Carsten Henkel, Jean-Yves Courtois, Robin Kaiser,
C. Westbrook, Alain Aspect

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Phase Shifts of Atomic de Broglie Waves at an Evanescent Wave Mirror

C. Henkel*, J.-Y. Courtois, R. Kaiser, C. Westbrook, and A. Aspect
Institut d'Optique Théorique et Appliquée, B.P. 147, F-91403 Orsay, France
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Abstract – A detailed theoretical investigation of the ref ection of an atomic de Broglie wave at an evanescent wave mirror is presented. The classical and the semiclassical descriptions of the ref ection process are reviewed, and a full wave-mechanical approach based on the analytical soution of the corresponding Schrödinger equation is presented. The phase shift at ref ection is calculated exactly and interpreted in terms of instantaneous ref ection of the atom at an effective mirror. Besides the semiclassical regime of ref ection describable by the WKB method, a pure quantum regime of ref ection is identif ed in the limit where the incident de Broglie wavelength is large compared to the evanescent wave decay length.

1. INTRODUCTION

Atomic mirrors are one of the key components in the f eld of atom optics [1]. In order to realize such a device, the use of evanescent waves appears very promising [2, 3]. In view of the future applications fo evanescent wave mirrors, a detailed investigation of their optical properties seems appropriate. For some basic purposes (for example, the def ection of an atomic beam), the characterization of the geometric optical properties of the evanescent wave mirror is suff cient. This can be achieved by treating the incident atom as classical particles and by deriving their classical trajectories (this is analogous to the calculation of light rays in conventional optics). For more elaborate purposes (atom interferometers [4], atomic cavities [5, 6]), knowledge of the wave-mechanical properties of the evanescent wave mirror is also required. One then needs to describe the atom by a de Broglie wave in order to estimate the phase shift experienced by the atom during ref ection at the mirror. A semiclassical derivation of this phase shift, based on the evaluation of the action integral along the classical atomic trajctories (WKB method), has been given by Opat *et al.* [7]. We present in this paper a more complete approach based on the analytical solution of the Schrödinger equation describing the interaction between the atom and the evanescent wave mirror in the regime of coherent atom optics (limit of negligible spontaneous emission). We interpret the atomic phase shift derived from the atomic wave function in terms of instantaneous ref ection at an effective mirror, which generalizes the one introduced in [7]. We distinguish between a semiclassical and a quantum regime of ref ection. In the semiclassical regime, realized at high incident energy, the Schrödinger and the WKB approach coincide, and the evanescent wave mirror behaves as a dephasing dispersive mirror, analogous to a dielectric mirror in conventional optics. By contrast, in the quantum regime of ref ection where the incident atomic de Broglie wavelength is large compared to the evanescent potential decay length, the evanescent wave mirror acts as a nondispersive inf nitely steep barrier, analogous to a metallic mirror in conventional optics. Finally, the ref ection process of an atomic wave packet incident on an evanescent wave mirror is discussed.

2. CLASSICAL AND SEMICLASSICAL DESCRIPTIONS OF THE REFLECTION PROCESS

Before turning to the full wave-mechanical treatment of atomic ref ection at an evanescent wave mirror, we start by reviewing the classical dynamics as well as the WKB description of the ref ection process. This will allow us to make a clear distinction between the semiclassical and the pure quantum features of atomic ref ection.

2.1. Presentation of the Model

We consider the simple case of a two-level atom normally incident of the surface ($z = 0$ plane) of an evanescent wave mirror.[1] Because we are interested in the regime of coherent

*Presently at Institut für Physik, Universität Potsdam, 14469 Potsdam, Germany, email Carsten.Henkel@quantum.physik.uni-potsdam.de
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[1]Because of the translational symmetry in the directions parallel to the mirror surface, the problem can be reduced to one dimension. This simplif cation holds for both the classical and the quantum viewpoints.

atom optics (limit of negligible spontaneous emission), we restrict ourselves to the limit of low saturation of the atomic transition, where the reactive part of the atom-evanescent wave coupling (light shift) is predominant over the dissipative part. We also assume that the detuning between the evanescent wave and the atomic frequency is properly chosen so that the atom ca be considered to follow adiabatically the optical potential associated with the light-shifted ground-state level. In this regime, all the physical phenomena can be accounted for by means of the Hamiltonian [3, 7]:

$$H = \frac{p^2}{2M} + \frac{p_{\max}^2}{2M}\exp(-2\kappa z), \quad (1)$$

which contains the atomic kinetic energy (f rst term) and the reactive part of the atom-f eld coupling (second term). In equation (1), p and $z \geq 0$ are the momentum and position of the atomic center of mass, M is the atomic mass, $p_{\max} > 0$ is the maximum momentum that can be ref ected by the optical potential barrier, and κ^{-1} is the characteristic decay length of the evanescent wave, of the order of the laser wavelength[2] (for a discussion of typical experimental parameters, see Appendix A). Note that when quantizing the atomic external degrees of freedom, one has to substitute the momentum and position operators P and Z for p and z in equation (1). The potential in equation (1) grows exponentially as $z \to -\infty$. We thus neglect any effects due to tunneling through the potential barrier to the physical mirror surfce.

2.2. Classical Dynamics of the Ref ection Process

Let us f rst consider the incident atom as a classical particle with asymptotic momentum $-p_\infty (0 < p_\infty < p_{\max})$. With an appropriate choice of time origin, the classical trajectory of the atom can be written [7] (Fig. 1):

$$z(t) = z_0 + \kappa^{-1}\ln\cosh(t/\tau_{\text{refl}}), \quad (2)$$

where

$$z_0 = \kappa^{-1}\ln(p_{\max}/p_\infty) \quad (3)$$

is the position of the turning point of the turning point of the trajectory (reached at $t = 0$), located about κ^{-1} in front of the mirror surface, and where

$$\tau_{\text{refl}} = M/\kappa p_\infty \quad (4)$$

is the time scale for the ref ection process and corresponds to the time taken to cross the thickness κ^{-1} of the optical potential at the asymptotic velocity p_∞/M (see Appendix A, the table for typical experimental values).

In the asymptotic region $z \gg \kappa^{-1}$ of vanishing optical potential, the atom is moving freely at constant velocity $\mp p_\infty/M$ along the asymptotes of the classical trajectory (see Fig. 1). These straight asymptotes intersect at the position

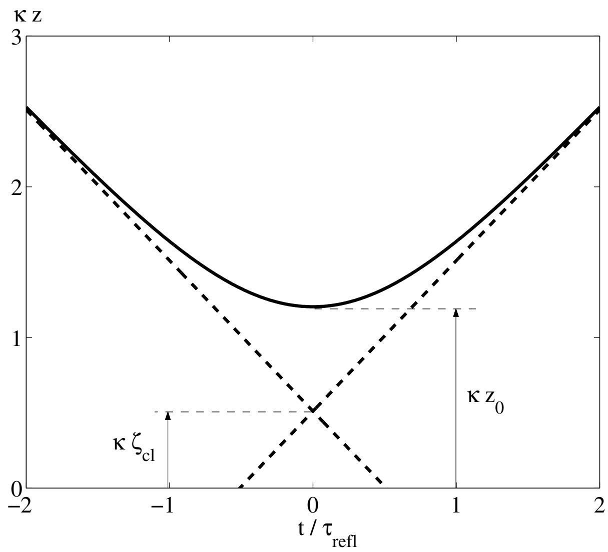

FIG. 1. Classical trajectory of an atom undergoing specular reglection at an evanescent wave mirror. The dimensionless atomic position κz is represented vs. t/τ_{refl} for the parameters $p_\infty = 3\hbar\kappa$ and $p_{\max} = 10\hbar\kappa$. The atom approaches the mirror surface at the minimum distance $z_0 \approx 1.2\kappa^{-1}$ and is ref ected on a time scale of the order of τ_{refl}. In the asymptotic region ($\kappa z \gg 1$) of vanishing optical potential, the atom propagates freely at the velocity $\pm p_\infty/M$, and the classical trajectory corresponds to straight asymptotes (dashed lines), which intersect at the position $\zeta_{\text{cl}} \approx 0.5\kappa^{-1}$.

$$\zeta_{\text{cl}}(p_\infty) = z_0(p_\infty) - \kappa^{-1}\ln 2, \quad (5)$$

which is shifted more deeply into the potential relative to z_0 by a quantity independent of the incident momentum (see Fig. 1). As far as the asymptotic classical dynamics of the atom is concerned, the evanescent wave mirror thus behaves as a f ctitious inf nitely steep barrier located at $\zeta_{\text{cl}}(p_\infty)$, at which the atom would experience an instantaneous ref ection. We name this barrier the classical effective mirror after [7].

2.3. The WKB Solution for the Evanescent Wave Mirror

Let us now consider the semiclassical description of the ref ection process. In this case, the atom is described by means of a wave function derived using the WKB approximation. In the classical allowed region $z > z_0$, this WKB wave function is given by [8]:

$$\psi_{\text{WKB}}(z) = \sqrt{\frac{4M}{p(z)}}\sin\left(\frac{\pi}{4} + \frac{1}{\hbar}\int_{z_0}^{z} p(z')dz'\right) \quad (6)$$

$$= \sqrt{\frac{4M}{p(z)}}\sin\left[\frac{\pi}{4} + \frac{p_\infty}{\hbar\kappa}(\text{artanh}(p(z)/p_\infty) - p(z)/p_\infty)\right],$$

[2]Note that, this estimate breaks down near the critical angle for the evanescent laser wave. In this case, the length scale κ^{-1} tends to inf nity.

where $p(z)$ is the classical momentum calculated from energy conservation:

$$p(z)^2 + p_{\max}^2 \exp(-2\kappa z) = p_\infty^2, \tag{7}$$

and where the phase $\pi/4$ results from the WKB connection formula, which matches the oscillating part of the wave function (6) to the decaying part in the classically forbidden region $z < z_0$. The normalization of the WKB wave function (6) has been chosen such that the incident and ref ected waves both have unit f ux independent of the asymptotic momentum.

In the asymptotic region, the atomic wave function is a superposition of two plane waves with wave vectors $k_\infty = \mp p_\infty/\hbar$, which correspond to the incident and ref ected waves. The phase shift that we are interested in at ref ection $\Delta\varphi_{\mathrm{WKB}}$ is related to the relative phase between these two plane waves. We def ne $\Delta\varphi_{\mathrm{WKB}}$ by writing the asymptotic WKB wave function in the form:

$$z \to +\infty : \psi_{\mathrm{WKB}}(z) \cong \sqrt{\frac{4M}{p_\infty}} \sin\left(\frac{1}{\hbar} p_\infty z + \frac{1}{2}\Delta\varphi_{\mathrm{WKB}}\right). \tag{8}$$

This def nition of the phase shift takes as reference ($\Delta\varphi_{\mathrm{WKB}} = 0$) a standing wave in front of an inf nitely steep barrier located at the origin $z = 0$. It is important to note that this def nition is somewhat arbitrary. In [7], for example, the reference is a standing wave in fromt of an inf nitely steep barrier located at the position $z = \zeta_{\mathrm{cl}}$ of the effective classical mirror [equation (5)]. Because ζ_{cl} depends on the incident atomic momentum, however, this phase reference is not absolute. With our def nition of the phase shift, $\Delta\varphi_{\mathrm{WKB}}$ is the phase correction in the situation where the evanescent optical potential is approximated by an inf nitely steep barrier located at the mirror surface [6]. By writing the asymptotic expansion of the WKB wave function (6) in the form (8), one obtains [7]

$$\begin{aligned} \Delta\varphi_{\mathrm{WKB}}(p_\infty) &= \frac{\pi}{2} - 2\frac{p_\infty}{\hbar\kappa}\left[1 + \ln\left(\frac{p_{\max}}{2p_\infty}\right)\right] \\ &= \delta\varphi_{\mathrm{WKB}} - 2p_\infty\zeta_{\mathrm{WKB}}/\hbar \end{aligned} \tag{9}$$

with

$$\delta\varphi_{\mathrm{WKB}} = \pi/2 \tag{10a}$$
$$\zeta_{\mathrm{WKB}}(p_\infty) = \zeta_{\mathrm{cl}}(p_\infty) + \kappa^{-1}. \tag{10b}$$

The order of magnitude of the WKB phase shift (9) is given by the ratio $p_\infty/\hbar\kappa$, which represents the phase shift associated with the free propagation of an atom of momentum p_∞, through the spatial extent κ^{-1} of the evanescent optical potential.

The physical interpretation of (10) becomes transparent if we write the asymptotic WKB wave function (8) as[3]

$$z \to +\infty : \quad \psi_{\mathrm{WKB}}(z) \cong \sqrt{\frac{4M}{p_\infty}} \sin\left(\frac{1}{2}\delta\varphi_{\mathrm{WKB}} + \frac{1}{\hbar}p_\infty(z - \zeta_{\mathrm{WKB}})\right), \tag{11}$$

which corresponds to a plane standing wave whose phase is f xed to the value $\frac{1}{2}\delta\varphi_{\mathrm{WKB}}$ at $z = \zeta_{\mathrm{WKB}}$. As far as the asymptotic WKB wave function is concerned, the evanescent wave mirror is thus equivalent to an effective dephasing mirror located at the position ζ_{WKB}, where the atomic wave is instantaneously ref ected and phase shifted by the amount $\delta\varphi_{\mathrm{WKB}}$ (as light on a mirror). By analogy with the classical case (Section 2.2), we call this mirror the WKB effective mirror. The dephasing character of this mirror results from the WKB phase factor $\pi/4$ and thus has a nonclassical origin. It holds as long as the WKB approximation remains valid, i.e., as long as the incident de Broglie wavelength is small compared to the decay length κ^{-1} of the evanescent optical potential [8]. Furthermore, the evanescent wave mirror is dispersive because of the dependence of ζ_{WKB} on p_∞. More precisely, the condition for the mirror to be dispersive is that $\partial^2\Delta\varphi_{WKB}/\partial p_\infty^2 \neq 0$: a linear dependence of $\Delta\varphi_{\mathrm{WKB}}$ on p_∞ can always be removed by an appropriate choice of absolute phase reference. Finally, it is interesting to note that the position ζ_{WKB} [equation (10b)] of the WKB effective mirror differs from the classical effective mirror position ζ_{cl} [equation (5)] by the quantity κ^{-1} independent of the incident atomic momentum. As a result, the classical and the WKB description of the ref ection process yield comparable physical pictures.

3. SCHRÖDINGER WAVE FUNCTION APPROACH

We now turn to the full wave-mechanical description of atomic ref ection at the evanescent wave mirror. This description is based on the analytical solution of the corresponding Schrödinger equation, which allows an exact calculation of the phase shift at ref ection.

3.1. Solution of the Stationary Schrödinger Equation

The full quantum description of atomic ref ection consists in solving exactly the stationary Schrödinger equation for the atomic wave function $\psi(z)$:

$$\left(-\hbar^2\frac{d^2}{dz^2} + p_{\max}^2\exp(-2\kappa z)\right)\psi(z) = p_\infty^2\psi(z). \tag{12}$$

We use the change of variable

[3]This decomposition separates the phase shift into a constant and an essentially linear term. Note that such an interpretation is not always unambiguous because one has to decompose $\Delta\varphi(p_\infty) = \delta\varphi - 2p_\infty\zeta/\hbar$, where $\delta\varphi \in [0, 2\pi]$ and ζ are weakly dependent on p_∞.

$$z \to u = \frac{p_{\max}}{\hbar\kappa}\exp(-\kappa z) \quad (13)$$

which takes advantage of the invariance of the Hamiltonian (1) under the transformation

$$\forall a, \qquad \begin{cases} z \to z + a \\ p_{\max} \to e^{2\kappa a} p_{\max} \end{cases}. \quad (14)$$

Equation (12) transforms into a Bessel-type equation:

$$\left(u^2 \frac{d^2}{du^2} + u\frac{d}{du} - (u^2 - \alpha^2)\right)\psi(u) = 0, \quad (15)$$

which only depends on one dimensionless parameter:

$$\alpha = p_\infty/\hbar\kappa. \quad (16)$$

The solutions of (15) are linear combinations of the Bessel functions $I_{\pm i\alpha}(u)$. Two boundary conditions impose a unique solution:

(i) The wave function must vanish in the limit $z \to -\infty$ (the probability of the atoms being in the region $z \le z_0$ inside the potential being small compared to the probability of being in the classically allowed region $z \ge z_0$).

(ii) In the asymptotic region $z \to +\infty$, the wave function is normalized in the same way as the WKB solution [equation (6)].

As shown in Appendix B, these conditions lead to the solution[4]

$$\psi_{\rm Schr}(z) = \sqrt{\frac{4M}{p_\infty}\frac{\pi\alpha}{\sinh(\pi\alpha)}}\frac{1}{2i}(I_{-i\alpha}(u(z)) - I_{i\alpha}(u(z))). \quad (17)$$

The Schrödinger wave function (17) and the corresponding WKB wave function (6) are represented in Fig. 2 as a function of the dimensionless parameter κz, in the case $p_\infty = 3\hbar\kappa$ and $p_{\max} = 10\hbar\kappa$. One sees that the wave functions are in good agreement in both the asymptotic region ($\kappa z \gg 1$) and far inside the optical potential ($\kappa z \ll 1$), but that they significantly differ around the classical turning point $\kappa z_0 \cong 1.2$ (where the WKB wave function actually diverges).

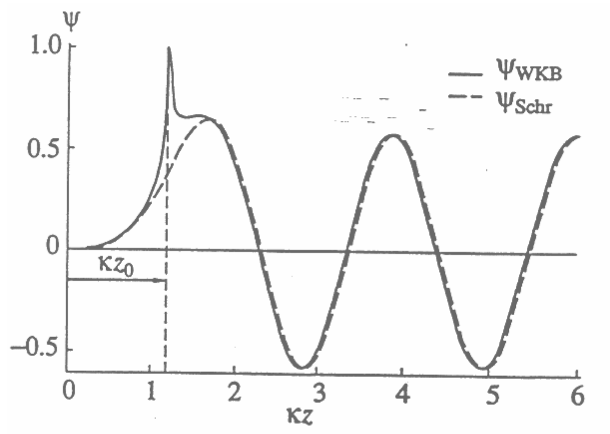

FIG. 2. Comparison between the WKB ($\psi_{\rm WKB}$) and the Schrödinger ($\psi_{\rm Schr}$) wave functions for the same parameters as in Fig.1. These wave functions coincide both in the asyptotic region ($\kappa z \gg 1$) and far inside the optical potential ($\kappa z \ll 1$) but significantly differ around the classical turning point z_0, where $\psi_{\rm WKB}$ diverges.

3.2. Phase Shift of the Schrödinger Wave Function

The exact solution (17) of the Schrödinger equation allows us to derive exactly the phase shift experienced by the atomic wave function at reflection on the evanescent wave mirror.

Following the definition (8) of the WKB phase shift at reflection, we define the Schrödinger phase shift $\Delta\varphi_{\rm Schr}$ by writing the asymptotic Schrödinger wave function (17) in the form:

$$z \to +\infty : \psi_{\rm Schr}(z) \cong \sqrt{\frac{4M}{p_\infty}}\sin\left(\frac{1}{\hbar}p_\infty z + \frac{1}{2}\Delta\varphi_{\rm Schr}\right). \quad (18)$$

By using (17) and the asymptotic expansion ($u \to 0$) of the Bessel functions $I_{\pm i\alpha}(u)$, one obtains (see Appendix B)

$$\Delta\varphi_{\rm Schr}(p_\infty) = -2\alpha\ln\left(\frac{p_{\max}}{2\hbar\kappa}\right) + 2\arg\Gamma(1 + i\alpha), \quad (19)$$

where Γ is the Euler gamma (factorial) function [9], and where $\arg\Gamma(1 + i\alpha)$ is the argument of the complex number $\Gamma(1 + i\alpha)$ defined as a continuous function of α.

The exact ($\Delta\varphi_{\rm Schr}$) and the semiclassical ($\Delta\varphi_{\rm WKB}$) phase shifts at reflection are represented in Fig. 3 as a function of the dimensionless parameter $\alpha = p_\infty/\hbar\kappa$. One can clearly distinguish between two limiting cases.

In the limit $\alpha \gg 1$ (high incident momentum), where the incident atomic de Broglie wavelength is small compared to the decay length κ^{-1} of the optical potential, the WKB and the Schrödinger approaches yield comparable phase shifts [8].

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[4] We have neglected any loss resulting from atomic tunneling to the mirror surface. However, the tunneling probability can be estimated by the flux of the wave function at $z = 0$. *Note added after publication:* To our knowledge, the exact solution (17) for the exponential barrier has been first derived by J. M. Jackson and N. F. Mott in 1932, *Proc. Roy. Soc. (London) Ser. A* **137**, 703.

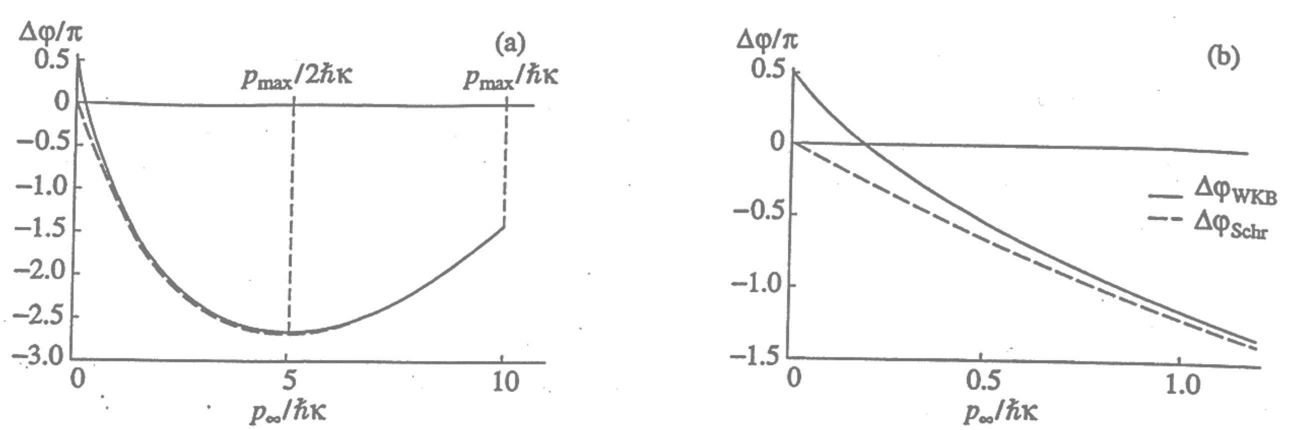

FIG. 3. Dependence of the WKB ($\Delta\varphi_{\mathrm{WKB}}$) and of the Schrödinger ($\Delta\varphi_{\mathrm{Schr}}$) phase shifts vs. the dimensionless incident atomic momentum $\alpha = p_\infty/\hbar\kappa$ for $p_{\max} = 10\hbar\kappa$. (a) $\Delta\varphi_{\mathrm{WKB}}$ and $\Delta\varphi_{\mathrm{Schr}}$ coincide in the limit of high incident atomic momentum ($\alpha \gg 1$), which thus corresponds to a semiclassical regime of ref ection. (b) In the limit of low incident atomic momentum ($\alpha \ll 1$), the semiclassical description of the evanescent wave mirror does not yield the correct phase shift at ref ection. This corresponds to a pure quantum regime of ref ection.

This corresponds to the semiclassical regime of atomic ref ection considered in Section 2.3. Using equation (19), it is possible to derive the f rst correction to the semiclassical phase shift. One thus f nds [9]

$$\alpha \gg 1 : \Delta\varphi_{\mathrm{Schr}}(p_\infty) = \Delta\varphi_{\mathrm{WKB}} - \frac{1}{6\alpha} + O(\alpha^{-3}). \quad (20)$$

In the limit $\alpha \ll 1$ (low incident momentum), where the incident atomic de Broglie wavelength is larger than the decay length of the evanescent optical potential, the WKB and the Schrödinger approaches yield different phase shifts. In particular, in the limit $\alpha \to 0^+$, $\Delta\varphi_{\mathrm{WKB}}$ tends toward $\pi/2$, whereas $\Delta\varphi_{\mathrm{Schr}}$ tends toward 0 (see Fig. 3b). The limit of low incident momentum thus corresponds to a pure quantum regime of ref ection, which can not be appropriately described in semiclassical terms. In order to obtain in this regime a representation of the evanescent wave mirror in terms of an effective mirror, we write equation (19) in a form analogous to (9). One thus f nds [9]

$$\begin{aligned} &\alpha \ll 1 : \\ &\Delta\varphi_{\mathrm{Schr}} \cong -2\alpha\left(\gamma + \ln\left(\frac{p_{\max}}{2\hbar\kappa}\right)\right) \\ &\quad = \delta\varphi_{\mathrm{Schr}} - 2p_\infty\zeta_{\mathrm{Schr}}/\hbar \end{aligned} \quad (21)$$

with the Euler constant $\gamma \cong 0.577$ and

$$\delta\varphi_{\mathrm{Schr}} \cong 0 \quad (22a)$$

$$\zeta_{\mathrm{Schr}}(p_\infty \ll \hbar\kappa) \cong \zeta_{\mathrm{cl}}(\hbar\kappa) + \gamma\kappa^{-1}. \quad (22b)$$

Equations (21) and (22) show that, in the quantum regime of ref ection, the evanescent wave mirror behaves as a nondephasing effective mirror located at the position $z \approx \zeta_{\mathrm{cl}}(\hbar\kappa)$ [equation (22b)], where the atomic wave is instantaneously ref ected (as light on a metallic mirror). The nondephasing character of the Schödinger effective mirror [equation (22a)] results from the fact that, on the spatial scale of the incident de Broglie wavelength, the evanescent optical potential appears to be an inf nitely steep barrier. In the quantum regime of ref ection, it is therefore legitimate to approximate the evanescent wave mirror by a hard barrier located at the position of the classical effective mirror for an asymptotic momentum $p_\infty \approx \hbar\kappa$. It is also interesting to note that, contrary to the semiclassical case, the position ζ_{Schr} of the Schrödinger effective mirror is essentially independent of p_∞. As a result, the evanescent wave mirror is no longer dispersive in the quantum regime.[5]

4. REFLECTION OF AN ATOMIC WAVE PACKET

In the experiments using effusive beams as a source of atoms, it is possible to describe the incident particles in terms of statistical mixtures of de Broglie waves having a welldef ned momentum (plane waves). In that case, the ref ection process at the evanescent wave mirror can be directly characterized using the results of the preceding sections. However, in some other situations (for example, when the incident atoms originate from an optical molasses where atom localization takes place [10]), it is more appropriate to describe the particles in terms of wave packets (superpositions of plane waves). In such a case, each partial plane wave of incident momentum p_∞ experiences a different phase shift at ref ection $\Delta\varphi_{\mathrm{Schr}}(p_\infty)$ (as given in Section 3.2), which shows up in a spatial shift of the center of the wave packet.

[5]It is perhaps surprising that the evanescent wave mirror is not despersive while the Schrödinger phase shift (21) depends linearly on the incident momentum. In fact, this dependence is related to the choice of reference for the phase shift. Thus, by taking as phase reference a standing wave in front of an inf nitely steep barrier located in $z \cong \zeta_{\mathrm{cl}}(\hbar\kappa) + \gamma\kappa^{-1}$, the phase shift at ref ection in the quantum regime would be independent of the incident momentum.

Let us consider an atomic wave packet incident on an evanescent wave mirror. In the asymptotic region, one may write the incident part of the atomic wave function $\psi_{\rm inc}(z,t)$ as

$$\psi_{\rm inc}(z,t) = \int dp_\infty \tilde{\psi}_{\rm inc}(p_\infty) \exp\left(-i\frac{p_\infty^2 t}{2M\hbar} - i\frac{p_\infty z}{\hbar}\right), \tag{23}$$

where $\tilde{\psi}_{\rm inc}(p_\infty)$ denotes the Fourier transform of $\psi_{\rm inc}$. During the ref ection process, each partial plane wave experiences a different phase shift $\Delta\varphi_{\rm Schr}(p_\infty)$ [equation (19)], so that in the asymptotic region, the ref ection part of the atomic wave function $\psi_{\rm ref}(z,t)$ reads

$$\psi_{\rm ref}(z,t) = -\int dp_\infty \tilde{\psi}_{\rm inc}(p_\infty) \times \times \exp\left(-i\frac{p_\infty^2 t}{2M\hbar} + i\frac{p_\infty z}{\hbar} + i\Delta\varphi_{\rm Schr}(p_\infty)\right), \tag{24}$$

where the minus sign in front of the integral results from our peculiar choise of phase origin.

By assuming that $\tilde{\psi}_{\rm inc}(p_\infty)$ is peaked around the average momentum $\bar{p}_\infty$, it is possible to characterize the position $z_{\rm wp}(t)$ of the center of the wave packet (23) or (24) via the method of stationary phase. One readily f nds

$$z_{\rm wp}^{\rm inc}(t) = -\frac{\bar{p}_\infty}{M}t, \tag{25a}$$

$$z_{\rm wp}^{\rm ref}(t) = -\frac{\bar{p}_\infty}{M}t - \hbar\left(\frac{\partial\Delta\varphi_{\rm Schr}(p_\infty)}{\partial p_\infty}\right)_{\bar{p}_\infty}, \tag{25b}$$

where $z_{\rm wp}^{\rm inc}(z_{\rm wp}^{\rm ref})$ denotes the position of the center of the incident (ref ected) wave packet. Equation (25) shows that, as far as the asymptotic wave packets are concerned, the evanescent wave mirror behaves as an inf nitely steep effective mirror located at the position $\zeta_{\rm wp}$ given by

$$\zeta_{\rm wp} = -\frac{\hbar}{2}\left(\frac{\partial\Delta\varphi_{\rm Schr}}{\partial p_\infty}\right)_{\bar{p}_\infty}. \tag{26}$$

Substituting for $\Delta\varphi_{\rm Schr}$ in (26) using (19) gives

$$\zeta_{\rm wp} = \kappa^{-1}(\ln(p_{\rm max}/2\hbar\kappa) - Re\Psi(1+i\bar{\alpha})), \tag{27}$$

where $\bar{\alpha} = \bar{p}_\infty/\hbar\kappa$ is the dimensionless parameter given by equation (16), and where Ψ is the digamma function def ned as

$$\Psi(x) = \partial\ln\Gamma(x)/\partial x. \tag{28}$$

As in the preceding section, we distinguish between two limiting regimes of ref ection of the atomic wave packet.

In the limit $\bar{\alpha} \gg 1$ (semiclassical regime of ref ection, see Section 3.2), where equation (27) reduces to

$$\bar{\alpha} \gg 1 : \zeta_{\rm wp} = \zeta_{\rm cl}(\bar{p}_\infty) + O(1/\bar{\alpha}^2), \tag{29}$$

the asymptotic wave packet appears to be instantaneously ref ected at an effective mirror located in $z = \zeta_{\rm cl}$ [equation (5)]. The ref ection process is thus analogous to that of a classical particle of incident momentum $\bar{p}_\infty$.[6] Note that the position of the effective mirror for the atomic wave packet corresponds to $\zeta_{\rm cl}$, and not to $\zeta_{\rm WKB}$ [equation (10b)].

In the limit $\bar{\alpha} \ll 1$ (quantum regime of ref ection), where equation (27) reads

$$\bar{\alpha} \ll 1 : \zeta_{\rm wp} \cong \zeta_{\rm Schr}(\bar{\alpha} \ll 1) \cong \zeta_{\rm cl}(\hbar\kappa) + \gamma\kappa^{-1}, \tag{30}$$

the evanescent wave mirror behaves as an inf nitely steep potential barrier located at the position $z \approx \zeta_{\rm cl}(\hbar\kappa) + \gamma\kappa^{-1}$, which instantaneously ref ects the atomic wave packet. The fact that this position coincides with that of the effective mirror for an atomic plane wave of incident momentum $\bar{p}_\infty$ [equation (22b)] is not surprising because in the quantum regime, the position of the effective mirror $\zeta_{\rm Schr}$ is independent of the incident atomic momentum (see Section 3.2). As a result, all partial plane waves of the wave packet are ref ected at the same barrier, and the wave packet behaves in the same way.

The position $\zeta_{\rm wp}$ of the effective mirror describing the ref ection process of the wave packet at the evanescent wave mirror is represented in Fig. 4 together with $\zeta_{\rm cl}$ as a function of the dimensionless parameter $\bar{\alpha} = \bar{p}_\infty/\hbar\kappa$. One clearly distinguishes between the semiclassical and the quantum regime of ref ection of the wave packet.

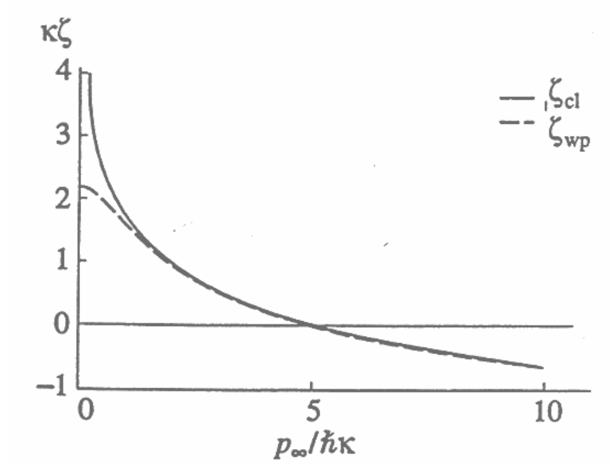

FIG. 4. Dependence of the positions of the wave packet ($\zeta_{\rm wp}$) and the classical ($\zeta_{\rm cl}$) effective mirrors vs. the dimensionless average incident momentum of the wave packet $\bar{\alpha} = \bar{p}_\infty/\hbar\kappa$ for $p_{\rm max} = 10\hbar\kappa$. $\zeta_{\rm cl}$ and $\zeta_{\rm wp}$ coincide in the semiclassical regime of ref ection ($\bar{\alpha} \gg 1$). By contrast, in the quantum regime of ref ection ($\bar{\alpha} \ll 1$), $\zeta_{\rm wp}$ tends toward a constant, whereas $\zeta_{\rm cl}$ tends to inf nity as $\bar{\alpha} \to 0^+$.

[6]Within the framework of the WKB approach, it can be shown that this result holds for any mirror potential vanishing at large z.

5. CONCLUSION

We have presented a detailed theoretical investigation of the ref ection process of an atomic de Broglie wave at an evanescent wave mirror in the regime of coherent atom optics. Our calculation of the atomic phase shift at ref ection using the exact solution of the corresponding Schrödinger equation has allowed us to identify two limiting regimes of ref ection. The semiclassical regime corresponds to incident de Broglie wavelengths much smaller than the decay length of the evanescent optical potential and can be satisfactorily accounted for by the WKB method. The evanescent wave mirror then behaves as a despersive dephasing mirror. In the quantum regime of ref ection, where the incident atomic de Broglie wavelength is larger than the decay length of the evanescent potential, the evanescent wave mirror behaves as a nondispersive hard potential barrier located in front of the actual evanescent wave mirror surface.

In experiments using either a supersonic beam or laser-cooled atoms accelerated by the earth gravity f eld, the minimum achievable atomic incident momentum is typically of the order of $10\hbar\kappa$. Under such conditions, the atomic ref ection process can always be accounted for in semiclassical terms. However, the experimental observation of atomic ref ection in the quantum regime seems feasible, using, for example, an evanescent wave mirror located at the summit of an atomic fountain where the atomic momentum approaches zero. Another, more challenging possibility would be to investigate the lowest bouncing modes of a gravitational cavity [6], which correspond to a de Broglie wavelength of the order of the decay length of the evanescent optical potential, and thus realize the quantum regime of ref ection for the bouncing atoms.

The exact derivation of the atomic wave function presented in this paper [equation(17)] can serve as a starting point for the investigation of many other effects. These include tunneling through the optical potential barrier, the inf uence of atomic internal states on the ref ection process, and, especially interesting, dipole-surface effects (such as the Van der Waals interaction), which may modify the potential we have assumed here, and hence also the phase shift at ref ection.

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APPENDIX A: EXPERIMENTAL INVESTIGATION OF THE REFLECTION PROCESS

We would like to comment in this appendix on the typical experimental parameters required to observe atomic ref ection on an evanescent wave mirror. The regime of coherent atom optics is realized provided that the probability of spontaneous emission during ref ection is negligible. It corresponds to the limit of small saturation of the atomic transition:

$$s = \frac{\Omega^2/2}{\Delta^2 + \Gamma^2/4} \ll 1, \tag{A1}$$

where $\Omega = -dE_0/\hbar$ is the resonant Rabi frequency that characterizes the coupling between the atomic dipole d and the evanescent f eld of maximum amplitude E_0 (at the evanescent wave mirror surface), Γ is the natural width of the atomic excited state, and $\Delta = \omega - \omega_A$ is the detuning between the frequency (ω) of the evanescent wave and the atomic (ω_A) frequency. It also requires that the incident atom follows adiabatically the optical potential associated with the light-shifted ground-state level. This is achieved in the limit of large detuning from resonance (we assume $\Delta > 0$, which allows an atom entering the optical potential in the ground-state to be ref ected at the evanescent wave mirror):

$$\Delta \gg \Gamma \tag{A2}$$

In the regime where (A1) and (A2) are fulf lled, it is possible to describe the atom ref ection process by means of the Hamiltonian (1). Note, however, that for a given laser intensity, the maximum ref ectible atomic momentum $p_{\max}$ decreases as the frequency detuning increases, as shown by the relation

$$\frac{p_{\max}^2}{2M} = \frac{1}{2}\hbar\Delta s \tag{A3}$$

In fact, by designing the evanescent wave mirror with the multilayer coating technique [12], it is possible fo fulf ll conditions (A1) and (A2) while simultaneously being able to ref ect atoms having high incident momentum ($p_{\max} \gg \hbar\kappa$). Typical experimental parameters are indicated in the table in the case of the D2 line of ^{85}Rb.

Typical experimental parameters for atomic ref ection at an evanescent wave mirror using the D2 line of ^{85}Rb atoms

Physical parameters	Notation	Typical value
Laser wavelength		780 nm
Natural linewidth	Γ	6 MHz
Frequency detuning from resonance	Δ	$5 \times 10^4\,\Gamma$
Maximum intensity of the evanescent wave	E_0^2	10^4 W/cm^2
Saturation parameter	s	6×10^{-4}
Maximal ref ected momentum	$p_{\max}$	$150\,\hbar\kappa$
Probability of spontaneous emission per ref ection for $p_\infty = p_{\max}$		2.5×10^{-3}
Probability of nonadiabatic departure from the light-shifted ground-state level for $p_\infty = p_{\max}$		$\leq 8 \times 10^{-15}$
Decay length of the evanescent optical potential	$1/\kappa^{-1}$	≈ 100 nm
Ref ection time for $p_\infty = p_{\max}$	τ_{refl}	$\approx 4\,\Gamma^{-1}$

APPENDIX B: SOLUTION OF THE STATIONARY SCHRÖDINGER EQUATION

The general solution of the Bessel-type Schrödinger equation (15) is a linear combination of the Bessel functions $I_{\pm i\alpha}(u)$. These function are def ned by [11]

$$I_{\pm i\alpha} = \sum_{n=0}^{\infty} \frac{1}{n!\,\Gamma(n+1\pm i\alpha)} \left(\frac{u}{2}\right)^{2n\pm i\alpha}, \tag{B1}$$

where Γ denotes the Euler gamma function. They both diverge as $z \to -\infty \Leftrightarrow u \to +\infty$ according to [11]

$$u \to +\infty : I_{\pm i\alpha}(u) \cong \frac{1}{\sqrt{2\pi u}} e^{u}(1 + O(1/u)). \tag{B2}$$

As a result, the only linear combination of $I_{\pm i\alpha}(u)$ satisfying the boundary condition (i) of Section 3.1 corresponds to the difference of $I_{i\alpha}(u)$ and $I_{-i\alpha}(u)$. This difference is proportional to the Bessel-K function:

$$K_{i\alpha} = \frac{\pi}{\sinh(\pi\alpha)} \frac{1}{2i}(I_{-i\alpha}(u) - I_{i\alpha}(u)), \tag{B3}$$

whose asymptotic expansion is [11]:

$$u \to +\infty : K_{i\alpha}(u) \cong \sqrt{\frac{\pi}{2u}} e^{-u}(1 + O(1/u)). \tag{B4}$$

Equation (B4) shows that the atomic wave function decays very rapidly (as $\exp[-(p_{\max}/\hbar\kappa)e^{-\kappa z}]$) inside the potential barrier.

We f nally consider the boundary condition (ii) of the Section 3.1. In the asymptotic region $z \to +\infty \Leftrightarrow u \to 0^{+}$, the expansion of $I_{\pm i\alpha}(u)$ is given by the f rst term of the series expansion (B1):

$$z \to +\infty : I_{\pm i\alpha}(u(z)) \cong \frac{1}{|\Gamma(1+i\alpha)|} \exp\Big(\mp i p_{\infty} z/\hbar \pm i\alpha \ln\left(\frac{p_{\max}}{2\hbar\kappa}\right) \mp i \arg\Gamma(1+i\alpha)\Big) \tag{B5}$$

with [9]:

$$|\Gamma(1+i\alpha)| = \sqrt{\frac{\pi\alpha}{\sinh(\pi\alpha)}}. \tag{B6}$$

Combining (B3), (B5), and (B6), one readly f nds that the only solution of equation (15) satisfying the boundary conditions (i) and (ii) of Section 3.1 is:

$$\psi_{\mathrm{Schr}}(z) = \sqrt{\frac{4M}{p_{\infty}} \frac{\pi\alpha}{\sinh(\pi\alpha)}} \frac{1}{2i}(I_{-i\alpha}(u(z)) - I_{i\alpha}(u(z))), \tag{B7}$$

or equivalently

$$\psi_{\mathrm{Schr}}(z) = \sqrt{\frac{4M}{\pi\hbar\kappa}} \sinh(\pi\alpha) K_{i\alpha}(u(z)). \tag{B8}$$

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