UNIVERSITÄT POTSDAM

Wirtschafts- und Sozialwissenschaftliche Fakultät

# STATISTISCHE DISKUSSIONSBEITRÄGE

## Nr. 37

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**Abstract**

This paper studies the persistence of daily returns of 21 German stocks from 1960 to 2008. We apply a widely used test based upon the modified R/S-Method by Lo [1991]. As an extension to Lux [1996] and Carbone *et al.* [2004] and in analogy to moving average or moving volatility, the statistics is calculated for moving windows of length 4, 8, and 16 years for every time series. Periods of persistence or long memory in returns can be found in some but not all time series. Robustness of results is verified by investigating stationarity and short memory effects.

# 1   Introduction

It is widely accepted that stock market returns do not exhibit persistence, see e.g. Lo [1991], Lux [1996] for the case of Germany, and recently Assaf [2008]. This assertion is based upon tests for persistence which apply to the complete time series. A major drawback of this approach is arbitrariness as to start and end date. Furthermore, when testing a time series for long memory the stability of the parameter should be studied carefully. If results characterize the whole time series, they should not vary very much if applied to windows of the same size but at different points in time, see e.g. Alexander [2001] or Zivot and Wang [2006]. However, in a recent study Hassler and Nautz [2008] analyse the EONIA spread and show that there is a change in persistence over time. Carbone *et al.* [2004], Cajueiro and Tabak [2004], and Silva *et al.* [2007] have applied rolling analysis techniques to financial time

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series. It was found that the persistence parameter changes significantly over time. The concept of changing Hurst exponent can also be found in the analysis of heartbeat dynamics, see e.g. Martinis *et al.* [2004].

It is conceivable that persistence is a property not so much of the stock but rather the beholder, i.e. the bid and ask side to the stock. A company owned by one type of investor, say an insurance company, can be sold to a very different type, e.g. a hedge funds that is very sensitive to slight losses of the company's market value. In the first case, changes in the stock price may not exert much pressure on the owner. This inertia damps reactions to market movements. However, in the second case a stop loss algorithm may be in place selling the stock right after a minor dip in value. Thus, investor behavior pertaining to the particular stock changes with time. Rolling analysis is able to detect this effect.

Long memory or synonymously persistence has severe consequences for practically very important fields such as risk measurement and derivatives pricing. Usually, a value at risk calculation scheme does not include autocorrelations of time series, see e.g. Jorion [2007]. Therefore, a simple scaling rule is used for applying results for one time scale to another one. This is not admissible for persistent time series. Moreover, long memory processes are no semi-martingales. Therefore common strategies for calculating prices of financial contracts fail, see e.g. Rogers [1997], Cheridito [2003] and Bender *et al.* [2007].

In this paper, we apply rolling analysis of persistence to 21 time series that have at least once belonged to the DAX and cover a time span of about 48 years. First, we count all periods for which a test for persistence rejects the null hypothesis (no persistence) at the 5%-level. Second, we count the maximum number of connected periods that show persistence. This number gives an indication as to the likeliness of an error of the first kind in the estimation. In addition, we address the question whether stock markets get more efficient over time. If this is the case, persistence should decrease over time and no new persistent periods should appear. The results obtained so far are then checked for robustness against short range autocorrelation, trends and heavy tails.

The rest of the paper ist organized as follows. In the next two sections long memory and some models are introduced. Section 4 discusses tests for long memory. In section 5, results for rolling analyis of persistence are presented. In section 6 we conclude.

# 2   Long Memory

A widely accepted definition of long memory is expressed in terms of the autocorrelation function $\rho_k$. A stationary process has long memory if there
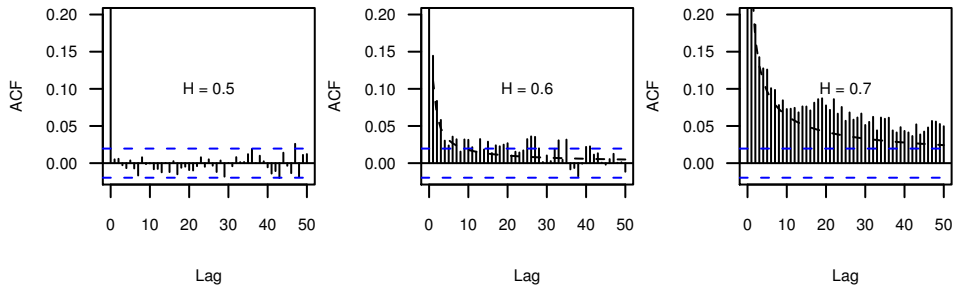
Figure 1: Autocorrelation function for Gaussian Series ($H = 0.5$), moderately persistent fractional Gaussian series ($H = 0.6$) and strongly persistent fractional Gaussian series ($H = 0.7$). Dashed lines: theoretical ACF. Length of series: 10.000 points.

exist $H \in [1/2, 1]$ and $c_\rho > 0$ such that

$$\lim_{k \to \infty} \frac{\rho_k}{c_\rho k^{2(H-1)}} = 1. \tag{1}$$

For $H > 1/2$ the autocorrelation function $\rho_k$ decays so slowly that the sum

$$\lim_{n \to \infty} \sum_{k=-n}^{n} |\rho_k|. \tag{2}$$

diverges. Recall that for short memory time series as for example ARMA processes the sum of autocorrelations is finite. In the long memory case, even small individual autocorrelations do collectively spoil statistical inference. In Fig. 1 we show the autocorrelation functions for an ordinary, an intermediately and a strongly persistent Gaussian process. The depicted sample paths corroborate the following two remarks. First, for processes with intermediate long memory, autocorrelations are very difficult to detect. The individual autocorrelations barely pass the threshold of significance (dashed line). Second, if long memory is present, estimated autocorrelations are mostly positive (theoretical autocorrelations must be positive). Shocks exerced on the time series at a certain moment in time persist in principle arbitrarily long, thus giving rise to the term persistence which is used synonymously with long memory.

Beran [1994] collects some qualitative features of long memory processes. The sample paths show the following properties.

- Long periods of observations at high levels are followed by long periods of observations at low levels and vice versa.

- At short time periods aperiodic cycles or trends are apparent.

- On the whole, the time series looks stationary.

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An introduction to the topic of aperiodic cycles can be found in Mandelbrot [1972]. Peters [1994] has devoted a large part of his more phenomenological monography to this phenomenon.

# 3   Models

There are two well studied theoretical models with long memory: fractional Brownian Motion and fractional ARIMA processes. The first one can be viewed as a generalization of Brownian motion. The second class can be looked upon as integrated ARMA processes. As for Brownian motion and the ARIMA processes the first one is continuous and the second are discrete. For a detailed survey of time series with long-memory confer e.g. Beran [1994] or Baillie [1996] and references given there.

**Continuous time: fractional Brownian motion**   Fractional Brownian motion can be defined in two ways: Either by fixing its properties, in particular its autocorrelation structure, or by an integral formulation based on ordinary Brownian motion. In the following, both ways are sketched briefly starting with the latter.

In order to motivate the integral formulation of fractional Brownian motion, we take a detour to fractional calculus of real variables, starting with the familiar Cauchy formula of repeated integration. Consider the operator $J$ on a well behaved function $f$, defined as

$$(Jf)(x) = \int_0^x f(t)dt.$$

$$(3)$$

The Cauchy formula of repeated application of $J$ reads

$$(J^n f)(x) = \frac{1}{\Gamma(n)} \int_0^x (x-t)^{n-1} f(t)dt. \tag{4}$$

where $\Gamma(n) = (n-1)!$ and $n \in \mathbb{N}$. Define a function $F$ such that $f = \frac{dF}{dx}$. Furthermore, set $n = H + 1/2$ with $H \in [\frac{1}{2}, 1[$. This leads to the equation

$$(J^{H+1/2}\frac{dF}{dx})(x) = \frac{1}{\Gamma(H+1/2)} \int_0^x (x-t)^{H-1/2} dF. \tag{5}$$

As can be shown only if $H = 1/2$ the usual derivative of equation (5) with respect to $x$ leads to a local property, i.e. evaluation of the function $f$ at $x$. In all other cases the derivative contains mixing of influences of the complete function $f$ on the interval $[0, t]$.

Now, reinterpret the integral in equation (5) as stochastic integral[1] and replace $F$ by Brownian motion $B$. Then – for $H = 1/2$ – the familiar stochastic integral formulation for Brownian motion is found.

$$B(t) = \int_0^t dB(s). \tag{6}$$

Setting $B_H := (J^{H+1/2} \frac{dB}{dt})$ leads to the defining equation of fractional Brownian motion[2]

$$B_H(t) = C \int_0^t (t-s)^{H-1/2} dB(s) \tag{7}$$

It was derived by Mandelbrot and van Ness [1968]. The constant $C$ is given by[3] $C = 1/\Gamma(H + 1/2)$. Choosing $C$ in this way provides the formal link to the Cauchy integral formula. There are different conventions, as e.g. Shiryaev [1999]

$$C = \sqrt{\frac{2H\Gamma(\frac{3}{2} - H)}{\Gamma(\frac{1}{2} + H)\Gamma(2 - 2H)}}, \tag{8}$$

in order to ensure $EB_H^2(1) = 1$.

Although the construction by analogy to fractional calculus is quite intriguing, Marinucci and Robinson [1999] have shown that the increments of the resulting time series are non-stationary due to the finite lower bound in the integral in eq. (7). In addition, Davidson and Hashimzade [2009] point out that due to the mixing of influences from the whole stochastic process on the interval $[0, t]$ the starting point of the time series must be at $-\infty$ in order to ensure elimination of the influence of the starting shock on the time series. Setting the lower bound of the integral at $-\infty$, i.e. adding the complete 'history' from $-\infty$ to the origin, and subtracting a constant as to fix the resulting path at the origin leads to fractional Brownian Motion of type I. It is stationary and overcomes the starting point problem. It can be written as

$$
\begin{aligned}
B_H(t) &= C \left\{ \int_{-\infty}^0 [(t-s)^{H-1/2} - (-s)^{H-1/2}] dB(s) \right. \\
&\quad \left. + \int_0^t (t-s)^{H-1/2} dB(s) \right\}.
\end{aligned}
\tag{9}
$$

where $C = 1/\Gamma(H+1/2)$. Different choices for $C$ are possible, as seen before. Davidson and Hashimzade [2009] have studied the properties of fBM of types I and II.

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[1] In an appropriate sense, see e.g. Beran [1994]; Shiryaev [1999]
[2] of type II, see below
[3] $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$.

The essential characteristics of fractional Brownian motion as defined in eq. (9) can be summarized in the property-based definition. Following Beran [1994], Fractional Brownian motion (fBm) is defined as follows.

**Definition 1 (fractional Brownian Motion)** *Let $B_H(t)$ with fixed $H \in (0,1)$ be a stochastic process with continuous path and the following properties*

1. *For every t $B_H(t)$ is a Gaussian random variable.*

2. $B_H(0) = 0$

3. $\mathrm{Cov}[B_H(t), B_H(s)] = \frac{\sigma^2}{2}\left[|s|^{2H} + |t|^{2H} - |t-s|^{2H}\right]$

4. $E[B_H(t) - B_H(s)] = 0$

*Then $B_H(t)$ is a fractional Brownian motion.*

The variance of an arbitrary increment is $\mathrm{Var}[B_H(t) - B_H(s)] = \sigma^2|t-s|^{2H}$. For $H = 1/2$ this expression coincides with the variance of an increment of ordinary Brownian motion. Note, that for any value of $H$ there exists exactly one process satisfying definition 1. This process is explicitly given by equation (9). In turn, definition 1 summarizes the key properties of the processes defined in eq. (9).

Calculating increments of fBm leads to fractional Brownian noise (fBn), the fractional version of Gaussian noise

$$\beta_H(t) = B_H(t) - B_H(t-1), \ t \geq 1. \tag{10}$$

As for fBm $\frac{1}{2} < H < 1$. Fractional Brownian noise is stationary. The autocorrelations for this process are given by

$$\rho(k) = \frac{1}{2}\left[|k+1|^{2H} + |k-1|^{2H} - 2|k|^{2H}\right]. \tag{11}$$

For large $k$ expansion of equation (11) in leading order in $1/k \to 0$ results in:
$$\rho(k) \sim \frac{1}{2}\left[H(2H-1)|k|^{2H-2}\right], \qquad k \to \infty \tag{12}$$

If $H \in ]1/2,1[$ this expression converges to zero too slowly as $k \to \infty$, such that
$$\sum_0^\infty \rho(k) = \infty.$$

According to definition (1) fBn is a long memory process. Mandelbrot and van Ness [1968] show that processes with autocorrelation function eq. (11) are self similar.

**Discrete time: Fractional ARIMA Processes**   The above definition of fractional Brownian noise is a continuous time representation. The counterpart in discrete time modelling is fractionally differenced white noise[4]. It was first introduced by Granger and Joyeux [1980] and Hosking [1981].

**Definition 2 (fractionally differenced white noise)** *Let $\epsilon_t$ be a process with $E(\epsilon_t) = 0$, $E(\epsilon_t^2) = \sigma^2$ and $E(\epsilon_t \epsilon_{t'}) = 0$ for $t \neq t'$ then the process $X_t$, defined by*

$$(1 - B)^d X_t = \epsilon_t. \tag{13}$$

*with d non-integer is called fractionally differenced white noise.*

The process is invertible for $d > -\frac{1}{2}$. For $d < \frac{1}{2}$ it is weakly stationary. Given this definition any ARMA process can be constructed with fractionally differenced increments, according to

$$\phi(B)(1 - B)^d Y_t = \theta(B)\epsilon_t \tag{14}$$

where $\phi$ and $\theta$ are the usual AR- and MA-polynomials. The fractional difference can be expressed as

$$(1 - B)^d = \sum_{k=0}^{\infty} \frac{\Gamma(d+1)}{\Gamma(k+1)\Gamma(d-k+1)} (-1)^k B^k \tag{15}$$

for any $d > -1$. The autocorrelation function of $X_t$ is

$$\rho(k) = \frac{\Gamma(k+d)\Gamma(1-d)}{\Gamma(k-d+1)\Gamma(d)}. \tag{16}$$

For $d > 0$, the autocorrelation is positive at all lags. As $k \to \infty$ the follwoing relation holds:

$$\rho(k) \sim \frac{\Gamma(1-d)}{\Gamma(d)} k^{2d-1}, \; k \to \infty \tag{17}$$

For $d > 0$ $\rho(k)$ decays so slowly with increasing lag that equation (16) it is not summable. Closing the section we note that comparing equations (17) and (12) suggests

$$H = d + \frac{1}{2} \tag{18}$$

which can also be rigorously shown, see e.g. Geweke and Porter-Hudak [1983].

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[4]Heuristically, fGn results from integrating fractionally and then differencing of order 1. This leads to a 'net' differenced process.

# 4   Tests

Testing for long memory has proven to be an intricate task. Many efforts have been undertaken with respect to discerning long memory from either short memory, see e.g. Lo [1991], Davidson and Sibbertsen [2009], or structural breaks, see e.g. Krämer *et al.* [2002], Krämer and Sibbertsen [2002], Hassler and Olivarez-Horn [2008], or trends, see e.g. Beran [1994], Hu *et al.* [2001]. However, although understanding of tests and deducible implications has improved considerably in the last years no technique free of arbitrariness has been proposed yet.

Hurst [1951] has proposed a non-parametric estimator for the long memory exponent $H$, which has been used by Mandelbrot and van Ness [1968], Mandelbrot and Wallis [1968, 1969] and refined mainly to improve robustness against short memory by Lo [1991]. This method will be described in detail below. Another non-parametric method, Detrended Fluctuation Analysis (DFA) studies the scaling behavior of the variance, Peng *et al.* [1994]; Cannon *et al.* [1997].

A semi-parametric estimation technique has been proposed by Geweke and Porter-Hudak [1983]. Variations have been proposed by Robinson [1995] Moulines and Soulier [1999] among others in order to discern between short and long memory. A remarkable work has been presented in Davidson and Sibbertsen [2009]. Its authors point towards a rigorous test to differentiate between long and short memory but also stress fundamental limitations of such tests. There exist several parametric methods based on maximum likelihood estimation. These are described in detail e.g. in Beran [1994] and Baillie [1996].

For the present study Lo's extension of Hurst's rescaled range method has been chosen by the authors for several reasons. Firstly, it is a non-parametric method that consumes relatively few computing power. Secondly, although there are many drawbacks with respect to robustness in the presence of short memory and trends, these properties are well studied and can be accounted for to a certain extent, see Lo [1991], Kunze [2009]. Thirdly, a simple test statistics is available and asymptotic quantiles are reasonable for time series of finite length. For a detailed analysis of size, power and robustness, especially in the context of finite time series see Kunze [2009].

In the following the method will be presented in some detail. A very thorough and accurate description is given by Lo [1991]. Let $\{x_i, \ i \in [1, 2 \ldots, N]\}$ be a discrete stationary time series with partial sums

$$X_{t,k} = \sum_{i=t}^{t+k} x_i. \tag{19}$$

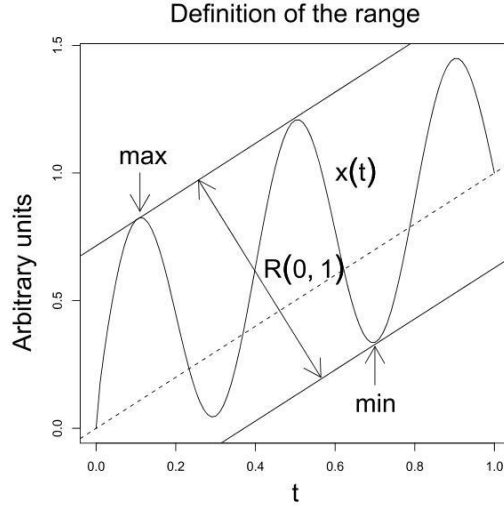The argument $t$ marks the starting point, $k$ the end point of the summation.

Figure 2: Definition of the range. Depicted is $R(t = 0, k = 1)$ in the continuous case. Maximum and minimum are measured from the trend-line (dotted line). The trend is therefore implicitly removed.

Given these preliminaries the range is defined as

$$R(t, k) = \max_{0 \leq i \leq k} \left[ X_{t,i} - \frac{i}{k} X_{t,k} \right] - \min_{0 \leq i \leq k} \left[ X_{t,i} - \frac{i}{k} X_{t,k} \right] \tag{20}$$

The term $\frac{1}{k} X_{t,k}$ is the empirical mean of the partial series $\{x_i, \ i \in [t, \ldots, t + k]\}$. The difference $X_{t,i} - \frac{i}{k} X_{t,k}$ is the deviation of the partial sum from the trend with index $i \leq k$. For $i = k$ the difference vanishes. Therefore the first term (maximum) is non-negative and the second term (minimum) is non-positive. It follows that, that $R(t, k)$ is non-negative. The concept is illustrated in Figure 2.

The expression $R(t, k)$ is normalized by the (ML-) estimator of the standard deviation for the partial series between $x_t$ and $x_{t+k}$:

$$S(t, k) = \left[ \frac{1}{k} \sum_{i=t+1}^{t+k} \left( x_i - \frac{1}{k} X_{t,k} \right)^2 \right]^{1/2}. \tag{21}$$

The quotient of $R(t, k)$ and $S(t, k)$

$$Q(t, k) = R/S = \frac{R(t, k)}{S(t, k)} \tag{22}$$

is known as classical R/S statistics. It is invariant under transformations $x_k \rightarrow c(x_k + m)$, $k \leq 1$. Therefore the statistics is independent of the first two moments of the distribution of $x_k$.

Table 1: Quantiles of the distribution of $Q(t,k)$ under $H_0$, (Gaussian noise) ($H = 0.5$, 10000 simulations)

| Nr. | H | Länge | 0.005 | 0.025 | 0.050 | 0.950 | 0.975 | 0.995 |
|-----|------|-------|--------|--------|--------|--------|--------|--------|
| 1 | 0.50 | 250 | 0.6699 | 0.7516 | 0.8003 | 1.6712 | 1.7931 | 2.0143 |
| 2 | 0.50 | 1000 | 0.6897 | 0.7797 | 0.8276 | 1.7037 | 1.8130 | 2.0758 |
| 3 | 0.50 | 10000 | 0.6985 | 0.7973 | 0.8483 | 1.7364 | 1.8591 | 2.1002 |
| 4 | 0.50 | $\infty$ | 0.7212 | 0.8094 | 0.8613 | 1.7473 | 1.8624 | 2.0977 |

The probability distribution of $Q(t,k)$ under the null hypothesis 'Gaussian noise' was found by Kennedy [1976] and Siddiqui [1976] based on work by Feller [1951]. With the notation $F_Q(x) = P(Q < x)$ it reads

$$F_Q(x) = 1 + 2 \sum_{k=1}^{\infty} (1 - 4k^2 x^2) \exp\left(-2\left(kx\right)^2\right), \qquad (23)$$

The null hypothesis should be rejected at the respective confidence level if the absolute value of $Q(t,k)$ is larger (smaller) than the appropriate quantile. Quantiles are given in Table 1

As has been shown by Lo [1991] short memory can lead to a considerable increase in empirical size of the test. This finding has led him to replace the denominator in $Q(t,k)$ by a heteroskedasticity and autocovariance consistent (HAC) estimator

$$
\begin{aligned}
\sigma_q^2(t,k) &= \frac{1}{k} \sum_{j=t}^{t+k} \left( x_j - \frac{1}{k} X_{j,k} \right)^2 \\
&+ \frac{2}{k} \sum_{j=1}^{q} \omega_j(q) \left\{ \sum_{i=j+t}^{t+k} \left( x_i - \frac{1}{k} X_{i,k} \right) \left( x_{i-j} - \frac{1}{k} X_{i-j,k} \right) \right\} \\
\omega_j(q) &\equiv 1 - \frac{j}{q+1} \qquad q < n.
\end{aligned}
\qquad (24)
$$

In choosing $\omega_j(q)$ he follows Newey and West [1987]. Different choices and a more general formulation of the summation weights can be found in Andrews [1991]. The modified statistics, also known as Lo's statistics is given by

$$Q_q(t,k) = R/S_q = \frac{R(t,k)}{\sigma_q(t,k)}. \qquad (25)$$

As has been shown by Lo, the quantiles in Table 1 are still valid. However, the choice of $q$, the truncation parameter determining the included lags in equation 24, is non-trivial. It is now understood that Lo [1991] has chosen $q$ in a way that renders the test very conservative, see e.g. Teverovsky *et al.*

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[1999]. Willinger *et al.* [1999] pointed out that the Hurst exponent of stock market returns is expected to be lower than 0.6. Finding the right trade-off between detection of long memory and excluding short-memory bias is therefore very delicate.

In order to account for short memory effects, the modified Lo statistic with lag 0, 5 and 21 days is used and presented. As has been pointed out by Kunze [2009], for time series with a length of 10.000 data points any autocorrelation (short or long memory) leads to a decreasing value for the Lo-statistics as the lag increases from zero. For a lag larger than 10 days a synthetic time series with $H = 0.6$ is not correctly identified as long memory. Interestingly, for increasing lag after passing a minimum Lo's statistics increases again. This effect can be seen in Lo [1991] and Lux [1996] but has not been recognized properly. Simulation studies have shown that lags in the vicinity of the length of the time series distort the statistics upwards and lead to an error of first kind, see Kunze [2009].

## 5 Empirical Results

We have investigated 21 time series of the German stock market who at least once belonged to the DAX (Deutscher Aktien-Index). Mostly, the time series start on January 5th 1960 and end on January 31st 2008. They have been transformed into daily percentage total return time series and are provided by the Karlsruher Kapitalmarktdatenbank[5] (KKMDB).

Throughout the study a time series is called persistent if the test statistics, equation (25), exceeds the 5% quantile of the Feller distribution, equation (23), i.e.

$$Q_q(t, k) = R/S_q > 1.7473. \qquad (26)$$

Persistence is measured for time windows of length fixed at 1000, 2000, and 4000 data points, corresponding to 4, 8, and 16 years. The pertaining value of the statistics is then attached to the last day in the time series. These windows are moved across the time series thus generating an analogue to a moving average or moving volatility. If not stated otherwise a lag of five days is chosen.

In Fig. 3 we show the modified R/S-statistics for Siemens. For a Gaussian series the expectation value for the R/S-statistics is $\sqrt{\pi/2} \approx 1.25$. As can be seen, the values for R/S vary strongly. Volatility of the underlying time series does not seem to influence the R/S statistics. For a window size of four years the null hypothesis 'no persistence' must be rejected in several periods. An oscillating pattern – reminiscent of regime switching – is apparent. Persistent periods are distributed roughly equally over the time

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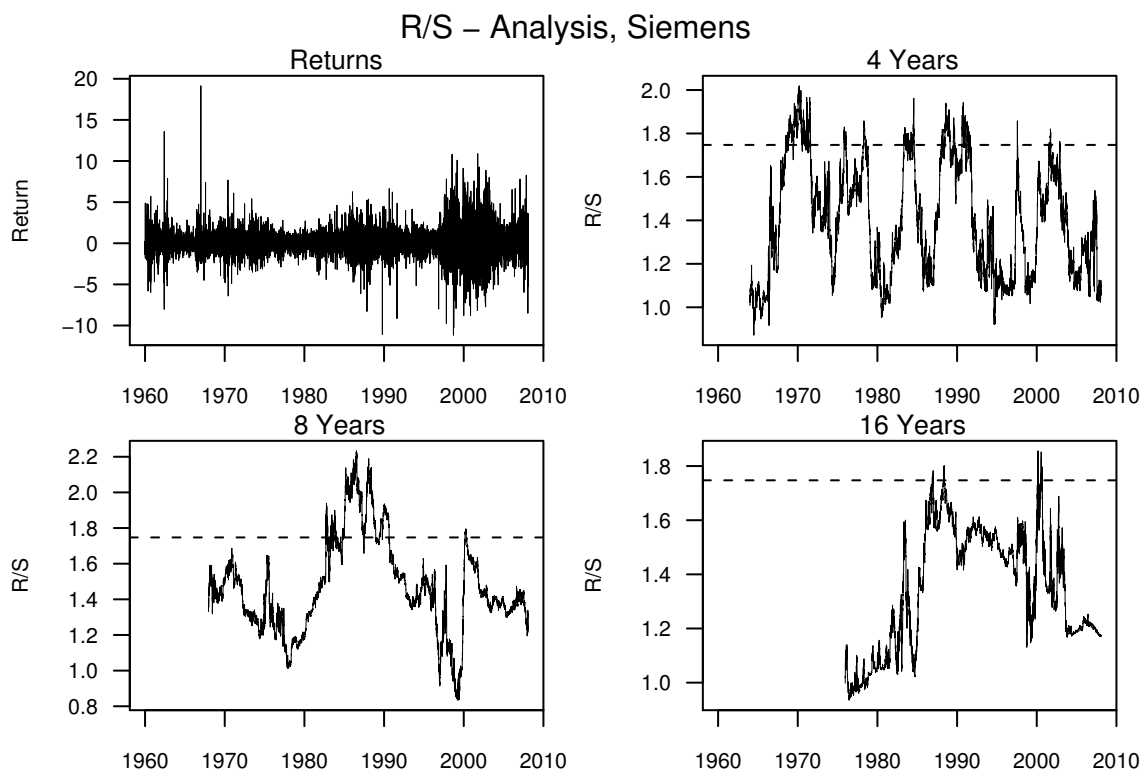[5]Homepage: http://fmi.fbv.uni-karlsruhe.de

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Figure 3: R/S-Analysis of Siemens. Lo's estimator with Lag 5 days. Top left: Total Returns. Top right: Four year window. Bottom left: Eight year window. Bottom right: Sixteen year window. Above the dashed line the null hypothesis must be rejected at the 5% level.
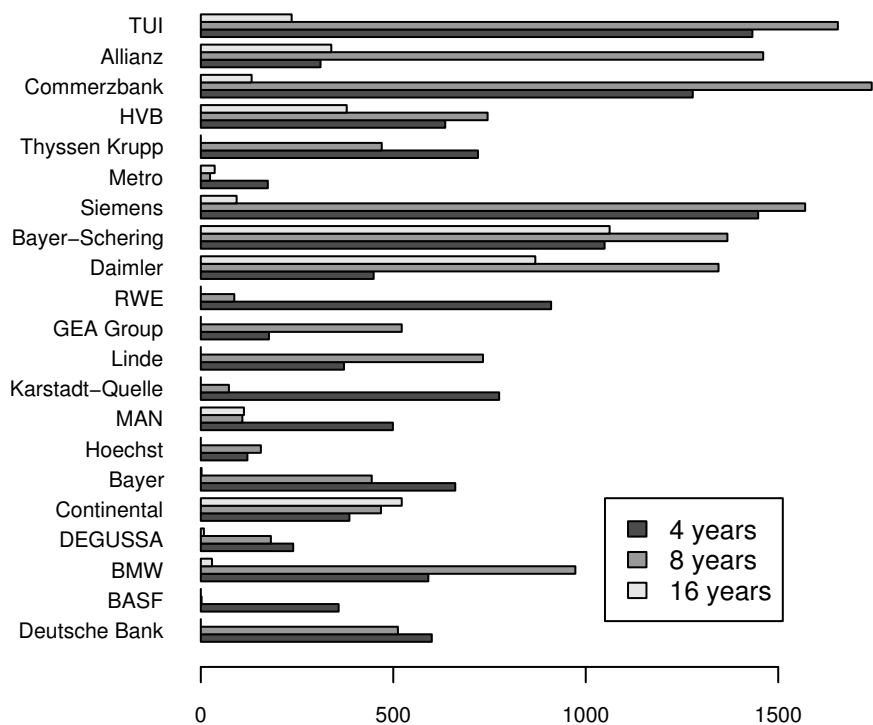
**Number of persistent periods**



Figure 4: Number of persistent periods for windows of size 4, 8, and 16 years.

axis. With a window size of eight years, most periods ending between 1980 and 1990 are persistent. For the period ending on July 15th 1985 the R/S-statistics reaches the maximum value of 2.2335. When the window size is set to 16 years only few periods of persistence are found. Judging from the graphical impression, for increasing window size the number of persistent periods decreases.

In order to study the properties of the 21 time series systematically, we count the number of persistent periods (5% level) for window sizes of 4,8, and 16 years and compare them in Fig. 4. If all periods were independent, at the 5% level there should be about 550, 500, and 400 detections of 'persistent' periods due to an error of the first kind for the 4, 8, and 16 year windows respectively. As can be seen, these numbers are exceeded in many cases. However, there are companies, for which the number of persistent periods is far below the threshold. These companies are Metro, Hoechst, Degussa, and BASF. Note, that two of the four companies belong to the chemical sector. Metro belongs to the retail sector and Degussa to industrial. On the

## Maximum number of connected persistent periods
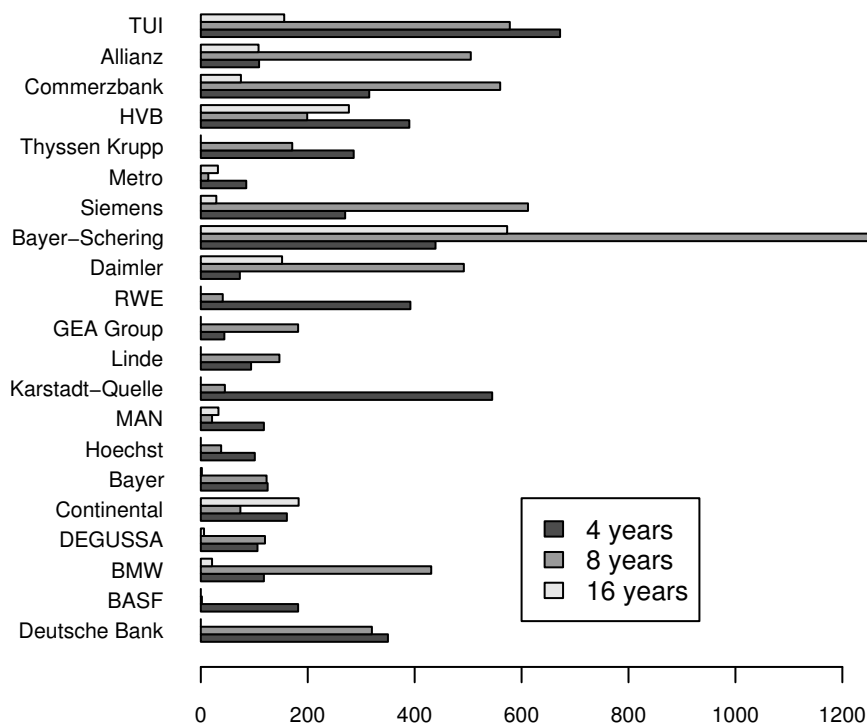


Figure 5: Maximum number of connected persistent periods.

other hand, there are companies for which the number of persistent periods are well above the respective thresholds for two or more window sizes. It is noteworthy that the automotive and large parts of the financial sector belong to this group.

In addition to counting the number of persistent periods, it is instructive to determine the largest number of connected periods of persistence. If this number ist large as compared to the total number of persistent periods, this is an encouraging fact concerning the reliability of results. For better comparability, the numbers are graphically presented in Fig. 5. One Company, Bayer-Schering, reaches more than 1200 connected persistent eight year windows. This corresponds to about five years in which all eight-year-windows are persistent. Apart from this outstanding case there are several companies for which more than 250 four or eight year windows are connected. All financial titles belong to this group.

Collecting results obtained so far, the assertion that stock market returns do not exhibit persistence needs reconsideration. There are periods where the null hypothesis "no persistence" must be rejected. However, as is well
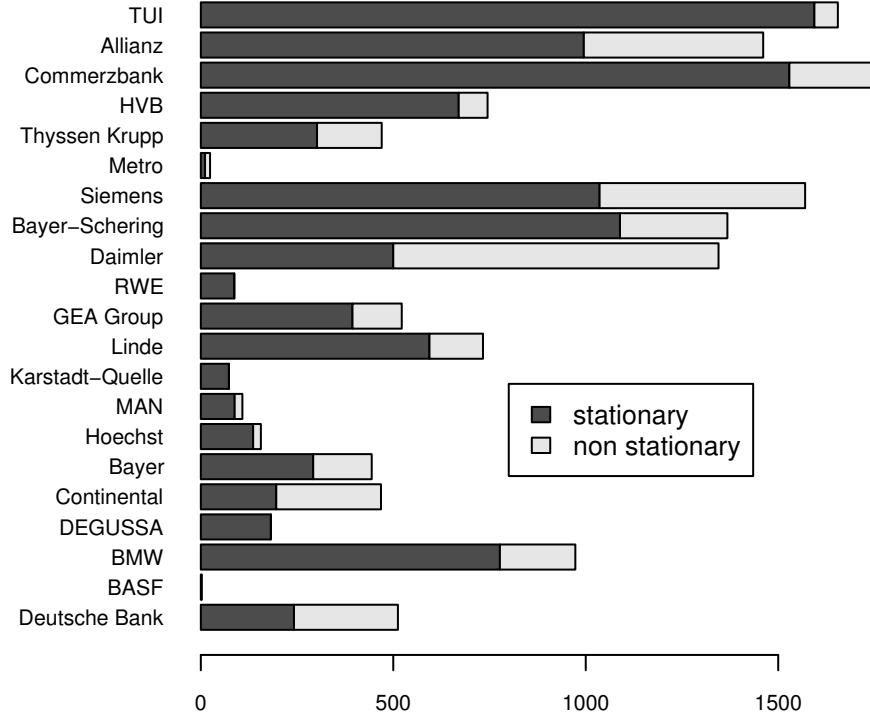
Figure 6: Persistent periods subdivided in stationary and non-stationary ones. Stationarity is defined according to the KPSS-test. Window size is 8 years, lag in modified R/S Test is 5 days.

known in the literature, non stationarity or short range autocorrelation may bias tests for long memory, see e.g. Lo [1991]; Hu *et al.* [2001]; Chen *et al.* [2002]; Craigmile *et al.* [2004]; Chen *et al.* [2005]; Winker and Jeleskovic [2007]; Davidson and Sibbertsen [2009]. Therefore, the remainder of the article is mostly devoted to robustness of results. We start with stationarity and test for robustness with two methods. First, we apply the KPSS test to the series under study, see Kwiatkowski *et al.* [1992]. Second, prior to testing for long memory, we apply a detrending filter to the time series.

**Stationarity** In the following, the KPSS-test is sketched briefly. Let $x_t$ be a time series with $t = 1, 2, \ldots, T$. It is assumed that the time series can be decomposed into a deterministic trend, a Brownian motion and a stationary error term:

$$x_t = \theta t + r_t + \epsilon_t \tag{27}$$

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**Persistence and stationarity – Original vs. Detrended**
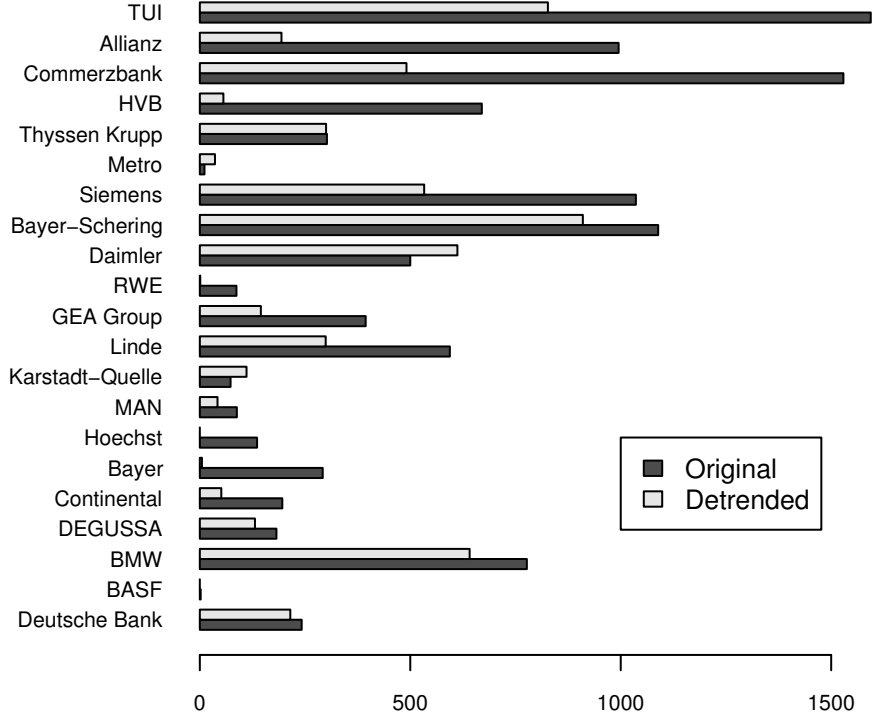


Figure 7: Number of periods classified as persistent and stationary (KPSS) compared to the according number of periods after detrending. Window size is 8 years, lag in modified R/S Test is 5 days.

The term $r_t$ represents Brownian motion:

$$r_t = r_{t-1} + u_t \qquad \text{with } u_t \sim \mathcal{F}(0, \sigma_u^2). \tag{28}$$

In this equation $\mathcal{F}$ is a distribution with vanishing mean and variance $\sigma_u^2$. The initial value $r_0$ is the intercept in equation (27). Under these assumptions the hypothesis "time series is stationary" corresponds to

$$\sigma_u^2 = 0 \tag{29}$$

In our analysis we restrict ourselves to the case of level stationarity around $r_0$, which implies setting $\theta = 0$ in equation (27) a priori. For further details as to the test-statistics and correction for heteroskedasticity refer to Kwiatkowski *et al.* [1992].

In Fig. 6 Persistent periods are subdivided in stationary and non-stationary ones. A period is called stationary if the KPSS-test does not
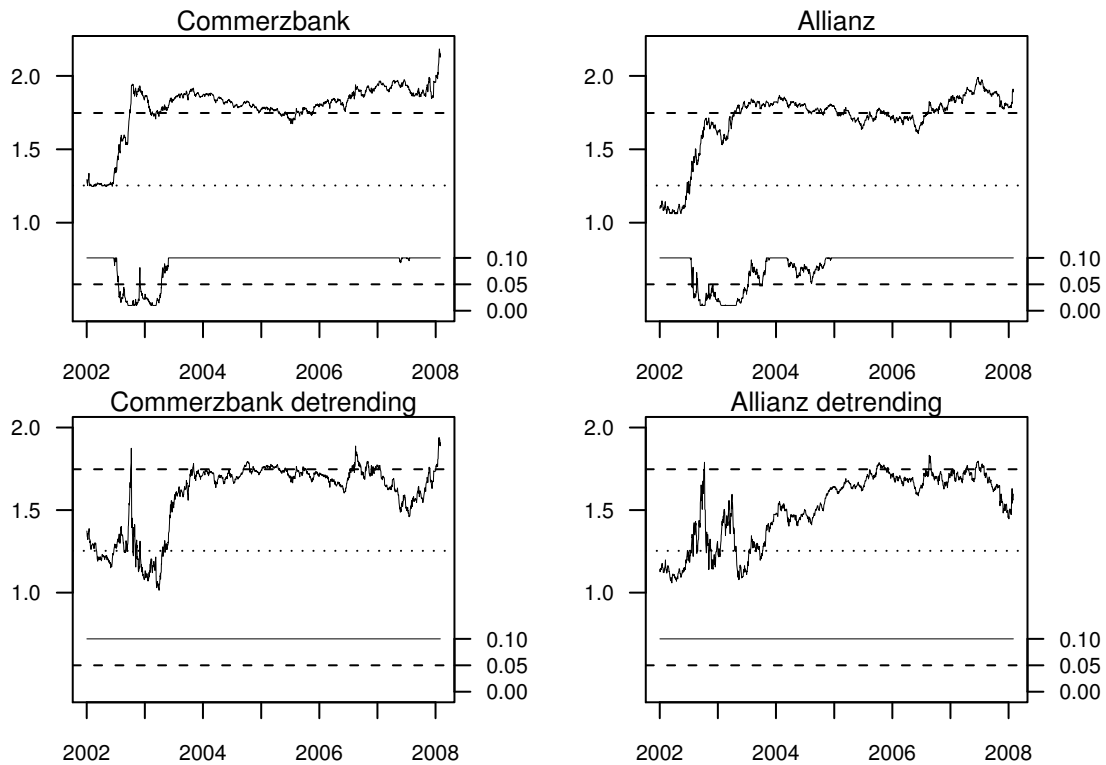
Figure 8: Modified R/S-Estimator eq. (25) with lag 5 for Commerzbank (left) and Allianz (right). The window size is 8 years. Left scale: modified R/S-statistics. Right scale: p-value of KPSS test for level stationarity. The upper dashed line denotes the threshold above which the null hypothesis must be rejected at the 5% level. The lower dashed line denotes the p-value below which the null hypothesis 'level stationarity' must be rejected.

reject the null hypothesis "level stationarity" at the 5% level. A substantial part of persistent periods is classified as stationary. However, in some cases, e.g. Daimler, more than half of the persistent periods is non stationary. On the other hand, for some companies such as TUI almost all persistent periods are classified stationary by the KPSS test.

After detrending the number of persistent periods is reduced considerably. The detrending filter consists of calculating residues of a regression against a linear trend. In Fig. 7 residuals are tested for persistence. Many persistent periods classified stationary by KPSS-Test (5% level) lack persistence after detrending. However, there are still time series where the number of peristent periods substantially exceeds the number of expected periods erroneously classified persistent at the 5% level, even after detrending.

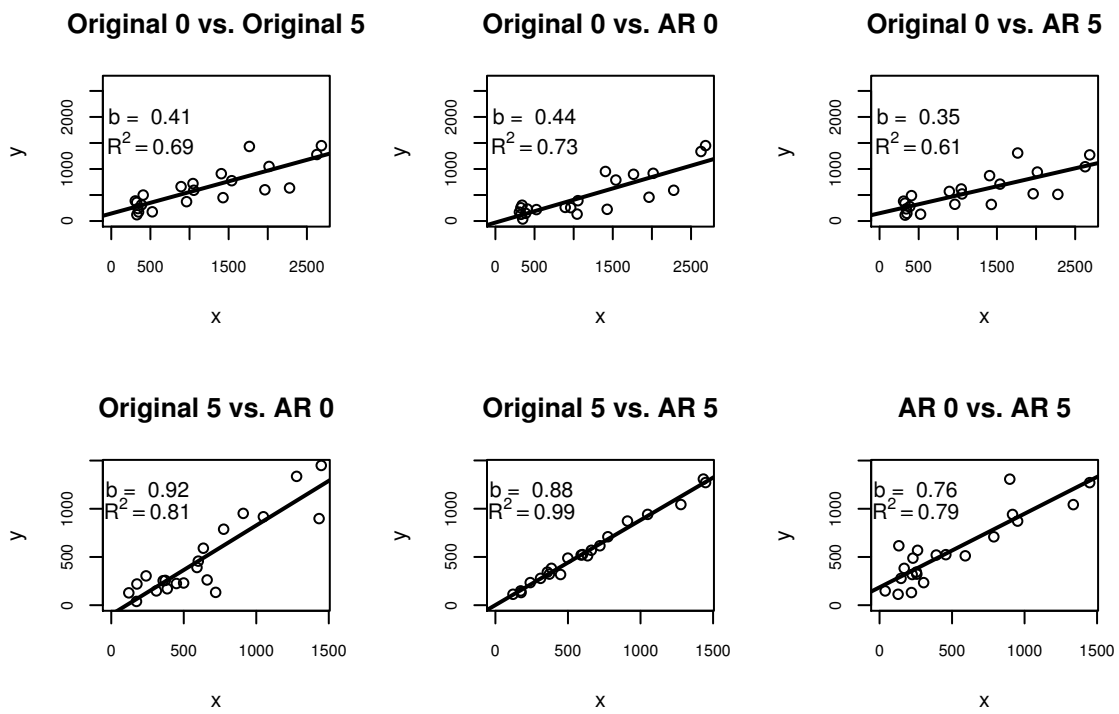In order to study these effects more in depth we have analysed Com-

Figure 9: Short Range Autocorrelation. Explanatory power of number of persistent periods in second time series (y-axis) through number of persistent periods in first time series (x-axis). Window size is 4 years. Time series denoted by AR are AR(1) residuals. Numbers (0 or 5) give lag in days of modified R/S-statistics.

merzbank and Allianz for recent years. In Fig. 8 the R/S - statistics for both companies is plotted versus the p-value of the KPSS-test both for the original and the detrended time series. First, it should be noted that there exist persistent and stationary periods even in recent years (upper row). Second, detrending reduces the modified R/S-statistics but does not eliminate persistence completely, especially in the case of Commerzbank. It can be observed that detrending has substantial effects where the KPSS test rejects the null hypothesis of stationarity. However, for periods where the KPSS test does not reject, even at the 10 % level, filtering does reduce the R/S statistics slightly. This slight reduction does not change the overall picture of the time series of the R/S statistics but as can be seen for both companies is enough that the null hypothesis "no persistence" cannot be rejected at the 5% level. Therefore, detrending the time series does make the test more conservative. However, at a small time scale persistent time series can be perceived as series with trends. When removing these "trends" with a detrending filter, persistence might also be removed thus reducing power. To that extent the detrending filter leads to inconclusive results. Nevertheless, rejection of null "stationarity" and effect of the filter correlate strongly.

A word should be said with respect to the increasing literature on the question whether financial markets get more and more efficient, see e.g. Cajueiro and Tabak [2004]; Silva *et al.* [2007]. It is remarkable that for two companies of the financial sector, almost all eight year windows ending after 2003 are classified persistent and stationary. This contradicts at least the simple random walk formulation of the efficient market hypothesis. In summary, results obtained in our study discourage the assumption of a "trend to efficiency".

**Autocorrelation**  In order to analyze the possible effect of short range autocorrelations on the test for persistence we have compared the number of persistent periods determined for the original time series with different lags with and without AR(1) filter. We perform a regression of the number of persistent time windows obtained with one technique on the number obtained with another one and interpret the results as follows. The closer the slope of the regression line to one and the closer $R^2$ to one the more similar the results of the compared techniques. In Fig. 9 the combinations under study are presented. The original time series is denoted by "Original" and the AR(1)-filtered time series is denoted by "AR". Lags are zero or five days.

First, consider the upper row. Classical R/S-statistics (original 0) gives more than twice the number of persistent periods found after correcting for short range autocorrelation either by filtering and/or by choosing a five day lag in the modified R/S statistics. This result is in line with results by Lo [1991]. There is strong support for the assumption that there is short
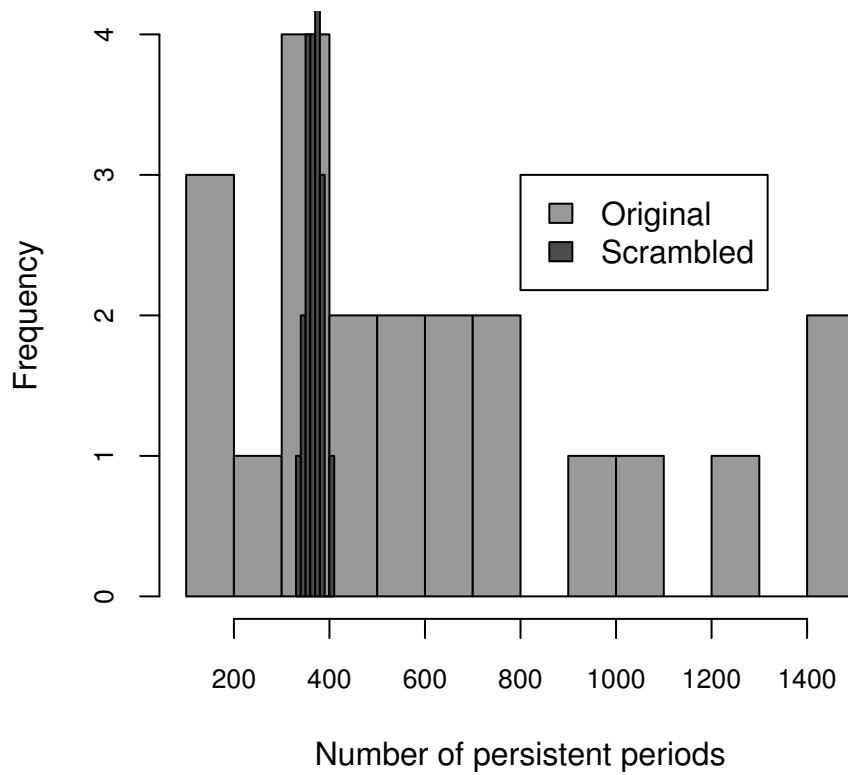
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Figure 10: Heavy tails give increased values of R/S-statistics, independent of the order of points in time. Randomizing order of time series removes stucture of distribution of persistent periods. Consequently, increased R/S-statistics is due to correlation structure of time series rather than heavy tails. Window size is four years, lag five days.

range autocorrelation which biases the plain R/S test. Now consider the lower row. The values obtained for the modified R/S-statistics with a lag of five days are almost invariant with respect to AR(1)-filtering, as can be seen in the left and middle diagrams. Finally, consider the lower right. A higher lag still reduces the number of persistent periods after filtering but much less than in the upper left diagram (without filter). We conclude that testing the original time series using the modified R/S test with lag 5 days is an appropriate approach precluding bias effects due to short range autocorrelations sufficiently.

**Heavy Tails**   We close the section with a consideration of leptokurtotic distributions of returns. Bouchaud and Potters [2000] among others mention these as possible origins for the R/S test to reject the null hypothesis. If the reason for rejection of the null lies in the individual distribution of returns then the result should not be influenced by randomizing the order of points in the time series (scrambling). However, Fig. 10 shows clearly that scrambling of the time series changes the pattern of the R/S estimation for the different accounts. The histogram for scrambled series classifies roughly 5% of the periods tested in one series reveal persistence. This is in line with the size of the test and should be considered insignificant[6]. The variance of the results is very small compared to the value without scrambling. We therefore conclude that the periods for which the null is rejected are persistent as opposed to leptokurtotic.

# 6   Conclusions

We have studied daily returns of 21 German companies (1960 to 2008) that belonged at least once to the stock price index DAX. Our main focus was rolling analysis of persistence in time windows of 4, 8, and 16 years. Using modified R/S analysis in conjunction with a 5% confidence level we found that some but not all time series exhibit persistent time windows, mostly for the 4 and the 8 year windows. In some cases, connected periods of persistence can last for several years. The fact that periods in recent years are persistent contradicts the assumption that markets get more efficient with time.

Robustness of results has been verified in three directions. First, robustness against non-stationarity has been studied. Second, effects of short range autocorrelations were considered. Finally, leptokurtosis as a source of rejection of the null has been ruled out.

In summary, previous work e.g. by Lux [1996], Lo [1991], and Carbone *et al.* [2004] was extended. We were able to show that returns of time series

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[6]Independence of samples is approximated by scrambling.

can exhibit persistence and not only volatilities or proxies thereof, such as squared returns.

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# 7 Appendix – Tables

In the following detailed tables of results are presented.
Tables 2, 3, 5, 6: The information left of the hyphen pertains to the window size, right of the hyphen to the lag.
Table 4: The information left of the hyphen pertains to the window size, right of the hyphen w – windows, cw – connected windows. Date is the end date of the longest connected period.

Table 2: Number of persistent time windows at 5% level

| Title | 4y-L0 | 4y-L5 | 4y-L21 | 8y-L0 | 8y-L5 | 8y-L21 | 16y-L0 | 16y-L5 | 16y-L21 |
|---|---|---|---|---|---|---|---|---|---|
| Deutsche Bank | 1963 | 600 | 19 | 1685 | 512 | 40 | 314 | 0 | 0 |
| BASF | 325 | 358 | 352 | 2 | 2 | 0 | 0 | 0 | 0 |
| BMW | 1054 | 591 | 932 | 1340 | 973 | 1055 | 49 | 29 | 6 |
| DEGUSSA | 341 | 240 | 72 | 239 | 182 | 30 | 13 | 8 | 0 |
| Continental | 307 | 386 | 315 | 105 | 468 | 772 | 82 | 522 | 363 |
| Bayer | 892 | 661 | 575 | 372 | 444 | 597 | 0 | 2 | 4 |
| Hoechst | 327 | 121 | 101 | 250 | 156 | 183 | 0 | 0 | 0 |
| MAN | 408 | 499 | 376 | 218 | 108 | 134 | 92 | 112 | 131 |
| Karstadt-Quelle | 1539 | 775 | 336 | 225 | 73 | 0 | 5 | 0 | 0 |
| Linde | 962 | 372 | 347 | 1037 | 733 | 200 | 0 | 0 | 0 |
| GEA Group | 524 | 177 | 32 | 1033 | 522 | 340 | 0 | 0 | 0 |
| RWE | 1406 | 910 | 722 | 331 | 87 | 181 | 0 | 0 | 0 |
| Daimler | 1428 | 449 | 128 | 2172 | 1345 | 726 | 1806 | 869 | 32 |
| Bayer-Schering | 2016 | 1049 | 581 | 1557 | 1368 | 1166 | 2093 | 1062 | 347 |
| Siemens | 2685 | 1448 | 654 | 2103 | 1570 | 701 | 759 | 93 | 0 |
| Metro | 349 | 174 | 159 | 25 | 24 | 6 | 96 | 36 | 13 |
| Thyssen Krupp | 1044 | 720 | 707 | 953 | 470 | 278 | 4 | 0 | 0 |
| HVB | 2278 | 635 | 31 | 1631 | 745 | 315 | 386 | 379 | 391 |
| Commerzbank | 2627 | 1278 | 86 | 2861 | 1743 | 906 | 698 | 132 | 12 |
| Allianz | 386 | 311 | 153 | 1828 | 1461 | 1186 | 796 | 339 | 164 |
| TUI | 1765 | 1433 | 1365 | 2527 | 1655 | 655 | 599 | 236 | 90 |

Table 3: Maximum number of adjacent persistent time windows at 5% level

| Title | 4y-L0 | 4y-L5 | 4y-L21 | 8y-L0 | 8y-L5 | 8y-L21 | 16y-L0 | 16y-L5 | 16y-L21 |
|---|---|---|---|---|---|---|---|---|---|
| Deutsche Bank | 620 | 350 | 10 | 764 | 320 | 31 | 147 | 0 | 0 |
| BASF | 182 | 182 | 193 | 1 | 2 | 0 | 0 | 0 | 0 |
| BMW | 256 | 118 | 123 | 462 | 431 | 436 | 30 | 21 | 3 |
| DEGUSSA | 179 | 106 | 19 | 132 | 120 | 18 | 8 | 6 | 0 |
| Continental | 161 | 161 | 164 | 18 | 74 | 189 | 34 | 183 | 147 |
| Bayer | 300 | 125 | 90 | 112 | 123 | 390 | 0 | 2 | 4 |
| Hoechst | 100 | 101 | 101 | 125 | 38 | 64 | 0 | 0 | 0 |
| MAN | 104 | 118 | 182 | 57 | 21 | 24 | 33 | 33 | 34 |
| Karstadt-Quelle | 558 | 545 | 153 | 52 | 45 | 0 | 3 | 0 | 0 |
| Linde | 370 | 94 | 105 | 393 | 147 | 105 | 0 | 0 | 0 |
| GEA Group | 194 | 44 | 11 | 463 | 182 | 92 | 0 | 0 | 0 |
| RWE | 390 | 392 | 341 | 53 | 41 | 43 | 0 | 0 | 0 |
| Daimler | 280 | 73 | 32 | 1110 | 492 | 243 | 338 | 152 | 23 |
| Bayer-Schering | 739 | 439 | 410 | 1303 | 1255 | 476 | 1285 | 573 | 291 |
| Siemens | 533 | 270 | 280 | 1883 | 612 | 468 | 289 | 29 | 0 |
| Metro | 112 | 85 | 57 | 15 | 14 | 6 | 40 | 32 | 13 |
| Thyssen Krupp | 401 | 286 | 400 | 657 | 171 | 94 | 4 | 0 | 0 |
| HVB | 1445 | 390 | 9 | 1078 | 199 | 135 | 386 | 277 | 386 |
| Commerzbank | 1405 | 315 | 16 | 1363 | 560 | 411 | 598 | 75 | 12 |
| Allianz | 201 | 109 | 109 | 663 | 505 | 515 | 298 | 108 | 122 |
| TUI | 682 | 672 | 1026 | 995 | 578 | 102 | 376 | 156 | 73 |

Table 4: All R/S - lags indicate persistence at 5% level

| Title | 4y-w | 4y-cw | 4y-date | 8y-w | 8y-cw | 8y-date | 16y-w | 16y-cw | 16y-date |
|---|---|---|---|---|---|---|---|---|---|
| Deutsche Bank | 19 | 10 | 26.07.88 | 40 | 31 | 20.01.86 | 0 | 0 | - |
| BASF | 294 | 181 | 22.11.70 | 0 | 0 | - | 0 | 0 | - |
| BMW | 365 | 112 | 19.10.75 | 653 | 431 | 30.08.70 | 6 | 3 | 01.05.83 |
| DEGUSSA | 41 | 19 | 19.03.72 | 30 | 18 | 18.10.71 | 0 | 0 | - |
| Continental | 162 | 161 | 30.12.63 | 104 | 18 | 12.12.06 | 82 | 34 | 04.06.98 |
| Bayer | 498 | 90 | 15.01.70 | 285 | 111 | 10.03.86 | 0 | 0 | - |
| Hoechst | 100 | 100 | 29.12.63 | 149 | 38 | 17.06.86 | 0 | 0 | - |
| MAN | 193 | 52 | 08.08.90 | 47 | 16 | 02.10.07 | 90 | 33 | 19.06.90 |
| Karstadt-Quelle | 336 | 153 | 16.02.69 | 0 | 0 | - | 0 | 0 | - |
| Linde | 137 | 94 | 14.01.64 | 138 | 105 | 16.01.68 | 0 | 0 | - |
| GEA Group | 25 | 11 | 10.12.91 | 264 | 92 | 29.01.75 | 0 | 0 | - |
| RWE | 673 | 341 | 20.12.04 | 53 | 30 | 03.12.07 | 0 | 0 | - |
| Daimler | 128 | 32 | 16.02.64 | 726 | 243 | 17.05.71 | 32 | 23 | 29.06.83 |
| Bayer-Schering | 541 | 410 | 06.06.67 | 1166 | 476 | 27.04.71 | 347 | 291 | 08.06.82 |
| Siemens | 645 | 265 | 30.06.69 | 701 | 468 | 21.01.85 | 0 | 0 | - |
| Metro | 108 | 57 | 09.10.75 | 6 | 6 | 15.03.87 | 13 | 13 | 20.11.86 |
| Thyssen Krupp | 489 | 286 | 14.03.68 | 213 | 94 | 07.12.81 | 0 | 0 | - |
| HVB | 31 | 9 | 20.12.67 | 314 | 135 | 31.08.03 | 377 | 277 | 26.12.06 |
| Commerzbank | 23 | 6 | 07.08.97 | 906 | 411 | 19.06.06 | 12 | 12 | 23.09.02 |
| Allianz | 132 | 108 | 24.10.05 | 966 | 325 | 17.10.06 | 49 | 15 | 19.03.03 |
| TUI | 923 | 671 | 01.06.65 | 324 | 102 | 05.03.75 | 90 | 73 | 20.04.83 |

Table 5: Persistence and stationarity at 5% level

| Title | 8y-L5 | stationary | non stationary |
|---|---|---|---|
| Deutsche Bank | 512 | 242 | 270 |
| BASF | 2 | 2 | 0 |
| BMW | 973 | 777 | 196 |
| DEGUSSA | 182 | 182 | 0 |
| Continental | 468 | 196 | 272 |
| Bayer | 444 | 292 | 152 |
| Hoechst | 156 | 136 | 20 |
| MAN | 108 | 88 | 20 |
| Karstadt-Quelle | 73 | 73 | 0 |
| Linde | 733 | 594 | 139 |
| GEA Group | 522 | 394 | 128 |
| RWE | 87 | 87 | 0 |
| Daimler | 1345 | 500 | 845 |
| Bayer-Schering | 1368 | 1089 | 279 |
| Siemens | 1570 | 1036 | 534 |
| Metro | 24 | 11 | 13 |
| Thyssen Krupp | 470 | 302 | 168 |
| HVB | 745 | 670 | 75 |
| Commerzbank | 1743 | 1529 | 214 |
| Allianz | 1461 | 995 | 466 |
| TUI | 1655 | 1594 | 61 |

Table 6: Persistence and stationarity at 5% level – detrending

| Titel | 8J-L5 | stationary | non stationary |
|---|---|---|---|
| Deutsche Bank | 215 | 215 | 0 |
| BASF | 0 | 0 | 0 |
| BMW | 641 | 641 | 0 |
| DEGUSSA | 131 | 131 | 0 |
| Continental | 51 | 51 | 0 |
| Bayer | 5 | 5 | 0 |
| Hoechst | 0 | 0 | 0 |
| MAN | 42 | 42 | 0 |
| Karstadt-Quelle | 111 | 111 | 0 |
| Linde | 299 | 299 | 0 |
| GEA Group | 145 | 145 | 0 |
| RWE | 1 | 1 | 0 |
| Daimler | 612 | 612 | 0 |
| Bayer-Schering | 910 | 910 | 0 |
| Siemens | 533 | 533 | 0 |
| Metro | 36 | 36 | 0 |
| Thyssen Krupp | 300 | 300 | 0 |
| HVB | 56 | 56 | 0 |
| Commerzbank | 491 | 491 | 0 |
| Allianz | 194 | 194 | 0 |
| TUI | 827 | 827 | 0 |

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