$\{0\} \times X$ corresponds to the vertex. In the following, X will be a C^∞ compact closed manifold. For the analysis of pseudodifferential operators on $C_t(X)$ we require a C^∞ structure with a singular point at the vertex. The classical way is to embed $C_t(X)$ in an Euclidean space or in a smooth manifold and to consider the induced structure on $C_t(X)$ near the vertex. For instance, if X is embedded in the unit sphere $\mathbb{S}^N$ of $\mathbb{R}^{N+1}$, then

$$C_t(X) \cong \{rx \in \mathbb{R}^{N+1} : \ r \geq 0, \ x \in X\},$$

which specifies a C^∞ structure with a conical point at the vertex. While topologically each one-point singularity is conical, the C^∞ structure with singularities does depend on the way in which $C_t(X)$ is embedded in a smooth manifold. Indeed, having embedded X in an open set $O \subset \mathbb{R}^N$ star-shaped with respect to the origin, we get

$$C_t(X) \cong \{r\, S(f(r)x) \in \mathbb{R}^{N+1} : \ r \geq 0, \ x \in X\} \tag{0.1}$$

where S is a diffeomorphism of O onto an open subset of $\mathbb{S}^N$, and $f(r)$ a positive C^∞ function on $\mathbb{R}_+$ bounded near $r = 0$. Under this embedding, $C_t(X)$ has a cusp at the origin provided that $f(r) \to 0$, as $r \to 0$. Any C^∞ structure with singularities on $C_t(X)$ determines a class of Riemannian metrics, a structure ring of C^∞ functions at $r = 0$ and a class of typical differential operators on the open "stretched" cone $\mathbb{R}_+ \times X$. And vice versa, either of these items specifies uniquely a C^∞ structure close to the vertex on $C_t(X)$. This can be demonstrated already by the analysis on the half-axis. If $\dim X = 0$, we have $C_t(X) = \overline{\mathbb{R}}_+$ with the one-point singularity $r = 0$ of $\overline{\mathbb{R}}_+$. For the conical singularity at $r = 0$, the structure ring consists of all functions infinitely differentiable up to $r = 0$. On the other hand, for the general embedding $\overline{\mathbb{R}}_+ \hookrightarrow \mathbb{R}^{N+1}$ given by (0.1), the structure ring at $r = 0$ consists of all functions of the form

$$F\left(r, \frac{1}{\delta'(r)}, \frac{\partial}{\partial r} e^{\delta(r)}, \mathbf{D} \log \frac{1}{\delta'(r)}, \mathbf{D}^2 \log \frac{1}{\delta'(r)}, \dots\right) \tag{0.2}$$

where $F(v_1, v_2, v_3, v_4, v_5, \dots)$ is a C^∞ function on all of $\overline{\mathbb{R}}_+ \times \mathbb{R} \times \mathbb{R} \times i\mathbb{R} \times \mathbb{R} \times \dots$, $t = \delta(r)$ is a diffeomorphism of $\mathbb{R}_+$ onto $\mathbb{R}$ satisfying $\delta'(r) = 1/rf(r)$ for small $r > 0$, and $\mathbf{D} = (1/\delta'(r))D$. Point singularities of cusp type were studied by Maz'ya and Plamenevskii [**MP78**], Schulze and Tarkhanov [**ST98**], Schulze, Sternin and Shatalov [**SSS98**], Rabinovich et al. [**RST97**], etc., in the latter paper under the aspect of calculus of pseudodifferential operators with slowly varying symbols. Special cuspidal singularities are also considered in the papers of Melrose and Nistor [**MN96**], Mazzeo and Melrose [**MM97**], et al., where the nature of symbols on compressed cotangent bundles is interpreted in the frame of corresponding vector fields. We refer the reader to these works for a more complete bibliography.

A Hausdorff topological space M is called a *manifold with point singularities* if there is a finite subset $S \subset M$ such that $M \setminus S$ is a paracompact manifold, and every $p \in S$ has a neighbourhood O which is homeomorphic to the cone $C_t(X)$ over a C^∞ compact closed manifold $X = X(p)$. If moreover M bears a C^∞ structure away from S and a C^∞ structure with singular points close to S, then M is said to be a C^∞ *manifold with point singularities*. We define the "stretched" manifold $\mathcal{M}$ associated with M by attaching the sets $[0,1) \times X(p)$, $p \in S$, to $M \setminus S$. Then $\mathcal{M}$ is a C^∞ manifold with boundary $\partial\mathcal{M} \cong \cup_{p \in S} X(p)$, and $\mathcal{M} \setminus \partial\mathcal{M}$ is diffeomorphic to $M \setminus S$.