financial autarky given in (2.12). This ensures stationarity under financial integration and, as shown below, that the steady state capital stock per efficient unit of labor is the same as in the long-run equilibrium under financial autarky (see, Barro and Sala-i-Martin 2004). Hence, financial integration only acts as an accelerator of domestic economic development.

<sup>&</sup>lt;sup>8</sup>See Appendix A.1.

<sup>&</sup>lt;sup>9</sup>The normalized capital stock per capital owner is given by  $\tilde{k}^{*c} = \frac{1}{n}\tilde{k}^*$ .

The opportunity to lend and borrow on the international level means that the domestic capital owner has to decide on capital and bond holdings. Since we consider the case of an initially poor country that accumulates a negative net foreign asset position under financial integration, implying capital flows from rich to poor, we define  $d_t^c$  as the capital owner's net debt to foreigners. The capital owner's budget constraint is then given by

$$c_t^c + k_{t+1}^c + R^* d_t^c = R_t k_t^c + d_{t+1}^c. (2.14)$$

The problem of the representative capital owner under financial integration is to maximize (2.7) subject to the budget constraint in (2.14). From the corresponding first-order conditions we obtain the two Euler equations

$$\tilde{c}_{t+1}^c = \frac{1}{1+q} \beta R_{t+1} \tilde{c}_t^c \tag{2.15}$$

and

$$\tilde{c}_{t+1}^c = \frac{1}{1+q} \beta R^* \tilde{c}_t^c. \tag{2.16}$$

Equation (2.15) is the Euler equation with respect to capital and equation (2.16) is the Euler equation with respect to bond holdings. Combining the two equations yields the important result  $R^* = R_{t+1}$ , which can be seen as the no-arbitrage condition of free capital flows. The no-arbitrage condition means that capital and bonds have to yield the same return in equilibrium since both are perfect substitutes from the agent's perspective. Furthermore, the no-arbitrage condition implies that the capital stock per efficient unit of labor under financial integration immediately jumps to its long-run level given in (2.13) regardless of the preferences of the domestic capital owner. In other words, we observe that the poor country accumulates a negative net foreign asset position under financial integration and that the domestic capital stock increases significantly. Finally, by defining  $a_t = k_t - d_t$  as the capital owner's net assets, we can solve for the optimal policy functions for normalized asset holdings and consumption under financial integration. For t > 0 we have  $^{11}$ 

$$\widetilde{a}_{t+1}^c = \frac{1}{1+q} \beta R^* \widetilde{a}_t^c \tag{2.17}$$

$$\widetilde{c}_t^c = (1 - \beta) R^* \widetilde{a}_t^c. \tag{2.18}$$

Given that  $R^*$  equals the common long-run steady state interest rate, consumption and savings immediately grow at rate g under financial integration.

 $<sup>^{10}</sup>$ The properties of the production function ensure this interior solution.

<sup>&</sup>lt;sup>11</sup>In period 0 we have  $\widetilde{a}_1^c = \frac{1}{1+g}\beta R_0\widetilde{a}_0^c$ ,  $\widetilde{c}_0^c = (1-\beta)R_0\widetilde{a}_0^c$  with  $\widetilde{a}_0^c = \widetilde{k}_0^c$  and  $R_0$  given (see Appendix A.2. for details).

#### 2.3 Effects of financial integration

We now employ the model to quantify the effects of financial integration. After an outline of the calibration, we start with the discussion of the welfare implications for the two groups of agents. For this, we consider two metrics of welfare. First, we compute the change in consumption that is needed to bring welfare under autarky to its level under financial integration in each period. That allows us to demonstrate the effects of financial integration over time. Second, we compute the permanent change in consumption that brings welfare under autarky to its level under financial integration. This allows us to summarize the overall effect of capital mobility in terms of average consumption change.

#### 2.3.1 Calibration

Regarding the model parameter values,  $\{\beta, \alpha, \delta, g\}$ , we stay close to the calibration of G&J in order to facilitate a direct comparison (see Table 2.1). The discount factor,  $\beta$ , is set to 0.96 and  $\alpha$  is set to 0.3, implying a labor income share of 0.7. The depreciation rate,  $\delta$ , equals 6 percent and the growth rate of productivity, g, is equal to 1.2 percent. These values lead to a long-run interest rate of  $R^* = 1.0542$ .

Table 2.1: Parameter Values

Parameter	Value	
discount factor	β	0.96
curvature of production	$\alpha$	0.3
depreciation rate	$\delta$	0.06
growth rate of productivity	g	0.012

In addition to the parameter values, we also have to define the initial level of the capital stock under financial autarky. This value is important since it determines the initial gap in interest rates that is closed under financial integration. We follow G&J who calculate the initial capital stock using data of 82 non-OECD countries in the year 1995. They find that the population-weighted average capital stock equals 41 percent of the capital stock that is reached in the long-run equilibrium. That means the average non-OECD country starts with less than half of the capital stock per worker that is reached immediately under financial integration. We use this as the benchmark specification, but also provide a sensitivity analysis with respect to changes in the initial conditions.

<sup>&</sup>lt;sup>12</sup>Specifically, G&J calculate a population-weighted average capital-to-output ratio of 1.4. The corresponding initial capital stock ratio then follows from the following formula,  $k_0/k^* = \left(\frac{k_0/y_0}{k^*/y^*}\right)^{\frac{1}{1-\alpha}} = 0.41$  with  $k^*/y^* = 2.63$ .

#### 2.3.2 Capital owners

The sign of the effect of financial integration for domestic capital owners is easily explained. Once trading barriers are removed, the poor country builds up a negative foreign asset position and the domestic capital stock increases until the domestic interest rate equals the world interest rate. Hence, interest rates equalize under financial integration so that there exist no arbitrage opportunities in equilibrium. But without opportunities for arbitrage profits, the only effect on the capital owner's income is the negative effect of the increased capital stock on the interest rate, leading to a process of lower consumption and savings that accumulates over time. Hence, domestic capital owners in the developing country will be worse off in the case of financial integration due to the downward adjustment of the interest rate.

To see this effect more clearly, we calculate the percentage change in consumption,  $\mu_t^c$ , that is required to equalize welfare under financial autarky and financial integration in each period. Formally,  $\mu_t^c$  is given by

$$\mu_t^c = \frac{c_t^{cI}}{c_t^{cA}} - 1,\tag{2.19}$$

where  $c_t^{cA}$ ,  $(c_t^{cI})$  denotes the capital owner's consumption under autarky (integration). By forward iteration of the Euler equations, we can use the fact that  $c_t^{cA} = (1 - \beta)\beta^t \prod_{j=0}^t R_j k_0^{cA}$ ,  $c_t^{cI} = (1 - \beta)\beta^t (R^*)^t R_0 k_0^{cI}$  and  $k_0^{cA} = k_0^{cI}$ , which allows us to express  $\mu_t^c$  as the ratio of the cumulative interest rates under financial integration and financial autarky

$$\mu_t^c = \frac{(R^*)^t}{\prod_{i=1}^t R_i} - 1. \tag{2.20}$$

Equation (2.20) highlights that the effects of financial integration for the domestic capital owner can be traced back to the differences in interest rates. In period 0, there is no difference in consumption. However, in the following periods, the negative effects of financial integration for the capital owner's income take place and cause the consumption gap to increase over time. This follows from the fact that the interest rate under financial autarky is higher than under financial integration as long as the economy under autarky is on its transition path with a lower capital stock but with a higher domestic return. This process continues until the economy under financial autarky reaches the long-run equilibrium where the interest rate is the same as under financial integration. Furthermore, equation (2.20) also shows that losses are larger when the initial capital stock is low because the larger is the gap between interest rates under financial autarky and financial integration.

Based on the theoretical discussion of the effects, we now quantify them. Figure 2.2 shows the results for the typical non-OECD country that starts with an initial capital stock ratio of less than 50 percent. Since we know that consumption under autarky will be higher than under financial integration, Figure 2.2 shows the decrease in consumption under autarky that equalizes welfare under autarky and integration.

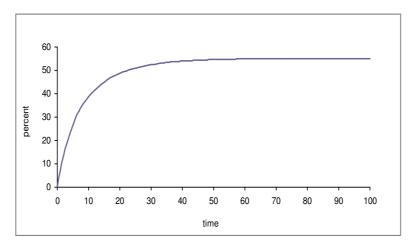


Figure 2.2: Capital Owner's Consumption Loss (in %)

As explained above, no reduction in consumption is needed in period 0. However, in the first period after financial integration, the required percentage decrease in consumption that is needed to avoid a loss in welfare under integration relative to autarky is roughly 8 percent. In the following periods, the gap further increases, albeit at a decreasing rate. The highest value is reached in the steady state and equals 55 percent. Hence, financial integration may have a very strong negative effect for capital owners in a typical non-OECD country.

We now turn to our second metric of welfare and calculate, in the same vein as G&J, the permanent change in consumption,  $\mu_c$ , which equalizes welfare under autarky and financial integration. Formally,  $\mu_c$  solves

$$\sum_{t=0}^{\infty} \beta^t \ln(c_t^{cA} - \mu_c c_t^{cA}) = U_c^I, \tag{2.21}$$

which finally leads to

$$\mu_c = 1 - \exp((1 - \beta)(U_c^I - U_c^A)).$$
 (2.22)

This expression has the advantage of summarizing the overall effect of financial integration in terms of a single indicator. For the average non-OECD country, we find that  $\mu_c$  is equal to 42 percent. This means that in a typical developing country, a permanent reduction of the representative capital owner's consumption of 42 percent is needed to equalize welfare under autarky and integration. In other words, domestic capital owners would spend roughly half of their consumption in order to avoid the removal of capital control barriers.

Finally, we want to illustrate the sensitivity of this result with respect to changes in the initial capital stock ratio. Figure 2.3 presents realizations of  $\mu_c$  for different capital stock ratios, starting with the ratio of the poorest countries in the sample of non-OECD countries and ending with the ratio of the richest countries.

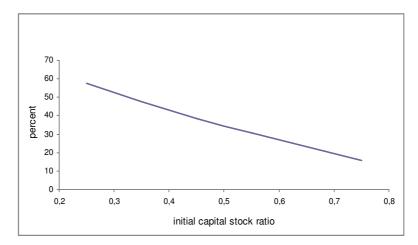


Figure 2.3: Capital Owner's Permanent Consumption Loss (in %)

Note: Figure 2.3 shows the capital owner's permanent consumption loss after liberalization,  $\mu_c$ , as a function of the initial capital stock ratio (relative to steady state).

Figure 2.3 shows that for the poorest countries, the permanent change in consumption is equal to 57 percent.  $\mu_c$  then falls to its lowest value of 16 percent which is the loss for domestic capital owners in a country starting with a relative capital stock of 75 percent. For the poorest countries, these losses to capital owners constitute a huge obstacle to financial market opening policies.

#### 2.3.3 Workers

The winners of free cross-border capital flows are workers. They benefit from the higher stock of capital which raises the wage rate above the level under financial autarky. Since workers consume their entire wage income, we can express the required change in consumption under autarky as

$$\mu_t^w = \frac{w_t^I}{w_t^A} - 1 = \left(\frac{k_t^I}{k_t^A}\right)^{\alpha} - 1, \tag{2.23}$$

where  $k_t^I(k_t^A)$  denotes capital per worker under integration (autarky).

Equation (2.23) shows that workers especially benefit in the first period after financial integration where the difference in capital stocks between autarky and integration is particularly high. However, in the following periods, the capital stock under autarky converges to its level under financial integration, implying the differences in consumption to decrease. In the steady state, where capital stocks are equal under autarky and integration, the consumption gap is zero.

Figure 2.4 shows the effects for the typical non-OECD country. The difference in consumption for workers is the highest in the first period after financial integration, reaching a maximum of around 27 percent. Thereafter, the economy under autarky closes the gap to the steady state in which the economy under financial integration

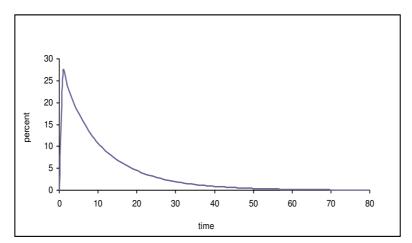


Figure 2.4: Worker's Consumption Gain (in %)

already is. Ten periods after financial market opening, the difference is about 11 percent while in the steady state, of course, the gap is zero. The comparison between these results and those of the capitalist shows the different dynamics caused by financial integration: While the workers' gain becomes smaller over time the capital owners' loss increases.

In the next step, we calculate the permanent increase in consumption under autarky,  $\mu_w$ , which equalizes welfare under both scenarios. Since workers gain in the case of free capital flows we can write

$$\sum_{t=0}^{\infty} \beta^t \ln(c_t^{wA} + \mu_w c_t^{wA}) = U_w^I, \tag{2.24}$$

which finally leads to

$$\mu_w = \exp((1 - \beta)(U_w^I - U_w^A)) - 1. \tag{2.25}$$

For the typical non-OECD country, the percentage permanent increase in consumption is 8 percent, which is significantly lower than the value of the capital owner's loss.

Finally, Figure 2.5 shows the sensitivity of  $\mu_w$  with respect to variations in the initial capital stock ratio under financial autarky. For the poorest countries, we observe a permanent change in consumption of 11.2 percent while it reduces to 2.6 percent for the richest.

The simulation results of the capital owner's loss and the worker's gain show that financial integration may have an enormous influence on consumption and, thereby, on welfare. The effects are drastically larger than in the homogenous agent approach of G&J where the permanent consumption gain is just 1.74 %. This comparison illustrates that even if the average net effect may be rather small, financial integration can create substantial gross effects.

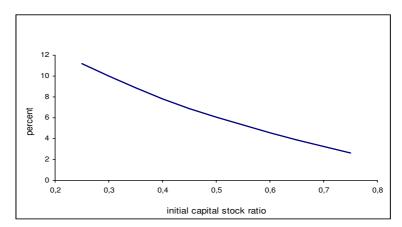


Figure 2.5: Worker's Permanent Consumption Gain (in %)

Note: Figure 2.5 shows the worker's permanent consumption gain after liberalization,  $\mu_c$ , as a function of the initial capital stock ratio (relative to steady state).

#### 2.3.4 Could a productivity catch-up process help the capital owners?

Apart from the direct effect on capital flows, financial integration is also associated with a positive impact on productivity, reflecting the transfer of knowledge between economically developed and economically less developed countries (e.g., Hoxha et al. 2009).<sup>13</sup> In their analysis, G&J address the question of how an increase in productivity affects the average welfare effect of financial integration. They find that in contrast to pure capital flows, the welfare effects caused by a productivity catch-up process are dramatically larger. For example, a typical developing country that starts with a technological level of roughly one quarter relative to the US gains 61 percent in terms of permanent consumption if financial integration leads to a reduction of the productivity gap of 25 percent. This clearly shows that a productivity catch-up process may dominate the effects of even large capital inflows.

Turning to our analysis, we are especially interested in the question whether such a process could help to reduce the capital owner's loss. We know from the previous discussion that capital owners suffer from the lower interest rate under financial integration. An increase in productivity seems therefore beneficial by increasing the domestic return to capital. However, the crucial point is that under financial integration the no-arbitrage condition

$$R_{t+1} = R^* (2.26)$$

still holds in equilibrium. Since the world interest rate is not affected by a change in the level of domestic productivity, the initial increase in the domestic interest rate is thus completely offset by a corresponding increase in the capital stock which brings the interest rate down to the world interest rate. Consequently, we may observe larger

 $<sup>^{13}</sup>$ See also Borensztein et al. (1998) and Baldwin et al. (2005) for a positive impact and Carkovic and Levine (2005) for a negative impact.

capital inflows, but domestic capital owners do not earn a higher return relative to the case where productivity remains unaffected.

We can thus state a very strong result: A productivity catch-up process which may be caused by financial integration does not offset the consumption and welfare losses of the domestic capital owners. For policymakers, this implies that no significant reduction in the capital owner's effort to defend capital control barriers can be expected.<sup>14</sup>

#### 2.4 Concluding remarks

The principal aim of this paper is to show that financial integration may have significant effects on consumption and welfare for different groups of society. One of our main findings is the dramatic loss of 42% of permanent consumption for domestic capital owners, contrasting with the relatively small net effects of free cross-border capital flows.

Of course, the complete polarization we present in our model is a very strong assumption, just like the one of a perfectly homogenous agent in the standard approach, but it emphasizes the importance of distributional effects and demonstrates the limited power of financial integration to increase overall welfare. Therefore, it adds to the debate about why financial integration has not yet delivered the main promises associated with it. We are aware of a trend towards more financial market opening (see, Mendoza et al. 2009a), but as Figure 2.1 has shown, there is still a number of poor countries with tight capital controls.

The simplicity of our framework makes it vulnerable to the critique of a too speedy rate of convergence which is a well-known weakness of standard neoclassical growth models (e.g., Hoxha et al. 2009; Barro and Sala-i-Martin 2004). Hoxha et al. (2009) estimate the implied autarky path of convergence from actual data and show that average welfare gains are substantially bigger if the speed of convergence is calibrated more realistically. Things would also change in our model if we assumed a slower rate of convergence, which would increase the worker's gain and reduce the capital owner's relative loss in the first periods. However, in the long-run, results would be the same and the order of magnitude of our results clearly shows that the distributional effects of financial integration play an important role in the ongoing debate on the effects of cross-border capital flows.

Another important point is that our approach provides a rationale for why many countries do not benefit from productivity catch-up processes. Productivity growth, which is regarded as one of the main sources of economic growth (see, Hall and Jones 1999), does not offset the negative effect of financial integration on capital owner's consumption. Thus, the incentive to oppose a capital market opening is not eliminated.

<sup>&</sup>lt;sup>14</sup>Of course, domestic workers will benefit from these catch-up effects due to the increase in the wage rate  $w_1 = A_1(1-\alpha)\tilde{k}^{\alpha}$ .

Institutions to redistribute the gains in net production which arise due to financial integration could solve the capital owner's problem, but their implementation may be hard to achieve in any political process. Furthermore, as shown by Prasad et al. (2006), nowadays there is even a tendency of capital to flow from poor to rich countries which may again affect the distributional effects of financial integration. Understanding this new pattern seems therefore important to fully understand the implications of capital account liberalization. We leave this for future research.

#### 2.5 Appendix

#### A.1. Financial autarky

Solving the capital owner's budget constraint in (2.8) forward and employing the transversality condition yields

$$k_t^c = \sum_{i=0}^{\infty} \left( \frac{c_{t+i}^c}{\prod_{j=0}^i R_{t+j}} \right). \tag{2.27}$$

Combining this with the forward iterated Euler equation (expressed in terms of percapita consumption)

$$c_{t+i}^c = \beta^i \prod_{j=1}^i R_{t+j} c_t^c, \tag{2.28}$$

we get the following expression for the capital stock

$$k_t^c = \sum_{i=0}^{\infty} \left( \frac{c_t^c \beta^i \prod_{j=1}^i R_{t+j}}{\prod_{j=0}^i R_{t+j}} \right). \tag{2.29}$$

Simplifying the terms in (2.29) and normalizing by labor productivity finally yields

$$\tilde{c}_t^c = (1 - \beta) R_t \tilde{k}_t^c \tag{2.30}$$

$$\widetilde{k}_{t+1}^c = \frac{1}{1+q} \beta R_t \widetilde{k}_t^c. \tag{2.31}$$

#### A.2. Financial integration

The stock of the domestic capital owner's assets under financial integration,  $a_t^c = k_t^c - d_t^c$ , can be determined in an analogous way as the capital stock under autarky. Using the optimality condition  $R_{t+1} = R^* \ \forall t \geqslant 1$  and the no-Ponzi game condition  $\lim_{T\to\infty} \frac{a_{t+T}^c}{R_t R^{*(T-1)}} = 0$  we get

$$a_t^c = \sum_{i=0}^{\infty} \left( \frac{c_{t+i}^c}{R_t (R^*)^i} \right).$$
 (2.32)

Using the forward iterated Euler equation  $c_{t+i}^c = \beta^i (R^*)^i c_t^c$  yields

$$a_t^c = \sum_{i=0}^{\infty} \left( \frac{\beta^i c_t^c (R^*)^i}{(R^*)^i R_t} \right). \tag{2.33}$$

Finally, normalizing by labor productivity, we get  $\forall t > 0$ 

$$\widetilde{c}_t^c = (1 - \beta) R^* \widetilde{a}_t^c \tag{2.34}$$

$$\widetilde{a}_{t+1}^c = \frac{1}{1+g} \beta R^* \widetilde{a}_t^c, \qquad (2.35)$$

and for t = 0

$$\widetilde{c}_0^c = (1 - \beta) R_0 \widetilde{k}_0^c \tag{2.36}$$

$$\widetilde{c}_0^c = (1 - \beta) R_0 \widetilde{k}_0^c \qquad (2.36)$$

$$\widetilde{a}_1^c = \frac{1}{1 + g} \beta R_0 \widetilde{k}_0^c. \qquad (2.37)$$

# Chapter 3

# Idiosyncratic Risk, Borrowing Constraints and Financial Integration<sup>1</sup>

#### Abstract

This paper examines under which conditions the puzzling observation of capital flows from poor to rich countries and accompanying changes in domestic economic development can be explained by the presence of financial market imperfections. Motivated by the mixed results from the literature, we employ an incomplete-markets model in which entrepreneurs face capital risk, earn risky profits and receive riskless wage income. Moreover, borrowing constraints simultaneously impede consumption smoothing and limit the access to external funds for scaling up production. We find that in the presence of uninsurable risk only and for plausible parameter values, capital does flow from poor to rich countries and that an increase in the interest rate leads to higher levels of capital and output in the steady state under financial integration. However, we also find that tight borrowing constraints and high persistence of shocks strongly affect the model predictions and lead to significantly tighter parameter restrictions. With these findings we contribute to the ongoing debate on the consequences of financial integration.

<sup>&</sup>lt;sup>1</sup>This paper was written in collaboration with Maik Heinemann. The authors would like to especially thank Christiane Clemens, Marius Clemens, Ulrich Eydam, Frank Heinemann, Jean Imbs, Vahagn Jerbashian, Tom Krebs, Wolfgang Strehl and Lutz Weinke for valuable comments and suggestions.

## 3.1 Introduction

The topic of international integration has clearly gained in importance over the last decades. Among others, Prasad et al. (2006, 2007) and Mendoza et al. (2009a) document a persistent increase in the volume of cross-border capital flows, which reflects the profound developments in the process of financial integration. However, existing discrepancies between the observed pattern on the one hand and the predictions from standard theory on the other hand have not been wiped away by the larger amount of capital flows and further, striking observations have emerged over time. In this respect, the so-called 'Lucas paradox' is one of the most prominent examples. In his influential paper, Lucas (1990) points to the fact that the share of capital flowing from rich to poor countries is significantly smaller compared to the predictions of the frictionless, neoclassical model. In recent times, this paradox has even intensified. Prasad et al. (2006, 2007) show that since the end of the 20th century, the average income of countries running current account surpluses has fallen below the average income of countries running current account deficits, implying that capital nowadays even flows from poor to rich countries.

In addition to the Lucas paradox, domestic economic development in the light of capital outflows challenges conventional wisdom. While the neoclassical model predicts a decrease of the domestic capital stock and output if the interest rate increases under financial integration, triggering capital outflows, Prasad et al. (2006, 2007) find periods of capital outflows from high-growth nonindustrial countries. Similarly, Sandri (2010) notes improvements in the current accounts of developing countries during periods of high per capita income growth. Finally, Gourinchas and Jeanne (2013) find a negative correlation between total factor productivity growth and net capital inflows for developing countries, which they call the 'allocation puzzle'.

The obvious failure of standard theory to explain the empirical pattern has given rise to a voluminous literature, seeking for explanations beyond the common assumptions of the frictionless, neoclassical model. Recently, special attention has been paid to the influence of financial market imperfections in the form of borrowing constraints or the presence of uninsurable idiosyncratic risks following from incomplete insurance markets. As pointed out by Mendoza et al. (2009a), countries still differ significantly in their level of financial development despite the persistent increase in the volume of cross-border capital flows. However, the questions whether and under what conditions the presence of uninsurable risks and borrowing constraints contributes to explain the empirical findings have not been completely resolved yet. In fact, the literature shows that the results are extremely diverse and strongly vary with the underlying source of risk (capital vs. income risk), the exact specification of borrowing constraints (household vs. production side), and further model assumptions. Hence, while progress has been made to understand some of the individual effects, we are still lacking a clear understanding of the rich interactional effects of the different types of financing constraints and risky income

components and therefore, of the joint overall effect of financial market imperfections on the process of financial integration. This, however, is important as the simultaneous presence of risks and different types of financing constraints can hardly be rejected based on available measures of financial market development. In this paper, we aim at filling this remaining gap in the literature. Our work relates to the line of research of Gertler and Rogoff (1990), Matsuyama (2005), Aoki et al. (2009), Buera and Shin (2009), Mendoza et al. (2009a,b), Sandri (2010), Angeletos and Panousi (2011), Song et al. (2011), Clemens and Heinemann (2013) and von Hagen and Zhang (2014).

The model we consider is an incomplete-markets model with two sectors of production and heterogeneous entrepreneurs. The model captures essential features from the related literature while providing a richer representation of the different effects of uninsurable risk and financing constraints. The model structure can be outlined as follows. In the final good sector, a homogenous good is produced under perfect competition with intermediate goods and labor as input factors. In the intermediate goods sector, firms operate under monopolistic competition and each firm, producing a single intermediate good, is owned and managed by one entrepreneur. The economy is populated by a continuum [0,1] of infinitely-lived households. Each household consists of one entrepreneur and is endowed with one unit of labor that is supplied inelastically to the perfectly competitive labor market. Entrepreneurs can invest in the own firm and can trade a riskless bond subject to a borrowing constraint. Idiosyncratic risk is introduced by stochastic fluctuations in the entrepreneur's productivity, capturing different kinds of business risk. The chosen model structure guarantees the existence of capital risk since investment has to be chosen before the idiosyncratic shock is realized, of a risky income component since profits fluctuate as well, and of a riskless income component given by the riskless wage income. Moreover, the borrowing constraints entrepreneurs face simultaneously impede consumption smoothing and restrict the access to external funds for scaling up individual production.

Contrary to main parts of the literature, we do not aim for a specific calibration strategy, but focus on understanding the possibly different model implications. From an economic point of view, we analyze the macroeconomic effects that have to be expected if financial integration takes place between countries that differ in the level of financial development, i.e. in the amount of risk that remains with the entrepreneurs and in the tightness of the borrowing constraint. From a more technical point of view, we show under which conditions, i.e. under which restrictions on the model parameters, the presence of uninsurable risks and borrowing constraints contributes to explaining the empirical findings and how these conditions may change with different model assumptions.

In order to keep track of the individual effects, we consider different scenarios, increasing the model complexity step by step. First, we consider a baseline scenario that focuses on the effects of the uninsurable capital risk, the risky profits and the riskless wage income. We find that in the baseline scenario, the model is in principle capable of contributing to explaining the empirical findings, but may also come to very different

conclusions. In particular, by comparing the steady state under financial autarky visà-vis financial integration, we find that it is in fact the financially less developed and initially poor country that builds up a positive net foreign asset position under financial integration and that the domestic capital stock and output may increase despite an increase of the interest rate.<sup>2</sup> However, we also find that the results may be exactly reverse; the outcome strongly varies with the underlying parametrization. Therefore, in a second step, we derive two rules of thumb that describe the parameter restrictions with high accuracy. In line with Angeletos (2007) and Angeletos and Panousi (2011), our two rules show that the elasticity of intertemporal substitution (EIS) has to exceed a certain threshold level in order to explain the empirical findings. For our baseline scenario, we find that the parameter restrictions remain moderate and are easily satisfied by empirically plausible values of the elasticity of intertemporal substitution. In other words, in the baseline scenario, we find a quite robust pattern of international capital flows from poor to rich countries.

In the second scenario, we increase the tightness of the debt limit, which means that borrowing constraints occasionally bind. On the one hand, borrowing constraints make it more difficult for agents to smooth consumption and lead to an increase in aggregate demand for the safe asset. On the other hand, borrowing constraints restrict the access to external funds for scaling up production and, even if not currently binding, discourage risky investment. The latter effects dampen the upward trend of the aggregate capital stock associated with lower interest rates and mean that the overall effect of borrowing constraints is generally ambiguous. However, we find that the saving effect is the dominant effect and that tighter borrowing constraints lead to tighter parameter restrictions, i.e. tighter restrictions on the elasticity of intertemporal substitution, compared to the first scenario. In particular, we find that in times of strong turmoil in financial markets, with an almost collapsing lending channel, the parameter restrictions become too tight in order to be satisfied by empirically plausible values. Put differently, we find that in the presence of severe borrowing constraints, financial integration may easily become an impediment for domestic economic development.

In the third and final scenario, we increase the persistence of shocks while keeping the unconditional variance at a constant level. A higher persistence of shocks increases the demand for the riskless asset and therefore amplifies the effects of the financial market imperfections. In almost all exercises we find that a higher persistence of shocks again leads to tighter parameter restrictions compared to the first two scenarios. This especially applies to moderate levels of the borrowing constraint. Hence, as an overall result, we find that with increasing model complexity, it becomes more difficult to contribute to explain the empirical pattern solely with the difference in financial development. From a more economic perspective, our results show that in the presence of different types of financing constraints and persistent risks, the response of international capital flows

<sup>&</sup>lt;sup>2</sup>In the following, we also refer to the steady state under financial autarky (integration) as the autarchic (integrated) steady state.

becomes more volatile and strongly reacts to even small deteriorations in financial market performance. This may also affect the society's support for financial liberalization, which we will discuss in more detail when analyzing the welfare implications.

The remainder of this paper is organized as follows. Section 3.2 provides a brief review of the relevant literature. Section 3.3 describes the model and Section 3.4 describes the benchmark parametrization. Section 3.5 introduces the different scenarios that compare the steady state under financial autarky vis-à-vis financial integration. Section 3.6 presents more details on transitory dynamics and discusses the welfare implications. Section 3.7 concludes. The appendix collects relevant proofs.

## 3.2 Literature review

The overview summarizes some of the opposing results arising due to variations in the source of the underlying risk and in the specification of borrowing constraints. We consider implications from both, closed- and open-economy settings. For a more detailed discussion of financial market imperfections and macroeconomic performance see, for example, Brunnermeier et al. (2012) or Gourinchas and Rey (2013).

Large parts of the analysis of financial market imperfections and financial integration build on the class of heterogeneous-agents, incomplete-markets models. In the standard incomplete-markets model, agents have to decide on an optimal consumption and savings path but face stochastic fluctuations in the income process.<sup>3</sup> The set of instruments to insure against income risk is restricted to a riskless asset and agents can only borrow up to an exogenous debt limit. Due to the presence of uninsurable income risk and borrowing constraints, agents engage in precautionary saving, which finally leads to a lower interest rate in the autarchic steady state (see Huggett 1993, Aiyagari 1994).

Mendoza et al. (2009b) study the welfare implications of financial integration in the presence of uninsurable income risk and borrowing constraints.<sup>4</sup> Differences in the level of financial development between countries are captured by differences in the tightness of the borrowing constraint. Mendoza et al. (2009b) show that if financial integration takes place, it is the financially less developed country that accumulates a positive net foreign asset position. However, since uninsurable income risk and borrowing constraints on the household side do not break the equality between the interest rate and the marginal product of capital, the financially less developed country is actually the rich country in terms of output under financial autarky, which means that capital flows from the rich to the poor country under financial integration. Mendoza et al. (2009b) assume exogenous differences in productivity levels between countries, which circumvent this result. However, these differences do not arise endogenously from the differences in financial market performance.

<sup>&</sup>lt;sup>3</sup>See Heathcote et al. (2009) or Ljungqvist and Sargent (2012).

 $<sup>^4</sup>$ See also Mendoza et al. (2009a) where investment risk is additionally included but without aggregate capital accumulation.

Contrary to the assumption of fluctuations in labor income, Angeletos (2007) emphasizes the importance of rate-of-return or capital risk and augments the neoclassical growth model to study the macroeconomic consequences of market incompleteness. Angeletos (2007) shows that in the presence of capital risk, the financially less developed country may also be the initially poor country under financial autarky. The fundamental difference compared to income risk is that capital risk also affects the demand for investment, thereby breaking the equality between the interest rate and the marginal product of capital. Consequently, as Angeletos and Panousi (2011) show, capital may flow from the financially less developed and initially poor country to the financially more developed and initially rich country under financial integration. However, as emphasized by Angeletos and Panousi (2011), even in the presence of capital risk, the deep structural model parameters have to satisfy certain conditions in order to explain the empirical findings.

Our approach most closely relates to Angeletos (2007) and Angeletos and Panousi (2011) by sharing the feature that entrepreneurs face capital risk and receive riskless wage income. However, entrepreneurs in our model also earn risky profits and we consider the case with occasionally binding borrowing constraints and with persistent effects of shocks. As we will show, these features may strongly affect the results from the baseline scenario.

Level effects of uninsurable investment risk in a closed-economy setting are also studied by Covas (2006) and Meh and Quadrini (2006). Meh and Quadrini (2006) consider different risk-sharing environments and find that the capital stock is lower if markets are incomplete. In contrast, Covas (2006) finds that the capital stock is higher under incomplete markets and shows that tighter borrowing constraints may further increase the difference between the complete and the incomplete markets case. Covas (2006), however, abstracts from any type of riskless income component beyond the safe asset.<sup>7</sup>

Finally, our approach also relates to the literature that focuses on the effects of financing constraints on the production side of the economy, e.g., Gertler and Rogoff (1990), Boyd and Smith (1997), Matsuyama (2005), Aoki et al. (2009), Buera and Shin (2009), Song et al. (2011), Benhima (2013), Clemens and Heinemann (2013), von Hagen and Zhang (2014), and Bacchetta and Benhima (2015).<sup>8</sup> As shown by Buera and Shin (2009) and Clemens and Heinemann (2010, 2013), financing constraints on the production side may help to overcome the result that the financially less developed

<sup>&</sup>lt;sup>5</sup>See Phelps (1962) and Levhari and Srinivasan (1969) for early discussions of the saving effect of risky returns and Sandmo (1970) for a comparison with income risk.

<sup>&</sup>lt;sup>6</sup>See also Angeletos and Calvet (2005, 2006) for a discussion with CARA preferences and endowment

risk.  $^{7}$ See also Covas and Fujita (2011) for a discussion of idiosyncratic and aggregate risk and Goldberg (2013) for a discussion of a credit crunch.

 $<sup>^8</sup>$ von Hagen and Zhang (2014) additionally distinguish between financial capital and foreign direct investments and von Hagen and Zhang (2011) compare the effects of limited commitment and incomplete markets.

country is also the rich country in traditional incomplete-markets models. Furthermore, in the presence of financing constraints, domestic output may increase under financial integration despite an increase of the interest rate. In our model, tight borrowing constraints also restrict the access to external financing, but increase the demand for the riskless asset as well. As we will show, this combination may lead to very different implications compared to an isolated consideration of financing constraints.

# 3.3 The model

We analyze the implications of financial liberalization in an incomplete-markets economy with two sectors of production and heterogeneous entrepreneurs. The economy structure can be outlined as follows.

Time is discrete and indexed by  $t \in [0, ..., \infty]$ . In the final good sector, a large number of perfectly competitive firms produce a homogeneous good, which can be used for consumption and investment. Input factors are intermediate goods and labor. In the intermediate goods sector, firms operate under monopolistic competition and each firm, producing a single intermediate good, is owned and managed by one entrepreneur.

The economy is populated by a continuum [0,1] of infinitely-lived households. Each household consists of one entrepreneur and is endowed with one unit of labor supplied inelastically to the perfectly competitive labor market. Since we assume perfect consumption sharing on the household level, we refer to the household and to the entrepreneur interchangeably. The entrepreneur invests in the own firm and can trade a riskless bond subject to a borrowing constraint. Idiosyncratic risk is introduced by stochastic fluctuations in the entrepreneur's productivity; a shortcut to capture different kinds of business risk. The model structure leads to the existence of capital risk since investment has to be chosen before the idiosyncratic shock is realized, of risky profits, and of a riskless income component given by the riskless wage income. Markets are incomplete so that full insurance against idiosyncratic risk is not obtainable. Furthermore, the borrowing constraints entrepreneurs face on bond holdings simultaneously impede consumption smoothing and restrict the access to external funds for scaling up individual production.

Under financial autarky, the bond market has to clear on the country-wide level, whereas under financial integration, bonds can be traded on the international level. We assume that the small economy we consider only differs with respect to the level of financial development from the rest of the world. In the baseline scenario, the level of financial development is determined by the amount of risk that cannot be insured through financial markets, and thus, remains with the entrepreneurs. In the second scenario, the level of financial development is also determined by the tightness of the

<sup>&</sup>lt;sup>9</sup>See also Buera and Shin (2011) for a discussion of the effects of increasing shock persistence, Buera et al. (2011) for a multi-sector analysis and Buera and Shin (2013).

borrowing constraint.<sup>10</sup> Depending on the scenario, a lower level of financial development means a larger amount of risk remaining with the entrepreneurs and/or a more tight debt limit. Note that we assume that financial integration takes place without financial development and that agents cannot simply bypass the domestic borrowing restrictions under financial integration.

# 3.3.1 Final good sector

In the final good sector of the small economy, the representative firm produces the homogenous good,  $Y_t$ , under perfect competition. Input factors are labor,  $L_t$ , and intermediate goods,  $x_{it}$ ,  $i \in [0,1]$ . Production takes place according to the following generalized production function

$$Y_t = L_t^{1-\alpha} \int_0^1 x_{it}^{\alpha} di, \quad 0 < \alpha < 1.$$
 (3.1)

Since  $\alpha$  is assumed to be less than one, intermediate goods are close but imperfect substitutes. The profit of the representative firm is given by

$$\Pi_t^F = Y_t - w_t L_t - \int_0^1 p_{it} x_{it} \, di, \tag{3.2}$$

where  $p_{it}$  denotes the price of intermediate good i and where the price of the final good is normalized to unity. Optimization yields the standard result that each input factor is paid according to its marginal product

$$w_t = (1 - \alpha) \frac{Y_t}{L_t} \tag{3.3}$$

$$p_{it} = \alpha x_{it}^{\alpha - 1} L_t^{1 - \alpha}. \tag{3.4}$$

#### 3.3.2 Household sector

Each household has preferences over consumption and maximizes discounted expected lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_{it}). \tag{3.5}$$

 $E_0$  is the expectation operator conditional on information at date t = 0, and  $0 < \beta < 1$  is the discount factor. Preferences regarding momentary consumption are standard and display constant relative risk aversion

$$U(c) = \begin{cases} \frac{c^{1-\rho}}{1-\rho} & \rho > 0, \ \rho \neq 1\\ \ln(c) & \rho = 1. \end{cases}$$
(3.6)

<sup>&</sup>lt;sup>10</sup>See Angeletos and Calvet (2006) and Angeletos and Panousi (2011) for a more detailed discussion of modeling financial markets.

The risky technology available to produce the intermediate good is given by

$$x_{it} = \theta_{it}k_{it}, \tag{3.7}$$

where  $k_{it}$  denotes the capital stock and  $\theta_{it}$  denotes the entrepreneur's stochastic productivity.  $\theta_{it}$  is assumed to be uncorrelated across agents but may be correlated over time. The household's budget constraint is given by

$$k_{it+1} + b_{it+1} + c_{it} = p_{it}x_{it} + (1 - \delta)k_{it} + R_t b_{it} + w_t, \tag{3.8}$$

where  $b_{it+1}$  denotes investment in the safe asset,  $R_t \equiv (1+r_t)$  is the gross riskless interest rate,  $w_t$  is the wage rate, and  $p_{it}x_{it}$  describes the income part from selling the intermediate good at chosen price  $p_{it}$ . The monopolistic optimization problem the household solves in each period is simple in this case since the capital stock is already installed at the beginning of period t. Consequently, the amount of the intermediate good produced and sold to the final good sector in period t is fixed after the realization of the individual productivity shock is observed. Using the demand function in (3.4) and using (3.7) we can express  $p_{it}x_{it}$  as  $p_{it}x_{it} = \alpha \bar{L}_t \tilde{\theta}_{it} k_{it}^{\alpha}$  with  $\tilde{\theta}_{it} \equiv \theta_{it}^{\alpha}$  and  $\bar{L}_t \equiv L_t^{1-\alpha}$ . The budget constraint then reduces to

$$k_{it+1} + b_{it+1} + c_{it} = \alpha \bar{L}_t \tilde{\theta}_{it} k_{it}^{\alpha} + (1 - \delta) k_{it} + R_t b_{it} + w_t.$$
 (3.9)

The representation in (3.9) indicates that the household essentially solves a consumption/savings problem as well as a portfolio choice problem between a riskless asset and a risky asset. To see the latter more clearly, note that we can decompose the household's income part from production,  $\alpha \bar{L}_t \hat{\theta}_{it} k_{it}^{\alpha}$ , into its two components capital income and profits. This separation follows from the fact that capital is the crucial input factor in production and that profits arise due to the monopolistic structure in the intermediate goods sector. Capital income is given by  $r_{it}^r k_{it}$  where the net return,  $r_{it}^r$ , measures the contribution of an additional marginal unit of capital, i.e.  $r_{it}^r \equiv \alpha^2 \bar{L}_t \tilde{\theta}_{it} k_{it}^{\alpha-1}$ . The net return will show up in the Euler equation for capital holdings in period t+1 and is risky because it depends on the entrepreneur's productivity which is subject to idiosyncratic shocks. The residual term are profits that follow from the monopolistic structure in the intermediate goods sector. Profits are given by  $\pi_{it} \equiv (1-\alpha)\alpha \bar{L}_t \hat{\theta}_{it} k_{it}^{\alpha}$  and shrink to zero if intermediate goods become perfect substitutes, which can be seen from the fact that  $\pi_{it} \to 0$  if  $\alpha \to 1$ . Furthermore, profits are risky as well since they also depend on the entrepreneur's stochastic productivity. Using this separation of capital income and profits, the household's budget constraint can finally be written as

$$k_{it+1} + b_{it+1} + c_{it} = R_{it}^r k_{it} + \pi_{it} + R_t b_{it} + w_t, \tag{3.10}$$

where  $R_{it}^r = (1 - \delta + r_{it}^r)$  is the gross return of capital. The representation in (3.10) highlights that the household essentially solves a portfolio choice problem between a

riskless asset (bond) and a risky asset (capital). Furthermore, it shows that, in terms of Sandmo (1970), entrepreneurs do not only face capital risk but also income risk where the latter is induced by the existence of risky profits. Clearly, capital and income risk in our model are not independent because both, the risky return and profits, depend on the same stochastic process. However, this separation plays an important role later on since only capital risk in addition to the borrowing constraint also affects the household's investment decisions, whereas income risk only leads to precautionary saving. This will become more clear from the household's first-order conditions.

Let the household's period t net worth,  $\omega_{it}$ , be defined as  $\omega_{it} \equiv R_{it}^r k_{it} + \pi_{it} + R_t b_{it} + w_t$ . Furthermore, let  $V_t(\omega_t, \tilde{\theta}_t)$  be the associated optimal value function. Then, the single household's optimization problem can be specified in terms of the following program<sup>11</sup>

$$V_t(\omega_t, \widetilde{\theta}_t) = \max_{c_t, b_{t+1}, k_{t+1}} \left\{ U(c_t) + \beta E\left[ V_{t+1}(\omega_{t+1}, \widetilde{\theta}_{t+1}) \mid \widetilde{\theta}_t \right] \right\}$$
(3.11)

$$s.t. \quad c_t + b_{t+1} + k_{t+1} = \omega_t \tag{3.12}$$

$$k_{t+1} \ge 0 ag{3.13}$$

$$b_{t+1} \geq -\overline{b}. \tag{3.14}$$

The constraint in (3.14) is the borrowing constraint the household faces on the safe asset. The tightness of the borrowing constraint is determined by the debt limit,  $\bar{b}$ . A lower value of  $\bar{b}$  means a lower amount the household can borrow to either smooth consumption or to scale up individual production.

The first-order conditions of the individual problem are given by

$$U'(c_{it}) = \beta R_{t+1} E_t \left[ U'(c_{it+1}) \right] + \lambda_{it}$$
 (3.15)

$$U'(c_{it}) = \beta E_t \left[ R_{it+1}^r U'(c_{it+1}) \right],$$
 (3.16)

where  $\lambda_{it}$  is the nonnegative Lagrange multiplier associated with the borrowing constraint and where  $R_{it+1}^r = 1 - \delta + \alpha^2 \bar{L}_{t+1} \tilde{\theta}_{it+1} k_{it+1}^{\alpha-1}$  is the gross return of capital in period t+1. Combining the two equations yields

$$E_t R_{it+1}^r - R_{t+1} = -\frac{Cov(U'(c_{it+1}), R_{it+1}^r)}{E_t U'(c_{it+1})} + \frac{\lambda_{it}}{\beta E_t U'(c_{it+1})}.$$
 (3.17)

Equation (3.17) shows that the presence of uninsurable capital risk and potentially binding borrowing constraints drives a wedge between the expected return of capital and the riskless interest rate. The first term on the right-hand side describes the

<sup>&</sup>lt;sup>11</sup>The subscript i is dropped in this definition for notational ease. Note that the time subscript attached to the value function indicates that the household's program is not only defined at steady states.

risk premium the household demands for bearing the uninsurable capital risk. Since  $Cov(U'(c_{it+1}), R_{it+1}^r)$  is negative, this expression is positive. The second term on the right-hand side additionally appears if the borrowing constraint binds in period t. Since  $\lambda_{it}$  is nonnegative, both terms positively contribute to the wedge and will play a key role in the further analysis.

Definition 1 below summarizes the equilibrium under financial autarky and financial integration from the perspective of the small economy. Under financial autarky, the bond market has to clear on the country-wide level, whereas under financial integration bonds can be traded on the international level. Note that the time index indicates that the equilibrium is not only defined at steady states where aggregate prices are constant over time, but also takes account of transitory dynamics.

**Definition 1** Given the initial distribution of households,  $\Psi_0(\omega, \widetilde{\theta})$ , a general competitive equilibrium under financial autarky is defined by a) a sequence of policy functions  $\left\{c_t(\omega, \widetilde{\theta}), k_{t+1}(\omega, \widetilde{\theta}), b_{t+1}(\omega, \widetilde{\theta})\right\}_{t=0}^{\infty}$ , b) a sequence of value functions  $\left\{V_t(\omega, \widetilde{\theta})\right\}_{t=0}^{\infty}$ , c) a sequence of prices  $\{R_t, w_t, p_t(i)\}_{t=0}^{\infty}$ , and d) a sequence of distributions  $\left\{\Psi_t(\omega, \widetilde{\theta})\right\}_{t=1}^{\infty}$ , such that, for all t

- 1. The policy functions described above solve the household's decision problem.
- 2. Intermediate goods and labor are paid according to their marginal product.
- 3. Aggregate quantities of consumption, capital, labor and bonds are the aggregation of individual quantities. For given prices markets clear, especially  $B_t = 0$  and  $L_t = 1$ .
- 4. The sequence of distributions is consistent with the initial distribution, the policy functions and the stochastic process for productivity.

A competitive equilibrium under financial integration is defined in a similar fashion. However, bonds can be traded on the international level given the world interest rate  $R^*$ . <sup>12</sup>  $B_t$  then represents the net foreign asset position of the small economy.

#### 3.4 Parametrization

In this section, we describe the benchmark parametrization, identify the financial parameters of the model and explain the differences between the three scenarios. In total, we have to assign seven parameter values,  $\{\alpha, \beta, \rho, \delta, \bar{b}, \rho_{\theta}, \sigma\}$ . We mainly choose standard values that are commonly considered in the literature.

In accordance with our discussion in the previous section,  $\sigma$  and  $\bar{b}$  are the formalization of a country's level of financial development.  $\sigma$  measures the level of uninsurable

<sup>&</sup>lt;sup>12</sup>Note that the interest rate under financial integration is determined by the large and financially more developed country that represents the rest of the word. Hence, interest rate differentials between countries under financial autarky are endogenously explained by differences in financial development.

risk and is defined as the standard deviation of  $\ln(\tilde{\theta})$ . We directly target the properties of  $\tilde{\theta} \equiv \theta^{\alpha}$  since it shows up as the relevant term in the household's budget constraint. As generally shown by Angeletos and Calvet (2006), a higher value of  $\sigma$  means a higher portion of risk that cannot be insured through financial markets and thus, remains with the entrepreneurs.<sup>13</sup> The debt limit,  $\bar{b}$ , defines the tightness of the borrowing constraint. A more tight borrowing constraint means a stronger impediment for entrepreneurs to either smooth consumption or to scale up individual production. A higher amount of risk remaining with the entrepreneurs and/or a more tight borrowing constraint means a lower level of financial development.

In each scenario, we consider the same values of  $\{\alpha, \beta, \rho, \delta, \sigma\}$  for our small and financially less developed benchmark economy. The parameter values are standard and commonly considered in the literature. The discount factor,  $\beta$ , is set to 0.96 and  $\alpha$  is set to 0.4, implying a labor income share of 0.6. The elasticity of intertemporal substitution,  $\vartheta = 1/\rho$ , is set to 2/3 which means that the parameter of relative risk aversion,  $\rho$ , equals 1.5. The depreciation rate,  $\delta$ , is set to a standard value of 0.08. In general, the productivity process is first-order Markov and defined as

$$\ln \theta_{t+1} = -\frac{\alpha}{1 + \rho_{\theta}} \frac{\sigma_{\epsilon}^2}{2} + \rho_{\theta} \ln \theta_t + \epsilon_{t+1}, \quad \epsilon \sim N(0, \sigma_{\epsilon}^2), \tag{3.18}$$

where  $\rho_{\theta}$  is the serial correlation parameter and where the specification of the constant term in (3.18) leads to the normalization  $E(\tilde{\theta}) = 1$ .  $\sigma_{\epsilon}^2$  is adjusted accordingly to ensure that  $\sigma$  is equal to 0.4 which is comparable to Covas (2006), Angeletos (2007) and Angeletos and Panousi (2011).

The three scenarios only differ with respect to the values of  $\rho_{\theta}$  and  $\bar{b}$ . In the baseline scenario, we focus on uninsurable risk only, assuming away tight borrowing constraints and persistent effects of shocks. That means,  $\rho_{\theta}$  controlling the persistence of shocks is set to zero and  $\bar{b}$  is equal to the Natural Debt Limit (NDL) defined as the maximum amount of repayable debt that is consistent with nonnegative consumption.<sup>14</sup> Differences in financial development between countries in the baseline scenario are solely captured by differences in the level of uninsurable risk. In the second scenario, we increase the tightness of the borrowing constraint and in the third scenario, we increase the persistence of shocks. Table 3.1 below summarizes the benchmark parameter values that are equal in all three scenarios.

<sup>&</sup>lt;sup>13</sup>See also Corneli (2009) and Angeletos and Panousi (2011).

<sup>&</sup>lt;sup>14</sup>Since  $x_{it} = \theta_{it}k_{it}$  is close to zero for bad realizations of  $\theta_{it}$ , we define the NDL in steady state as  $NDL \equiv w/(R-1)$ .

Table 3.1: Benchmark Parameter Values

Parameter		Value
discount factor	β	0.96
curvature of production (final good sector)	$\alpha$	0.4
depreciation rate	$\delta$	0.08
elasticity of intertemporal substitution	$\vartheta=1/\rho$	2/3
standard deviation of $\ln(\widetilde{\theta})$	$\sigma$	0.4

# 3.5 Results

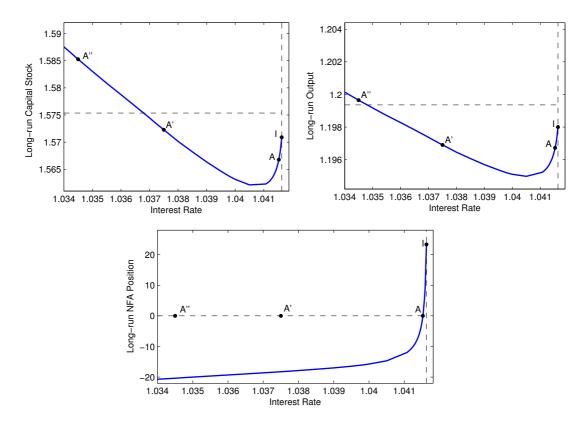
#### 3.5.1 Overview

In this section, we describe our three scenarios. In the baseline scenario, we focus on uninsurable risk, in the second scenario we increase the tightness of the borrowing constraint, and in the third scenario we additionally allow for persistent effects of shocks. Apart from these differences, however, the focus remains the same and relates to the questions of interest, i.e. explaining the direction of international capital flows and accompanying changes in domestic economic development. In particular, we are interested in two main features a model should have in order to contribute to explaining the empirical pattern. First, the financially less developed country should not only display a lower interest rate in the initial equilibrium under financial autarky, but also lower levels of capital and output, i.e. it should also be the economically less developed country. This result ensures that when financial integration takes place, it is in fact the initially poor country that builds up a positive net foreign asset position, which explains the Lucas paradox. Second, the increase in the interest rate from the perspective of the financially less developed country should lead to a higher aggregate capital stock and a higher output level in the steady state under financial integration rather than to lower levels as predicted by standard theory. This result ensures that, at least from a steady state comparison, we observe a positive correlation between economic growth, higher interest rates and capital outflows. We will analyze under which conditions the presence of uninsurable risk and borrowing constraints contributes to explaining these findings and how the conditions may change with different model assumptions. In the baseline scenario, where our model closely relates to Angeletos and Panousi (2011), we derive two rules of thumb that describe the parameter restrictions with high accuracy.

#### 3.5.2 Scenario 1: Uninsurable risk

Figure 3.1 shows the main results for our baseline scenario. The solid blue lines in Figure 3.1 show the long-run relationship between selected macroeconomic variables and the interest rate for our small and financially less developed benchmark economy. The steady state under financial autarky, where the bond market has to clear on the country-wide level, is indicated by point A and the steady state under financial integration, where bonds can be traded on the international level, is indicated by point I. The horizontal and vertical dashed lines in Figure 3.1 indicate the autarchic steady state of the large and financially more developed country that represents the rest of the world. Entrepreneurs in the financially more developed country face a lower level of uninsurable risk ( $\sigma = 0.2$ ) which reflects the superior insurance opportunities provided by the financial sector. Since the financially more developed country is assumed to be sufficiently large, it determines the interest rate under financial integration ( $\mathbb{R}^{Int} = 1.04163$ ).

Figure 3.1: Main Results Baseline Scenario



Note: Point A shows the autarchic steady state of the financially less developed benchmark economy and point I shows the steady state under financial integration. The horizontal and vertical dashed lines show the autarchic steady state of the large and financially more developed country representing the rest of the world. Point A' and A" show two possible alternative equilibria for the financially less developed country that, in a similar fashion, are obtained under different parametrizations.

Inspecting Figure 3.1 leads to a number of interesting results. First, Figure 3.1 shows that an equilibrium like point A is exactly the starting point that is needed in order to explain the empirical findings. Point A means that our financially less developed benchmark economy does not only display a lower interest rate in the initial equilibrium under financial autarky, but also lower levels of the domestic capital stock and output, i.e. it is also the economically less developed country. This can be seen from comparing the position of point A (autarchic steady state financially less developed country) with the positions of the horizontal and vertical dashed lines (autarchic steady state financially more developed country). The lower interest rate in the financially less developed country can be traced back to the higher level of risk that entrepreneurs have to bear. The higher level of risk leads to a higher precautionary saving demand for the riskless asset, which forces the interest rate to fall in order to clear the bond market under financial autarky. This risk-induced saving effect is well-known in the literature and occurs under both capital and income risk (see Aiyagari 1994; Angeletos 2007). However, the lower capital stock and the lower output level observed in the financially less developed country crucially depend on the existence of capital risk (see Angeletos 2007). As Equation (3.17) in Section 3.3 shows, capital risk drives a wedge between the expected return of the risky asset and the riskless interest rate since the entrepreneurs demand a risk premium if full insurance is not provided. This wedge makes it possible to observe a lower interest rate and a lower capital stock in the financially less developed country. This result is exactly what is needed in order to explain the Lucas paradox, i.e. the fact that it is the initially poor country that accumulates a positive net foreign asset position so that capital flows from the poor to the rich country under financial integration. To see this more clearly, consider a financial market liberalization reform that removes the trading barriers between the two countries after both countries have reached the autarchic steady state. The financially less developed country will converge to its new steady state under financial integration that is indicated by point I in Figure 3.1. Inspecting the position of point I in the lower panel shows that the financially less developed country features a positive net foreign asset position in the integrated steady state, implying that capital flows from the less to the financially more developed country under financial integration. 15 This result is driven by the fact that the interest rate increases from the perspective of the financially less developed country. The interest rate increases because it is determined by the financially more developed country that displays a higher interest rate in the autarchic steady state due to the lower level of  $^{16}$  Consequently, since capital flows from the financially less developed to the finan-

<sup>&</sup>lt;sup>15</sup>The general pattern of transitory dynamics presented in Section 3.6 shows that the evolution of the net foreign asset position towards the integrated steady state is a gradual and monotone process. In particular, in all exercises considered, we find that the financially less developed country facing an increase in the interest rate under financial integration runs a persistent series of current account surpluses along the transition path. See also Angeletos and Panousi (2011).

<sup>&</sup>lt;sup>16</sup>In a two-country framework with similar weight on each country, one would expect that the common interest rate under financial integration settles at a level between both autarchic steady state interest rates. However, qualitatively, the effect that the interest rate increases from the perspective of the

cially more developed country under financial integration, the financially less developed country has to be the initially poor country in order to explain that capital does indeed flow from the poor to the rich country. That is exactly the case in the initial equilibrium indicated by point A.

The second important property that can be inferred from point A refers to the consequences of financial integration for domestic economic development. Although the riskless bond is the only asset that is traded on the international level, the change in the interest rate also affects the domestic capital stock and output. According to standard theory, the increase in the interest rate from the perspective of the financially less developed country should lead to a lower capital stock and a lower output level in the integrated steady state. This follows from the usual opportunity-cost effect stating that investing in one asset becomes less attractive if the return of the other asset increases. However, the upper panels in Figure 3.1 show that the financially less developed country displays a higher capital stock and a higher output level in the integrated steady state (point I) compared to the autarchic steady state (point A). This can be seen from the fact that point A is located on the increasing part of the long-run capital and output functions, i.e. on the increasing part of the blue lines. The result that the capital stock and output may increase with the interest rate is driven by a second effect that exists in the presence of capital risk and that relates to the agents' willingness to take risk (see Angeletos and Panousi 2011). Due to diminishing absolute risk aversion, entrepreneurs are willing to increase investment in the risky asset, i.e. to build up the capital stock, when they become richer. Since entrepreneurs become richer under financial integration by building up their positive net foreign asset position, the wealth effect stimulates capital accumulation when the interest rate increases above its autarchic steady state level. Though the accumulation of wealth is a gradual process, which means that the capital stock may initially fall during the transition, the wealth effect may finally dominate the opportunity-cost effect so that the capital stock and output are higher in the integrated steady state.<sup>17</sup> That is exactly the case when starting from an equilibrium like point A.

In summary, the effects of uninsurable capital risk that are described by Angeletos and Panousi (2011) may also be preserved in the presence of income risk that is induced by the existence of risky profits in our model. However, as indicated by point A' and A" in Figure 3.1, the model predictions in the baseline scenario may also be quite different. First, assume that the autarchic steady state of the financially less developed country is given by point A' rather than by point A. We will show in the next section that such an equilibrium exists under different parametrizations. Point A' in the upper panels in Figure 3.1 means that a small increase in the interest rate leads to a lower long-run

financially less developed country remains the same as in our exercise.

 $<sup>^{17}\</sup>mathrm{See}$  Section 3.6 for a discussion of the transitory dynamics.

<sup>&</sup>lt;sup>18</sup>Note that the blue lines in Figure 3.1 themselves change with different parameter values. However, the main characteristics we refer to, i.e. the U-shaped form and the fact that the blue line may lie below the horizontal dashed line, are preserved.

capital stock and a lower long-run output level, because in the close neighborhood of point A', the opportunity-cost effect dominates the wealth effect. Consequently, an equilibrium like A' means that the model fails to explain a boost in long-run domestic economic development from the perspective of the financially less developed country. Furthermore, point A" shows that the model may even fail to explain the Lucas paradox. This follows from the fact that in point A" the financially less developed country is the initially rich country under financial autarky, which means that capital flows from the rich to the poor country under financial integration.

Given these opposing outcomes, how can we find the conditions under which the model is capable of contributing to explaining the empirical findings and under which the model may fail? In principle, this is a cumbersome task since the model has no closed-form solution and numerous simulations have to be conducted. However, we partly overcome this problem by deriving two rules of thumb that explain the required parameter restrictions with high accuracy.<sup>19</sup> We will present the two rules in the next section.

#### Two rules of thumb

The first rule of thumb relates to the Lucas paradox. The rule describes the condition guaranteeing that the financially less developed country is also the initially poor country in the autarchic steady state so that it is in fact the initially poor country that accumulates a positive net foreign asset position under financial integration. According to Figure 3.1, the first rule may lead to an equilibrium like point A but does not yet rule out an equilibrium like point A'. Therefore, we derive a second rule that describes the condition guaranteeing that the long-run domestic capital stock and long-run output necessarily increase with the interest rate. If this condition is satisfied, then the financially less developed country is not only the poor country in the autarchic steady state, but an increase in the interest rate implied by financial integration also leads to a higher capital stock and higher output in the integrated steady state. In order to derive our first rule, we assume that entrepreneurs in the financially more developed country can completely insure against idiosyncratic risk, i.e. markets in the financially more developed country are assumed to be complete. However, as shown in Appendix B, the complete markets assumption is not restrictive in this case so that both rules can also be applied to the general case where entrepreneurs in both countries suffer from incomplete markets as in Figure 3.1. In short, our two rules can be stated as follows.

 $<sup>^{19}</sup>$ We refer to our rules as rules of thumb since their derivation is partly based on a model comparison. See Appendix A for details.

Rule of Thumb 1 In the autarchic steady state, levels of the aggregate capital stock and output are lower in the economy with incomplete markets than in the case of complete markets if and only if

$$\vartheta > \frac{\widehat{\phi}}{2 - \widehat{\phi}}, \qquad \widehat{\phi} \equiv \frac{\alpha - \delta \frac{K^*}{\alpha Y^*}}{1 - \delta \frac{K^*}{V^*}}, \qquad (3.19)$$

where production is approximated by  $Y^* = (K^*)^{\alpha}$ .

Rule of Thumb 2 Based on the autarchic steady state of the economy with incomplete markets, the long-run capital stock and long-run output necessarily increase with any increase of the interest rate if and only if

$$\vartheta > \frac{\widehat{\phi}}{1 - \widehat{\phi}} , \qquad \widehat{\phi} \equiv \frac{\alpha - \delta \frac{K^*}{\alpha Y^*}}{1 - \delta \frac{K^*}{Y^*}}, \qquad (3.20)$$

where production is approximated by  $Y^* = (K^*)^{\alpha}$ .

In both rules,  $K^*$  denotes the aggregate capital stock and  $Y^*$  denotes aggregate output, and both are evaluated at the autarchic steady state. We provide a detailed description of how to derive the two rules in Appendix A. The general idea is to start with a simpler two-period model version and to use the results provided by Angeletos (2007) and Angeletos and Panousi (2011) in order to understand how the condition changes between the two-period model and the infinite-horizon model.

What does the first rule of thumb show us? The first rule shows that the structural parameters of the model have to satisfy a certain condition in order to guarantee that the financially less developed country is also the initially poor country in the autarchic steady state. More specifically, the first rule shows that the elasticity of intertemporal substitution,  $\vartheta$ , has to exceed a certain threshold level. The fact that the elasticity of intertemporal substitution is a key parameter of the model is well-known from Ak-type models that capture the effects of uninsurable capital risk.<sup>20</sup> Intuitively, the presence of capital risk leads to a lower risk-adjusted return and agents' response to this change crucially depends on their attitude towards intertemporal substitution. In fact, our model comes close to an Ak-model if  $\alpha$  is close to unity and our first rule of thumb confirms the well-known result that the elasticity of intertemporal substitution has to be greater than unity in order to ensure that a lower level of financial development also leads to a lower level of economic development (c.f., Weil 1990; Obstfeld 1994). However, if  $\alpha$  is less than unity, two additional effects exist in our model. First, entrepreneurs earn riskless wage income and second, they also face income risk due to the existence of risky profits. The income risk tends to tighten the parameter restrictions by increasing the precautionary demand for saving. In contrast, the riskless wage income tends to

<sup>&</sup>lt;sup>20</sup>The fact that the elasticity of intertemporal substitution rather than the parameter of relative risk aversion is the key parameter of the model is also discussed intensively by Angeletos (2007) and Angeletos and Panousi (2011).

loosen the parameter restrictions by reducing the percentage drop of consumption in times when the risky asset pays off poorly, thereby weakening the need for precautionary savings. In order to assess the overall effect, and to evaluate the performance of our first rule, we can compute the final threshold level for the elasticity of intertemporal substitution.<sup>21</sup> Table 3.2 shows the results for our financially less developed benchmark economy. The numbers in the second column are the predicted threshold levels that are derived from our first rule and the numbers in the third column are the actual threshold levels that are derived from simulations. The numbers in parentheses indicate the prediction errors.

Table 3.2: Under-accumulation of Capital: Threshold Levels

Depreciation rate	First rule of thumb	Model solutions
0.001	0.244	0.238
	(0.006)	
0.04	0.125	0.122
	(0.003)	
0.08	0.083	0.076
	(0.007)	

Table 3.2 shows two main results. First, the second column shows that our first rule is able to predict the actual threshold levels with high accuracy. This follows from the fact that the prediction errors that are reported in the second column are rather small. Further simulations suggest that this result is also robust to changes in  $\alpha$  and in the discount factor  $\beta$ . Second, and equally important, the third column shows that the threshold level for the elasticity of intertemporal substitution is very low for plausible values of the depreciation rate, which is generally in line with Angeletos (2007). For  $\delta = 0.08$ , a value of the elasticity of intertemporal substitution slightly larger than 0.076 is already sufficient to ensure that the financially less developed country is the initially poor country in the autarchic steady state and that capital flows from the poor to the rich country under financial integration.<sup>22</sup> This means that even in the presence of income risk, the combination of uninsurable capital risk and riskless wage income explains the Lucas paradox quite well. Introducing capital risk makes it possible to observe a lower capital stock and a lower interest rate in the financially less developed country and the riskless wage income ensures that the restrictions on the structural

<sup>&</sup>lt;sup>21</sup>According to our first rule, we compute the values of  $\vartheta$  satisfying  $\vartheta = \widehat{\phi}/(2-\widehat{\phi})$ , i.e. condition (3.19) holds with equality. For these critical values, the capital stock should be exactly the same under complete and incomplete markets. Larger values of  $\vartheta$  then lead to  $\vartheta > \widehat{\phi}/(2-\widehat{\phi})$ , such that, according to our first rule, the capital stock and output should be lower in the economy suffering from incomplete markets.

<sup>&</sup>lt;sup>22</sup>In terms of the parameter of relative risk aversion,  $\rho$ , we obtain the restriction  $\rho < 13$ .

parameters are quite loose. Even if we reduce the share of the riskless wage income to 0.4, the threshold level for the elasticity of intertemporal substitution does not exceed 0.15 in case the depreciation rate equals 8 percent. Since large parts of the empirical literature suggest a value of the elasticity of intertemporal substitution close to unity (see Angeletos 2007), the parameter restrictions remain very moderate.

Next, we consider how the restrictions on the elasticity of intertemporal substitution change if we additionally demand that the long-run capital stock and long-run output necessarily increase with the interest rate. If this condition holds, then the financially less developed country is not only the poor country in the autarchic steady state but an increase in the interest rate implied by financial integration also leads to a higher capital stock and a higher output level in the integrated steady state. Inspecting the second rule in (3.20) shows that the threshold level for the elasticity of intertemporal substitution will most probably increase compared to our first rule. This result is intuitive since the second condition rules out an equilibrium like point A' in Figure 3.1, which is more restrictive than what is required under our first rule. In order to assess the overall effect, we compute the corresponding threshold levels for the elasticity of intertemporal substitution that are predicted by our second rule and that are obtained from simulations. The results appear in Table 3.3, and numbers in parentheses indicate the prediction errors.

Table 3.3: Long-run Effects Capital and Output: Threshold Levels

Depreciation rate	Second rule of thumb	Model solutions
0.001	0.645	0.556
	(0.089)	
0.04	0.287	0.286
	(0.001)	
0.08	0.184	0.182
	(0.002)	

First, we observe that the prediction errors reported in the second column are again rather small. Consequently, also our second rule is able to describe the properties of the model with high accuracy. Second, the numbers in the third column confirm the conjecture that the threshold level is higher compared to our first rule. For  $\delta=0.08$ , the threshold level more than doubles from 0.08 to 0.18. However, since the threshold level is still below 0.2, and large parts of the empirical literature suggest a value of the elasticity of intertemporal substitution close to unity, we can conclude that in the baseline scenario, an equilibrium like point A in Figure 3.1 is in fact the most likely outcome for a broad range of plausible parameter values. Nevertheless, it is important to know that the empirical estimates of the elasticity of intertemporal substitution are

far from being uniform. Dacy and Hasanov (2011), for example, find a much smaller value of the elasticity of intertemporal substitution of around 0.2. Their finding raises the issue of how the parameter restrictions may change when tight borrowing constraints and persistent effects of shocks are taken into account. We address these issues in the next sections.

# 3.5.3 Scenario 2: Borrowing constraints

In the second scenario, we relax the assumption that entrepreneurs may in principle borrow up to the Natural Debt Limit (NDL) and consider more tight borrowing constraints. As it can already be inferred from the first-order conditions derived in Section 3.3, borrowing constraints affect entrepreneurs in two different ways. Since, in bad times, agents need to borrow to finance both, consumption and the capital stock of the individual firm, borrowing constraints impede consumption smoothing and limit the access to external funds for scaling up individual production. Since the corresponding effects tend to work in opposite directions as discussed below, the ultimate effect of borrowing constraints is generally ambiguous and borrowing constraints may either tighten or weaken the parameter restrictions compared to the baseline scenario.

Figure 3.2 provides first insight into the effects of borrowing constraints. Similar to Figure 3.1, the blue lines show the long-run relationship between selected macroe-conomic variables and the interest rate for the financially less developed benchmark economy. The steady state under financial autarky is indicated by point A and the steady state under financial integration is indicated by point I. However, different from the baseline scenario, entrepreneurs in the benchmark economy considered here face a borrowing limit of  $\bar{b} = 1.2$ , which roughly corresponds to the average net income in the steady state under financial autarky.<sup>23</sup>

At first view, Figure 3.2 shows that the main characteristics of the baseline scenario carry over to the case where borrowing constraints play a significant role. In particular, we observe that it is still possible to obtain an equilibrium like point A in Figure 3.2 that leads to the same implications as in the baseline scenario, i.e. of capital flowing from poor to rich countries and a boost in long-run domestic economic development despite an increase of the interest rate. However, Figure 3.2 also shows that there are some differences compared to the baseline scenario that reflect the saving and investment effect of borrowing constraints. We investigate these differences in the following.

The saving effect of borrowing constraints refers to the households' demand for the riskless asset and can be inferred from the lower panel in Figure 3.2. In particular, by comparing the position of point A with the corresponding position in the baseline scenario, we find that the autarchic steady state interest rate in our financially less developed benchmark economy decreases with tight borrowing constraints. Though the difference is quantitatively moderate, the lower interest rate indicates that tighter

<sup>&</sup>lt;sup>23</sup>Huggett (1993) suggests a debt limit of one year's average endowment to be reasonable.

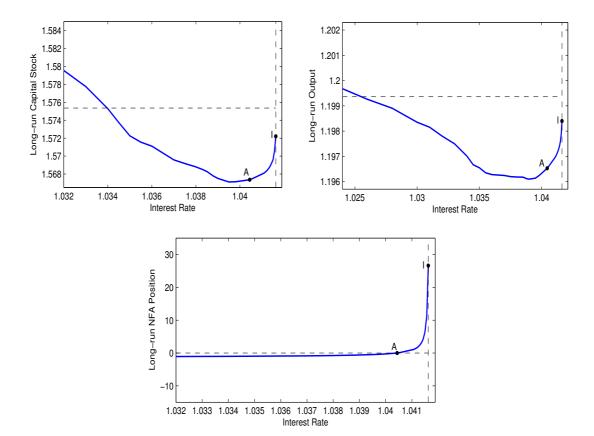


Figure 3.2: Aggregate Results: Borrowing Constraints

Note: Point A shows the autarchic steady state of the financially less developed benchmark economy and point I shows the steady state under financial integration. Compared to the baseline scenario, entrepreneurs in the benchmark economy considered here face a borrowing limit of  $\bar{b}=1.2$ . The horizontal and vertical dashed lines show the autarchic steady state of the large and financially more developed country that represents the rest of the world. The financially more developed country is identical to the baseline scenario.

borrowing constraints lead to a higher aggregate demand for the riskless asset.<sup>24</sup> The higher aggregate demand for the riskless asset can be explained by two effects of borrowing constraints on the individual level. In a mechanical way, borrowing constraints reduce the supply of the riskless asset on the individual level by limiting the amount households can borrow. Furthermore, households who are currently not constrained but whose level of net worth is rather low save more compared to the case where only the NDL is imposed. This higher individual demand for the riskless asset mainly follows from a stronger necessity to self-insure against the idiosyncratic risk. Since both, the lower supply and the higher demand on the individual level, lead to an increased demand for the riskless asset on the aggregate level, the interest rate has to decrease with

<sup>&</sup>lt;sup>24</sup>In the baseline scenario, the autarchic steady state interest rate in the financially less developed benchmark economy is equal to 4.15%. If the debt limit is equal to  $\bar{b} = 1.2$ , the interest rate decreases to 4.04%.

tighter borrowing constraints in order to clear the bond market under financial autarky. The observed saving effect of borrowing constraints is similar to standard incompletemarkets models, in which tight borrowing constraints also lead to a lower interest rate in the autarchic steady state (e.g., Huggett 1993). Furthermore, if a lower interest rate is the only effect of borrowing constraints, we can expect that tighter borrowing constraints lead to tighter parameter restrictions compared to the baseline scenario. To see this more clearly, note that a lower interest rate level pushes the location of the corresponding autarchic steady state capital stock to the left on the long-run capital function, i.e. to the left on the blue line in the upper left panel of Figure 3.2. Consequently, it becomes less likely that the financially less developed country ends up being the poor country under financial autarky, which is crucial to explain that it is the poor country accumulating a positive net foreign asset position under financial integration. Moreover, it becomes also less likely that the autarchic steady state capital stock of the financially less developed country is located on the increasing part of the long-run capital function so that financial integration necessarily leads to a boost in long-run domestic economic development. Hence, if the only effect of borrowing constraints is to lead to a lower interest rate, we can expect that tighter borrowing constraints lead to tighter parameter restrictions in order to maintain an equilibrium like point A. However, in our case, borrowing constraints also affect entrepreneurs' investment decisions, which changes the relation between aggregate capital accumulation and the interest rate. We analyze this effect in the following.

One of the main changes in the relation between aggregate capital accumulation and the interest rate that is implied by borrowing constraints can be seen with the help of the following example. In the baseline scenario, the capital stock in the financially less developed country reaches the same level as in the financially more developed country if the interest rate is equal to 3.68%. However, if we introduce tight borrowing constraints, the financially less developed country remains the poor country until the interest rate decreases by more than 8%. This can be seen in Figure 3.2 by the intersection point of the blue line and the dashed horizontal line. Consequently, we observe that tight borrowing constraints dampen aggregate capital accumulation so that a lower interest rate under tight borrowing constraints does not necessarily mean a higher aggregate capital stock. From this we can conclude that the investment effect of borrowing constraints weakens or even outweighs the saving effect of borrowing constraints: Even if the interest rate is lower in the steady state under financial autarky, the financially less developed country may still be the poor country under financial autarky and the capital stock may still be located on the increasing part of the long-run capital function. In other words, borrowing constraints do not necessarily lead to tighter parameter restrictions compared to the baseline scenario since the saving and investment effect work in opposite directions. The reason behind this opposing effect of capital accumulation can be seen again most clearly on the individual level. Since borrowing constraints do not only affect consumption smoothing, but also limit the options to scale up individual production, entrepreneurs who are currently constrained invest less compared to the case where only the NDL is imposed. Furthermore, entrepreneurs who are currently not constrained but whose level of net worth is rather low, choose a lower capital stock as well. Intuitively, the riskless asset is more suitable than the risky asset to transfer resources to those states where the borrowing constraint may become binding and entrepreneurs are willing to adjust their portfolio in favor of the riskless asset.<sup>25</sup> Consequently, investment levels are lower in case of tight borrowing constraints, either because borrowing constraints are binding today or are expected to become binding in the near future.

In sum, what can be inferred from Figure 3.2 are two opposing effects of borrowing constraints. On the one hand, borrowing constraints lead to a lower interest rate in the autarchic steady state compared to the case where only the NDL is imposed. On the other hand, borrowing constraints also affect investment and a lower interest rate does not necessarily mean a relatively higher capital stock. Therefore, the overall effect of borrowing constraints is generally ambiguous and borrowing constraints may either tighten or weaken the parameter restrictions compared to the baseline scenario. In order to assess the overall effect, we consider several debt limits and compare the corresponding threshold levels for the elasticity of intertemporal substitution.

Table 3.4: Under-accumulation of Capital - Borrowing Constraints

Debt Limit	Threshold Level
NDL	0.076
twice average net income	0.111
average net income	0.154
20 percent of average net income	0.417

Table 3.4 shows the results that again refer to the Lucas paradox. Specifically, Table 3.4 shows the threshold levels the elasticity of intertemporal substitution has to exceed in order to ensure that the financially less developed benchmark economy is also the poor economy in the autarchic steady state compared to the financially more developed economy where markets are assumed to be complete. The threshold level in the first row of Table 3.4 is known from the baseline scenario, while the other threshold levels refer to more tight borrowing constraints. The tightest borrowing constraint in Table 3.4 means that agents can only borrow up to approximately 20 percent of the average net income in the autarchic steady state.

Though the discussion above has shown that the overall effect of borrowing constraints is generally ambiguous, Table 3.4 shows that the threshold level monotonically increases with the tightness of the borrowing constraint. That means, we find that the saving effect of borrowing constraints dominates the investment effect so that tighter

<sup>&</sup>lt;sup>25</sup>Angeletos and Panousi (2011) also briefly mention this effect of borrowing constraints, but borrowing constraints do not play any role in their theoretical model.

borrowing constraints lead to tighter parameter restrictions in order to explain the Lucas paradox. Compared to the baseline scenario, the threshold level is twice as large if the debt limit is equal to the average net income in the autarchic steady state and becomes roughly 5 times larger when entrepreneurs can only borrow up to approximately 20 percent of the average net income in the autarchic steady state. Similarly, by setting  $\alpha$  to 0.6, we find that the threshold level monotonically increases from 0.147 when only the NDL is imposed to 0.455 when the debt limit corresponds to 20 percent of the average net income in the autarchic steady state. However, are these results just driven by the fact that markets are assumed to be complete in the financially more developed country?

Figure 3.3: Threshold Levels, Changes in Risk and Debt Limits

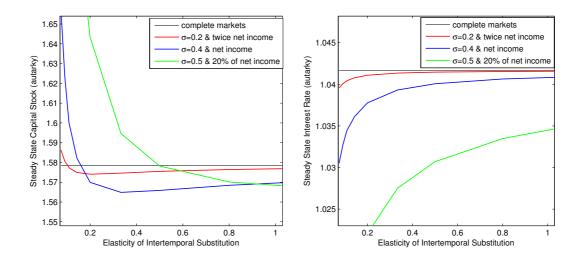


Figure 3.3 shows that this is not the case and that the same pattern also occurs if entrepreneurs in both countries suffer from uninsurable risk and borrowing constraints. The left panel of Figure 3.3 shows the relationship between the elasticity of intertemporal substitution and the corresponding autarchic steady state capital stock for several combinations of uninsurable risk and borrowing constraints. The key insight is that the intersection point of any two lines determines the threshold level the elasticity of intertemporal substitution has to exceed in order to ensure that the financially less developed country is also the poor country in the autarchic steady state. For example, Figure 3.3 shows that the elasticity of intertemporal substitution has to be larger than 0.2 in order to ensure that the financially less developed country represented by the blue line displays a lower capital stock compared to the financially more developed country represented by the red line. Comparing the green and the blue line shows that the corresponding threshold level is even around 0.9. Recall that in the baseline scenario, a value of the elasticity of intertemporal substitution slightly larger than 0.076 already

<sup>&</sup>lt;sup>26</sup>The observation of tighter parameter restrictions is in line with Covas (2006).

<sup>&</sup>lt;sup>27</sup>If the borrowing limit is approximately equal to 10 percent of the average net income in the autarchic steady state, the threshold level is even around 0.8.

ensures that the financially less developed country is the poor country in the autarchic steady state so that capital flows from the poor to the rich country under financial integration. Hence, we find that borrowing constraints may lead to significantly tighter parameter restrictions in order to explain the Lucas paradox.

Next, we consider how borrowing constraints affect the parameter restrictions from the baseline scenario if we additionally demand that the long-run domestic capital stock and long-run output necessarily increase with the interest rate.

Table 3.5: Long-run Effects Capital and Output - Borrowing Constraints

Debt Limit	Threshold Level
NDL	0.182
twice average net income	0.435
average net income	0.556
20 percent of average net income	2.5

Table 3.5 shows the corresponding threshold levels for the financially less developed benchmark economy. If the elasticity of intertemporal substitution is larger than the threshold level, the financially less developed benchmark economy is not only the poor economy in the autarchic steady state, but the increase in the interest rate implied by financial integration also necessarily leads to a higher capital stock and a higher output level in the integrated steady state. Based on the same borrowing limits considered before, we observe that the threshold level monotonically increases with the tightness of the borrowing constraint. In particular, we find that for the tightest debt limit presented in Table 3.5, the elasticity of intertemporal substitution has to be larger than 2.5, which constitutes a drastic increase compared to the baseline scenario. Hence, borrowing constraints may also lead to significantly tighter parameter restrictions in order to explain higher levels of capital and output in the integrated steady state. <sup>28</sup>

In summary, our results show that if borrowing constraints do not only affect the production side of the economy, but also affect households through the consumer credit channel, we observe stronger parameter restrictions compared to the baseline scenario. Whether these restrictions may turn out to be too strong depends on the question of interest and the tightness of the borrowing constraint. By exclusively focusing on the Lucas paradox, uninsurable risk and borrowing constraints may still explain capital flows from poor to rich countries as long as the borrowing constraints are not too tight in both countries. For example, the point at which the blue line and the red line intersect in Figure 3.3 shows that the threshold level for the elasticity of intertemporal substitution remains around 0.2. However, as the green line shows, the threshold level

 $<sup>^{28} \</sup>rm Similarly,$  if we set  $\alpha$  to 0.6, we find that the threshold level monotonically increases from 0.385 when only the NDL is imposed to 2.5 when the debt limit approximately equals 20 percent of the average net income in the autarchic steady state.

easily rises to 0.9 or higher when borrowing constraints become very tight. Turning to the question of higher levels of capital and output in the integrated steady state, we observe a similar effect of borrowing constraints. When only the NDL is imposed, a value of the elasticity of intertemporal substitution slightly larger than 0.182 already ensures that the financially less developed economy necessarily features higher levels of capital and output in the integrated steady state compared to the autarchic steady state. However, Table 3.5 shows that the threshold level already increases to 0.56 if entrepreneurs face a debt limit approximately equal to the average net income in the autarchic steady state. Tighter debt limits may even require that the elasticity of intertemporal substitution is larger than 2 and, thus, at least twice as large as suggested by main parts of the empirical literature. Hence, we can conclude that especially very tight borrowing constraints strongly affect the parameter restrictions to explain the observed pattern of international capital flows and accompanying changes in domestic economic development. From a slightly different perspective, our results show that if borrowing constraints are very tight in financially less developed countries, it becomes less likely that these countries experience an increase in capital and output under financial integration. Hence, understanding the effects of borrowing constraints is crucial for understanding the implications of financial integration.

#### 3.5.4 Scenario 3: Persistence of shocks

In the last scenario, we focus on the effects associated with changes in the properties of the underlying productivity process. So far, productivity is described by a simple i.i.d. shock and current levels of the individual return and of profits do not affect entrepreneurs' expectations regarding future developments. However, though this assumption is widely used for analytical reasons, the empirical literature emphasizes the existence of substantial persistence in labor earnings risk and in business income risk. For example, Storesletten et al. (2004) analyze the properties of labor earnings and find idiosyncratic risk to be highly persistent with an annual autocorrelation coefficient of 0.95. Similarly, DeBacker et al. (2012) find strong persistence in business income risk from privately held businesses.

In order to take account of the empirical facts, we relax the assumption of  $\rho_{\theta} = 0$  in the following and allow shocks to have persistent effects. We are especially interested in the question of how a higher level of persistence affects the parameter restrictions from the previous scenarios. Therefore, we consider several levels of the persistence parameter,  $\rho_{\theta}$ , and compute the corresponding threshold levels for the elasticity of intertemporal substitution. We show the results first and then provide some intuition.

Table 3.6 shows the results that again refer to the Lucas paradox. Specifically, Table 3.6 shows the threshold levels the elasticity of intertemporal substitution has to exceed in order to ensure that the financially less developed benchmark economy is also the poor economy in the autarchic steady state compared to the financially more developed

Persistence parameter $(\rho_{\theta})$	Threshold Level
0	0.076
0.3	0.110
0.5	0.159
0.7	0.278

Table 3.6: Under-accumulation of Capital - Persistence

country where markets are assumed to be complete. The threshold level in the first row of Table 3.6 refers to the baseline scenario, while the other threshold levels refer to higher levels of the persistence parameter. The highest level of  $\rho_{\theta}$  is equal to 0.7 and leads to substantial persistence of productivity shocks.

Inspecting the results in Table 3.6 shows that a higher level of persistence leads to a higher threshold level for the elasticity of intertemporal substitution. In other words, we observe that a higher level of persistence leads to tighter parameter restrictions in order to explain the Lucas paradox. Compared to the baseline scenario, the threshold level doubles if  $\rho_{\theta}$  is equal to 0.5 and becomes more than three times larger if  $\rho_{\theta}$  is equal to 0.7. Although the increase remains moderate in absolute terms, the strong relative increase of the threshold level indicates a strong impact of persistent effects of shocks.

In order to understand the above results, it is useful to consider the saving effect associated with a higher level of the persistence parameter. Similar to tighter borrowing constraints, a higher level of persistence leads to a higher demand for the riskless asset in the financially less developed benchmark economy. This follows from the fact that a higher level of  $\rho_{\theta}$  means that recovering from bad shocks takes longer and agents try to protect themselves by increasing their financial wealth. However, since the bond market has to clear on the country-wide level under financial autarky, the interest rate decreases in the financially less developed country. The lower interest rate then finally stimulates capital accumulation so that tighter parameter restrictions are needed in order to ensure that the financially less developed country remains the poor country under financial autarky. Hence, similar to the case of tight borrowing constraints, the saving effect of a higher level of persistence leads to tighter parameter restrictions in order to explain the Lucas paradox. Furthermore, the effects become even more pronounced if we consider the joint impact of persistence and borrowing constraints. For example, in the previous scenario, the threshold level for the elasticity of intertemporal substitution is equal to 0.15 if entrepreneurs face a debt limit equal to the average net income in the autarchic steady state. However, if we set  $\rho_{\theta}$  to 0.7, the threshold level more than doubles and equals 0.37. Hence, a higher level of persistence strongly interacts with the effects of financial market imperfections and leads to tight parameter restrictions even at moderate levels of the borrowing constraint. The result of tighter parameter restrictions is also in line with the intuition that the average agent, who does

not hold bonds in equilibrium under financial autarky, increases his investment in the risky asset in order to account for the higher precautionary saving demand (see, Covas 2006).

Finally, we address the question of how a higher level of persistence affects the parameter restrictions if we additionally demand that the long-run capital stock and long-run output necessarily increase with the interest rate. If this condition is satisfied, then the financially less developed country is not only the poor country under financial autarky, but the increase in the interest rate implied by financial integration also leads to a higher capital stock and higher output in the integrated steady state.

Table 3.7: Long-run Effects Capital and Output - Persistence

Persistence Parameter $(\rho_{\theta})$	Threshold Level
0	0.182
0.3	0.278
0.5	0.417
0.7	0.625

Table 3.7 shows the results for the financially less developed benchmark economy. Based on the same values of  $\rho_{\theta}$  considered before, we observe that the threshold level for the elasticity of intertemporal substitution increases with the level of persistence. If  $\rho_{\theta}$  is equal to 0.5, the elasticity of intertemporal substitution has to be larger than 0.4 to ensure that the financially less developed country features a higher capital stock and higher output in the integrated steady state. The threshold level associated with  $\rho_{\theta} = 0.7$  is even equal to 0.625 and highlights the strong impact of persistent effects of shocks. Recall that, in the baseline scenario, a value of the elasticity of intertemporal substitution slightly larger than 0.182 already ensures that financial integration leads to a boost in long-run domestic economic development from the perspective of the financially less developed country.

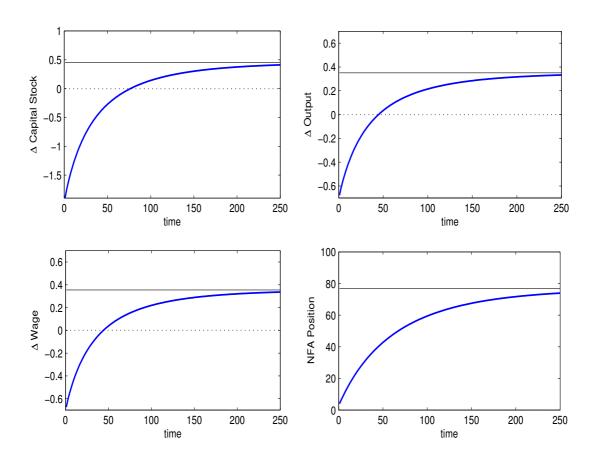
In summary, our results from the second and third scenario show that both borrowing constraints and persistent effects of shocks may strongly affect the parameter restrictions that are required in order to explain the Lucas paradox and a boost in long-run domestic economic development. Though not all combinations of borrowing constraints and persistence reverse the results from the baseline scenario, we observe significantly tighter parameter restrictions at very tight borrowing constraints and at high levels of persistence. Therefore, our results show that in a quantitative application, a more careful consideration of the country-specific characteristics is needed in order to assess the implications of financial integration. These stronger requirements are not well understood from an isolated consideration of capital risk that underestimates the existence of tight parameter restrictions. With these results at hand, we conclude the discussion of our three scenarios.

# 3.6 Transitory dynamics and welfare implications

In this section, we briefly consider the transitory dynamics between steady states from the perspective of the financially less developed country. First, it remains to show that the accumulation of financial wealth is a gradual and monotone process that also shapes the short-run dynamics of other variables towards the integrated steady state. Second, we want to assess the welfare implications of financial integration, which requires to take account of the entire transition path.

## 3.6.1 Transitory dynamics

Figure 3.4: Transitory Dynamics, First Exercise



Note: Figure 3.4 shows the transitory dynamics for the first exercise in which financial integration leads to higher levels of capital, output and the wage in the steady state under financial integration. The steady state under financial integration, relative to financial autarky, is indicated by the solid black lines.

Figure 3.4 shows the adjustment paths of selected macroeconomic variables between the steady state under financial autarky and financial integration from the perspective of the financially less developed country. Entrepreneurs in the economy considered here face a debt limit equal to the average net income in the autarchic steady state,  $\sigma$  is equal to 0.4,  $\rho_{\theta}$  is equal to 0.7 and the other parameter values are identical to the baseline scenario. Note that for these parameter values, we know that they satisfy the above derived parameter restrictions. Apart from the net foreign asset position, the dynamics of aggregate variables over time are shown as percentage deviations from the corresponding autarchic steady state level. The net foreign asset position is measured relatively to the initial output level.

The interest rate under financial integration is determined by the large and financially more developed country that represents the rest of the world. We assume that entrepreneurs in the financially more developed country can borrow up to roughly twice the average net income in the autarchic steady state, which leads to a 5.7 percent higher net interest rate compared to the financially less developed country. When barriers to trading the riskless asset are removed this gap is closed immediately.

The lower right panel of Figure 3.4 shows that the financially less developed country immediately starts to accumulate a positive net foreign asset position once trading barriers are removed. The monotone increase of the net foreign asset position means that the financially less developed country runs a series of current account surpluses along the transition path and finally reaches its long-run asset position.<sup>29</sup> As mentioned earlier, this pattern is not specific to the exercise considered here, but generally arises if financial integration is associated with an increase of the interest rate. Hence, the direction of international capital flows between countries can directly be inferred from the initial differential in interest rates.

The dynamics of the net foreign asset position also influence the short-run dynamics of the domestic aggregate capital stock. When trading barriers are removed, the capital stock of the financially less developed country initially falls below its autarchic steady state level but immediately starts to recover. Since the elasticity of intertemporal substitution is just large enough to ensure that the long-run capital stock increases with the interest rate, the capital stock eventually reaches its higher level in the steady state under financial integration. The observed short-run behavior of the aggregate capital stock is similar to Angeletos and Panousi (2011) and is driven by the different timing of the opportunity-cost and the wealth effect. The higher interest rate under financial integration immediately reduces entrepreneurs' incentive to invest in the own firm, while the accumulation of wealth, stimulating investment in the following periods, is a gradual process. The fact that the adjustment of the capital stock takes some time is known from the related literature (e.g., Angeletos and Panousi 2011; Clemens and Heinemann 2013) and shows that the potential benefits of financial integration do not occur instantaneously.

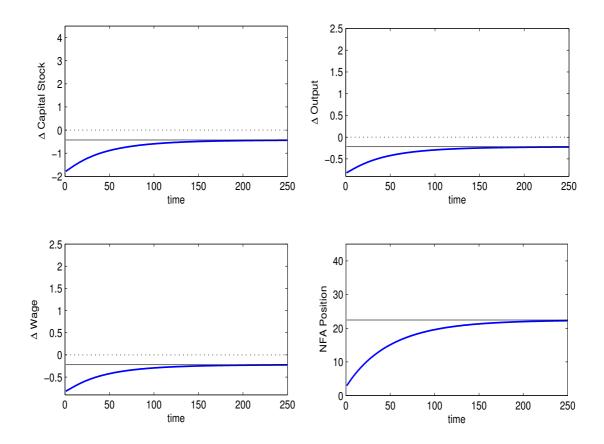
Finally, output and the wage rate largely follow the adjustment path of the aggregate capital stock, but both variables return to their autarchic steady state levels somewhat

 $<sup>^{29}</sup>$ Note that the considered example serves to show the general pattern and does not target a specific asset-to-GDP ratio.

faster. The faster recovery is driven by the improvement in production efficiency that is implied by the process of financial integration. Specifically, the accumulation of wealth under financial integration helps entrepreneurs to partly overcome the borrowing constraint and also reduces the differences in the risk premium implied by uninsurable capital risk. In other words, we observe that financial integration leads to an increase in total factor productivity.<sup>30</sup>

In order to facilitate the understanding of the following welfare analysis, we also briefly consider a second exercise in which financial integration leads to a lower long-run capital stock and a lower output level. We obtain such a long-run development by reducing the elasticity of intertemporal substitution from 0.67 to 0.25. Figure 3.5 shows the corresponding results.

Figure 3.5: Transitory Dynamics, Second Exercise



Note: Figure 3.5 shows the transitory dynamics for the second exercise in which financial integration leads to lower levels of capital, output and the wage compared to the situation of financial autarky. The steady state under financial integration, relative to financial autarky, is indicated by the solid black lines.

 $<sup>^{30}</sup>$ See also Buera and Shin (2009) and Clemens and Heinemann (2013) for a discussion of financial integration and productivity.

The key difference compared to the first exercise is that now, the wealth effect associated with a higher interest rate is not strong enough to finally outweigh the opportunity-cost effect. Consequently, from the perspective of the financially less developed country, the increase of the interest rate under financial integration does not lead to a boost in long-run domestic development. Instead, Figure 3.5 shows that the capital stock, output and the wage rate remain permanently below their autarchic steady state levels once the barriers to trading the riskless asset are removed. Since this development is quite different compared to the first exercise where the wage, capital and output reach higher levels in the integrated steady state, we will compare the corresponding welfare implications.

#### 3.6.2 Welfare implications

We measure the individual welfare effect of financial integration as the proportional change in consumption that is required to leave each household indifferent between financial autarky and financial integration. Our measure takes account of the transition path between the two steady states. Formally, for each household with net worth,  $\omega$ , and productivity,  $\tilde{\theta}$ , the individual welfare effect,  $g(\omega, \tilde{\theta})$ , measures the change in consumption that equates the value functions under financial autarky and financial integration, i.e.  $g(\omega, \tilde{\theta})$  solves

$$\left(1 + g(\omega, \widetilde{\theta})\right)^{1-\rho} = V^{I}(\omega, \widetilde{\theta}) / V^{A}(\omega, \widetilde{\theta}), \tag{3.21}$$

where  $V^A(\omega, \tilde{\theta})$  and  $V^I(\omega, \tilde{\theta})$  are the value functions under financial autarky and financial integration, respectively.<sup>31</sup> A positive value of g reflects a welfare gain of financial integration compared to the situation of remaining in the steady state under financial autarky.

Figure 3.6 shows the results for our first exercise in which financial integration leads to higher levels of capital and output in the integrated steady state. Specifically, Figure 3.6 shows the individual welfare effect of financial integration for households with average productivity but with different levels of net worth defining the household's total income. We observe that households with low net worth experience a welfare loss under financial integration while households with high net worth experience a welfare gain. Quantitatively, the welfare loss for poor households remains moderate with roughly 0.05 percent of consumption, whereas welfare gains are much more pronounced for households with high net worth. In short, Figure 3.6 shows that the poor lose while the rich win from a financial market liberalization reform. This result also applies to other productivity levels and is in line with Mendoza et al. (2009b), Angeletos and Panousi (2011), and depending on the tightness of the credit constraint, Clemens and Heinemann (2013).

In the case of log-utility, (3.21) reads  $1 + q(\omega, \widetilde{\theta}) = \exp((1 - \beta)(V^I(\omega, \widetilde{\theta}) - V^A(\omega, \widetilde{\theta})))$ .

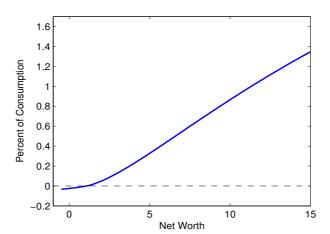


Figure 3.6: Individual Welfare Effects, First Exercise

In order to understand the above results, it is useful to firstly consider the interest rate effect of financial integration for poor households. In the first exercise, the interest rate increases from the perspective of the financially less developed country once trading barriers are removed. The higher interest rate is beneficial for savers but simultaneously means an increase in the cost of borrowing. Since the poorest households borrow a substantial amount under financial autarky, they strongly suffer from the higher interest rate under financial integration and hence, experience a welfare loss.

Moving to higher levels of net worth, financial integration turns out to be beneficial. According to Figure 3.6, households with a net worth larger than 1.1 are better off in the case of financial integration. This result is remarkable since not all households are savers and gain from the higher interest rate. In fact, up to a level of net worth equal to 2.65, households still borrow, albeit at a decreasing rate. Consequently, the welfare gains of these households must be crucially influenced by the dynamics of the wage, eventually reaching higher levels under financial integration than under financial autarky. This result becomes more evident if we consider our second exercise in which financial integration leads to lower levels of the capital stock, output and the wage compared to the situation of financial autarky. The corresponding welfare effects are presented in Figure 3.7 and, again, refer to households with average productivity.

Although a one-to-one comparison of individual numbers between the two exercises is restricted due to the change in the elasticity of intertemporal substitution, we observe some interesting differences between the groups of agents who gain from financial integration. First, Figure 3.7 shows that households in the second exercise have to be about four times richer in order to benefit from financial integration. Moreover, we do not observe households who borrow in the autarchic steady state and gain from financial integration, but we do observe households who save and lose from financial integration. This follows from the fact that in the second exercise, households with a net worth larger than 2.86 already become savers, whereas a positive welfare effect of financial

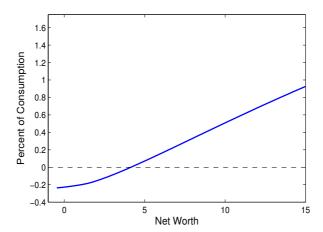


Figure 3.7: Individual Welfare Effects, Second Exercise

integration is only associated with a net worth larger than 4. Hence, in the second exercise, only households who save a substantial amount benefit from financial integration while in the first exercise, where output and the wage reach higher levels under financial integration, a larger group of agents experience a welfare gain. Consequently, we can conclude that the effects of financial integration on domestic economic development are not only of interest to describe the empirical pattern, but also may have significant welfare implications, at least for some members of society.

Finally, we compute the aggregate welfare effect of financial integration to answer the question whether the individual welfare gains or losses dominate on the aggregate level. Following Mendoza et al. (2009b), we compute the aggregate welfare effect, G, according to

$$(1+G)^{1-\rho} = \int_{\omega,\widetilde{\theta}} V^{I}(\omega,\widetilde{\theta}) \Psi^{A}(\omega,\widetilde{\theta}) / \int_{\omega,\widetilde{\theta}} V^{A}(\omega,\widetilde{\theta}) \Psi^{A}(\omega,\widetilde{\theta}), \qquad (3.22)$$

where  $\Psi^A(\omega, \tilde{\theta})$  is the stationary distribution over households under financial autarky. G can be interpreted as the required proportional change in consumption that makes a utilitarian welfare planner indifferent between financial autarky and financial market liberalization. In our first exercise, we find an aggregate welfare gain for the financially less developed country of 0.13 percent of consumption, while in our second exercise, we find an aggregate welfare loss for the financially less developed country of 0.11 percent of consumption. In other words, we find an aggregate welfare gain when the capital stock, output and the wage reach higher levels under financial integration while we observe an aggregate welfare loss when financial integration leads to permanently lower levels of the capital stock, output and the wage. Although we do not want to claim that the differences in aggregate welfare between the two exercises are completely determined by the observed differences in output and the wage, our results are clearly in line with the related literature. Angeletos and Panousi (2011), for example, find a positive aggregate welfare effect if financial integration leads to a boost in domestic economic development,

whereas Mendoza et al. (2009b) find a negative aggregate welfare effect if financial integration impedes domestic economic development. Hence, understanding the conditions under which financial integration stimulates domestic economic development is important for understanding the welfare implications of financial integration.

#### 3.7 Concluding remarks

In this paper, we examined under which conditions the observed pattern of international capital flows and accompanying changes in domestic economic development can be explained by the presence of uninsurable idiosyncratic risks and borrowing constraints. Motivated by the mixed results from the literature, we employed a heterogeneous-agent model that does not only encompass capital risk and a riskless income component, but also income risk due to the existence of risky profits. Furthermore, borrowing constraints simultaneously impede consumption smoothing and restrict the access to external funds for scaling up individual production. We considered different scenarios, increasing the model complexity step by step. In the baseline scenario, we focused on the effects of the capital risk, the risky profits, and the riskless wage income. We found that in the baseline scenario, the restrictions on the model parameters that are required to explain capital flows from poor to rich countries and a boost in long-run domestic economic development are moderate and easily satisfied by empirically plausible parameter values. As in Angeletos and Panousi (2011), we emphasized the combination of capital risk and the riskless wage income. Introducing capital risk makes it possible to observe a lower capital stock and a lower interest rate in the financially less developed country and the riskless wage income ensures that the parameter restriction remain quite loose. This combination works quite well even in the presence of income risk that leads to additional precautionary savings. We derived our results with the help of two rules of thumb that describe the underlying parameter restrictions with high accuracy and that can also be applied to other types of models considered in the literature.

In the second scenario, we considered the case with tight borrowing constraints. On the one hand, borrowing constraints make it more difficult for agents to smooth consumption and lead to an increase in aggregate demand for the safe asset. On the other hand, borrowing constraints restrict the access to external funds for scaling up production and, even if not currently binding, discourage risky investment. However, we find that the saving effect is the dominant effect and that tighter borrowing constraints lead to tighter parameter restrictions compared to the baseline scenario. In particular, we find that in times of strong turmoil in financial markets, with an almost collapsing lending channel, the model predictions change drastically and indicate that financial integration may easily become an impediment for domestic economic development. This result is not only important for explaining the empirical pattern, but also has significant welfare implications, at least for those members of society who do not benefit from an increase of the interest rate under financial integration.

In the third and final scenario, we increased the persistence of shocks. A higher level of persistence increases the demand for the riskless asset and therefore, amplifies the effects of the financial market imperfections. In almost all exercises, we find that a higher persistence of shocks again leads to tighter parameter restrictions compared to the first two scenarios. This especially applies to moderate levels of the borrowing constraint. Hence, as an overall result, we can conclude that a very careful consideration of the country specific characteristics is needed in order to fully understand the implications of financial integration. Even if two countries are characterized by a similar level of financial development, the implications of financial integration may be very different, depending on the dominant source of risk, the exact specification of borrowing constraints or the persistence of risk. This paper provides a basis for understanding these specific differences in financial market performance.

There are several interesting ways in which the current analysis could be extended with regard to future research. For example, we do not assume that the process of financial liberalization is accompanied by improvements in financial market performance, resulting in less tight borrowing constraints or better insurance opportunities.<sup>32</sup> Such improvements seem likely to occur once countries get access to the advanced financial instruments provided by developed countries. Allowing for this interplay of financial market performance and financial integration may relax the required parameter restrictions and, thereby, may further improve the ability of incomplete-markets models to explain the empirical pattern. Furthermore, improvements in financial market performance may increase the political support for financial liberalization among the population, thereby reducing the need for implementing redistribution measures. We leave investigating this channel for future research.

 $<sup>^{32}</sup>$ Financial market liberalization may also be accompanied by other types of reforms. See Buera and Shin (2009) for a discussion.

#### 3.8 Appendix

#### 3.8.1 Appendix A. Derivation of the two rules of thumb

In order to derive our two rules of thumb, we proceed along the following steps. First, we simplify our model by reducing the time horizon to two periods and by assuming that all households start with the same level of net worth. The first simplification relates to the discussion of Krusell and Smith (2006), emphasizing the usefulness of the two-period setup for conducting pilot studies within the class of incomplete-markets models. The second simplification means that choices of capital and bonds are identical across all households so that aggregation is easily obtained. Within the simplified framework, we compare the outcome under incomplete markets and complete markets and derive the condition guaranteeing that the capital stock and output are lower in the incomplete markets case. Subsequently, we derive a corresponding condition for a simplified model version sharing the main features of Angeletos (2007) and Angeletos and Panousi (2011), i.e. capital risk and riskless wage income, but no further risky income component.<sup>33</sup> Based on the comparison of the two conditions and the conditions described in Angeletos (2007) and Angeletos and Panousi (2011), we finally derive the two rules of thumb that refer to the steady state of the underlying infinite-horizon model.

#### Step 1. Two-period model - incomplete markets

The simplified household's optimization problem under incomplete markets is given by

$$\max_{c_t, k_{t+1}, b_{t+1}} U(c_t) + \beta E_t U(c_{it+1})$$
(3.23)

$$s.t. c_t = \omega_t - k_{t+1} - b_{t+1} (3.24)$$

$$c_{it+1} = \omega_{it+1} \tag{3.25}$$

$$\omega_{it+1} = b_{t+1}R_{t+1} + k_{t+1}R_{it+1}^r + w_{t+1} + \pi_{it+1}, (3.26)$$

where the subscript i is dropped whenever optimal choices are the same across households. By assumption,  $\omega_t$  is equal across all households, whereas the individual level of net worth in the final period t+1 depends on the realization of the idiosyncratic shock. The decomposition of  $\omega_{it+1}$  is in line with the expression in (3.10) and again shows the existence of capital risk and profits constituting a risky income component. The risky return is given by  $R_{it+1}^r \equiv 1 - \delta + \alpha^2 \tilde{\theta}_{it+1} k_{t+1}^{\alpha-1}$  and risky profits are given by  $\pi_{it+1} \equiv (1-\alpha)\alpha\tilde{\theta}_{it+1}k_{t+1}^{\alpha}$ , exploiting the fact that  $L_{t+1} = 1$  holds in equilibrium.

<sup>&</sup>lt;sup>33</sup>The specification of investment risk in Angeletos and Panousi (2011) slightly differs from Angeletos (2007), but the main results we refer to are comparable. See also Angeletos and Panousi (2009).

The first-order conditions are given by

$$U'(c_t) = \beta R_{t+1} E_t [U'(c_{it+1})]$$
 (3.27)

$$U'(c_t) = \beta E_t[R_{it+1}^r U'(c_{it+1})], \qquad (3.28)$$

where (3.27) is the Euler equation for bond holdings and (3.28) is the Euler equation for capital. Note that, since the optimal choice of risky investment is the same across all households,  $k_{t+1}$  denotes the individual as well as the aggregate capital stock. Using this result and the fact that  $E(\tilde{\theta}) = 1$ , the expression for the wage reduces to  $w_{t+1} = (1-\alpha)k_{t+1}^{\alpha}$ . Finally, since market clearing of the bond market implies  $\int_0^1 b_{t+1} di = 0$  and  $b_{t+1}$  is the same across all households, the capital Euler equation becomes a function of  $k_{t+1}$  alone

$$U'(\omega_t - k_{t+1}) = \beta E_t [R_{it+1}^r U'(k_{t+1} R_{it+1}^r + w_{t+1} + \pi_{it+1})]. \tag{3.29}$$

#### Step 2. Two-period model - complete markets

In the complete markets case, households can completely insure against idiosyncratic risk. That means, instead of earning a risky return and risky profits as under incomplete markets, households receive the expected value of the risky return and of risky profits in each state in period t+1. This follows from the fact that the risk is purely idiosyncratic and complete markets allow for perfect risk pooling. Hence, the optimal choice of capital under complete markets is without risk and the capital Euler equation is given by

$$U'(\omega_t - k_{t+1}) = \beta E_t[R_{t+1}^r]U'(k_{t+1}E_t[R_{t+1}^r] + w_{t+1} + E_t[\pi_{t+1}]), \tag{3.30}$$

where the subscript i attached to the return and to profits is dropped in order to emphasize that the expected value of the return and of profits is identical across all households, i.e.  $E_t[R_{it+1}^r] = E_t[R_{t+1}^r]$  and  $E_t[\pi_{it+1}] = E_t[\pi_{t+1}]$ . Equation (3.30) highlights the fact that the complete markets case corresponds to a deterministic setting where the riskless return and riskless profits are equal to their expected values. In other words, the comparison of the optimal choice of capital between (3.29) and (3.30) corresponds to the discussion of how agents respond to the introduction of risk.

#### Step 3. Comparing the Euler equations

In order to simplify the comparison between (3.29) and (3.30), the right-hand side of (3.29) is approximated using a second-order Taylor expansion around the point  $(E_t[R_{t+1}^r], E_t[\pi_{t+1}])$ . For small risks, (3.29) simplifies to

<sup>&</sup>lt;sup>34</sup>See Baiardi et al. (2014) for a related analysis.

$$U'(\omega_t - k_{t+1}) = \beta E_t [R_{t+1}^r] U'(\cdot)$$
(3.31)

+ 
$$\frac{1}{2} \beta \left[ E_t \left[ R_{t+1}^r \right] U'''(\cdot) k_{t+1}^2 + 2 U''(\cdot) k_{t+1} \right] \sigma_{R_{t+1}^r}^2$$
 (3.32)

$$+ \frac{1}{2} \beta E_t [R_{t+1}^r] U'''(\cdot) \sigma_{\pi_{t+1}}^2$$
 (3.33)

$$+ \beta \left[ E_t \left[ R_{t+1}^r \right] U'''(\cdot) k_{t+1} + U''(\cdot) \right] \sigma_{R_{t+1}^r \pi_{t+1}}, \tag{3.34}$$

with  $(\cdot) \equiv (k_{t+1}E_t[R_{t+1}^r] + w_{t+1} + E_t[\pi_{t+1}])$ .  $\sigma_{R_{t+1}^r}^2$  is the conditional variance of the return,  $\sigma_{\pi_{t+1}}^2$  is the conditional variance of profits and  $\sigma_{R_{t+1}^r\pi_{t+1}}$  is the conditional covariance between the two. The term on the right-hand side in (3.31) is equal to the corresponding expression under complete markets in (3.30), and the additional terms in (3.32)-(3.34) capture the influence of risk. Based on this representation, the following relation between the optimal choice of  $k_{t+1}$  under complete and incomplete markets can be derived.

**Lemma 1** Let  $k_{t+1}^*$  denote the solution in the incomplete markets case. Then, the capital stock is the same under complete and incomplete markets if and only if the additional terms in (3.32)-(3.34) evaluated at  $k_{t+1}^*$  sum up to zero. Moreover, the capital stock in the incomplete markets case is lower (larger) than in the complete markets case if and only if the additional terms in (3.32)-(3.34) evaluated at  $k_{t+1}^*$  sum up to a value strictly lower (larger) than zero.

*Proof of Lemma 1.* For notational ease, we introduce the following definitions

$$f(k_{t+1}) \equiv U'(\omega_t - k_{t+1}) \tag{3.35}$$

$$g(k_{t+1}) \equiv \beta E_t[R_{t+1}^r]U'(\cdot) \tag{3.36}$$

$$h(k_{t+1}) \equiv \frac{1}{2} \beta \left[ E_t \left[ R_{t+1}^r \right] U'''(\cdot) k_{t+1}^2 + 2 U''(\cdot) k_{t+1} \right] \sigma_{R_{t+1}^r}^2$$

$$+ \frac{1}{2} \beta E_t \left[ R_{t+1}^r \right] U'''(\cdot) \sigma_{\pi_{t+1}}^2$$

$$+ \beta \left[ E_t \left[ R_{t+1}^r \right] U'''(\cdot) k_{t+1} + U''(\cdot) \right] \sigma_{R_{t+1}^r \pi_{t+1}}^r,$$

$$(3.37)$$

with  $(\cdot) \equiv (k_{t+1}E_t[R_{t+1}^r] + w_{t+1} + E_t[\pi_{t+1}])$  and where it is explicitly indicated that all functions depend on  $k_{t+1}$ . Then, (3.31)-(3.34) can be paraphrased as

$$f(k_{t+1}) = g(k_{t+1}) + h(k_{t+1}), (3.38)$$

whereas in the complete markets case, according to (3.30) we have

$$f(k_{t+1}) = g(k_{t+1}). (3.39)$$

Let  $k_{t+1}^*$  denote the solution in the incomplete and  $\hat{k}_{t+1}$  denote the solution in the complete markets case.

If  $h(k_{t+1}^*) = 0$  then  $f(k_{t+1}^*) = g(k_{t+1}^*)$ . Since  $\hat{k}_{t+1}$  satisfies  $f(\hat{k}_{t+1}) = g(\hat{k}_{t+1})$  as well, it follows that  $k_{t+1}^* = \hat{k}_{t+1}$ .

If  $h(k_{t+1}^*) < 0$  then  $f(k_{t+1}^*) < g(k_{t+1}^*)$ . Since  $f(k_{t+1})$  is continuous and strictly increasing in  $k_{t+1}$  and  $g(k_{t+1})$  is continuous and strictly decreasing in  $k_{t+1}$ , it follows that  $f(k_{t+1}^*) < g(k_{t+1}^*)$  and  $f(\hat{k}_{t+1}) = g(\hat{k}_{t+1})$  implies  $k_{t+1}^* < \hat{k}_{t+1}$ .

If  $h(k_{t+1}^*) > 0$  then  $f(k_{t+1}^*) > g(k_{t+1}^*)$ . Since  $f(k_{t+1})$  is continuous and strictly increasing in  $k_{t+1}$  and  $g(k_{t+1})$  is continuous and strictly decreasing in  $k_{t+1}$ , it follows that  $f(k_{t+1}^*) > g(k_{t+1}^*)$  and  $f(\hat{k}_{t+1}) = g(\hat{k}_{t+1})$  implies  $k_{t+1}^* > \hat{k}_{t+1}$ .

The other way around it follows from  $k_{t+1}^* = \hat{k}_{t+1}$  that  $h(k_{t+1}^*) = 0$  since  $f(k_{t+1}^*) = g(k_{t+1}^*)$ .

If  $k_{t+1}^* < \hat{k}_{t+1}$ , it follows that  $f(k_{t+1}^*) < g(k_{t+1}^*)$  since  $f(k_{t+1})$  is continuous and strictly increasing in  $k_{t+1}$ ,  $g(k_{t+1})$  is continuous and strictly decreasing in  $k_{t+1}$  and  $f(\hat{k}_{t+1}) = g(\hat{k}_{t+1})$ . Consequently, since  $k_{t+1}^*$  satisfies  $f(k_{t+1}^*) = g(k_{t+1}^*) + h(k_{t+1}^*)$ , it follows that  $h(k_{t+1}^*) < 0$ .

If  $k_{t+1}^* > \hat{k}_{t+1}$ , it follows that  $f(k_{t+1}^*) > g(k_{t+1}^*)$  since  $f(k_{t+1})$  is continuous and strictly increasing in  $k_{t+1}$ ,  $g(k_{t+1})$  is continuous and strictly decreasing in  $k_{t+1}$  and  $f(\hat{k}_{t+1}) = g(\hat{k}_{t+1})$ . Consequently, since  $k_{t+1}^*$  satisfies  $f(k_{t+1}^*) = g(k_{t+1}^*) + h(k_{t+1}^*)$ , it follows that  $h(k_{t+1}^*) > 0$ . End of proof.

Before we proceed with the derivation of our final rule, we can use the expressions in (3.32)-(3.34) to briefly discuss the differences in the influence of the underlying sources of risk. Initially, we focus exclusively on the effect of income risk being captured by the expression in (3.33). In our model, income risk is implied by the existence of risky profits but the same expression also occurs if, for simplicity,  $\pi_{t+1}$  is treated as a stochastic endowment component with variance  $\sigma^2_{\pi_{t+1}}$ . According to (3.33) and Lemma 1, the well-known result applies that uninsurable income risk generates precautionary savings and, consequently, leads to over-accumulation of capital if  $U'''(\cdot) > 0$  (see e.g., Leland 1968; Kimball 1990). Since this condition holds in our case, uninsurable income risk alone would necessarily lead to larger levels of capital and output in the financially less developed country with incomplete markets.

Next, we focus exclusively on capital risk so that only (3.32) shows up in the incomplete markets case and the model economy becomes closely related to Levhari and Srinivasan (1969) and Sandmo (1970).<sup>35</sup> Inspecting (3.32) shows that the ultimate effect of uninsurable capital risk on the choice of  $k_{t+1}$  is ambiguous. On the one hand, a precautionary saving effect exists as well, but on the other hand, the term  $2U''(\cdot)$   $k_{t+1}$  is negative if agents are risk-averse.<sup>36</sup> Intuitively, and in contrast to income risk, agents can reduce the extent to which their resources are exposed to potential losses by consuming more and saving less in the case where risk is associated with risky returns. Consequently, capital risk leads to lower levels of capital and output in the economy with incomplete markets if the latter effect dominates.

<sup>&</sup>lt;sup>35</sup>See also Rothschild and Stiglitz (1971) and for a separate analysis of both types of risk see Eeckhoudt and Schlesinger (2008).

<sup>&</sup>lt;sup>36</sup>In general, the sign of  $2U''(\cdot)$   $k_{t+1}$  also depends on whether agents are savers  $(k_{t+1} > 0)$  or borrowers  $(k_{t+1} < 0)$ . In our case, however, only  $k_{t+1} > 0$  is relevant.

Finally, we assume that both types of risk exist simultaneously and if  $\sigma_{R_{t+1}^r\pi_{t+1}} \neq 0$ , all terms in (3.32)-(3.34) show up in the approximation. In general, the appearance of the covariance in (3.34) implies another ambiguous effect on the choice of  $k_{t+1}$  and the sign of the covariance reflects the extent to which the risky asset can be used to hedge against the underlying income risk.<sup>37</sup> In our model,  $\sigma_{R_{t+1}^r\pi_{t+1}}$  is positive and second moments, evaluated at  $k_{t+1}^*$ , are given by

$$\sigma_{R_{t+1}^r}^2 = \alpha^4 (k_{t+1}^*)^{2\alpha - 2} \sigma_{\widetilde{\theta}_{t+1}}^2 
\sigma_{\pi_{t+1}}^2 = (1 - \alpha)^2 \alpha^2 (k_{t+1}^*)^{2\alpha} \sigma_{\widetilde{\theta}_{t+1}}^2 
\sigma_{R_{t+1}^r \pi_{t+1}}^r = (1 - \alpha) \alpha^3 (k_{t+1}^*)^{2\alpha - 1} \sigma_{\widetilde{\theta}_{t+1}}^2,$$
(3.40)

where  $\sigma_{\widetilde{\theta}_{t+1}}^2$  denotes the variance of  $\widetilde{\theta}_{t+1}$ .

Based on the expressions in (3.40) and Lemma 1 we can now derive the final condition guaranteeing that the capital stock and output are lower under incomplete markets than under complete markets. Specifically, according to Lemma 1, the capital stock and output are lower under incomplete markets than under complete markets if and only if

$$[E_{t} [R_{t+1}^{r}] U'''(\cdot) (k_{t+1}^{*})^{2} + 2 U''(\cdot) k_{t+1}^{*}] \alpha^{4} (k_{t+1}^{*})^{2\alpha - 2} \sigma_{\widetilde{\theta}_{t+1}}^{2}$$

$$+ E_{t} [R_{t+1}^{r}] U'''(\cdot) (1 - \alpha)^{2} \alpha^{2} (k_{t+1}^{*})^{2\alpha} \sigma_{\widetilde{\theta}_{t+1}}^{2}$$

$$+ 2[E_{t} [R_{t+1}^{r}] U'''(\cdot) k_{t+1}^{*} + U''(\cdot)] (1 - \alpha) \alpha^{3} (k_{t+1}^{*})^{2\alpha - 1} \sigma_{\widetilde{\theta}_{t+1}}^{2} < 0,$$

$$(3.41)$$

where  $(\cdot) \equiv (k_{t+1}^* E_t[R_{t+1}^r] + w_{t+1} + E_t[\pi_{t+1}]).$ 

Since  $k_{t+1}^* > 0$ , getting rid of  $\alpha^2 (k_{t+1}^*)^{2\alpha} \sigma_{\widetilde{\theta}_{t+1}}^2 > 0$  and collecting terms yields

$$E_t[R_{t+1}^r]U'''(\cdot)[\alpha^2+1-2\alpha+\alpha^2+2\alpha-2\alpha^2] \ + \ 2U''(\cdot)(k_{t+1}^*)^{-1}[\alpha^2+\alpha-\alpha^2] \ < 0, \ (3.42)$$

and finally

$$-\frac{1}{\alpha} \frac{U'''(\cdot)}{U''(\cdot)} (E_t[R_{t+1}^r] k_{t+1}^*) < 2. \tag{3.43}$$

Applying  $U''(x) = -\rho x^{-\rho-1}$  and  $U'''(x) = \rho(\rho+1)x^{-\rho-2}$  leads to

$$\left(\frac{1}{\vartheta} + 1\right) \frac{1}{\alpha} \frac{k_{t+1}^* E_t[R_{t+1}^r]}{k_{t+1}^* E_t[R_{t+1}^r] + w_{t+1} + E_t[\pi_{t+1}]} < 2,\tag{3.44}$$

where  $\vartheta$  is the elasticity of intertemporal substitution  $(\vartheta=1/\rho)$ . Since production of the final good simplifies to  $Y_{t+1}^*=(k_{t+1}^*)^\alpha$  and  $K_{t+1}^*\equiv\int_0^1k_{t+1}^*di=k_{t+1}^*$ , we get  $w_{t+1}+E_t[R_{t+1}^r]k_{t+1}^*+E_t[\pi_{t+1}]=Y_{t+1}^*+(1-\delta)K_{t+1}^*$  and  $k_{t+1}^*E_t[R_{t+1}^r]=\alpha^2Y_{t+1}^*+(1-\delta)K_{t+1}^*$ . Plugging these expressions into (3.44) leads to the following final condition:

<sup>&</sup>lt;sup>37</sup>For a more rigorous and simultaneous treatment of both types of risk in cases where income and the return are exogenously given, see Li (2012) and Baiardi et al. (2014).

**Lemma 2** Levels of the aggregate capital stock and output are lower in the economy with incomplete markets compared to the complete markets case if and only if <sup>38</sup>

$$\vartheta > \frac{\phi}{2 - \phi}, \qquad \phi \equiv \frac{\alpha + (1 - \delta) \frac{K_{t+1}^*}{\alpha Y_{t+1}^*}}{1 + (1 - \delta) \frac{K_{t+1}^*}{Y_{t-1}^*}},$$
 (3.45)

where production simplifies to  $Y_{t+1}^* = (K_{t+1}^*)^{\alpha}$ .

## Step 4. A two-period model version of Angeletos (2007) and Angeletos and Panousi (2011)

In this step, we derive a respective condition for the case when the two-period model shares the main features of Angeletos (2007) and Angeletos and Panousi (2011).<sup>39</sup> Comparable to our model economy, each household owns a private firm, receives riskless wage income by supplying one unit of labor, and freely trades a riskless bond. However, no intermediate goods sector exists, but all firms produce the same final good using capital and labor as inputs. Each firm owner invests capital, and optimal employment is chosen after the capital stock has been installed and the contemporaneous idiosyncratic shock has been observed. Assuming that individual production in period t+1 takes place according to  $y_{it+1} = \theta_{it+1}^{\alpha} k_{it+1}^{\alpha} l_{it+1}^{1-\alpha}$ , capital income becomes linear in the capital stock. With  $ln(\theta) \sim N(-\sigma^2/2, \sigma^2)$  and identical initial conditions, the household's maximization problem under incomplete markets reads

$$\max_{c_t, k_{t+1}, b_{t+1}} U(c_t) + \beta E_t U(c_{it+1})$$
(3.46)

$$s.t. c_t = \omega_t - k_{t+1} - b_{t+1} (3.47)$$

$$c_{it+1} = \omega_{it+1} \tag{3.48}$$

$$\omega_{it+1} = b_{t+1}R_{t+1} + \pi_{it+1} + w_{t+1} + (1-\delta)k_{t+1}, (3.49)$$

where the subscript i is dropped whenever optimal choices are the same across all households.  $\delta$  is the depreciation rate,  $k_{t+1}$  and  $b_{t+1}$  denote risky and riskless investment, respectively, and  $w_{t+1}$  is the wage rate.  $\pi_{it+1}$  is defined as

$$\pi_{it+1} \equiv \max_{l_{it+1}} \theta_{it+1}^{\alpha} k_{t+1}^{\alpha} l_{it+1}^{1-\alpha} - w_{t+1} l_{it+1}, \tag{3.50}$$

where optimal employment maximizes  $\pi_{it+1}$  state by state, leading to

 $l_{it+1} = \left(\frac{1-\alpha}{w_{t+1}}\right)^{\frac{1}{\alpha}} \theta_{it+1} k_{t+1}$ . Using this result,  $\omega_{it+1}$  simplifies to

$$\omega_{it+1} = b_{t+1}R_{t+1} + k_{t+1}R_{it+1}^r + w_{t+1}, \tag{3.51}$$

<sup>&</sup>lt;sup>38</sup>Note that the condition described in (3.45) is defined for the interesting case of  $(2 - \phi) > 0$ .

<sup>&</sup>lt;sup>39</sup>Regarding Angeletos (2007), we refer to the model without extensions (p.5f.).

where  $R^r_{it+1} \equiv 1 - \delta + \theta_{it+1} \alpha \left(\frac{1-\alpha}{w_{t+1}}\right)^{\frac{1-\alpha}{\alpha}}$  is the risky return. Hence, in this case, households only face capital risk where capital income,  $k_{t+1}R^r_{it+1}$ , is linear in the individual capital stock.

Bond market clearing implies  $\int_0^1 b_{t+1} di = b_{t+1} = 0$  so that the capital Euler equation in the incomplete markets case is given by

$$U'(\omega_t - k_{t+1}) = \beta E_t [R_{it+1}^r U'(k_{t+1} R_{it+1}^r + w_{t+1})]. \tag{3.52}$$

Since labor market clearing implies  $\int_0^1 l_{it+1} di = 1$ , we finally get  $w_{t+1} = (1 - \alpha)k_{t+1}^{\alpha}$ .

In the complete markets case, households completely insure against the idiosyncratic risk and receive the expected value of the risky return in each state in period t+1. The resulting capital Euler equation under complete markets is thus given by

$$U'(\omega_t - k_{t+1}) = \beta E_t[R_{t+1}^r]U'(k_{t+1}E_t[R_{t+1}^r] + w_{t+1}), \tag{3.53}$$

where the subscript i attached to the return is dropped in order to emphasize that the expected value of the individual return is identical across all households, i.e.  $E_t[R_{it+1}^r] = E_t[R_{t+1}^r]$ .

Since only capital risk exists in the incomplete markets case, the right-hand side of equation (3.52) is approximated using a second-order Taylor expansion around the point  $(E_t[R_{t+1}^r])$ . For small risks this leads to

$$U'(\omega_t - k_{t+1}) = \beta E_t [R_{t+1}^r] U'(\cdot)$$
(3.54)

+ 
$$\frac{1}{2} \beta \left[ E_t \left[ R_{t+1}^r \right] U'''(\cdot) k_{t+1}^2 + 2 U''(\cdot) k_{t+1} \right] \sigma_{R_{t+1}^r}^2,$$
 (3.55)

with  $(\cdot) \equiv k_{t+1} E_t[R_{t+1}^r] + w_{t+1}$ .

Let  $k_{t+1}^*$  denote the solution in the incomplete markets case. By analogy with Lemma 1, the capital stock and output are lower in the incomplete compared to the complete markets case if and only if

$$[E_t [R_{t+1}^r] U'''(\cdot) (k_{t+1}^*)^2 + 2 U''(\cdot) k_{t+1}^*] \sigma_{R_{t+1}^r}^2 < 0.$$
 (3.56)

Since  $\sigma_{R_{t+1}^r}^2 k_{t+1}^* > 0$ , this simplifies to

$$-\frac{U'''(\cdot)}{U''(\cdot)}E_t[R_{t+1}^r]k_{t+1}^* < 2. (3.57)$$

Applying  $U''(x) = -\rho x^{-\rho-1}$  and  $U'''(x) = \rho(\rho+1)x^{-\rho-2}$  leads to

$$\left(\frac{1}{\vartheta} + 1\right) \frac{E_t[R_{t+1}^r]k_{t+1}^*}{E_t[R_{t+1}^r]k_{t+1}^* + w_{t+1}} < 2,\tag{3.58}$$

where  $\vartheta$  is the elasticity of intertemporal substitution ( $\vartheta=1/\rho$ ). Since aggregate production simplifies to  $Y_{t+1}^*=(k_{t+1}^*)^{\alpha}$  and  $K_{t+1}^*\equiv\int_0^1k_{t+1}^*di=k_{t+1}^*$ , it follows that

 $E_t[R_{t+1}^r]k_{t+1}^* = (1-\delta)K_{t+1}^* + \alpha Y_{t+1}^*$  and  $E_t[R_{t+1}^r]k_{t+1}^* + w_{t+1} = (1-\delta)K_{t+1}^* + Y_{t+1}^*$ . Plugging these expressions into (3.58) yields the following final condition:

**Lemma 3** In the simplified two-period model sharing the main features of Angeletos (2007) and Angeletos and Panousi (2011), levels of the aggregate capital stock and output are lower in the economy with incomplete markets compared to the complete markets case if and only if

$$\vartheta > \frac{\kappa}{2 - \kappa} , \qquad \kappa \equiv \frac{\alpha + (1 - \delta) \frac{K_{t+1}^*}{Y_{t+1}^*}}{1 + (1 - \delta) \frac{K_{t+1}^*}{Y_{t+1}^*}}, \qquad (3.59)$$

where aggregate production simplifies to  $Y_{t+1}^* = (K_{t+1}^*)^{\alpha}$ .

Comparing our condition in (3.45) with the condition in (3.59) shows that both share a similar structure, but  $\phi$  and  $\kappa$  differ from each other when  $\delta < 1$ . That means, in the relevant case without complete depreciation, the conditions to observe a lower capital stock and a lower output level in the financially less developed country differ between the models.

#### Step 5. The first rule of thumb

While the analysis so far has focused on the simplified model structure, the next step compares the results described in Lemma 3 with the condition explaining the behavior of aggregate variables in the autarchic steady state of the underlying infinite-horizon model. According to Angeletos (2007, p.11), and assuming the same Cobb-Douglas specification on the individual level as in the simplified two-period model version, the following condition applies for plausible parameter values:

In the model considered by Angeletos (2007), autarchic steady state levels of the capital stock and output are lower in the economy with incomplete markets than in the case of complete markets if and only if

$$\vartheta > \frac{\widehat{\kappa}}{2 - \widehat{\kappa}} , \qquad \widehat{\kappa} \cong \frac{\alpha - \delta \frac{K^*}{Y^*}}{1 - \delta \frac{K^*}{Y^*}}, \qquad (3.60)$$

where  $Y^* = (K^*)^{\alpha}$  in case of a Cobb-Douglas specification.

Even though condition (3.60) refers to the steady state of the infinite-horizon model, whereas condition (3.59) explains the influence of uninsurable capital risk in the simplified two-period model, they share a similar structure.<sup>40</sup> The only significant difference, which appears in the expressions of  $\kappa$  and  $\hat{\kappa}$ , indicates a different role played by the depreciation rate  $\delta$ . The difference, however, seems intuitively plausible due to the

<sup>&</sup>lt;sup>40</sup>In Angeletos (2007), capital risk also includes depreciation risk. However, taking account of this feature in the two-period version does not change the condition in (3.59). Consequently, the implications derived from comparing (3.59) with (3.45) and (3.59) with (3.60) also remain unaffected.

different nature of the steady state and of the final period in the two-period model. In the two-period model, aggregate consumption in the final period equals aggregate production and  $\kappa$  is equal to  $\alpha$  if capital completely depreciates. The corresponding wage share equals  $(1-\alpha)$  and the income share of capital is equal to  $\alpha$ . To obtain the same relation in the steady state of the infinite-horizon model and to get the same value of  $\hat{\kappa}$ , the depreciation rate has to converge to zero so that no additional resources are needed to keep the aggregate capital stock at its steady state level. Hence, the "natural" counterpart of  $\delta=1$  in the two-period model is the case of  $\delta=0$  in the steady state of the infinite-horizon model.

Since the comparison of (3.59) with (3.60) shows how the condition changes between the simplified two-period model and the underlying infinite-horizon model, we can use this information to finally derive our first rule of thumb. In the last step, we simply derive a new variable,  $\hat{\phi}$ , such that the difference between  $\hat{\phi}$  and  $\phi$  in (3.45) reflects the observed difference between  $\hat{\kappa}$  in (3.60) and  $\kappa$  in (3.59). Roughly speaking, we simply replace the term  $(1 - \delta)$  in (3.45) with its counterpart  $-\delta$ . Applying this last step leads to our first rule of thumb described in (3.19).

#### Step 6. The second rule of thumb

The derivation of the second rule builds on Angeletos and Panousi (2011) and the above described results. According to Figure 3.1, the second rule describes the condition guaranteeing that the long-run capital stock and long-run output necessarily increase with the interest rate. In the model considered by Angeletos and Panousi (2011), the following condition ensures that in the neighborhood of the autarchic steady state, the wealth effect of a higher interest rate dominates the opportunity-cost effect:<sup>41</sup>

$$\vartheta > \frac{\widehat{\kappa}}{1 - \widehat{\kappa}}, \qquad \widehat{\kappa} \cong \frac{\alpha - \delta \frac{K^*}{Y^*}}{1 - \delta \frac{K^*}{Y^*}}, \qquad (3.61)$$

where  $Y^* = (K^*)^{\alpha}$  in case of a Cobb-Douglas specification.

Comparing the condition in (3.60) with the condition in (3.61) shows that both share a similar structure. The only difference is that in the denominator of the first term in (3.60) there appears a '2', whereas in (3.61) there appears a '1'. Hence, we can use this similarity of the two conditions in order to derive our second rule. Specifically, we simply adjust our first rule in (3.19) to match the observed difference between (3.60) and (3.61). Applying this step leads to our second rule of thumb that is described in (3.20).

 $<sup>^{41}</sup>$ We slightly change the presentation compared to Angeletos and Panousi (2011, p.874) to facilitate the further discussion.

#### 3.8.2 Appendix B. Baseline scenario: The general case

Figure 3.8: Threshold Level of the Elasticity of Intertemporal Substitution: The General Case

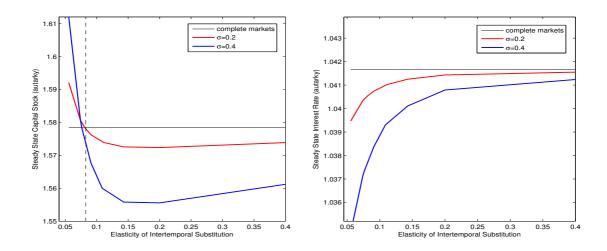


Figure 3.8 shows the general case for the baseline scenario where entrepreneurs in both countries suffer from uninsurable risk. The left panel of Figure 3.8 shows the relationship between the elasticity of intertemporal substitution,  $\vartheta$ , and the corresponding capital stock in the steady state under financial autarky for three economies that only differ in the level of uninsurable risk. The black line refers to the complete markets case, the red line refers to the economy with  $\sigma = 0.2$  and the blue line refers to the benchmark economy with  $\sigma = 0.4$ . Apart from the differences in the level of uninsurable risk, the economies are identical to the benchmark economy.

The dashed vertical line in the left panel of Figure 3.8 shows the threshold level for the elasticity of intertemporal substitution predicted by our first rule of thumb. According to our first rule, the capital stock in the steady state under financial autarky should be lower in the benchmark economy than in the economy with complete markets if  $\vartheta$  exceeds that threshold level. The actual threshold level, in turn, is determined by the intersection point of the blue line and the black complete markets line. As Figure 3.8 shows, our first rule of thumb is able to predict the actual threshold level with high accuracy.

The key insight from the left panel of Figure 3.8 is that the intersection point of the blue line and the red line, which determines the threshold level for the elasticity of intertemporal substitution in the general case where entrepreneurs in both economies suffer from uninsurable risk, is almost identical to the threshold level that is determined by the intersection point of the blue line and the black complete markets line. From this it follows that the predictions of our first rule of thumb that are derived under the assumption that markets are complete in the financially more developed country also carry over to the general case with frictions in both countries. In particular, in all

cases considered, we find that if the elasticity of intertemporal substitution exceeds the threshold level that is predicted by our first rule, the financially less developed country features a lower capital stock and a lower output level in the autarchic steady state compared to any financially more developed country, and not only compared to the ideal case of complete markets.

#### 3.8.3 Appendix C. Computational issues

Steady state, financial autarky

- 1. We start with an initial guess for the wage and for the interest rate.
- 2. We place a grid over the state space of net worth with more grid points being allocated to lower levels of net worth. The productivity process described in (3.18) is approximated by a five-state Markov chain using the method of Rouwenhorst (1995).
- 3. We compute the policy functions for consumption, capital holdings and bond holdings by value function iteration. Between grid points, we use linear interpolation.
- 4. The stationary distribution of households is computed using the policy functions and the transition matrix of the productivity process. Aggregate quantities are calculated by adding up weighted individual demands.
- 5. Within a first loop, the guess for the wage, and within a second loop, the guess for the interest rate are updated. The procedure is repeated until the market clearing conditions (labor and bond market) are satisfied, except for a tolerably small approximation error.

#### Steady state, financial integration

The procedure to compute the long-run equilibrium under financial integration for the small and financially less developed country is similar to the steps outlined above. However, bonds can be traded on the international level given a fixed interest rate that is determined by the large and financially more developed country.

#### Transitional Equilibrium

- 1. In a first step, we solve for the initial (autarchic) and the final (integrated) steady state, following the steps outlined above.
- 2. The number of transition periods, T, is chosen. T is set sufficiently large to ensure that the integrated steady state is, approximately, reached in T periods.

- 3. Based on an initial guess for the time path of the wage rate, the sequence of policy functions is computed by iterating backwards in time, starting from period t = T 1.
- 4. The sequence of distributions is computed by moving forward in time, starting from the initial autarchic steady state. Aggregate quantities are calculated.
- 5. The initial guess for the time path of the wage rate is updated and the procedure is repeated from step 3 until the labor market clearing condition is satisfied at each point in time, except for a tolerably small approximation error.

### Chapter 4

# Incomplete Information, Financial Market Imperfections, and Aggregate Saving<sup>1</sup>

#### Abstract

This paper examines the effects of incomplete information about idiosyncratic shocks in a heterogeneous-agents, incomplete-markets model. Agents in the economy are subject to permanent and transitory productivity shocks but cannot distinguish the two components due to a limited information set. We find that incomplete information does not only have a strong quantitative effect on aggregate saving, but also a qualitatively ambiguous effect that varies with the interest rate. At low interest rates, aggregate asset holdings of the population are larger under complete information, whereas at higher interest rates, aggregate asset holdings are larger under incomplete information. We show that the ambiguous behavior of aggregate saving can be traced back to the interplay of incomplete information and the financial market imperfections. The no-borrowing constraint breaks the symmetry in the agent's saving behavior based on a good or bad shock, whereas the income risk leads to a higher precautionary saving demand under incomplete information. In general equilibrium, we find that the aggregate capital stock is higher under incomplete information under plausible parametrizations.

<sup>&</sup>lt;sup>1</sup>I especially thank Maik Heinemann for his inspiration, valuable comments and suggestions. I also thank Michael Burda, Ulrich Eydam, Janine Hart and Lutz Weinke for their helpful comments and advice.

#### 4.1 Introduction

This paper investigates the importance of incomplete information about idiosyncratic shocks in a heterogeneous-agents, incomplete-markets model.<sup>2</sup> Agents in the economy are subject to two types of productivity shocks but cannot distinguish the two components due to a limited information set. We are especially interested in the question of how incomplete information affects aggregate saving with a special focus lying on the interplay of incomplete information and financial market imperfections. Our analysis takes account of incomplete insurance markets and borrowing constraints and covers a partial as well as a general equilibrium perspective. We find that in this setting, incomplete information does not only have a strong quantitative effect on aggregate saving but also a qualitatively ambiguous effect. Dependent on the interest rate level and, thereby, dependent on the strength of the financial frictions, aggregate savings may be around 25 percent lower but also up to 40 percent higher under incomplete information than in the complete information case. These strong quantitative and qualitative effects broaden the view that incomplete information has a solely monotone effect on aggregate saving.

Our analysis contributes to the large and long-standing literature that casts doubt on the common assumption of complete information, i.e. that agents are able to observe and to distinguish each type of income shock. As already pointed out by Goodfriend (1992) and Pischke (1995), not all information is instantaneously available and collecting information is costly. Therefore, agents may not be able or willing to attribute each change in income to the respective underlying income shock. The literature in this field shows that understanding the agent's information set is not only important for explaining individual behavior, but also takes a big step towards understanding the observed pattern of aggregate variables. Pischke (1995) and Ludvigson and Michaelides (2001) for instance show that the agent's inability to distinguish between individual and aggregate income shocks contributes to explaining the excess smoothness and sensitivity of aggregate consumption. In this paper, we too find a strong impact of incomplete information; however, our focus is more directed towards the interaction between incomplete information and financial market imperfections and the joint impact on aggregate saving.

The model we consider is an Aiyagari-type heterogeneous-agents, incomplete-markets model (see Aiyagari 1994). Agents in the economy are subject to idiosyncratic labor productivity shocks, which expose each of them to an uninsurable income risk. Agents can only partially self-insure against the income risk by accumulating a stock of assets that pay out a risk-free return. Furthermore, asset holdings are bounded from below by a no-borrowing constraint. As it is well-known, these assumptions lead agents into precautionary saving. Regarding the properties of the individual productivity process, we follow the work by Kuhn (2013) and assume that agents are subject to two types of

 $<sup>^2</sup>$ See Heathcote et al. (2009) for a comprehensive overview of incomplete-markets models and Ljungqvist and Sargent (2012) for a textbook treatment.

productivity shocks: permanent and transitory shocks.<sup>3</sup> This specification is in line with several studies confirming that earnings risk exhibits both a permanent and a transitory component (e.g., MaCurdy 1982; Meghir and Pistaferri 2004; Blundell et al. 2008). Under the common assumption of complete information the agent can distinguish between permanent and transitory income shocks, whereas under incomplete information the agent cannot distinguish the two components. The limited information set reflects the fact that collecting information is costly and that not all information is instantaneously available. Given that the agent cannot distinguish between permanent and transitory shocks under incomplete information, the agent has to use a simpler, single-shock model to predict future productivity growth. Technically, and in line with Pischke (1995) and Ludvigson and Michaelides (2001), the process of productivity growth under incomplete information looks just like an MA(1) process to the agent and the agent uses the MA(1)representation to predict future developments. However, what is even more important to note is that due to the limited information set under incomplete information, the agent underestimates the effect of a permanent shock and overestimates the effect of a transitory shock. This property crucially drives the differences in saving between the complete and the incomplete information case.

To fully assess the effects of incomplete information, we compare the aggregate saving behavior of the population under complete and incomplete information from both, a partial and a general equilibrium perspective. In general equilibrium, the labor market clears and the aggregate supply of capital equals the aggregate demand for capital. In partial equilibrium, the interest rate and the wage rate are treated as free parameters. We find that the effect of incomplete information on aggregate saving is not only quantitatively substantial, but also qualitatively ambiguous and varies with level of the interest rate. At low interest rates, aggregate asset holdings are larger under complete information, whereas at higher interest rates, closer to the general equilibrium outcome, aggregate asset holdings are larger under incomplete information. An immediate implication of this result is that the choice of the interest rate level in a partial equilibrium application or, similarly, the choice between the partial and the general equilibrium framework becomes a critical factor: Depending on whether the interest rate is rather high or low, aggregate savings may be considerably under- but also considerably overestimated under the common assumption of complete information.

To better understand the ambiguous effects, we then compare aggregate asset holdings under complete and incomplete information at different levels of cash-on-hand.<sup>4</sup> The level of cash-on-hand is of particular interest because it indicates how strongly the agents are affected by the financial market imperfections. If the agents are sufficiently rich, they are less concerned about the no-borrowing constraint and the missing

<sup>&</sup>lt;sup>3</sup>Permanent income shocks also play a crucial role in, e.g., Deaton (1991), Constantinides and Duffie (1996), Carroll (1997, 2009, 2011), and Szeidl (2013). Kuhn (2013), however, proves the existence of a recursive competitive equilibrium in an Aiyagari-type economy with permanent and transitory income shocks.

<sup>&</sup>lt;sup>4</sup>Cash-on-hand is defined as the sum of the current wage income and current financial wealth.

insurance markets since their future income is mainly derived from riskless asset holdings. With decreasing levels of wealth, however, the influence of the no-borrowing constraint and the labor income risk increases. By focusing on variables normalized by labor productivity, we show that the observation of larger aggregate asset holdings under complete information can be explained by the interplay of incomplete information and the no-borrowing constraint. The no-borrowing constraint breaks the symmetry in the agent's saving behavior based on a good or bad shock, and agents save more based on a good shock under complete information. Furthermore, we find that the observation of larger aggregate asset holdings under incomplete information can be explained by the interplay of incomplete information and the income risk. We isolate the income-risk channel and show that agents have a higher precautionary saving demand under incomplete information. Finally, we find that if cash-on-hand becomes extremely high, no systematic difference exists in aggregate saving between the complete and the incomplete information case. This result emphasizes that the interplay of incomplete information and financial market imperfections, rather than the assumption of incomplete information alone, drives the observed differences between the complete and the incomplete information case.

Finally, we test the robustness of the results with respect to different parametrizations. We find that the ambiguous behavior of aggregate saving is preserved under different assumptions concerning the ratio of the permanent and the transitory shock, and the parameter of relative risk aversion. Furthermore, in all cases considered, we find that in general equilibrium, the aggregate capital stock remains to be higher under incomplete information. In this respect, incomplete information may contribute to improve the outcome of standard incomplete-markets models. As pointed out by Aiyagari (1994), the increase in aggregate saving that can be generated in a standard incomplete-markets model with complete information is rather moderate for plausible parametrizations.<sup>5</sup> In our model, however, we find that due to the higher precautionary saving demand, aggregate asset holdings are larger under incomplete information than under complete information in general equilibrium and at interest rates in the close neighborhood. Therefore, incomplete information may explain a high saving rate without requiring a particularly high variance of the structural shocks, a result which is broadly in line with Wang (2004) and Luo et al. (2015).

As mentioned above, there is a large and still growing literature that deals with the implications of informational frictions. The assumption that agents cannot distinguish income shocks by type has been applied in several studies to explain the empirical pattern of aggregate consumption. Pischke (1995) considers the idea in a model in which agents behave in accordance with the permanent income hypothesis. Assuming that agents cannot distinguish between individual and aggregate income shocks, Pischke (1995) finds that incomplete information contributes to explain the observed smoothness

<sup>&</sup>lt;sup>5</sup>Guvenen and Smith (2014) further show that the amount of risk commonly assumed in calibrated macroeconomic models with incomplete markets is too large compared to the data.

and volatility of aggregate consumption.<sup>6</sup> Ludvigson and Michaelides (2001) study the informational setup of Pischke (1995) in a model with buffer-stock behavior. They find that only the incomplete-information version of the buffer-stock model is capable of simultaneously producing some smoothing of aggregate consumption and a robust correlation between consumption growth and lagged income growth. In this paper, we consider a similar approach of modeling incomplete information as Pischke (1995) and Ludvigson and Michaelides (2001). However, we focus on the interplay of incomplete information and financial market imperfections and the joint influence on aggregate saving.

Our paper also relates to the work by Wang (2004) and Luo et al. (2015). Wang (2004) considers a model in which the agent's exogenous labor income is subject to shocks with different degrees of persistence and volatility. The agent does not observe the different shocks directly and the Kalman filter is used to estimate the individual components. By assuming away binding borrowing constraints and due to constant absolute risk averse utility, Wang (2004) shows analytically that the agent's demand for saving is higher when the agent takes the effects of estimation risk into account. Luo et al. (2015) extend the analysis of Wang (2004) by introducing robustness and by studying the general equilibrium effects in a Huggett-type economy (see, Huggett 1993). Our approach differs from Wang (2004) and Luo et al. (2015) in that we assume constant relative risk averse utility, include borrowing constraints, and consider an Aiyagari-type economy with production. While aggregate savings may be larger under incomplete information in our model as well, we also show that the results may be reversed due to the presence of the no-borrowing constraint.

Finally, there are also a number of papers that focus on advance information in contrast to the assumption of incomplete information (e.g., Kaplan and Violante 2010; Guvenen and Smith 2014; Singh and Stoltenberg 2017). Singh and Stoltenberg (2017), for instance, consider a model with limited commitment and study the influence of informative signals on consumption risk sharing. In this paper, however, we follow the opposite route and focus on the effects of incomplete information on aggregate saving within the standard incomplete-markets model framework.

The remainder of this paper is organized as follows. Section 4.2 introduces the model. Section 4.3 describes the benchmark parametrization. Section 4.4 shows the effects of incomplete information from a partial and a general equilibrium perspective and provides some robustness considerations. Section 4.5 concludes.

<sup>&</sup>lt;sup>6</sup>See also Hryshko (2014) for a related analysis with negatively correlated permanent and transitory income shocks and Demery and Duck (2000) for an empirical investigation.

#### 4.2 The model

We consider an incomplete-markets model with aggregate production and incomplete information about idiosyncratic income shocks. The model structure can be outlined as follows. Time is discrete and indexed by  $t \in [0, 1, ..., \infty]$ . The economy is populated by a continuum [0,1] of agents, indexed by i. Agents have preferences over consumption and maximize discounted expected lifetime utility. Agents can save using an asset that pays out a risk-free return, but no further insurance against income risk is provided. Furthermore, following the standard approach in the literature, we assume that asset holdings are bounded from below (e.g., Huggett 1993; Aiyagari 1994). We impose the no-borrowing constraint  $a_{it+1} \geq 0$ , where  $a_{it+1}$  denotes individual asset holdings.

In addition to receiving income from savings, agents also receive labor income by supplying one unit of labor inelastically to the competitive labor market. Agents, however, differ with respect to the realization of idiosyncratic shocks to the labor productivity which expose each of them to an uninsurable income risk. Following Kuhn (2013), we assume that agents are subject to both permanent and transitory shocks. This specification is in line with several studies showing that earnings risk exhibits both a permanent and a transitory component (e.g., Meghir and Pistaferri 2004; Blundell et al. 2008). The key feature of the model is that agents under complete information can distinguish between permanent and transitory shocks, whereas under incomplete information agents cannot distinguish the two components. In the latter case, changes in log labor productivity look just like an MA(1) process to the agent and the agent uses the MA(1) representation to predict future developments. We will explain the differences in the information sets in more detail below.

As a technical aside, note that the presence of permanent income shocks generally means that the cross-sectional variance increases over time. To keep the distribution of the economy nevertheless stationary, we follow Constantinides and Duffie (1996) and Kuhn (2013) and assume that every agent faces a positive probability of death in every period. An agent who dies is then replaced by a newborn agent whose initial labor productivity is normalized to unity. This random reset event prevents labor productivity from growing without bounds in the economy. Also note that the assumption of a positive probability of death requires the redistribution of the financial wealth of agents who die. Following Constantinides and Duffie (1996), we introduce a simple bequest scheme and allocate current resources to the next generation of newborn agents. Specifically, we assume that a newborn agent receives the current average asset holdings of the last generation. This simple bequest scheme facilitates the partial equilibrium analysis that accompanies our discussion of the general equilibrium.

<sup>&</sup>lt;sup>7</sup>Following Kuhn (2013), an intuitive interpretation is an economy with different labor market cohorts. Every period, new workers enter the labor market while some current workers randomly drop out. Hence, death should not necessarily be associated with physical death, which provides us with some flexibility regarding the parametrization of the model.

<sup>&</sup>lt;sup>8</sup>Note that for our considered parametrizations, we do not observe larger problems arising from the fact that the distribution from which the endowment of the newborn agent is drawn is not exogenously

#### The individual problem

Agents in the economy have preferences over consumption and maximize discounted expected lifetime utility. Formally, the objective function of each agent i is given by

$$E_0 \left[ \sum_{t=0}^{\infty} \tilde{\beta}^t (1-\chi)^t U(c_{it}) \right], \tag{4.1}$$

where  $c_{it}$  denotes consumption,  $E_0$  is the expectation operator conditional on information at date  $t = 0, 0 < \tilde{\beta} < 1$  is the discount factor, and  $\chi$  is the constant probability of death. Following Kuhn (2013), we assume that at the end of period t, every agent draws a survival shock,  $\phi_{it+1}$ , from a binomial distribution. A draw of  $\phi_{it+1} = 1$  is associated with survival from period t to period t+1.

The agent's preferences regarding momentary consumption are standard and display constant relative risk aversion

$$U(c) = \begin{cases} \frac{c^{1-\rho}}{1-\rho} & \rho > 0, \ \rho \neq 1\\ \ln(c) & \rho = 1. \end{cases}$$
(4.2)

The agent's budget constraint is given by

$$c_{it} + a_{it+1} = R \, a_{it} + w \, m_{it}, \tag{4.3}$$

where  $a_{it}$  denotes asset holdings at the beginning of period t,  $m_{it}$  denotes labor productivity,  $R \equiv (1+r)$  is the gross interest rate, and w is the wage rate. Since no risk exists on the aggregate level, the wage and the interest rate are constant in the stationary equilibrium.

As outlined above, agents experience permanent and transitory shocks to the labor productivity. The process of labor productivity,  $m_{it}$ , can thus be written as the product of a permanent component, modeled as a random walk in logs, and a transitory component, modeled as a white noise process. That means, permanent shocks lead to a permanent change in labor productivity, whereas transitory shocks are assumed to have a pure one-period effect. Formally, labor productivity is given by

$$m_{it+1} = \gamma_{it+1}\tau_{it+1} \tag{4.4}$$

$$\gamma_{it+1} = \gamma_{it} \exp(\eta_{it+1}) \qquad \eta \stackrel{iid}{\sim} N(0, \sigma_{\eta}^2)$$

$$\tau_{it+1} = \exp(\theta_{it+1}) \qquad \theta \stackrel{iid}{\sim} N(0, \sigma_{\theta}^2),$$

$$(4.5)$$

$$\tau_{it+1} = \exp(\theta_{it+1}) \qquad \theta \stackrel{na}{\sim} N(0, \sigma_{\theta}^2),$$
 (4.6)

where  $\eta$  denotes the permanent and  $\theta$  denotes the transitory shock.

fixed under the proposed bequest scheme. See Kuhn (2013) for a more detailed discussion.

To simplify the above expressions, we can combine the three equations to obtain the stationary process of individual labor productivity growth

$$\Delta \ln m_{it+1} = \eta_{it+1} + \theta_{it+1} - \theta_{it}.$$
 (4.7)

Note that the process shown in (4.7) highlights the differences between the two types of idiosyncratic shocks. While the transitory shock only leads to a one-period change in labor productivity, the effect of the permanent shock lasts over the subsequent periods.

Based on the process in (4.7), we can now discuss the differences between the complete and the incomplete information case. Under complete information, the agent knows the structure of the process in (4.7) and can distinguish between permanent and transitory shocks. That means, the agent can use the actual income process to predict future developments. This is the common assumption in the incomplete-markets literature. Now, consider again the process in (4.7), but assume that the agent only observes changes in labor productivity but cannot distinguish between the individual components. The limited information set reflects the fact that not all information is instantaneously available and that collecting information is costly (cf. Goodfriend 1992; Pischke 1995). Given that the agent cannot distinguish between permanent and transitory shocks under incomplete information, the process of productivity growth to her looks just like a moving-average process of order one. Intuitively, this follows from the fact that the agent under incomplete information has to use a simpler, single-shock model to predict future productivity growth. To see this more formally, note that due to stationarity, the process of productivity growth also has a single Wold representation of the form

$$\Delta \ln m_{it+1} = \psi(L)u_{it+1},\tag{4.8}$$

where  $\psi(L)$  is the lag polynomial. Since the autocovariance function of  $\Delta$  ln  $m_{it+1}$  truncates after the first lag as implied by (4.7), the lag polynomial reduces to  $\psi(L) = 1 + \psi L$ , which defines the MA(1) process. Of course, the shocks under incomplete information are actually driven by the innovations to the permanent and the transitory component, but since the agent cannot distinguish them under incomplete information, the best the agent can do is to recognize that the unobserved components model in (4.7) has an MA(1) representation. Thus, the observable process under incomplete information can be written as

$$\Delta \ln m_{it+1} = u_{it+1} + \psi u_{it},$$
 (4.9)

where  $\psi$  is the MA parameter and  $\sigma_u^2$  denotes the variance of the compound shock u. The values of  $\psi$  and  $\sigma_u^2$  follow from the properties of the process in (4.7) and are

<sup>&</sup>lt;sup>9</sup>Our approach of modeling incomplete information, in the spirit of Muth (1960), closely relates to Pischke (1995) and Ludvigson and Michaelides (2001), with the difference being that we focus on purely idiosyncratic income shocks. See also Deaton (1991) for a related discussion.

obtained from matching the autocovariances between (4.7) and (4.9). Defining the ratio  $q \equiv \sigma_\eta^2/\sigma_\theta^2$ , we get  $\sigma_u^2 = -\sigma_\theta^2/\psi$  and  $\psi = (-(2+q) + \sqrt{q^2+4q})/2$  as the invertible

The MA(1) process in (4.9) indicates the differences in the agent's expectations regarding future developments compared to the complete information case. Since -1  $< \psi < 0$ for plausible values of q, the agent assumes that every shock in period t has a persistent but not a completely permanent effect. This is an intuitive result; since the agent cannot distinguish between permanent and transitory shocks under incomplete information, the agent simply assumes that every shock lies in between a pure permanent and a pure transitory shock. The expected persistence is determined by the value of  $\psi$ , which in turn depends on the ratio q. That means, the agent under incomplete information underestimates the effect of a purely permanent shock and overestimates the effect of a purely transitory shock. This observation will be important in explaining the differences in aggregate saving between the complete and the incomplete information case.

In summary, the agent under complete information solves the following decision problem

$$\max_{\{c_{it}, a_{it+1}\}} E_0 \left[ \sum_{t=0}^{\infty} \tilde{\beta}^t (1 - \chi)^t U(c_{it}) \right]$$

$$s.t. c_{it} + a_{it+1} = Ra_{it} + wm_{it}$$
(4.10)

$$s.t. \quad c_{it} + a_{it+1} = Ra_{it} + wm_{it} \tag{4.11}$$

$$a_{it+1} \ge 0 \tag{4.12}$$

$$m_{it+1} = m_{it} \exp(\eta_{it+1} + \theta_{it+1} - \theta_{it})$$
(4.13)

$$a_{i0}, m_{i0}, \theta_{i0}$$
 given. (4.14)

Under incomplete information, the agent's decision problem is given by

$$\max_{\{c_{it}, a_{it+1}\}} E_0 \left[ \sum_{t=0}^{\infty} \tilde{\beta}^t (1 - \chi)^t U(c_{it}) \right]$$
 (4.15)

$$s.t. \quad c_{it} + a_{it+1} = Ra_{it} + wm_{it} \tag{4.16}$$

$$a_{it+1} \ge 0 \tag{4.17}$$

$$m_{it+1} = m_{it} \exp(u_{it+1} + \psi u_{it})$$
 (4.18)

$$a_{i0}, m_{i0}, u_{i0}$$
 given.  $(4.19)$ 

Comparing the two programs shows that the only difference lies in the specification of the labor productivity process. While the agent under complete information derives her optimal policy functions for consumption and asset holdings based on the process in (4.13), the agent under incomplete information derives her optimal policy functions based on the process in (4.18).

#### Production

Production in the economy takes place under perfect competition. The representative firm produces the single final good according to a standard Cobb-Douglas production function

$$Y_t = K_t^{\alpha} L_t^{1-\alpha},\tag{4.20}$$

where  $K_t$  denotes the aggregate capital stock and  $L_t$  denotes labor in productivity units, i.e. the inelastic labor supply multiplied by labor productivity and aggregated over all agents. The firm chooses K, L to maximize profits. The first-order conditions are given by

$$r = \alpha \left(\frac{K}{L}\right)^{(\alpha-1)} - \delta \tag{4.21}$$

$$w = (1 - \alpha) \left(\frac{K}{L}\right)^{\alpha}, \tag{4.22}$$

where  $\delta$  is the constant depreciation rate. Since the first-order conditions have to hold in equilibrium, we can express the wage rate as a function of the interest rate

$$w = (1 - \alpha) \left(\frac{\alpha}{r + \delta}\right)^{\frac{\alpha}{1 - \alpha}}.$$
 (4.23)

#### General equilibrium

Definition 2 below summarizes the stationary general equilibrium in the economy under complete and under incomplete information. In the partial equilibrium analysis, we treat the wage and the interest rate as free parameters.

**Definition 2** A stationary competitive general equilibrium under complete information is defined as:

- 1. Given equilibrium prices  $w^*$  and  $r^*$ , the policy functions for consumption and asset holdings solve the agent's optimization problem in (4.10)-(4.14).
- 2. Given equilibrium prices  $w^*$  and  $r^*$ , the firm's demand for capital and labor solve (4.21)-(4.22).
- 3. Markets clear at equilibrium prices w\* and r\*. The firm's demand for capital equals the aggregate capital supply of agents and the firm's demand for labor equals the aggregate effective labor supply of agents.
- 4. The stationary distribution of agents over individual asset holdings, labor productivity and the current transitory shock is the fixed point of the law of motion which is consistent with the agent's policy functions, equilibrium prices and the endowment scheme for newborn agents.

A stationary competitive general equilibrium under incomplete information is similarly defined. However, the agent's policy functions for consumption and asset holdings solve the optimization problem in (4.15)-(4.19).

#### 4.3 Parametrization and solution

As it is well-known from incomplete-markets models, the properties of the model can be evaluated only numerically. For our benchmark economy, we mainly choose standard parameter values which are commonly considered in the related incomplete-markets literature. In Section 4.4.3, we then provide some robustness considerations.

#### **Production**

Production in the economy takes place according to a standard Cobb-Douglas production function with capital share equal to  $\alpha$ . In line with the empirical evidence, we set  $\alpha$  to 0.36. The depreciation rate,  $\delta$ , is set to a standard value of 10 percent.

#### Income process and Probability of Death

The two parameters of the underlying productivity process are the standard deviation of the permanent and the standard deviation of the transitory shock. These parameters also define the properties of the MA(1) model. In line with Carroll (1997) and Kaplan and Violante (2010), we set the standard deviation of the permanent shock,  $\sigma_{\eta}$ , to 0.1. The standard deviation of the transitory shock,  $\sigma_{\theta}$ , however, is more controversial. Kaplan and Violante (2010), for instance, suggest a value around 0.22, whereas Ludvigson and Michaelides (2001) consider a value of 0.1. For our benchmark economy we choose a value in between those numbers and set  $\sigma_{\theta}$  to 0.14. In Section 4.4.3 we also consider the case with  $\sigma_{\theta} = 0.1$ . Note that the productivity process, combined with the positive probability of death, leads to some interesting implications and restrictions on the model parameters. Recall that an agent who dies is replaced by a newborn agent at the same index with initial labor productivity normalized to unity. Thus, the combined law of motion for productivity can be written as

$$m_{it+1} = \begin{cases} a_{it+1}m_{it} & with \ prob. \ 1 - \chi \\ 1 & with \ prob. \ \chi, \end{cases}$$

$$(4.24)$$

where  $a_{it+1} \equiv \exp(\eta_{it+1} + \theta_{it+1} - \theta_{it})$ . The process shown in (4.24) is generally known as stochastic multiplicative process with reset events (e.g., Manrubia and Zanette 1999; Nirei and Aoki 2016). According to Manrubia and Zanette (1999), this process exhibits a stationary power-law probability distribution and drawing from Nirei and Aoki (2016), the corresponding Pareto exponent,  $\lambda$ , is determined by

$$(1 - \chi)E(a_{it}^{\lambda}) = 1. \tag{4.25}$$

The existence of a power-law distribution nicely relates the assumption of permanent income shocks to the current debate on the income and wealth distribution which, from a theoretical perspective at least, is mainly based on models featuring uninsurable capital risk (see, Nirei and Aoki 2016). Although a detailed discussion of similarities

and differences between our model and models featuring capital risk is beyond the scope of this paper, we have to account for the implied parameter restrictions. Solving analytically for  $\lambda$  and imposing the restriction  $\lambda > 1$  to ensure that the mean remains finite leads to the condition

$$\sqrt{\frac{-2\ln(1-\chi)}{\sigma_{\eta}^2 + 2\sigma_{\theta}^2}} > 1. \tag{4.26}$$

Given our choices of  $\sigma_{\eta}$  and  $\sigma_{\theta}$  mentioned above, we set the probability of death,  $\chi$ , to 3.5 percent, which satisfies the condition in (4.26). The value of  $\chi$  is slightly larger but still comparable to the 2.9 percent considered by Kuhn (2013).

#### **Preferences**

The objective function of the agent depends on three parameters, the parameter of relative risk aversion,  $\rho$ , the discount factor,  $\tilde{\beta}$ , and the probability of death,  $\chi$ . Given our choice for  $\chi$  mentioned above, we set  $\tilde{\beta}$  to 0.9948, so that the effective discount factor,  $\beta \equiv \tilde{\beta}(1-\chi)$ , is equal to 0.96. Finally, and in line with Aiyagari (1994) and Krusell and Smith (1997), we set the parameter of relative risk aversion equal to unity (log-utility). In Section 4.4.3 we also consider the case with  $\rho = 2$ . Table 4.1 below summarizes the benchmark parameter values.

Table 4.1: Benchmark Parameter Values

Parameter		Value
curvature of production	$\alpha$	0.36
depreciation rate	$\delta$	0.1
discount factor	$ ilde{eta}$	0.9948
parameter of relative risk aversion	ho	1
standard deviation permanent shock	$\sigma_{\eta}$	0.1
standard deviation transitory shock	$\sigma_{ heta}$	0.14
probability of death	χ	0.035

#### 4.3.1 Computational methods

The goal of the next section is to analyze the aggregate saving behavior under complete and incomplete information. Since we want to compare a number of stationary solutions (partial and general equilibrium), it is useful to firstly reduce the computational burden by slightly reformulating the agent's optimization problem. In a first step, we introduce

the variable cash-on-hand,  $x_{it}$ , which summarizes the agent's total resources, i.e. we define cash-on-hand as  $x_{it} \equiv Ra_{it} + wm_{it}$ . The agent's budget constraint then simplifies to

$$c_{it} + a_{it+1} = x_{it}. (4.27)$$

In a second step and similar to King et al. (1988), Deaton (1991), and Kuhn (2008), we normalize variables by labor productivity. Normalized cash-on-hand, for instance, is defined as  $\tilde{x}_{it} \equiv x_{it}/m_{it}$ , where the on top of the variables indicates the normalization. Given this transformation, we then compute the agent's optimal policy functions by iterating on the first-order conditions associated with the agent's optimization problem. Specifically, the policy function for normalized consumption under complete information,  $\tilde{c}(\tilde{x}, \theta)$ , is computed by finding the fixed point to the following equation 11

$$\widetilde{c}(\widetilde{x},\theta) = \min \left\{ \widetilde{x}, \left( \widetilde{\beta}(1-\chi)RE_t \left[ \left( \exp(\eta' + \theta' - \theta) \right)^{-\rho} \ \widetilde{c}(\widetilde{x}',\theta')^{-\rho} \right] \right)^{-\frac{1}{\rho}} \right\}. \tag{4.28}$$

Equation (4.28) shows that consumption is equal to cash-on-hand if the no-borrowing constraint becomes binding and is otherwise determined by the standard, albeit normalized, Euler equation. Note that in both cases, normalized cash-on-hand evolves according to the following law of motion

$$\widetilde{x}' = \frac{R}{\exp(\eta' + \theta' - \theta)} (\widetilde{x} - \widetilde{c}) + w. \tag{4.29}$$

Under incomplete information, we apply a similar transformation, taking into account the difference in the observed productivity process. The policy function for normalized consumption is a function of normalized cash-on-hand,  $\tilde{x}$ , and the compound shock, u, which reflects the differences in the information sets, i.e.  $\tilde{c}(\tilde{x},u)$ . With the policy functions at hand, we then simulate the economy under complete and incomplete information to obtain the long-run stationary aggregate asset holdings. For this, we follow the simulations steps described by Aiyagari (1994) and Kuhn (2008). First, we use the policy functions under complete and incomplete information to simulate the individual consumption and saving behavior for a large number of households and for a large number of periods. Second, we discard the first periods and average over several observations of the cross-sectional asset distribution to obtain the long-run aggregate asset holdings. We find that in all simulations, a total number of 15000 periods and of 100000 households leads to robust results.

 $<sup>^{10}</sup>$ See Deaton (1991) and Kuhn (2013) for an extensive discussion of the approach of iterating on the Euler equation.

<sup>&</sup>lt;sup>11</sup>We omit subscript t for all variables in the current period and we let the prime denote the value of variables one period ahead.

#### 4.4 Results

This section shows the main results and is divided into three parts. First, we compare the aggregate saving behavior under complete and incomplete information from a partial and a general equilibrium perspective. Second, we untangle the underlying effects by exploiting the observed variation in the level of cash-on-hand. Finally, we discuss the robustness of our main results with respect to different parametrizations.

#### 4.4.1 Aggregate saving under complete and incomplete information

Figure 4.1: Aggregate Saving and Capital Demand

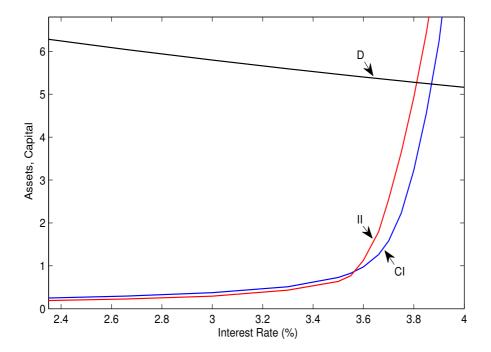


Figure 4.1 shows the aggregate saving behavior under complete and incomplete information for our benchmark economy. The blue line, labeled CI (Complete Information), shows the stationary aggregate asset holdings of the population under the common assumption of complete information and the red line, labeled II (Incomplete Information), shows the stationary aggregate asset holdings of the population under the assumption of incomplete information. We obtain the two saving functions by computing the long-run aggregate asset holdings at different interest rates and corresponding wage rates. The black line, labeled D, shows the firm's demand for capital. The capital demand curve can be derived analytically from the firm's profit maximization problem, taking into account that the stationary aggregate labor supply in the economy is equal to 1.18. The intersection point of the CI and the D curve determines the stationary general equilibrium under complete information and the intersection point of the II and the D curve determines the stationary general equilibrium under incomplete information. In

general equilibrium, the labor market clears and the aggregate supply of capital equals the firm's demand for capital. If the interest rate does not lead to market clearing, we interpret the results in terms of a stationary partial equilibrium with an exogenously fixed interest rate and an exogenously determined income process. The partial equilibrium framework is commonly considered in the consumption literature (e.g., Deaton 1991; Ludvigson and Michaelides 2001) and therefore is essential to our analysis.

At first view, Figure 4.1 shows that the aggregate saving behavior under complete information, i.e. the CI curve, is generally in line with what is known from traditional incomplete-markets models.<sup>12</sup> We observe that for our benchmark parametrization, aggregate asset holdings remain at a rather low level if the interest rate stays below 3.5 percent, but significantly increase if the interest rate gets closer to the rate of time preferences, i.e. if  $\beta(1+r)$  gets close to 1. The increase in aggregate saving reflects the well-known result that agents are willing to build up a buffer-stock of saving in order to protect themselves against the underlying income risk.

Turning to the II curve in Figure 4.1 firstly shows that the general shape of the aggregate saving function is preserved under the assumption of incomplete information. Similar to the complete information case, we observe that aggregate asset holdings remain at a low level if the interest rate stays below 3.5 percent, but significantly increase at higher interest rates. However, despite these similarities, Figure 4.1 also shows that there are some profound differences between the complete and the incomplete information case. At low interest rates, we observe that the CI curve lies above the II curve, which means that aggregate asset holdings are larger under complete information, whereas at high interest rates, the II curve lies above the CI curve, which means that aggregate asset holdings are larger under incomplete information. Hence, incomplete information has an effect on aggregate saving and even more, the effect tends to be ambiguous. Quantitatively, the differences between the complete and the incomplete information case are substantial. For instance, if we consider an interest rate of 2.66 percent in our benchmark economy, aggregate asset holdings are around 30 percent higher under complete information than under incomplete information. In contrast, if we consider an interest rate of 3.85 percent, aggregate asset holdings are around 40 percent higher under incomplete information than under complete information. An immediate implication of these high numbers is that the choice of the interest rate level in a partial equilibrium application, or similarly, the choice between the partial and the general equilibrium framework, becomes a critical factor: Depending on whether the interest rate is rather high or low, aggregate savings may be considerably under-but also considerably overestimated under the common assumption of complete information. Note that both, the low and the high interest rate case are of relevance when related to the consumption and saving literature. Ludvigson and Michaelides (2001), for instance, consider an interest rate of 2 percent in their buffer-stock model of saving with

 $<sup>^{12}</sup>$ See Aiyagari (1994) for a discussion of persistent, but transitory shocks and Kuhn (2013) for a discussion of permanent shocks.

indistinguishable aggregate and idiosyncratic income shocks.<sup>13</sup> According to Figure 4.1, this relates to the case where aggregate asset holdings are larger under complete information. In contrast, when focusing on interest rate levels that are closer to the general equilibrium outcome of an economy with aggregate production, Figure 4.1 indicates that it seems more likely to observe that aggregate asset holdings are larger under incomplete information. In sum, the ambiguous effect of incomplete information makes estimating the agents' saving behavior a more complex task.

Finally, having a closer look at the general equilibrium outcome, we observe that the difference in the aggregate capital stock between the complete and the incomplete information case remains quantitatively rather moderate. This result, however, is mainly driven by the flat capital demand curve and much larger differences arise in the close neighborhood. As described above, aggregate savings are around 40 percent higher under incomplete information than under complete information if we fix the interest rate at 3.85 percent, which is a value in between the general equilibrium under complete and incomplete information. In this perspective, incomplete information may contribute to explaining a high saving rate without requiring a particularly high variance of the underlying structural shocks. As pointed out by Aiyagari (1994), the increase in aggregate saving that can be generated in a standard incomplete-markets model with complete information is in contrast rather low. However, before we can elaborate on this result, we firstly have to understand the ambiguous behavior of aggregate saving.

#### 4.4.2 Untangling the effects

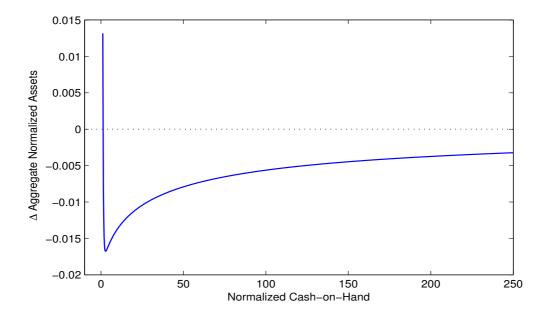
To better understand the effects of incomplete information, we now consider the results presented in Figure 4.1 from a slightly different angle. In particular, while Figure 4.1 compares aggregate asset holdings under complete and incomplete information at different levels of the interest rate, we now compare aggregate asset holdings at different levels of cash-on-hand. The idea underlying this change in perspective is based on the following two observations. First, Figure 4.1 shows that there is a strong co-movement between changes in the interest rate level and in the level of cash-on-hand. This simply follows from the fact that cash-on-hand itself is mainly determined by asset holdings. We can infer from Figure 4.1 that a stationary equilibrium given a low interest rate also means a low average level of cash-on-hand due to low average asset holdings. With increasing interest rates, however, aggregate asset holdings increase and thereby the average level of cash-on-hand. This applies to both, the complete and the incomplete information case. Second, and even more important, the level of cash-on-hand is of particular interest because it shows more clearly how strongly the agents are affected by the financial market imperfections, i.e. by the no-borrowing constraint and the missing

<sup>&</sup>lt;sup>13</sup>Ludvigson and Michaelides (2001) refer to an impatience condition that influences the choice of the interest rate level.

 $<sup>^{14}</sup>$ Recall from Section 4.3.1 that cash-on-hand defines the agent's resources, i.e. financial wealth plus labor income.

insurance markets. When the agents are sufficiently rich, they are less concerned about the no-borrowing constraint and the missing insurance markets since their future income is mainly derived from riskless asset holdings. With decreasing levels of cash-on-hand, however, the influence of the borrowing constraint and the missing insurance markets increases. In order to exploit this variation in the influence of the financial market imperfections, we compute the difference in aggregate asset holdings between the complete and the incomplete information case at different levels of cash-on-hand. For this, we firstly use the agent's optimal policy functions to compute individual decisions at different levels of normalized cash-on-hand and then we aggregate the individual decisions.<sup>15</sup> Note that due to the normalization of the policy functions, the obtained variables are also normalized variables, i.e. normalized by labor productivity. Nevertheless, we observe that the results presented in Figure 4.2 possess the same characteristics as observed in Figure 4.1. We investigate them in the following.

Figure 4.2: Differences in Asset Holdings



Note: Figure 4.2 shows the difference in aggregate normalized asset holdings between the complete and the incomplete information case at different levels of normalized cash-on-hand. A positive number means that aggregate normalized asset holdings are larger under complete information, whereas a negative number means that aggregate normalized asset holdings are larger under incomplete information.

First, Figure 4.2 shows that at very low levels of normalized cash-on-hand aggregate normalized asset holdings are larger under complete information than under incomplete information. This result is in line with Figure 4.1, showing that aggregate asset holdings are larger under complete information if the interest rate and thereby, the average level of cash-on-hand are relatively low. We will show that this result can be explained

 $<sup>^{15}\</sup>mathrm{See}$  the Appendix for details.

by the interplay of incomplete information and the no-borrowing constraint, with the no-borrowing constraint breaking the symmetry in the agent's saving behavior. Second, Figure 4.2 shows that with increasing normalized cash-on-hand, aggregate normalized asset holdings become larger under incomplete information than under complete information. This result is also in line with the behavior of aggregate saving observed in Figure 4.1 and we will show that it can be explained by the interplay of incomplete information and the income risk, leading to a higher precautionary saving demand under incomplete information. Finally, Figure 4.2 also shows that at very high levels of normalized cash-on-hand, the differences in aggregate saving between the complete and the incomplete information case steadily decline. Although we do not observe such high levels of cash-on-hand in Figure 4.1, the third result is quite interesting and is thus considered first.

The result that the differences in aggregate saving at very high levels of cash-on-hand steadily decline is of interest because it highlights the importance of the financial market imperfections for understanding the effects of incomplete information. Recall from above that the main effect of a very high level of cash-on-hand is that agents are less concerned about the presence of the no-borrowing constraint and the missing insurance markets. Hence, what Figure 4.2 shows is that when financial market imperfections do not play a crucial role, there is no systematic difference in aggregate saving between the complete and the incomplete information case. Of course, this result does not mean that agents stop making forecast errors under incomplete information, but it means that when agents become relatively rich and the policy functions become sufficiently linear, the individual errors tend to cancel out. In other words, we can conclude that the differences in aggregate asset holdings that arise at lower levels of cash-on-hand are driven by the interplay of incomplete information and financial market imperfections rather than by the assumption of incomplete information alone.

With this result at hand, we now consider the first-mentioned observation from Figure 4.2 that aggregate normalized asset holdings are larger under complete information than under incomplete information when the level of normalized cash-on-hand is very low. To understand this outcome, we turn to the individual level and directly compare the policy functions for normalized asset holdings under complete and incomplete information at the lowest level of normalized cash-on-hand. Recall that, apart from cash-on-hand, the policy function under complete information also depends on the transitory shock,  $\theta$ , whereas the policy function under incomplete information depends on the compound shock, u. The difference arises from the limited information set under incomplete information. Exploiting this variation and understanding the interplay with the borrowing constraint is the aim of the exercise presented in Figure 4.3.

<sup>&</sup>lt;sup>16</sup>We also considered even higher levels of normalized cash-on-hand than presented in Figure 4.2, confirming the result that the difference in aggregate normalized asset holdings between the complete and the incomplete information case steadily approaches zero.

<sup>&</sup>lt;sup>17</sup>The lowest value of normalized cash-on-hand in Figure 4.2 is equal to 1.095.

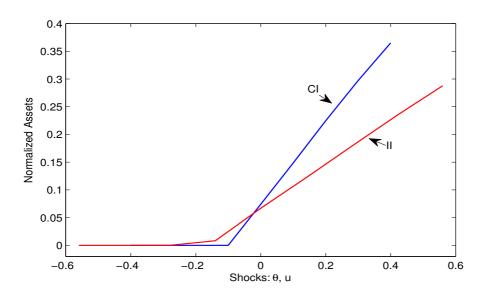


Figure 4.3: Normalized Asset Holdings under the Borrowing Constraint

The blue line in Figure 4.3, labeled CI, shows normalized asset holdings under complete information for different realizations of the transitory shock,  $\theta$ , and the red line, labeled II, shows normalized asset holdings under incomplete information for different realizations of the compound shock, u. Although we cannot think of a particular realization of the transitory shock,  $\theta$ , leading to the same realization of the compound shock, u, we can think of different agents under complete and incomplete information observing a shock of similar size. Keeping that in mind, Figure 4.3 shows that based on a good shock greater than zero, agents save more under complete information than under incomplete information. This reflects the differences in the information sets between the complete and the incomplete information case. Applied to a bad shock less than zero, we would expect that agents save less under complete information than under incomplete information to maintain the symmetry in the saving behavior. Figure 4.3 confirms this line of reasoning, but only for negative shocks that are close to zero. At very bad shocks, however, we observe that the no-borrowing constraint becomes binding. The fact that the no-borrowing constraint becomes binding follows from the fact that we consider the lowest level of normalized cash-on-hand and contributes to explaining why asset holdings are larger under complete information on the aggregate level. Due to the presence of the no-borrowing constraint, agents receiving a bad shock under complete information cannot borrow more than agents receiving a bad shock under incomplete information. But since, based on a good shock, agents save more under complete information than under incomplete information, it follows that aggregate asset holdings are larger under complete information than under incomplete information. In simple terms, the no-borrowing constraint breaks the symmetry in the agents' saving behavior based on a good or bad shock. Since the no-borrowing constraint plays a particularly important role at low levels of cash-on-hand, either because it is currently binding or

is expected to bind in the near future, we can conclude that the observation of larger aggregate asset holdings under complete information in Figure 4.1 and in Figure 4.2 can be explained by the described interplay of incomplete information and the no-borrowing constraint.

Finally, we consider the last observation from Figure 4.2 that aggregate normalized asset holdings are larger under incomplete information than under complete information when the level of normalized cash-on-hand is neither very low nor very high. An intuitive explanation for this result already follows from the meaning of cash-on-hand being neither very low nor very high. Not very low means that the no-borrowing constraint does not play a crucial role, whereas not very high means that the agents are still concerned about the fluctuations in labor income. Hence, the dominant channel that remains in this case is the income risk channel, which indicates that agents have a higher precautionary saving demand under incomplete information.<sup>18</sup> A more formal statement for this result can be made by noticing that the variance of the compound shock, u, is greater than the sum of the variances of the transitory shock,  $\theta$ , and the permanent shock,  $\eta$ . This relation holds because the parameter values are determined by matching the unconditional autocovariances between (4.7) and (4.9). For our benchmark parametrization we get that  $\sigma_n^2 + \sigma_\theta^2$  is equal to 0.03, whereas  $\sigma_u^2$  is equal to 0.04. Hence, from the agents' perspective, the variance of the uninsurable income shock is greater under incomplete information, which means that agents accumulate more savings in order to cope with the higher uninsurable income risk.

As a final exercise, we can briefly consider a simpler, two-period model version to highlight the above result. The two-period model has the advantage that we can completely eliminate the influence of the no-borrowing constraint by setting the initial level of normalized cash-on-hand sufficiently high so that the no-borrowing constraint does not bind. Figure 4.4 shows the results for the two-period model.<sup>19</sup>

The blue line, labeled CI, shows aggregate normalized asset holdings under complete information and the red line, labeled II, shows aggregate normalized asset holdings under incomplete information. The black line in Figure 4.4, called the risk-neutral line, shows aggregate normalized asset holdings under the assumption that agents do not respond to the income risk, which means that agents do not engage in precautionary saving. The risk-neutral line is derived from a first-order approximation of the Euler equation.<sup>20</sup> Note that the fact that the risk-neutral line refers to both the complete and the incomplete information case again shows that there is no systematic difference in aggregate saving when financial market imperfections do not play a crucial role.

 $<sup>^{18}</sup>$ See also Wang (2004), who finds that the agent's precautionary saving demand is higher when the agent takes the effects of estimation risk into account.

<sup>&</sup>lt;sup>19</sup>Note that we set  $\sigma_{\eta}$  to 0.14 and  $\sigma_{\theta}$  to 0.196 in the two-period model. The higher standard deviations are only used to highlight the effects, but do not affect the general pattern. In particular, we get the same ratio  $q \equiv \sigma_{\eta}^2/\sigma_{\theta}^2 = 0.51$  as for our benchmark values of  $\sigma_{\eta}$  and  $\sigma_{\theta}$ .

<sup>&</sup>lt;sup>20</sup>Specifically, we derive the risk-neutral line by replacing the expression  $E_tU'(c_{it+1})$  on the right-hand side of the Euler equation with the first-order approximation  $U'(E_t[c_{it+1}])$ .

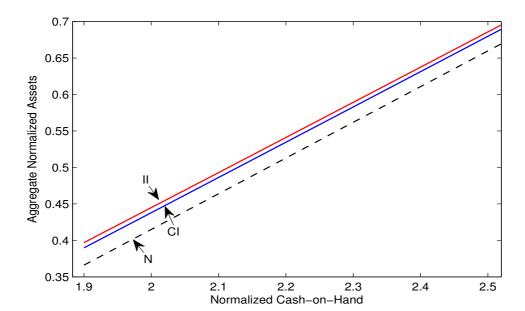


Figure 4.4: Aggregate Normalized Asset Holdings - Two-Period Model

Inspecting the positions of the three lines in Figure 4.4 shows that the blue and the red line lie above the risk-neutral line at all levels of normalized cash-on-hand. This is precisely the precautionary saving effect, meaning that the agents save more when they do care about the income risk.<sup>21</sup> However, what is even more important is that the increase in aggregate saving is greater under incomplete information than under complete information. This follows from the fact that the red line also lies above the blue line at all levels of normalized cash-on-hand. This difference is of particular interest because it confirms the conclusion that the observation of larger aggregate normalized asset holdings under incomplete information can be explained by a higher precautionary saving demand.

In summary, we find that the observed ambiguous behavior of aggregate saving between the complete and the incomplete information case can be traced back to the interplay of incomplete information and the financial market imperfections. The noborrowing constraint breaks the symmetry in the agent's saving behavior and favors the observation of larger aggregate asset holdings under complete information, whereas the interplay of incomplete information and the income risk leads to a higher precautionary saving demand under incomplete information. With these results at hand, we discuss the robustness of our main results and especially of the general equilibrium outcome in the next section.

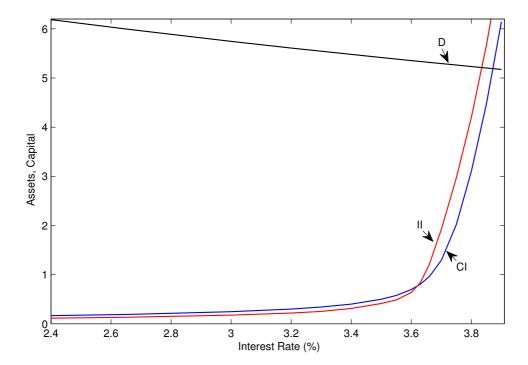
 $<sup>^{21}</sup>$ See Leland (1968) for an extensive discussion of precautionary saving.

#### 4.4.3 Sensitivity analysis

In this section, we return to our main results that are presented in Figure 4.1 and test the model performance with respect to different parametrizations. In particular, we consider a different ratio of the variances of the permanent and the transitory shock and we increase the parameter of relative risk aversion. As the main result, we find that the general pattern remains unaffected; the ambiguous behavior of aggregate saving between the complete and the incomplete information case is preserved and the capital stock remains to be higher in the general equilibrium under incomplete information.

In the first exercise, we consider the case where the ratio  $q \equiv \sigma_{\eta}^2/\sigma_{\theta}^2$  is exactly equal to unity. That means, shocks become equally likely to be either permanent or transitory, which makes them even less informative for the agent under incomplete information. To obtain q = 1, we follow Ludvigson and Michaelides (2001) and reduce the standard deviation of the transitory shock,  $\sigma_{\theta}$ , from 0.14 to 0.1. Figure 4.5 shows the results for q = 1. The blue line, labeled CI, shows aggregate asset holdings under complete information and the red line, labeled II, shows aggregate asset holdings under incomplete information. The black line, labeled D, shows the firm's demand for capital.

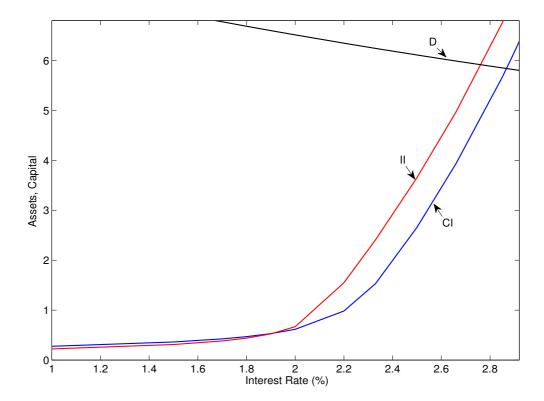
Figure 4.5: Aggregate Saving and Capital Demand: q = 1



At first view, Figure 4.5 shows that the change in the ratio of the variances does not affect the general pattern. Similar to our benchmark economy, we observe that aggregate asset holdings are larger under complete information as long as the interest rate stays below 3.6 percent, but increase more strongly under incomplete information at higher levels of the interest rate. The latter result then also implies that the aggregate

capital stock in general equilibrium is higher under incomplete information. Note that the observed robustness of the general equilibrium outcome is also in line with our previous analysis. As shown in Figure 4.2, aggregate asset holdings are larger under complete information only if cash-on-hand remains at a very low level. This, however, is unlikely to happen in general equilibrium because the supply of capital has to equal the firm's demand for capital. That means, the market clearing condition simply prevents that aggregate savings and thereby, the average level of cash-on-hand, remain at a very low level. Consequently, in all cases considered, we find that the aggregate capital stock is higher under incomplete information in general equilibrium. Finally, Figure 4.5 also shows that the differences in aggregate saving between the complete and the incomplete information case remain quantitatively substantial at various levels of the interest rate. For instance, if we fix the interest rate at 3.85 percent, aggregate asset holdings are around 27 percent higher under incomplete information than under complete information. Furthermore, if we consider an interest rate of 2.66 percent, aggregate asset holdings are around 40 percent higher under complete information than under incomplete information. These numbers are in line with our benchmark economy and again, highlight the strong influence of incomplete information on aggregate saving.

Figure 4.6: Aggregate Saving and Capital Demand:  $\rho = 2$ , q = 1



In the second exercise, we consider the effects associated with a higher value of the parameter of relative risk aversion. Specifically, we increase  $\rho$  from 1 to 2, while keeping the ratio q at the new value of 1. Figure 4.6 shows the results. At first, we observe that the higher value of  $\rho$  leads to a lower interest rate level at which the saving

functions under complete and incomplete information intersect. The interest rate is approximately equal to 1.9 percent, which is a significantly lower value compared to Figure 4.1 or Figure 4.5. This effect is driven by the fact that aggregate savings already increase significantly at lower levels of the interest rate due to the higher value of  $\rho$ . Apart from this change, however, Figure 4.6 also shows that the general pattern remains the same as in all previous exercises. At low interest rates, aggregate asset holdings are larger under information, whereas at higher interest rates aggregate asset holdings are larger under incomplete information. Finally, in the general equilibrium, the aggregate capital stock is higher under incomplete information.

# 4.5 Concluding remarks

In this paper, we have examined the effects of incomplete information about idiosyncratic shocks in a heterogeneous-agents, incomplete-markets model. Agents in the economy are subject to permanent and transitory productivity shocks, but cannot distinguish the two components due to a limited information set. We have considered both, a partial and a general equilibrium perspective, to fully understand how incomplete information affects aggregate saving.

Our results showed that incomplete information has an ambiguous effect on aggregate saving when compared to the common assumption of complete information. Solving for the stationary aggregate asset holdings at different interest rates, we found that incomplete information leads to lower aggregate asset holdings compared to the complete information case when the interest rate is rather low, but leads to larger aggregate asset holdings compared to the complete information case when the interest rate is rather high. In general equilibrium we found that the aggregate capital stock is higher under incomplete information for plausible parametrizations.

To understand the ambiguous results, we then studied the differences in aggregate asset holdings at different levels of cash-on-hand. We found that a central role is played by the interplay of incomplete information and financial market imperfections. While the no-borrowing constraint breaks the symmetry in the agent's saving behavior and favors the accumulation of higher savings under complete information, the interplay of incomplete information and the income risk leads to a higher precautionary saving demand under incomplete information. All in all, our results show that incomplete information plays a complex role within the incomplete-markets model framework. The differences in aggregate saving compared to the complete information case are not only quantitatively substantial, but are also qualitatively ambiguous and vary with the strength of the different types of financial market imperfections. Therefore, we can expect that incomplete information also plays an important role in other applications that follow from our analysis. For example, consider the business cycle implications. Since we have shown that incomplete information has a strong impact on aggregate saving in stationary equilibrium, we can certainly expect that incomplete information also

has a strong impact on aggregate saving over the business cycle where the fraction of borrowing-constrained agents varies over time. Since transitional dynamics are generally more complex, it is important to know that incomplete information leads to ambiguous effects and is more than just another source of risk. Furthermore, we have shown that the effects of incomplete information strongly depend on the level of financial development. Since there is still substantial variation in the level of financial development between countries (e.g., Mendoza et al. 2009a), incomplete information may contribute to explain country-specific differences in consumption and savings and, thereby, maybe even the observed pattern of international capital flows. We leave investigating these research questions to future research.

# 4.6 Appendix

# Figure 4.2: Computations

To compute the difference in aggregate saving between the complete and the incomplete information case at different levels of (normalized) cash-on-hand, we proceed along the following steps. First, we compute the agent's optimal policy function for normalized consumption under complete information.<sup>22</sup> The policy function is a function of normalized cash-on-hand and of the transitory shock. Then, starting at the lowest level of normalized cash-on-hand, we multiply the optimal consumption level associated with a particular realization of the transitory shock with the probability that this shock occurs and sum over all probability-weighted consumption levels.<sup>23</sup> Since the probability of observing a particular shock determines the share of agents actually receiving that shock in a sufficiently large economy, this number gives us information about the aggregate normalized consumption level given that all agents are currently associated with the lowest level of normalized cash-on-hand. We repeat this calculation at different levels of normalized cash-on-hand and repeat the whole procedure for the incomplete information case, taking into account the difference in the observed productivity process. Based on these calculations, we then compute the differences in aggregate normalized asset holdings between the complete and the incomplete information case. The final results appear in Figure 4.2.

 $<sup>^{22}</sup>$ We set the net interest rate at 3.85 percent, which is slightly larger than the interest rate in general equilibrium under incomplete information and slightly lower than the interest rate in general equilibrium under complete information.

<sup>&</sup>lt;sup>23</sup>Since we use a discrete approximation of the shocks in the numerical procedure, we describe our calculations based on discrete realizations of the shocks.

# Chapter 5

# Financing of Government Spending in an Incomplete-Markets Model: The Role of Public Debt<sup>1</sup>

#### Abstract

We use a heterogeneous-agents, incomplete-markets model to analyze the influence of different types of fiscal rules on the response of key macroeconomic variables to a government spending shock. In particular, we distinguish between a debt-intensive fiscal rule that leads to a strong temporary increase in public debt and a non-debt-intensive rule that keeps the primary deficit small. We find that the debt-intensive fiscal rule contributes to stabilizing consumption and leisure in the first periods following the increase in government spending, whereas the non-debt-intensive fiscal rule leads to a faster recovery of consumption, leisure, capital and output in later periods. The observed differences can be traced back to the different responses of wealth-poor agents who are affected most by the financial market imperfections. Regarding optimal debt policy, we find that the debt-intensive fiscal rule leads to the largest aggregate welfare benefit and that the individual welfare gain is particularly high for wealth-poor agents with low productivity. With these findings we contribute to the renewed debate on debt policy and financial market imperfections.

<sup>&</sup>lt;sup>1</sup>This paper was written in collaboration with Maik Heinemann. We especially thank Rainald Borck, Atanas Christev, Ulrich Eydam, Frank Heinemann and William Peterman for valuable comments and suggestions.

# 5.1 Introduction

In the aftermath of the 2008-09 financial crisis, fiscal policy in general and debt policy in particular have regained a lot of interest among the political and the academic profession. The sharp increase of debt-to-GDP ratios observed in the US economy and most of the European countries has led to a renewed debate about the role of public debt and about the question to what extent a high level of public debt can be justified on the grounds of optimal debt policy. Notably and certainly based on the observed turmoil in financial markets, special emphasis in this respect has been placed on the influence of financial market imperfections such as borrowing constraints or the presence of uninsurable idiosyncratic risk. Since the work by Woodford (1990), Aiyagari and McGrattan (1998) and Flodén (2001), it has been known that public debt plays a special role in the presence of borrowing constraints and uninsurable idiosyncratic risk that goes beyond the classical tax-smoothing argument. Given a constant level of government spending, a higher level of public debt effectively relaxes the borrowing constraint, thereby increasing the flexibility of the private sector to respond to variations in income and spending opportunities. Aiyagari and McGrattan (1998) conclude from a calibrated incomplete-markets model that the liquidity effect of public debt justifies a positive and pronounced level of public debt in the long-run equilibrium.

However, a series of recent studies comes to the conclusion that the negative effects associated with a high level of public debt may also play a more dominant role, which drastically changes the view on optimal debt policy. Vogel (2014), Dyrda and Pedroni (2016) and Röhrs and Winter (2016), among others, reconsider the role of public debt within the incomplete-markets model framework and find that the optimal long-run level is negative rather than positive. Röhrs and Winter (2016) emphasize that a stronger influence of the crowding out effect of public debt is crucial for explaining their results.<sup>2</sup> A higher level of public debt increases the interest rate, thereby crowding out private capital and lowering output and consumption. Furthermore, the recent literature also puts larger emphasis on transitory dynamics and shows that, once they are accounted for, optimal debt policy becomes even less clear. Dyrda and Pedroni (2016) find that implementing the policy that maximizes welfare in the long-run equilibrium leads to a welfare loss of 6.4% once transitory effects are accounted for. Similarly, Röhrs and Winter (2017) find that the transitional welfare costs associated with reducing government debt more than offset the long-run benefits. As it stands, the role of public debt and its interplay with financial market imperfections is far from being ultimately resolved.

<sup>&</sup>lt;sup>2</sup>Desbonnet and Kankanamge (2016) show that different assumptions on long-term growth may also affect the sign of the optimal level of debt. Peterman and Sager (2017) emphasize the influence of introducing a life cycle.

<sup>&</sup>lt;sup>3</sup>See also Desbonnet and Weitzenblum (2012) for a discussion of transitory dynamics given a positive optimal long-run level of public debt.

In this paper, we contribute to the ongoing debate on public debt by studying its role in a heterogeneous-agents, incomplete-markets model. Similar to Dyrda and Pedroni (2016) and Röhrs and Winter (2017), we emphasize the importance of transitory dynamics and the associated welfare effects. However, we deviate from the main line of the aforementioned literature that mainly assumes a constant level of government spending when deriving optimal debt policy. In this paper, we allow for changes in government spending and focus on the optimal time paths of public debt and taxes following a government spending shock.<sup>4</sup> Thereby, we take account of the fact that fiscal policy in many countries is not only characterized by periods of constant government spending, but also by episodes of substantial expansion. Mountford and Uhlig (2009), for example, identify a series of substantial positive government spending shocks in the US economy.<sup>5</sup> Changes in government spending and thereby the question of financing have become an even more important issue after the recent financial crisis when many countries have launched a substantial fiscal stimulus package. Thus, we are especially interested in the questions of how debt policy in the presence of financial market imperfections affects the response of the economy to a government spending shock, whether a temporary strong increase in public debt can be justified on the grounds of optimal debt policy and how the individual welfare gains and losses are distributed among the population.

We follow the work by Aiyagari and McGrattan (1998) and Flodén (2001) and consider a heterogeneous-agents, incomplete-markets model with a government sector. The government raises taxes and issues debt securities to finance the expenditures on government consumption. The economy is populated by a continuum of agents who choose an optimal consumption, saving and working plan subject to idiosyncratic labor income shocks and a borrowing constraint. Agents are ex-ante identical but differ in financial wealth over time due to the incomplete insurance markets. The model is calibrated to capture key characteristics of the US economy following the work by Trabandt and Uhlig (2011), Dyrda and Pedroni (2016) and Röhrs and Winter (2017). Starting from the steady state equilibrium of the economy, we consider the following fiscal policy scenario. The government announces a temporary increase in government spending, which eventually requires a corresponding increase in lump-sum taxation. However, following a simple fiscal rule, the government is allowed to temporarily increase the level of public debt. The only restriction we impose is that public debt follows a stable difference equation in each period and finally returns to the initial steady state level.

To assess the impact of debt policy, we compare the response of the economy to the government spending shock under different parametrizations of the fiscal rule. The fiscal rule is called debt-intensive if public debt strongly increases in the first periods following the increase in government spending. The fiscal rule is called non-debt-intensive if the primary deficit remains rather small in all periods. Note that in the absence of financial

<sup>&</sup>lt;sup>4</sup>See the end of this section for further literature.

<sup>&</sup>lt;sup>5</sup>Alesina and Ardagna (2010) consider a broader set of OECD countries and point to more than 90 periods of fiscal stimuli.

market imperfections, i.e. in the complete-markets case, the economy's response to the government spending shock does not depend on the underlying fiscal rule. However, in the presence of borrowing constraints and uninsurable idiosyncratic risk, we observe significant differences in the responses of consumption, leisure, capital and output between the two fiscal rules. On impact and in the first periods of transition, the drop in consumption and leisure is more moderate under the debt-intensive fiscal rule than under the non-debt-intensive fiscal rule. As time passes, however, the pattern changes. The recovery of consumption, leisure, capital and output is slower under the debt-intensive fiscal rule than under the non-debt-intensive fiscal rule. Hence, the possible benefits of raising the level of public debt in the first periods come along with lower consumption and leisure in later periods.

To understand the observed impact of debt policy, we have to consider the interplay of public debt and financial market imperfections. In the presence of borrowing constraints and uninsurable idiosyncratic risk, Ricardian equivalence<sup>6</sup> fails to hold and agents with low financial wealth respond differently to a change in either the lump-sum tax or in public debt. Compared to a tax increase that leads to an unavoidable negative wealth effect, an increase in public debt allows borrowing-constrained agents to mitigate the negative effect on consumption. This explains the more moderate drop in aggregate consumption and leisure under the debt-intensive fiscal rule in the first periods of transition. However, in later periods, taxes have to increase even under the debt-intensive fiscal rule. Furthermore, the sluggish increase in aggregate saving leads to a higher interest rate and, thereby, to a lower capital stock along the adjustment path. Both effects contribute to explain why the recovery of consumption, leisure, capital and output is slower under the debt-intensive fiscal rule.

The observed differences in the adjustment paths of aggregate variables do not offer a simple answer to the question of which type of fiscal rule maximizes aggregate welfare. On the one hand, agents may prefer the debt-intensive fiscal rule due to its stabilizing effect on consumption and leisure in the first periods of transition. On the other hand, agents may also prefer the non-debt-intensive fiscal rule due to the faster recovery of consumption, leisure, capital and output in later periods of transition. Comparing the aggregate welfare effects between the two fiscal rules, we find that aggregate welfare is higher under the debt-intensive fiscal rule than under the non-debt-intensive fiscal rule. Although the differences in aggregate welfare remain quantitatively rather small, this result indicates that a temporary strong increase in public debt in response to the increase in government spending may be justified on the grounds of optimal debt policy. Finally, we also study the individual welfare effects in greater detail to identify those members of society who benefit most from implementing the debt-intensive fiscal rule. We find that the individual welfare gain is particularly high for wealth-poor agents with low productivity, who suffer most from the missing insurance markets and the borrowing constraints. This result is in line with the observed differences in aggregate

<sup>&</sup>lt;sup>6</sup>See Barro (1974) on the Ricardian equivalence theorem.

consumption and again, highlights the importance of financial market imperfections for understanding the role of public debt.

Our findings contribute to the renewed debate on optimal debt policy and financial market imperfections. In addition to the aforementioned literature, there are a few papers focusing on related fiscal policy experiments in heterogeneous-agents, incomplete-markets economies. Heathcote (2005) considers a temporary tax cut in a heterogeneous-agents model with stochastic taxes. He finds that the boost to consumption is larger and that the investment stimulus is smaller when financial markets are incomplete. In line with our findings, Heathcote (2005) emphasizes the importance of borrowing-constrained agents in explaining the results. Kaplan and Violante (2014) develop a model with a low-return liquid asset and a high-return illiquid asset and study the 2001 tax rebate episode. They find that the consumption responses are in line with the empirical evidence and are of larger magnitude compared to the standard one-asset case.

Challe and Ragot (2011) and Brinca et al. (2016) study the responses of consumption and output to a government spending shock. Challe and Ragot (2011) consider an analytically tractable liquidity-constrained economy and find that a debt-financed increase in government spending may lead to an increase in private consumption due to the liquidity effect of public debt. We find, however, that under more standard assumptions on labor supply and capital accumulation, private consumption is more likely to decrease in response to the government spending shock.<sup>8</sup> Brinca et al. (2016) study the effects of wealth inequality on the magnitude of fiscal multipliers in an OLG model. They find that the fiscal multiplier is highly sensitive to the fraction of borrowing-constrained agents and to the average level of wealth. Our results contribute to the findings of Brinca et al. (2016) by showing that, in the presence of financial market imperfections, the fiscal multiplier is also quite sensitive to the time paths of the lump-sum tax and public debt. In particular, we find that the fiscal output multiplier is smaller under the debt-intensive fiscal rule than under the non-debt-intensive fiscal rule. The differences arise from the different response of wealth-poor agents to a change in either the lump-sum tax or in government debt.

Finally, in two recent studies, Angeletos et al. (2016) and Le Grand and Ragot (2017) consider the Ramsey problem of optimal taxation and liquidity provision in the presence of idiosyncratic risk and borrowing constraints. Angeletos et al. (2016) highlight that issuing more public debt eases the underlying financial friction, but also tightens the government budget by raising the interest rate on public debt. Le Grand

<sup>&</sup>lt;sup>7</sup>See also Oh and Reis (2012) for a discussion of optimal transfers and Viegas and Ribeiro (2013) for an assessment of two Dutch fiscal adjustment episodes. Furthermore, our work also relates to the literature on optimal taxation in representative-agent economies including, among others, Barro (1979), Lucas and Stokey (1983), Aiyagari et al. (2002) and Bhandari et al. (2017). While this literature highlights the importance of tax smoothing and insurance against aggregate shocks, it mainly neglects the influence of uninsurable idiosyncratic risk and borrowing constraints.

 $<sup>^8</sup>$ Furthermore, Challe and Ragot (2011) do not assess the welfare implications of different types of fiscal rules.

and Ragot (2017) develop a truncation theory for incomplete-markets economies with aggregate shocks. Applied to a technology shock, Le Grand and Ragot (2017) find that public debt optimally decreases, whereas capital taxes increase on impact. Compared to our work, both papers share the advantage of providing analytical results on optimal debt policy. However, compared to Angeletos et al. (2016), our approach allows for more flexible and standard assumptions on preferences and heterogeneity. Furthermore, capital accumulation plays a central role in our model.<sup>9</sup> Compared to Le Grand and Ragot (2017), we focus on a government spending shock and the impact of changes in the time path of public debt.<sup>10</sup>

The remainder of this paper is organized as follows. Section 5.2 introduces the model. Section 5.3 describes the calibration strategy. Section 5.4 presents the fiscal policy experiment, discusses the welfare implications and provides some robustness considerations. Section 5.5 concludes.

# 5.2 The model

#### 5.2.1 Overview

To analyze the influence of debt policy in the presence of financial market imperfections, we follow the work by Aiyagari and McGrattan (1998) and Flodén (2001) and consider a heterogeneous-agents, incomplete-markets model with a government sector. Time is discrete and indexed by  $t \in [0,1,...,\infty]$ . The economy is populated by a continuum [0,1] of agents, indexed by i. Agents choose an optimal consumption, saving and working plan subject to idiosyncratic labor productivity shocks and a borrowing constraint. Agents are ex-ante identical but differ in financial wealth over time as financial markets are incomplete with respect to the idiosyncratic income shocks. Production in the economy takes place under perfect competition. Firms own a constant-returns-to-scale technology that uses capital and labor to produce one homogenous good each period. The homogenous good can be used for private consumption, government consumption and investment. The government in the economy finances its expenditures on consumption and debt repayment by a lump-sum tax and the issuance of new bonds. We assume that the government follows a simple fiscal rule to determine the time paths of public debt and taxes. The fiscal rule strongly simplifies the computation of the transition dynamics while still providing substantial variation in the response of public debt. Note that if financial markets are assumed to be complete, the model boils down to a deterministic variant of a standard RBC model in which Ricardian equivalence holds. This relation allows us to highlight the importance of financial market imperfections for understanding the role of public debt.<sup>11</sup>

<sup>&</sup>lt;sup>9</sup>See Angeletos et al. (2013) for a discussion of collateral constraints on the production side.

<sup>&</sup>lt;sup>10</sup>In a former version of the paper, Le Grand and Ragot also briefly consider a government spending shock, but use a more specific model setup.

<sup>&</sup>lt;sup>11</sup>Furthermore, our results also directly relate to the discussion of wealth inequality, financial market imperfections and fiscal multipliers launched by Brinca et al. (2016).

## 5.2.2 The individual problem

Agents in the economy have preferences over consumption and hours worked and maximize discounted expected lifetime utility. Formally, the objective function of each agent i is given by

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t U(c_{it}, n_{it}) \right], \tag{5.1}$$

where  $c_{it}$  denotes consumption,  $n_{it}$  denotes hours worked,  $E_0$  is the expectation operator conditional on information at date t = 0, and  $0 < \beta < 1$  is the discount factor. The momentary utility function, U(c, n), takes the following form

$$U(c,n) = \frac{c^{1-\rho}}{1-\rho} - \chi \frac{n^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}},$$
(5.2)

where  $\rho$  is the parameter of relative risk aversion,  $\gamma$  is the Frisch elasticity of labor supply, and  $\chi$  is the weight on the disutility of labor. The specification of the utility function is quite standard and in line with the related literature (e.g., Brinca et al. 2016).

At the beginning of each period, agents receive idiosyncratic shocks to their labor productivity. Markets are incomplete so that perfect insurance is not obtainable. Agents can only partially self-insure against idiosyncratic risk by accumulating a stock of assets that pay out a risk-free return. Asset holdings consist of private capital, which the agent rents to the representative firm, government bonds and bonds that are issued by borrowing agents. Since all assets yield the same return in equilibrium, we do not separate them on the individual level and denote the agent's overall asset holdings by  $a_{it}$ . When the agent becomes a net borrower,  $a_{it}$  becomes negative. In addition to facing incomplete insurance markets, agents also face a borrowing limit on assets that further restricts them in their ability to smooth consumption over time. Asset holdings are bounded from below by the constraint  $a_{it+1} \geq \overline{b}$ , where  $\overline{b}$  denotes the exogenous borrowing limit. We set the value of  $\overline{b}$  in the next section in accordance with the Survey of Consumer Finances.

Let  $V_t(a_t, e_t)$  be the optimal value function for an agent with asset holdings  $a_t$  and log labor productivity  $e_t$ .<sup>12</sup> Then the agent's optimization problem can be specified in terms of the following program

$$V_t(a_t, e_t) = \max_{c_t, a_{t+1}, n_t} \left\{ U(c_t, n_t) + \beta E \left[ V_{t+1}(a_{t+1}, e_{t+1}) \mid e_t \right] \right\}$$
 (5.3)

$$s.t. \quad c_t + a_{t+1} = R_t a_t + w_t \exp(e_t) n_t - T_t \tag{5.4}$$

$$a_{t+1} \geq \bar{b} \tag{5.5}$$

$$c_t \geq 0, \ 0 \leq n_t \leq 1,$$
 (5.6)

 $<sup>^{12}</sup>$ The index i is dropped for notational ease.

where  $R_t \equiv (1+r_t)$  is the gross risk-free interest rate,  $w_t$  is the wage rate, and  $T_t$  denotes the lump-sum tax. The time subscript attached to the value function indicates that the agent's problem is time-dependent because public debt and taxes as well as aggregate prices  $\{w_t, r_t\}$  temporarily change in response to the government spending shock.

#### **5.2.3** Firms

Production in the economy takes place under perfect competition. The representative firm produces final output according to a standard Cobb-Douglas production function

$$Y_t = K_t^{\alpha} N_t^{1-\alpha},\tag{5.7}$$

where  $K_t$  denotes the aggregate capital stock and  $N_t$  denotes labor in productivity units, i.e. the elastic labor supply multiplied by labor productivity and aggregated over all agents. The firm chooses  $K_t$ ,  $N_t$  to maximize profits. The first-order conditions are given by

$$r_t = \alpha \left(\frac{K_t}{N_t}\right)^{(\alpha-1)} - \delta \tag{5.8}$$

$$w_t = (1 - \alpha) \left(\frac{K_t}{N_t}\right)^{\alpha}, \tag{5.9}$$

where  $\delta$  is the constant depreciation rate. Since the first-order conditions have to hold in equilibrium, we can express the wage rate as a function of the interest rate

$$w_t = (1 - \alpha) \left(\frac{\alpha}{r_t + \delta}\right)^{\frac{\alpha}{1 - \alpha}}.$$
 (5.10)

#### 5.2.4 The government

The government finances its expenditures on consumption and debt repayment by tax revenues and the issuance of new bonds. Formally, the government budget constraint is given by

$$G_t + R_t B_t = B_{t+1} + T_t, (5.11)$$

where  $G_t$  denotes expenditures on government consumption,  $B_t$  denotes public debt at the beginning of period t and  $T_t$  is the lump-sum tax. In line with the related literature, we assume that the lump-sum tax evolves according to the following simple fiscal rule<sup>13</sup>

$$T_t = \overline{T} + \phi \left( B_{t+1} - \overline{B} \right), \tag{5.12}$$

where  $\overline{T}$  is the lump-sum tax in steady state,  $\overline{B}$  is the level of public debt in steady state and  $\phi$  is the fiscal policy parameter. Put into words, the fiscal rule says that taxes in period t respond to changes in public debt with strength captured by the policy

<sup>&</sup>lt;sup>13</sup>See Uhlig (2010), Challe and Ragot (2011), and Drautzburg and Uhlig (2015).

parameter  $\phi$ . To assess the implications for the time path of public debt, we combine (5.11) and (5.12) to obtain the following difference equation

$$B_{t+1} = C + \frac{1+r_t}{1+\phi} B_t + \frac{1}{1+\phi} G_t, \tag{5.13}$$

where  $C \equiv (1/(1+\phi))(\phi \overline{B} - \overline{T})$ . Equation (5.13) shows that public debt will remain stationary as long as the policy parameter  $\phi$  exceeds the net interest rate  $r_t$ .<sup>14</sup> This condition is relatively easy to satisfy by imposing an adequate lower bound on  $\phi$ . If stationarity is ensured, then changes in  $\phi$  allow us to model substantial variation in the time path of public debt in order to depict different regimes of debt policy. In particular, equation (5.13) leads to a quite natural distinction between what we call a debt-intensive and a non-debt-intensive fiscal rule. According to equation (5.13), if  $\phi$  is less than unity, then the increase in public debt in response to the government spending shock is stronger than the increase in taxes. This leads to an adjustment path with a temporary strong increase in public debt and a limited impact response of taxes. Although real world systems are more complex than our fiscal rule, this pattern is in line with the empirical evidence. <sup>15</sup> In what follows, we call a fiscal rule with  $\phi < 1$ debt-intensive to account for the substantial increase in public debt. Conversely, if  $\phi$  is greater than unity, then the primary budget deficit remains rather small in all periods. This leads to an adjustment path where taxes move closely with government spending while public debt quickly returns to its initial value. Accordingly, we call a fiscal rule with  $\phi > 1$  non-debt-intensive.

Figure 5.1 provides an example of the two regimes of debt policy under the two fiscal rules. The left panel shows the case of a debt-intensive fiscal rule with  $\phi$  equal to 0.2, and the right panel shows the case of a non-debt-intensive fiscal rule with  $\phi$  equal to 2. In line with the empirical evidence, we assume that the underlying government spending shock exhibits some persistence but dies out after a couple of periods. As outlined above, the left panel shows that the debt-intensive fiscal rule means a very strong increase in public debt in response to the government spending shock and a limited impact response of taxes. In particular, we observe that the increase in public debt in the first periods is much stronger than the increase in taxes. Consequently, it takes a considerable amount of time for public debt to return to the initial level. In contrast, the right panel shows that the non-debt-intensive fiscal rule only leads to a small increase in public debt while taxes move closely with government spending. Consequently, public debt returns much faster to the initial level than it is observed in the left panel. Hence, Figure 5.1 confirms that reasonable changes in the fiscal policy parameter allow us to model substantial

<sup>&</sup>lt;sup>14</sup>We demand that  $\phi$  exceeds the net interest rate in each period to rule out any cycles of increasing public debt and increasing interest rates that may arise under weaker restrictions. In terms of Leeper (1991), we focus on passive fiscal policy only.

<sup>&</sup>lt;sup>15</sup>Blanchard and Perotti (2002), for example, document a limited impact response of taxes.

<sup>&</sup>lt;sup>16</sup>The initial increase in government spending equals 5 percent of steady state output of the benchmark economy as specified in the next section.

0.07 0.07 **Public Debt Public Debt** 0.06 0.06 Tax Tax Gov. Spending Gov. Spending 0.05 0.05 Fax. Public Debt Fax, Public Debt 0.04 0.04 0.03 0.03 0.02 0.02 0.0  $\phi = 0.2$ 0.01 0 0 0 10 20 30 40 10 20 30 40

Figure 5.1: Differences in Public Debt and Taxes

time

Note: Figure 5.1 shows the adjustment path for public debt (beginning of period t) and for the lump-sum tax under different parametrizations of the fiscal rule. The left (right) panel shows the case of  $\phi = 0.2$  ( $\phi = 2$ ). The interest rate is kept fixed at 3 percent.

time

variation in the response of public debt. In our fiscal policy scenario described after the next section, we will exploit the variation between the two fiscal rules to analyze the economy's response to the government spending shock. As benchmark parametrization of the debt-intensive fiscal rule, we follow the example shown in the left panel of Figure 5.1 and set  $\phi$  equal to 0.2. This value satisfies the condition that public debt still follows a stable difference equation in each period and finally returns to the initial steady state level. Equally important, the value captures the limited impact response of taxes quite well. Blanchard and Perotti (2002), for example, study the responses of output and taxes to a government spending shock in a VAR model. Their results indicate that net taxes increase on impact by approximately 0.13 dollars when government spending rises by one dollar. When combining equation (5.12) and (5.13), we get that for  $\phi$  equal to 0.2, taxes increase on impact by 0.166 dollars, which is fairly close to the empirical value. As benchmark parametrization of the non-debt-intensive fiscal rule, we follow the example shown in the right panel of Figure 5.1 and set  $\phi$  equal to 2. As described above, this allows us to contrast the empirically plausible pattern generated by the debtintensive fiscal rule with the alternative regime of keeping the primary deficit small.<sup>17</sup> Definition 3 below summarizes the general equilibrium in this economy.

<sup>&</sup>lt;sup>17</sup>We allow for a slight increase in public debt even under the non-debt-intensive fiscal rule to show that our results do not require a too extreme time path of taxes. Nevertheless, in Section 5.4.3, we also consider the case where taxes move one-to-one with government spending.

**Definition 3** Given the initial distribution of agents over asset holdings and labor productivity,  $\Psi_0(a, e)$ , a general competitive equilibrium is defined by a) a sequence of policy functions  $\{c_t(a, e), n_t(a, e), a_{t+1}(a, e)\}_{t=0}^{\infty}$ , b) a sequence of value functions  $\{V_t(a, e)\}_{t=0}^{\infty}$ , c) a sequence of prices  $\{r_t, w_t\}_{t=0}^{\infty}$ , d) a sequence of policies  $\{G_t, T_t, B_{t+1}\}_{t=0}^{\infty}$ , and e) a sequence of distributions  $\{\Psi_t(a, e)\}_{t=1}^{\infty}$ , such that, for all t

- 1. The policy functions described above solve the agent's optimization problem in (5.3)-(5.6).
- 2. The firm's demand for capital and labor solve (5.8)-(5.9).
- 3. Aggregate quantities of consumption, labor, capital and government bonds are the aggregation of individual quantities. Markets clear for given prices, especially  $K_t + B_t = A_t$  and  $N_t^s = N_t^d$ .
- 4. Taxes and public debt evolve according to (5.12)-(5.13).
- 5. The sequence of distributions is consistent with the initial distribution, the agent's policy functions and the stochastic process for productivity.

## 5.3 Calibration

We calibrate the initial steady state equilibrium of the model economy to replicate key properties of the US economy relevant for our fiscal policy scenario. The considered time period is from 1995 to 2007 in order to capture pre-crisis developments. Regarding the capital-to-output ratio and the investment-to-output ratio, we use the numbers provided by Dyrda and Pedroni (2016) based on data from the NIPA tables. The tightness of the borrowing constraint is set in accordance with the Survey of Consumer Finances (SCF) evaluated by Wolff (2010). The debt-to-output ratio and the expenditure-to-output ratio are taken from Trabandt and Uhlig (2011). Table 5.1 below summarizes the target values and chosen parameter values.

**Preferences.** The objective function of the agent depends on four parameters, the parameter of relative risk aversion,  $\rho$ , the Frisch elasticity of labor supply,  $\gamma$ , the weight on the disutility of labor,  $\chi$ , and the discount factor,  $\beta$ . We set the parameter of relative risk aversion and the Frisch elasticity of labor supply in accordance with the literature. The parameter of relative risk aversion is set at 1.5 and the Frisch elasticity is set at 0.82. Although there is still a considerable debate about the Frisch elasticity in the literature, a value of around one is considered as an empirically plausible benchmark specification (e.g., Brinca et al. 2016, Dyrda and Pedroni 2016, Röhrs and Winter 2017). The weight on the disutility of labor is chosen to generate an empirically plausible allocation of time endowment between work and leisure. In the stationary equilibrium,

average hours worked is close to 0.33 so that approximately 1/3 of time endowment is devoted to working. For the discount factor,  $\beta$ , we target a capital-to-output ratio of 2.72.

Borrowing Constraint and Income Process. The tightness of the borrowing constraint is set in accordance with the Survey of Consumer Finances (several years) evaluated by Wolff (2010). We discipline the borrowing constraint to generate an empirically plausible share of agents with zero or negative asset holdings in the stationary equilibrium. According to Wolff (2010), the average share of households with zero or negative net worth for the period from 1995 to 2007 equals 18 percent. The level of  $\bar{b}$  we choose matches this number relatively well, while also ensuring that the borrowing constraint remains sufficiently tight.

The stochastic process for log labor productivity,  $e_t$ , is modeled as an AR(1) process with autocorrelation coefficient  $\rho_e$  and standard deviation of the idiosyncratic shocks  $\sigma$ ,

$$e_{t+1} = \rho_e \, e_t + \eta_{t+1}, \quad \eta \sim N(0, \sigma^2).$$
 (5.14)

In line with the estimation results from Storesletten et al. (2004), we set  $\rho_e$  at 0.9 and  $\sigma$  at 0.19. In the numerical procedure used to solve the model, we approximate the stochastic process in (5.14) with a five-state Markov chain following the method proposed by Rouwenhorst (1995).<sup>18</sup> As a robustness exercise, we also consider a more flexible approach and set the transition probabilities individually.<sup>19</sup> This allows us to model a higher level of wealth inequality in the steady state equilibrium. However, we find that our main results are quite robust to changes in the specification of the labor productivity process. In particular, we find that increasing the level of wealth inequality does not change the result whether the debt-intensive or the non-debt-intensive fiscal rule has the largest aggregate welfare benefit.

Technology and Fiscal Policy. Production in the economy takes place under a standard Cobb-Douglas production function with capital share equal to  $\alpha$ . In line with the empirical evidence, we set  $\alpha$  at 0.36. For the depreciation rate,  $\delta$ , we target an investment-to-output ratio of 0.27.

Regarding the level of public debt and government expenditures in steady state, we use the numbers provided by Trabandt and Uhlig (2011). According to Trabandt and Uhlig (2011), the average debt-to-output ratio for the period from 1995 to 2007 is equal to 0.63 and the expenditure-to-output ratio is equal to 0.18. Both numbers are frequently used in the literature as reasonable initial values for conducting fiscal policy experiments.

<sup>&</sup>lt;sup>18</sup>We also slightly adjust the obtained values such that E[(exp(e))] = 1.

<sup>&</sup>lt;sup>19</sup>We mainly follow the approach by Röhrs and Winter (2017). See Section 5.4.3 for details.

Table 5.1: Parameter Values of the Benchmark Economy

Parameter		Value	Target	Data	Model
parameter of relative risk aversion	ρ	1.5	-	-	-
Frisch elasticity	$\gamma$	0.82	-	-	-
weight on disutility of labor	χ	22	avg. hours worked	0.33	0.35
discount factor	$\beta$	0.954	capital/output	2.72	2.71
curvature of production	$\alpha$	0.36	-	-	-
depreciation rate	δ	0.10	investment/output	0.27	0.27
AR(1) coefficient	$ ho_e$	0.9	-	-	-
standard deviation of shocks	$\sigma$	0.19	-	-	-
debt limit	$\overline{b}$	-0.15	frac. agents $a \leq 0$	0.18	0.163
gov. expenditures to output	G/Y	0.18	-	-	-
lump-sum tax to output	T/Y	0.20	public debt/output	0.63	0.63

# 5.4 Results

This section is divided into three parts. First, we describe our fiscal policy scenario that focuses on the influence of debt policy on macroeconomic activity following a temporary spending shock. Second, we examine the welfare implications and derive optimal debt policy. In the last part, we discuss the robustness of our main results with respect to changes in the benchmark calibration, including changes in the income process.

#### 5.4.1 Impulse responses

To understand the impact of debt policy on the economy's response to a government spending shock, we consider the following fiscal policy scenario. Initially, the benchmark economy, as specified in the previous section, stays in the stationary equilibrium. The interest rate in steady state is equal to 3.27 percent, output is equal to 0.60 and the capital-to-output ratio is equal to 2.71. Then, the government undertakes an unanticipated increase in government spending. Following Challe and Ragot (2011), we assume that the increase in period t=0 is equivalent to 5 percent of steady state output.

<sup>&</sup>lt;sup>20</sup>In other words, we consider a so-called "MIT shock", i.e. an unexpected deviation of the economy from the steady state equilibrium. See Boppart et al. (2017) on how MIT shocks can be used to compute equilibria in heterogeneous-agent models in which aggregate shocks are expected to have non-zero variances.

In the following periods, government expenditures gradually return to the initial steady state level that is finally reached after 7 periods. Once the change in government spending is announced, the government has to decide on the fiscal rule to determine the time paths of public debt and taxes. Since we are mainly interested in comparing the effects of a strong and a weak increase in public debt, we consider the two fiscal rules that have been introduced in Figure 5.1. The debt-intensive fiscal rule with  $\phi = 0.2$  leads to a strong increase in public debt in response to the change in government spending, whereas the non-debt-intensive fiscal rule with  $\phi = 2$  keeps the primary deficit small. Note that we assume that agents can foresee the time paths of public debt and taxes under both fiscal rules. The perfect foresight assumption simplifies the computation of the transition dynamics and is commonly used in the incomplete-markets literature (e.g., Brinca et al. 2016). <sup>21</sup>

Figure 5.2 shows the economy's response to the government spending shock under the two fiscal rules. The red lines in Figure 5.2 indicate the debt-intensive fiscal rule and the blue lines indicate the non-debt-intensive fiscal rule. Apart from government spending, changes in aggregate variables over time are expressed as percentage deviations from the corresponding steady state level. Changes in government spending are measured relatively to the steady state output level.

At first view, Figure 5.2 shows that the responses of private consumption, hours worked, capital and output look qualitatively similar under both fiscal rules and that they are in line with what is known from a standard representative-agent, complete-markets model.<sup>22</sup> Consumption and capital fall in response to the government spending shock, whereas hours worked and output increase on impact.<sup>23</sup> The observed pattern follows from the fact that the increase in government spending ultimately means an increase in taxation under both fiscal rules. However, the above result does not mean that debt policy has no influence on the economy. What Figure 5.2 actually shows is that the two fiscal rules lead to significant quantitative differences in the adjustment paths of aggregate variables. These differences that would not appear in the complete-markets setup because Ricardian equivalence holds, establish a role for public debt in the presence of financial market imperfections. We investigate these differences in the following.

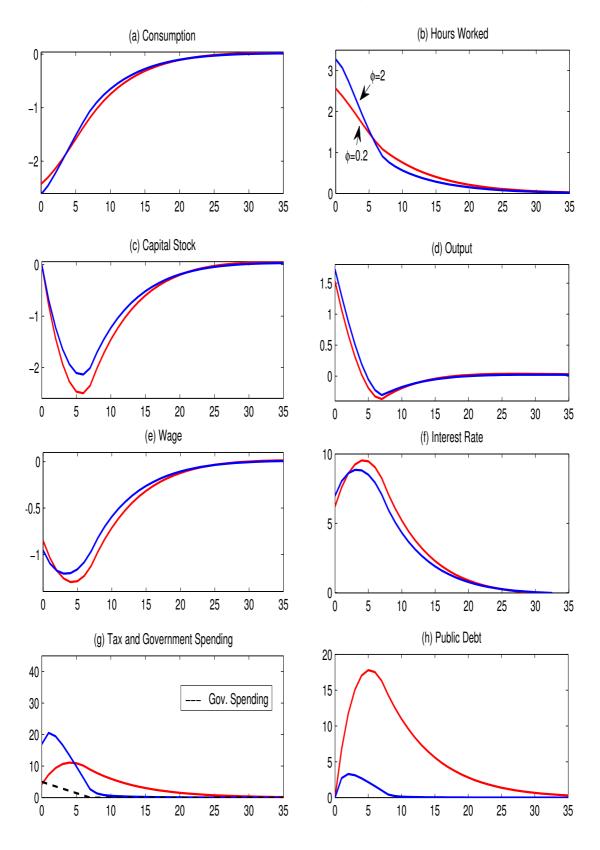
Regarding private consumption, we observe that the debt-intensive fiscal rule provides a stabilizing effect in the first periods following the change in government spending. This follows from the observation that in the first periods consumption losses are smaller under the debt-intensive fiscal rule than under the non-debt-intensive fiscal rule. The stabilizing effect is quantitatively strongest in the initial period in which the fall in

<sup>&</sup>lt;sup>21</sup>See the Appendix for further details on the numerical procedures.

<sup>&</sup>lt;sup>22</sup>See, for example, Uhlig (2010) on the fiscal multiplier in an RBC model.

<sup>&</sup>lt;sup>23</sup>We are aware of the fact that there is still an ongoing debate about the response of private consumption to a government spending shock, reflecting the different views of neoclassical and Keynesian theory (e.g., Challe and Ragot 2011). However, in this paper and in line with the related literature, we focus on the *financing side* of government spending, not on the desired effects of spending per se.

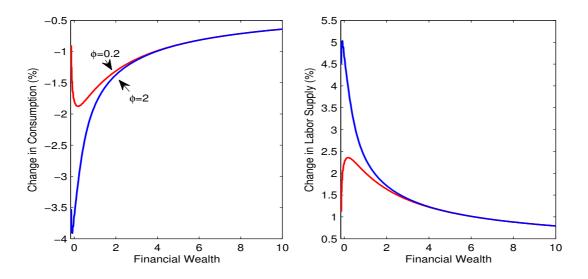
Figure 5.2: Impulse Responses (debt-intensive (red) / non-debt-intensive (blue))



consumption is approximately 7 percent lower under the debt-intensive fiscal rule than under the non-debt-intensive rule. However, we also observe that the pattern changes along the transition path. In particular, we observe that the recovery of consumption is slower under the debt-intensive fiscal rule than under the non-debt-intensive fiscal rule. That means the smaller losses in consumption in the first periods under the debt-intensive fiscal rule come at the price of higher losses in later periods. Regarding hours worked, we observe a similar effect of debt policy. In the first periods following the change in government spending, the increase in hours worked is more moderate under the debt-intensive fiscal rule than under the non-debt-intensive fiscal rule. However, despite the lower initial increase, hours worked return to the initial steady state level under the debt-intensive fiscal rule more slowly. Thus, the debt-intensive fiscal rule has a stabilizing effect on consumption and leisure in the first periods of transition, whereas the non-debt-intensive fiscal rule leads to a faster recovery of consumption and leisure in later periods of transition.

To better understand the differences in consumption and hours worked, we have to consider the interplay of public debt and financial market imperfections. For this it is useful to focus on the initial period in which the change in government spending takes place and in which government expenditures take the highest value. Under the debt-intensive fiscal rule we observe a stronger increase in public debt in response to the increase in government spending, whereas under the non-debt-intensive fiscal rule we observe a stronger increase in the lump-sum tax. If Ricardian equivalence holds, this different effect of the debt-intensive and the non-debt-intensive fiscal rule does not matter for the response of private consumption. Agents under the debt-intensive fiscal rule understand that taxes are bound to increase in the future and they are willing to cut back consumption and to increase savings, meeting the higher supply of government bonds. Consequently, if Ricardian equivalence holds, the response of agents to the government spending shock does not depend on the underlying fiscal rule. However, as it is well-known, when financial market imperfections come into play, agents become restricted in their options to allocate resources over time. This especially holds true for wealth-poor agents who are particularly exposed to the income risk, with the borrowing constraint currently binding or expected to become binding in the near future. Consequently, in the presence of financial market imperfections, the response of wealth-poor agents to the government spending shock does depend on the change in current income and, thereby, on the underlying fiscal rule. Figure 5.3(a) shows this effect in greater detail. Figure 5.3(a) compares the change in individual consumption in the first period in which the increase in government spending takes place between the debtintensive and the non-debt-intensive fiscal rule. We keep the interest rate and wage rate fixed at steady state levels to abstract from general equilibrium effects. In line with the reasoning outlined above, Figure 5.3(a) shows that the response of wealth-poor agents to the government spending shock differs between the two fiscal rules. In particular, we observe that wealth-poor agents under the debt-intensive fiscal rule do not cut back consumption at the same rate as they are forced to under the non-debt-intensive fiscal rule due to the increase in the lump-sum tax. This is precisely the described stabilizing effect of the debt-intensive fiscal rule, meaning that the debt-intensive fiscal rule allows wealth-poor agents to mitigate the loss in consumption. Finally, Figure 5.3 also shows that the differences in consumption and labor supply vanish at higher levels of financial wealth. Hence, rich agents behave more in line with Ricardian Equivalence, which confirms the conclusion that the observed differences in aggregate consumption and hours worked arise from the different response of wealth-poor agents and, thereby, from the interplay of debt policy and financial market imperfections.

Figure 5.3: Differences in Consumption and Labor Supply



Note: Figure 5.3 shows the change in consumption and labor supply of agents with low productivity in the first period in which government expenditures take the highest value. Changes are expressed as percentage deviations from the initial steady state level. The red (blue) line shows the case of the (non-) debt-intensive fiscal rule with  $\phi=0.2$  ( $\phi=2$ ). Note that the interest rate and wage rate are kept fixed at steady state levels.

With the above insight into the effects on consumption and hours worked, we return to Figure 5.2 to examine the differences in capital and output in more detail. As Figure 5.2(c) shows, the debt-intensive fiscal rule leads to a lower capital stock along the adjustment path compared to the non-debt-intensive fiscal rule. Different from the effects on consumption and hours worked that means both, a larger decline of capital in the first periods and a slower recovery of capital in later periods under the debt-intensive fiscal rule. Quantitatively, the differences in capital between the two fiscal rules become substantial along the transition path. Under the debt-intensive fiscal rule, the capital stock falls by 2.5 percent before finally recovering, whereas under the non-debt-intensive fiscal rule leads to a 19 percent larger drop in the capital stock compared to the non-debt-intensive fiscal rule. The differences in capital between the two fiscal

rules are best understood by considering the adjustment path of the interest rate and with it, the supply of government bonds. Naturally, the debt-intensive fiscal rule leads to a higher supply of government bonds along the adjustment path, which requires a stronger stimulus to aggregate saving. Based on the described unwillingness of wealth-poor agents to reduce consumption, this means that the interest rate has to increase to higher levels under the debt-intensive fiscal rule. The higher interest rate finally means that demand for capital is lower under the debt-intensive fiscal rule.<sup>24</sup> This crowding out effect of public debt has been of particular interest within the recent incomplete-markets literature (e.g., Röhrs and Winter 2017) and is also an important channel in our fiscal policy scenario.

Finally, it is worth considering the differences in output between the two fiscal rules in greater detail. As Figure 5.2(d) shows, the debt-intensive fiscal rule leads to a lower increase in output in the first periods of transition and a stronger decrease in output in later periods of transition compared to the non-debt-intensive fiscal rule. That means the debt-intensive fiscal rule leads to a smaller fiscal output multiplier. Technically, the smaller output multiplier observed under the debt-intensive fiscal rule is driven by the lower increase in hours worked and the stronger drop in the capital stock. As described above, the differences in hours worked and capital between the two fiscal rules arise from the different responses of wealth-poor agents to a change in either the lump-sum tax or in government debt. By addressing the influence of debt policy on fiscal multipliers, our results also relate to the recent findings of Brinca et al. (2016). Brinca et al. (2016) show in an incomplete-markets OLG model that the fiscal multiplier is highly sensitive to the fraction of borrowing-constrained agents and to the average level of wealth. Our results relate by showing that the presence of borrowing-constrained agents also makes the fiscal multiplier quite sensitive to the time paths of the lump-sum tax and public debt. In particular, we observe that the impact multiplier is around 13 percent higher under the non-debt-intensive fiscal rule than under the debt-intensive fiscal rule. Of course, the magnitude of fiscal multipliers also depends on other factors like, for example, the existence of distortionary taxes. However, our results show that when focusing exclusively on the interplay of debt policy and financial market imperfections, a fiscal rule that leads to a temporary strong increase in public debt tends to reduce the expansion of output.

In summary, our results show that changes in debt policy have a significant impact on the responses of consumption, leisure, capital and output to the positive government spending shock. As a rule of thumb, it can be said that if the government allows for a temporary strong increase in public debt, i.e. implements a debt-intensive fiscal rule, we observe a stabilizing effect on consumption and leisure in the first periods following the change in government spending. In contrast, if the government keeps the primary deficit small, i.e. implements a non-debt-intensive fiscal rule, we observe a faster recovery of

<sup>&</sup>lt;sup>24</sup>Furthermore, as recently emphasized by Angeletos et al. (2013) and Angeletos et al. (2016), a higher interest rate also means an increase in the tax burden of servicing the outstanding debt.

consumption and leisure in later periods. Furthermore, if the government keeps the primary deficit small, we observe a larger fiscal output multiplier. In the next section, we address the question how the differences in debt policy affect aggregate and individual welfare.

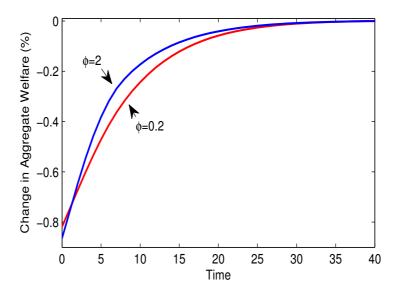
## 5.4.2 Welfare and optimal policy

The observed differences in the adjustment paths of aggregate variables do not offer a simple answer to the question of which type of fiscal rule maximizes aggregate welfare. On the one hand, agents may prefer a debt-intensive fiscal rule that leads to a temporary strong increase in public debt in response to the increase in government spending and that has a stabilizing effect on consumption and leisure in the first periods of transition. On the other hand, agents may also prefer a non-debt-intensive fiscal rule that keeps the primary deficit small and leads to a faster recovery of consumption and leisure in later periods of transition. To assess the welfare consequences, we compute for both fiscal rules the change in aggregate welfare between the case when the economy remains in the stationary equilibrium and the case when the government spending shock occurs at t=0. We also report welfare changes at later points in time along the transition to gain more information about the temporal pattern. Note however, that the overall welfare effect is measured at period t=0 and takes account of the entire transition path. Formally, we define a change in aggregate welfare,  $\Delta W_t$ , according to

$$\Delta W_t = \left(1 - \frac{\int_{a,e} V_t(a,e)\Psi_t(a,e)}{\int_{a,e} V_{ss}(a,e)\Psi_{ss}(a,e)}\right) * 100, \tag{5.15}$$

where  $V_t(a, e)$  is the optimal value function at time t and  $\Psi_t(a, e)$  is the distribution of agents over asset holdings and productivity at time t given that the government spending shock occurs at t=0.  $V_{ss}(a, e)$  and  $\Psi_{ss}(a, e)$  are the optimal value function and distribution, respectively given that the economy remains in the stationary equilibrium.

Figure 5.4 shows the results for the two fiscal rules in our benchmark economy. First, we observe that an increase in government spending leads to an aggregate welfare loss irrespective of the underlying fiscal rule. This result is not surprising since aggregate consumption decreases and the average fraction of time devoted to working increases in response to the government spending shock. More importantly, however, when taking account of the entire transition path at t=0, we observe that the drop in aggregate welfare is more moderate under the debt-intensive fiscal rule than under the non-debt-intensive fiscal rule. In other words, we find that implementing the debt-intensive fiscal rule leads to a larger aggregate welfare benefit than implementing the non-debt-intensive fiscal rule. Quantitatively, the difference in aggregate welfare between the two fiscal rules remains rather moderate. If we take account of the entire transition path, the difference in aggregate welfare between the two fiscal rules means a difference in



permanent consumption of around 0.1 percent.<sup>25</sup> The small quantitative effect reflects the fact that both fiscal rules provide their benefits that pay off at different points in time along the transition path. However, despite the small quantitative effect, we can conclude that agents prefer the debt-intensive fiscal rule on the aggregate level.

Having found that aggregate welfare is higher under the debt-intensive fiscal rule, we next address the question how the individual welfare gains of implementing the debt-intensive fiscal rule are distributed among the population. Since agents are affected differently by changes in debt policy, dependent on the level of financial wealth and of labor productivity, agents will also benefit differently from implementing the debt-intensive fiscal rule. To assess the individual welfare effects, we compute the change in individual welfare between the debt-intensive and the non-debt-intensive fiscal rule, taking into account the entire transition path. Formally, we define a change in individual welfare,  $\Delta w(a, e)$ , according to

$$\Delta w(a, e) = \left(1 - \frac{V^D(a, e)}{V^{ND}(a, e)}\right) * 100, \tag{5.16}$$

where  $V^D(a,e)$  and  $V^{ND}(a,e)$  are the optimal value functions under the debt-intensive fiscal rule and the non-debt-intensive fiscal rule, respectively. A positive number means that implementing the debt-intensive fiscal rule leads to a welfare gain.

 $<sup>^{25}\</sup>text{To}$  translate the difference in aggregate welfare between the two fiscal rules into differences in permanent consumption, we consider the following calculations. First, we compute the constant consumption level equal for each agent,  $c_d$ , that leads to the same aggregate welfare as observed under the debt-intensive fiscal rule. Formally,  $c_d$  solves  $c_d = \left[(1-\beta)(1-\rho)W_d\right]^{1/(1-\rho)}$ , where  $W_d$  denotes aggregate welfare under the debt-intensive fiscal rule. Following the same steps, we then compute the constant consumption level equal for each agent,  $c_{nd}$ , that leads to the same aggregate welfare as observed under the non-debt-intensive fiscal rule. By taking the difference between  $c_{nd}$  and  $c_d$ , we get an estimate of how much permanent consumption differs between the two fiscal rules.

Figure 5.5: Individual Welfare Effects

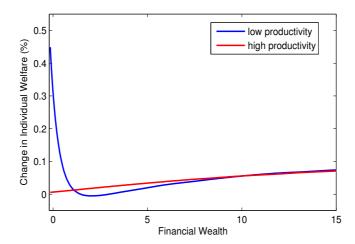


Figure 5.5 shows the results for agents with low and high labor productivity. First, we observe that the individual welfare gain of implementing the debt-intensive fiscal rule is especially high for wealth-poor agents with low productivity. This result confirms the intuition that agents who suffer most from the financial market imperfections also benefit most from the stabilizing effect of the debt-intensive fiscal rule. Wealth-poor agents with high productivity also experience a welfare gain, but the differences are quantitatively smaller. Agents with high productivity are slightly less affected by the borrowing constraint and, thus, benefit less from the debt-intensive fiscal rule. Moving to higher levels of financial wealth, Figure 5.5 shows that also rich agents prefer the debt-intensive fiscal rule to the non-debt-intensive fiscal rule. This result may seem surprising as rich agents behave in line with Ricardian equivalence and, thus, should be indifferent between the two fiscal rules. However, rich agents benefit from the fact that the interest rate has to increase to higher levels under the debt-intensive fiscal rule due to the sluggish increase in aggregate saving. That means rich agents benefit from the interest rate effect of the debt-intensive fiscal rule. Finally, Figure 5.5 also shows that the debt-intensive fiscal rule does not lead to a welfare gain for all members of society. Agents who are neither very poor nor very rich but have a low level of productivity experience a small welfare loss under the debt-intensive fiscal rule. Agents who are neither very poor nor very rich do not particularly benefit from either the stabilizing or the interest rate effect of the debt-intensive fiscal rule, but strongly suffer from the larger drop of the wage rate under the debt-intensive fiscal rule. Hence, while the interest rate effect of the debt-intensive fiscal rule leads to a welfare gain for rich agents, the associated wage effect leads to a welfare loss for agents with moderate wealth. The fact that not all agents may benefit from the debt-intensive fiscal rule naturally raises the question which type of fiscal rule has the greater political support, i.e. which type of fiscal rule is preferred by more than 50 percent of the population. Although the results in Figure 5.5 point in favor of the debt-intensive fiscal rule, the outcome may be reverse if the majority of agents turns out to be neither too poor nor too rich in order to benefit from the debt-intensive fiscal rule. Computing the individual welfare effects for all agents, we find that around 68 percent of the population experience a welfare gain under the debt-intensive fiscal rule while around 32 percent of the population experience a small welfare loss. Although the share of agents experiencing a small welfare loss is not nil, we can conclude that the vast majority of agents prefers the debt-intensive fiscal rule to be implemented. Among the biggest winners are the wealth-poor agents with low productivity who benefit most from the stabilizing effect of the debt-intensive fiscal rule.

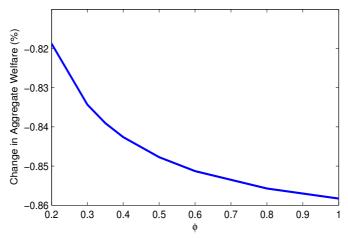
In summary, our results show that when isolating the interplay of debt policy and financial market imperfections, a temporary strong increase in public debt in response to the increase in government spending can be justified on the grounds of optimal debt policy. The debt-intensive fiscal rule leads to a larger aggregate welfare benefit compared to the non-debt-intensive fiscal rule, and also means an individual welfare gain for the vast majority of agents. Furthermore, as Figure 5.6 shows, the benchmark parametrization of the debt-intensive fiscal rule with  $\phi = 0.2$  is also preferred to any other parametrization of the debt-intensive fiscal rule with a higher value of  $\phi$ . From this perspective, our results provide some rationale for the observed pattern of public debt.

Having established the above results, it also seems interesting to briefly compare them to the related literature in this field. Though different fiscal policy experiments have been considered, the observed support for the debt-intensive fiscal rule seems to form a common pattern. For example, Röhrs and Winter (2017) consider the transition path between two stationary equilibria where the latter is associated with a lower level of public debt. The authors show that optimal fiscal policy means that the required increase in taxes should be smooth in the beginning of the transition, getting more pronounced only in later periods. Although we consider a different fiscal policy experiment, our debt-intensive fiscal rule plays a very similar role by allowing for a smooth increase in taxes in the first periods of transition, thereby providing a stabilizing effect for poor agents with low financial wealth. This motive becomes even more clear from the work of Heathcote (2005) who considers a temporary tax cut financed by higher public debt. Heathcote (2005) shows that the implied increase in private consumption under this policy is particularly strong for wealth-poor agents at the borrowing constraint. Finally, Desbonnet and Weitzenblum (2012) show that when transitional dynamics are taken into account, the optimal level of public debt may be higher than in a pure steady state comparison. What connects these different experiments is that they emphasize the short-run impact of public debt, acting as stabilizer for poor agents who are affected most by the financial market imperfections. Furthermore, the results show that the short-run effects of public debt may be very different from the optimal long-run level of public debt.<sup>26</sup> However, what is also a common feature is that there exists a trade-off

<sup>&</sup>lt;sup>26</sup>This becomes most clear from the fact that the optimal long-run level of public debt that is derived

between the different regimes of debt policy. This follows from the fact that higher public debt today means a pronounced increase in taxes in later periods of transition. As we have shown, the differences in aggregate welfare between the debt-intensive and the non-debt-intensive fiscal rule are quantitatively rather small, reflecting the fact that both policies provide their benefits. Hence, small changes in the valuation of the two fiscal rules may lead to big changes regarding the optimal time paths of public debt and taxes. We share this result with the related literature showing that optimal debt policy in incomplete-markets models can vary strongly and is closely tied to the underlying model assumptions. To address this issue, we provide some robustness considerations in the next section.

Figure 5.6: Aggregate Welfare and Debt-Intensive Fiscal Rule



Note: Figure 5.6 shows the change in aggregate welfare between the case where the economy remains in the stationary equilibrium and the case where the government spending shock occurs in period t=0 for different parametrizations of the debt-intensive fiscal rule. The results show that aggregate welfare decreases with higher values of  $\phi$  as the welfare loss associated with the government spending shock increases.

#### 5.4.3 Sensitivity analysis

To assess the robustness of the results, we repeat our fiscal policy experiment for different model specifications and parametrizations. First, we modify the stochastic process for labor productivity along the lines of Röhrs and Winter (2017), which leads to higher wealth inequality in the stationary equilibrium. Second, we allow for more pronounced differences in the time paths of public debt and taxes and, third, we consider a higher level of the discount factor. As a main result, we find that our policy implications remain qualitatively robust; a strong temporary increase in public debt in response to the government spending shock still leads to the largest aggregate welfare benefit.<sup>27</sup>

from a steady state comparison is negative in our benchmark economy as in Röhrs and Winter (2017). Hence, the support for the debt-intensive fiscal rule is not driven by preferences for a higher long-run level of public debt.

 $<sup>^{27}</sup>$ We also considered higher initial debt-to-output ratios (100%, 120%) and find that the aggregate welfare effects remain robust. Furthermore, we also find that the aggregate welfare effects are robust

#### Income process and wealth inequality

As emphasized by Röhrs and Winter (2017), changes in the wealth and earnings distribution may have a strong impact on the optimal quantity of public debt. Röhrs and Winter (2017) verify this effect with respect to the long-run equilibrium and the adjustment path given a constant level of government spending. In our first exercise, we want to test whether changes in the wealth distribution also have a strong impact on the optimal time paths of public debt and taxes following a government spending shock. For this, we largely adopt the specification of the labor productivity process proposed by Röhrs and Winter (2017). The vector of productivity states, s, and the corresponding transition matrix,  $\Pi$ , are shown below.

$$s = [0.32, 0.6, 1.2, 8], \qquad \Pi = \begin{bmatrix} 0.940 & 0.040 & 0.020 & 0.000 \\ 0.034 & 0.816 & 0.150 & 0.000 \\ 0.001 & 0.080 & 0.908 & 0.011 \\ 0.100 & 0.015 & 0.060 & 0.825 \end{bmatrix}$$
(5.17)

Compared to our benchmark AR(1) process, the productivity process in (5.17) is characterized by a larger difference between the lowest and highest state of productivity and a higher probability of moving from the highest state today to the lowest state tomorrow. These two features generate a strong saving motive even for wealth-rich agents, which crucially drives the unequal distribution of financial wealth. As Table 5.2 shows, the top 20 percent of the population in the stationary equilibrium own more than 85 percent of total wealth, whereas the poorest 40 percent of the population own less than 1 percent of total wealth. Compared to our benchmark economy this means a higher concentration of financial wealth in the top quintile and a larger share of agents who have hardly any resources to buffer a change in current income. The latter result is especially important for our fiscal policy scenario since it means a larger share of agents who are affected by the borrowing constraint.

Table 5.2: Stationary Distribution of Financial Wealth

	Bottom $40.0\%$	3rd~20.0%	$2\mathrm{nd}\ 20.0\%$	Top $20.0\%$
Financial Wealth (%)	0.11%	4.52%	8.72%	86.65%

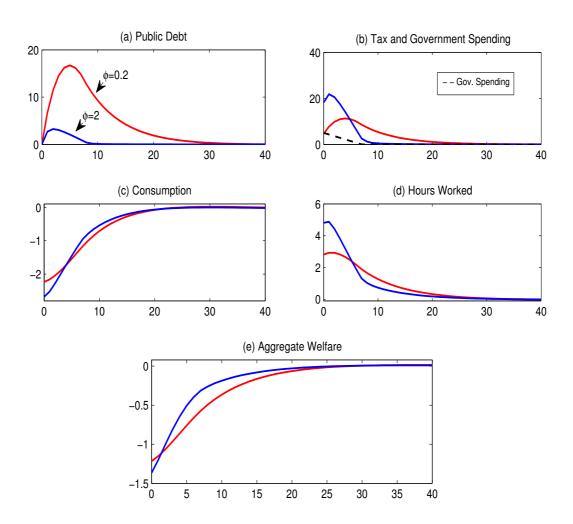
Figure 5.7(c) and Figure 5.7(d), respectively show the responses of private consumption and hours worked to the government spending shock under the new productivity process.<sup>28</sup> We observe that, despite the changes in the wealth distribution, the general

to the introduction of a proportional labor income tax in replacement of the lump-sum tax. However, since we are mainly interested in the interplay of debt policy and financial market imperfections, we stick to the lump-sum tax approach and focus on the robustness exercises outlined above.

<sup>&</sup>lt;sup>28</sup>Note that we leave other parameter values unchanged to facilitate the comparison with the benchmark economy.

pattern remains largely the same as in the benchmark economy. The debt-intensive fiscal rule provides a stabilizing effect on consumption and leisure in the first periods following the change in government spending, whereas the non-debt-intensive fiscal rule leads to a faster recovery of consumption and leisure in later periods. Quantitatively, however, the differences between the two fiscal rules become more pronounced compared to the benchmark economy. For instance, the drop in consumption in the initial period is around 16 percent lower under the debt-intensive fiscal rule than under the non-debt-intensive fiscal rule. In the benchmark economy, the difference is quantitatively smaller and equals 7 percent. The stronger quantitative effect reflects the fact that the fraction of borrowing-constrained agents increases under the new productivity process, which amplifies the influence of debt policy.

Figure 5.7: Alternative Productivity Process



Note: Panel (a)-(d) show the adjustment paths of public debt, taxes, private consumption and hours worked, respectively for the productivity process in (5.17). Panel (e) shows the corresponding aggregate welfare effects that are computed according to equation (5.15).

Figure 5.7(e) shows the corresponding aggregate welfare effects that are computed according to equation (5.15). Similar to the responses of consumption and hours worked, we observe that the aggregate welfare effects are qualitatively robust to the change in the productivity process. Specifically, when taking into account the entire transition path at t=0, aggregate welfare remains to be higher under the debt-intensive fiscal rule. Quantitatively, the difference in aggregate welfare between the two fiscal rules becomes more pronounced compared to the benchmark economy. Under the new productivity process, the difference in aggregate welfare between the two fiscal rules means a difference in permanent consumption of around 0.3 percent. In the benchmark economy, the difference is quantitatively smaller and equals 0.1 percent. Thus, we find that changes in the wealth distribution have an effect on the optimal time paths of public debt and taxes, but they do not change the general pattern. In particular, we do not find evidence that higher wealth inequality tends to reduce the optimal level of public debt. If at all, we find stronger support for the debt-intensive fiscal rule which reflects the fact that a larger share of agents benefits from the stabilizing effect of the debt-intensive fiscal rule.

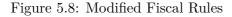
#### Fiscal rules

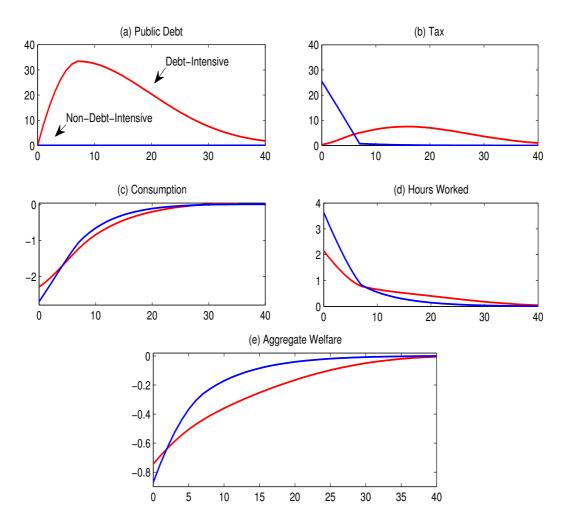
The second exercise aims at showing that the derived welfare effects are not tied to the functional form of the underlying fiscal rule and persist under more pronounced differences in the time paths of public debt and taxes. For this, we modify the non-debt-intensive fiscal rule to capture the extreme case of taxes moving one-to-one with changes in government spending while public debt remains at the initial steady state level.<sup>29</sup> The debt-intensive fiscal rule is modified to capture the other extreme case where taxes barely move in the first periods of transition. Note that this specification of the debt-intensive fiscal rule violates the restriction that public debt follows a stable difference equation for these first periods.

Figure 5.8(c) and Figure 5.8(d), respectively show the responses of private consumption and hours worked to the government spending shock under the two modified fiscal rules. Similar to the previous exercise, we observe that the general pattern remains largely unaffected, but the differences between the two fiscal rules are more pronounced compared to the benchmark economy. For instance, the drop in consumption in the initial period is 15 percent lower under the debt-intensive fiscal rule than under the non-debt-intensive fiscal rule. In the benchmark economy, the difference is quantitatively smaller and equals 7 percent. The stronger quantitative effect is not surprising in this case as the two fiscal rules strongly amplify the influence of debt policy.

Figure 5.8(e) shows that the robust behavior of consumption and hours worked also carry over to the aggregate welfare effects. When taking account of the entire transition path at t=0, aggregate welfare remains to be higher under the debt-intensive fiscal rule. Furthermore, we observe that the aggregate welfare gain of implementing the debt-intensive fiscal rule increases compared to the benchmark economy. The stronger

<sup>&</sup>lt;sup>29</sup>This outcome is, approximately, obtained when  $\phi$  becomes extremely large, approaching infinity.





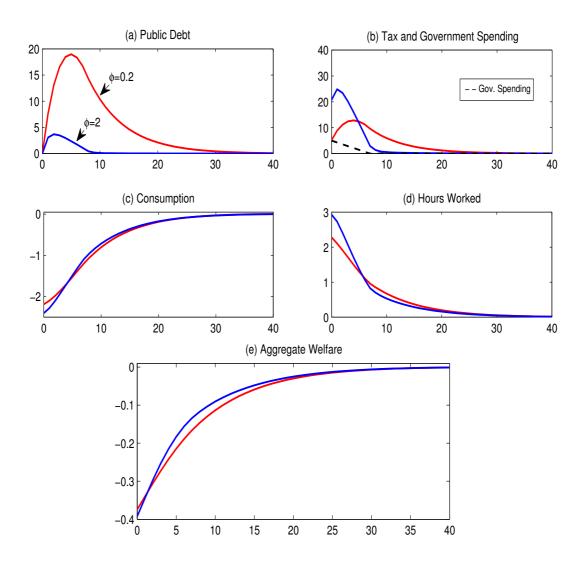
Note: Panel (a)-(d) show the adjustment paths of public debt, taxes, private consumption and hours worked, respectively for the two modified fiscal rules. Panel (e) shows the aggregate welfare effects that are computed according to equation (5.15).

support for the debt-intensive fiscal rule reflects the increased benefits of raising the level of public debt. In sum, we find that our policy implications are robust to more pronounced differences in the time paths of public debt and taxes and, thus, are not tied to the specific form of the underlying fiscal rule.

#### Discount factor

In the last exercise, we test the robustness of the aggregate welfare effects with respect to an increase in the discount factor,  $\beta$ . A higher value of  $\beta$  means a higher weight of future periods in the agent's utility function. This case is of interest as it may shift the results in favor of the non-debt-intensive fiscal rule whose benefits especially pay off in later periods of transition. Compared to our benchmark economy, we increase  $\beta$  from

Figure 5.9: Increase in  $\beta$ 



Note: Panel (a)-(d) show the adjustment paths of public debt, taxes, private consumption and hours worked, respectively for the higher value of  $\beta$ . Panel (e) shows the aggregate welfare effects that are computed according to equation (5.15).

0.954 to 0.98. The other parameter values are kept at their benchmark levels.

Figure 5.9(e) shows that the higher value of  $\beta$  in fact brings the two fiscal rules closer together. Compared to the benchmark economy, the difference in permanent consumption between the two fiscal rules is cut by more than half. However, we also observe that the balancing effect is not strong enough to change the well-known pattern. As in all previous exercises, we observe that agents still prefer the debt-intensive fiscal rule on the aggregate level.

# 5.5 Concluding remarks

In this paper we employed a heterogeneous-agents, incomplete-markets model, which we calibrated to match key characteristics of the US economy. We used the model to analyze the influence of debt policy on the economy's response to an increase in government spending. We found that a debt-intensive fiscal rule that leads to a temporary strong increase in public debt has a stabilizing effect on consumption and leisure in the first periods following the change in government spending. In contrast, a non-debt-intensive fiscal rule that keeps the primary deficit small leads to a faster recovery of consumption, leisure, capital and output in later periods of transition. The observed differences in the adjustment paths of aggregate variables can be traced back to the different response of wealth-poor agents to a change in either the lump-sum tax or in government debt. We also studied the individual and aggregate welfare effects to determine the optimal fiscal rule and to identify those members of society who benefit most from implementing the optimal fiscal rule. We found that the debt-intensive fiscal rule leads to the largest aggregate welfare benefit and receives support from the vast majority of agents, the benefits being particularly high for wealth-poor agents with low productivity. Thus, from a political economy perspective, our results provide some rationale for the observed pattern of public debt. However, one has to also bear in mind that there are a number of factors that counteract a too optimistic view of high public debt. First, the differences in aggregate welfare between the fiscal rules are quantitatively rather small. Hence, if other elements that are commonly associated with a high level of public debt, e.g. an increase in the default risk, play a dominant role, the benefits of implementing the debt-intensive fiscal rule may be overturned. Second, there is a conflict between two potential targets of fiscal policy. While the debt-intensive fiscal rule in our benchmark economy leads to the larger aggregate welfare benefit, the non-debt-intensive fiscal rule leads to the larger fiscal output multiplier. Dependent on the weight of each target, a more balanced time path of public debt and of taxes may be preferable. Finally, our model economy shows a rather simple structure with respect to the available tax instruments. While the simple framework allows us to highlight the interplay of debt policy and financial market imperfections, it certainly does not cover all aspects that may influence the design of optimal debt policy. As the related literature shows, optimal debt policy in heterogeneous-agents economies can vary greatly. Nevertheless, despite all these limitations and cautious interpretations, our results show that when isolating the interplay of debt policy and financial market imperfections, a temporary increase in public debt in response to an increase in government spending may be justified on the grounds of optimal debt policy.

# 5.6 Appendix

## **Numerical Methods**

Solving the model requires to compute the initial stationary equilibrium and the adjustment paths of aggregate variables following the government spending shock. We use standard methods for both parts. To compute the stationary equilibrium, we start with an initial guess of the interest rate and wage rate. Next, we place a grid over the state space of current asset holdings with more grid points being allocated to lower levels of financial wealth. The agent's policy functions for consumption, next period's asset holdings and labor supply are computed by Euler equation based policy function iteration.<sup>30</sup> With the policy functions at hand, we compute the invariant distribution of agents over asset holdings and productivity to obtain the new values of aggregate variables. Finally, we update the guess of the interest rate and wage rate and repeat the procedure until markets clear, except for a tolerably small approximation error. Once the stationary equilibrium has been recovered, we can compute the economy's response to the government spending shock. For this, we firstly specify the government spending shock and choose the number of transition periods, T. We set T sufficiently large to allow the economy to return to the steady state equilibrium in T periods. Next, we make an initial guess for the time paths of the interest rate and wage rate and compute the sequence of policy functions by iterating backwards in time, starting from period t = T-1. The sequence of distributions and of aggregate variables are computed by moving forward in time. Finally, the initial guess for the time paths of the interest rate and wage rate are updated and the procedure is repeated until markets clear at each point in time, except for a tolerably small approximation error.

 $<sup>^{30}</sup>$ See Coleman (1990) and Deaton (1991) for an exposition of the approach of iterating on the Euler equation.

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# Eidesstattliche Erklärung

Ich versichere an Eides statt, dass meine hinsichtlich der früheren Teilnahme an Promotionsverfahren gemachten Angaben richtig sind und, dass die eingereichte Arbeit oder wesentliche Teile derselben in keinem anderen Verfahren zur Erlangung eines akademischen Grades vorgelegt worden sind. Ich versichere darüber hinaus, dass bei der Anfertigung der Dissertation die Grundsätze zur Sicherung guter wissenschaftlicher Praxis der DFG eingehalten wurden, die Dissertation selbstständig und ohne fremde Hilfe verfasst wurde, andere als die von mir angegebenen Quellen und Hilfsmittel nicht benutzt worden sind und die den benutzten Werken wörtlich oder sinngemäß entnommenen Stellen als solche kenntlich gemacht wurden. Einer Überprüfung der eingereichten Dissertation bzw. der an dieser Stelle eingereichten Schriften mittels einer Plagiatssoftware stimme ich zu.