Clumping in Hot Star Winds
W.-R. Hamann, A. Feldmeier & L.M. Oskinova, eds.
Potsdam: Univ.-Verl., 2008
URN: http://nbn-resolving.de/urn:nbn:de:kobv:517-opus-13981

Observational overview of clumping in hot stellar winds

Anthony F.J. Moffat
Université de Montréal, Canada

In the old days (pre ∼1990) hot stellar winds were assumed to be smooth, which made life fairly easy and bothered no one. Then after suspicious behaviour had been revealed, e.g. stochastic temporal variability in broadband polarimetry of single hot stars, it took the emerging CCD technology developed in the preceding decades (∼1970-80's) to reveal that these winds were far from smooth. It was mainly high-S/N, time-dependent spectroscopy of strong optical recombination emission lines in WR, and also a few OB and other stars with strong hot winds, that indicated all hot stellar winds likely to be pervaded by thousands of multiscale (compressible supersonic turbulent?) structures, whose driver is probably some kind of radiative instability. Quantitative estimates of clumping-independent mass-loss rates came from various fronts, mainly dependent directly on density (e.g. electron-scattering wings of emission lines, UV spectroscopy of weak resonance lines, and binary-star properties including orbital-period changes, electron-scattering, and X-ray fluxes from colliding winds) rather than the more common, easier-to-obtain but clumping-dependent density-squared diagnostics (e.g. free-free emission in the IR/radio and recombination lines, of which the favourite has always been Hα). Many big questions still remain, such as: What do the clumps really look like? Do clumping properties change as one recedes from the mother star? Is clumping universal? Does the relative clumping correction depend on $\dot{M}$ itself?

1 Preamble

Between 1978 and 1995 when it was active, the International UV Explorer satellite (IUE) made great strides in the area of hot-star winds, leading to the discovery, mostly in strong resonant lines, of large-scale wind structures. These were referred to as discrete absorption components (DACs), seen mainly in the absorbing column between the observer and the projected disk of the star. Their variation with time (more or less periodically) is commonly believed to be due to large-scale corotating interactive regions (CIR), as seen in the Solar wind. In hot-star winds, CIRs are probably produced by some kind of rotating inhomogeneity at the surface of the star: non-radial pulsations or local magnetic loops come to mind, which then perturb the rotating wind (e.g. Cranmer & Owocki 1996). It is possible that all hot-star winds contain CIRs, but this remains to be proven (e.g. difficult in WR stars?).

But what we now normally refer to as clumping in hot-star winds is something that appears to be much more widespread and pervasive in all hot-star winds than DACs/CIRs. While many different phenomena pointed towards the presence of some kind of granulation in the winds, it took high-S/N, moderately-high time-resolved spectroscopy as the final clue to say that virtually all parts of all hot-star winds are stochastically inhomogeneous on various (mainly small) scales. This was first revealed in time-resolved optical spectra of WR stars, where their strong, broad emission lines probe the whole wind simultaneously, not just the column along the line-of-sight. It was also in the optical where mostly recombination lines are seen and where it is relatively easy to obtain very high S/N with CCD detectors, unlike the photon-counting systems used in the UV from space.

2 Indirect evidence for clumping

One of the first indications that hot-star winds are not smooth arose from a study of the light-curve eclipses of the Rosetta Stone among WR + O binaries, V444 Cygni by Cherepashchuk et al. (1984). While the primary eclipse (WR in front) shape changes little in passing from optical to near-IR wavelengths, the secondary minimum (O star blocking the WR star) becomes significantly deeper and wider in the IR. This can only occur if most of the IR emission comes from density-squared dependent free-free emission processes in the bright WR wind which must be clumpy, compared to linear density-dependent electron scattering in the optical. If the WR wind were smooth, this would not make a difference. However, one sees a big difference, although the actual nature of the clumpy wind-structure could not be deduced from these data alone. Nevertheless, the clumping must be something that pervades most of the wind where IR-dominated f-f emission originates.

Another phenomenon that had been noticed for some time is electron-scattering wings seen to the red of strong WR emission lines. This is also a contributing factor making WR-star emission-line radial velocities (RVs) more positive than systemic. A clear recent case is WR111, a bright, single Galactic WC5 star, with high-quality available optical and UV spectra, for which Hillier & Miller (1999) deduce a wind filling-factor of f = 0.1 (1 = non-clumped). Assuming the observed lines to be optically thin, this leads to the need for revision of the mass-loss rate down by a factor $1/f^{0.5} \approx 3$. However, this phenomenon yields no obvious constraint on the nature of the clumps (e.g. shells could produce the same effect as stochastic clumps).

In the X-ray domain several clues have arisen, athough the lack so far of detected stochastic variability in X-ray flux from any single hot star (e.g. Berghöfer & Schmitt 1995) might be considered as evidence for absence of clumping. However in fact, the apparently general lack of stochastic X-ray variability is likely to be quite compatible with small-scale multi-clumping, given the limited sizes of X-ray telescopes combined with the relatively low X-ray fluxes, that make detection of the expected <1% variability difficult. On the other hand, massive X-ray binaries (MXRB) containing OB stars sometimes exhibit slow X-ray flickering on timescales of 10's of minutes. A good example of this is the out-of-eclipse X-ray lightcurve of 2U1700-37 = HD153919, O6Iaf + NS, P = 3.41d (Haberl et al. 1989). The apparently random flickering could very well be due to rarer large clumps being accreted onto the NS or its accretion disk, although this has never been analyzed in this vein. Perhaps the X-ray flaring of eta Car could also de due to similar circumstances (Corcoran & Moffat, in prep.). More recently, X-ray spectroscopy has revealed that OB-star winds must have reduced mass-loss rates (although not necessarily porous) in order to account for the line shapes (Owocki & Cohen 2006).

As for photometric and polarimetric variability, the former is less obvious to relate to inhomogeneous winds, since other factors can be at play, such as pulsations, rotation modulation, etc. However, in (linear, where electron scattering is key) polarization, localized inhomogeneities can more easily account for the observed variability, even if it does occur at a very small level. The best examples are for WR stars, which have been extensively and systematically monitored in linear polarization (e.g. Drissen et al. 1987; St-Louis et al. 1987), although some single hot stars of other types have also revealed stochastic variability (e.g. P Cyg: Taylor et al. 1991). At two extremes are the WR stars WR111, WC5, which did not vary during 6 weeks in linear polarization significantly more than the instrumental noise, ∼0.01%, and WR40, WN8, which showed stochastic variations in Stokes' Q and U up to ∼1%. The latter is as large as the strictly periodic variability seen in the strongest-modulated, short-period WR + O binaries, where the free electrons that do the polarizing are mostly associated with the WR wind as a whole, rather than in large clumps. However, it is known that WN8 stars are also the most highly intrinsically *photometric* variables (up to ∼10%) among all WR stars, possibly due in large part to stellar pulsations affecting the winds (Lefèvre et al. 2005).

From the coherent polarimetric variability of WR + O systems, one can also derive mass-loss rates, independent of wind clumping (St-Louis et al. 1988). Along with mass-loss rates based on electron-scattering wings, these were among the first observations to suggest that previous density-squared based $\dot{M}$'s were overestimated by factors of 2-5. The best example of this is V444 Cyg, where $\dot{M}(WR)/(10^{-5} M_{\odot}/yr)$ is 2 or more from thermal radio and IR lines, and 0.7 from polarimetry (from two independent ways: orbital modulation and eclipses) and dynamical period change. I.e. we see the same factor ∼3 here as for the electron-scattering wings in WR111.

However, all of these observations can only be related indirectly to clumping. We now look at more direct lines of evidence.

3 Direct evidence for clumping

Ideally, one would like to actually have enough spatial resolution to image wind clumps directly. To resolve clumps of typical size ≈1 $R_{\odot}$ at a distance of close O stars (∼ 1000 pc) would require microarcsec resolution. However, as clumps expand with the wind flow, they will be much larger if they survive into the outer wind. It is tempting to associate the stochastic structures seen in some WR ring nebulae as due to wind clumps. The best example of this is M1-67 around WR124 (WN8h) (Grosdidier et al. 1998), where the "young" WR wind is just starting to ram into the slow wind of the previous LBV (or possibly RSG) stage. However, one can probably not distinguish between new clumps from the interaction and those left from the expanding wind, although the numerous as-yet unexplained small (∼1") expanding Hα-emitting knots are quite intriguing in the context of wind clumping.

Perhaps the chances are better in resolved thermal radio images of hot-star winds. The best example of this is probably WR147, a WN8 star (accompanied by a 0.6" separated OB companion and a NT wind-wind interaction region between the 2 stars) whose thermal wind has been resolved by the Merlin array (Williams et al. 1997). In this image, one does see knots of several degrees across (as seen from the central star), as expected if the largest clumps survived out this far, i.e. $\sim$4000 stellar radii. This is only slightly larger than in the simulations of Runacres & Owocki (2002).

Possibly the most convincing evidence for clumpy structures in hot-star winds comes from time-resolved spectroscopic monitoring of WR emission lines (Schumann & Seggewiss 1975 - although in a binary, where the interpretation could be ambiguous; Moffat et al. 1988; McCandliss et al. 1988; Robert 1992). These now well-known spectral series show clear stochastic subpeaks of various sizes moving in a direction from line centre to line edge, interpreted as wind clumps whose projected velocities we see as they propagate with the outflow. Although the stars are not resolved, one reaches effectively a poor-man's resolution with the aid of Doppler expansion of the wind, which cannot unambiguously locate the structures other than in a ring of constant angle from the central star. Nevertheless, **this was the first time that it became clear that small-scale wind-structures are essentially universal at least in the strong winds of WR stars.**

4 Clump characteristics

Early analysis of the 9 WR stars for which adequate series of spectral time-series were available used gaussian fits and wavelet analysis of the spectral subpeaks, assuming each to be associated with a clump. General salient properties of the substructures can be summarized as follows (Moffat & Robert 1992, 1994; Moffat et al. 1994):

- $\sim$10 subpeaks are seen at any given time, on average
- relative line variability is typically a few percent
- the width of the subpeak features is FWHM $\sim$2-5 Å (100 - 300 km/s)
- they are seen at all times and last typically $\sim$10 hours
- within the errors, they always move away from line centre
- they are much narrower near line centre
- lines of lower ionization potential tend to be more variable
- the level of variability tends to follow the same wavelength-dependent profile as the emission-line itself, but P Cyg absorption edges tend to be more variable

Assuming most lines to be optically thin, the second-last point above implies that the instabilities tend to grow as they move from the inner to the outer wind, as expected in hydrodynamic simulations at the time of Owocki (1994). The last point suggests that *the wind is the clumps*, i.e. all clumps follow on average the same expansion law and formation profile in the wind.

Using wavelets allowed one to probe down to smaller substructures propagating across the line profiles. It was found that in general, real repeating substructures tend to follow a simple power law relation between substructure flux and substructure width in velocity space: $f \propto \sigma_v^{2.0\pm0.4}$. Non-repeating substructures both in the line and in the neighbouring continuum followed a flatter-slope power law, as expected if due to noise associated with a constant detection limit in substructure height $h = f/\sigma_v$. Furthermore, a less-reliable but feasible relation was found for the substructure lifetimes: $\tau \propto f^{0.8\pm0.2}$.

One of the main signatures of turbulence is the presence of scaling laws among the various structures in a turbulent medium. This is quite evident in the ISM for GMC strucures, which reveal strong evidence for supersonic, compressible turbulence, driven ultimately by fluctuations in the local gravitational field (Henriksen 1991). In that case, one expects typical clump lifetimes of $\tau \sim l/\sigma_v \sim 1/\sqrt{G\rho}$, i.e. the free-fall time in virial equilibrium. Also, for constant gas pressure, $P \sim \rho\sigma_v^2 \sim$ const. ¿From these, it follows that $\rho \propto 1/l$ (i.e. opacity ρl = const) and $\sigma_v \propto l^{1/2}$. Unfortunately, ρ and l are not directly observable, but one can also use τ and $f \propto \rho^2 l^3$ for an ideal gas, from which follow that $f \propto l \propto \sigma^2$ and $\tau \propto l^{1/2} \propto f^{1/2}$. These are quite close to what is observed in WR winds, as noted in the previous paragraph!

Furthermore, one can also look at the numbers of substructures of different flux level or mass, such that $\mathrm{N}(f)df = N(\mathrm{m})\mathrm{dm}$. From the wavelet analysis of the observed emission-line fluctuations of the WR stars, one finds that $\mathrm{N}(f) \propto f^{\alpha}$, with typical α = -1.9 ±0.2. Then with a clump mass m $\propto \rho l^3 \propto l^2 \propto f^2$ one finds $N(\mathrm{m}) \propto \mathrm{m}^{\gamma}$, where $\gamma = (\alpha - 1)2 = -1.5 \pm 0.1$, like in GMCs on average (Stutzki 1994)!

The similarity of these scaling laws for the clumps to what one sees in GMCs leads one to believe that the clumpy structure of hot-star winds is also a result of full-scale anisotropic turbulence, with scaling laws like those seen in the fractalized ISM. The only real difference is that in the winds, the driver is radiative instabilities, while in the ISM it is small-scale fluctuating gravity. As R.N. Henriksen always used to remind me (see also Henriksen 1994), **"In astrophysical turbulent media, it often doesn't matter what the physics is - the result is always about the same!"**. Indeed, the presence of scaling laws in a medium is a good signature of turbulence, with lots of structure and subscale elements, both nested and not nested in superscale elements (Henriksen 1991).

Later, Sébastien Lépine (1998) in his PhD thesis re-analyzed the same repeated spectra for the 9 WR stars using Mexican-hat wavelets that Carmelle Robert analyzed using gaussians in her thesis (Robert 1992). From this new work (Lépine & Moffat 1999, with even more intense data for the single WC8 star WR135: Lépine et al. 2000), it became clear that the individual subpeaks seen moving across spectral emission lines are not due to individual clumps. This may not matter regarding the above results based on the opposite assumption, given the nested nature of turbulence! Nevertheless, the newer work represents a fresh, more rigorous look at the problem.

In particular, Lépine & Moffat (1999) found from their mean wavelet power-spectral analysis that the dominant scale occurs at mean line-of-sight velocities of $\sim$100 km/s for all 9 analyzed WR stars, except WR134 with $\sim$350 km/s, due to a velocity component from rotating CIRs in that case. In their analysis, Lépine & Moffat made line-profile variation (LPV) simulations assuming a β-expansion law for the wind occupied by a large number of discrete wind-emission elements (DWEEs), whose fluxes obeyed a power-law distribution within the line-emitting region (LER). In all cases, at least 10^4 DWEEs were needed to simulate the observed LPVs in each WR star, assuming that there are no other components besides DWEEs in the wind. Interestingly, the number of simulated spectral subpeaks was $\sim$10 on average, but varying with the actual line width, as the actual data show. By varying the anisotropy of the simulated emitting DWEEs in the ratio of radial to lateral emission and velocity dispersion, they found that the simulations which best matched the observations were with isotropic emission but larger radial than transverse velocity dispersion by a factor 4. This latter value is actually probably much higher, as a result of projection effects, as shown by Dessart & Owocki (2002). This means that the velocity dispersion occurs mostly in the radial direction, where in fact the bulk of the radiative force (and instabilities associated with it) occur.

Lépine & Moffat (1999) also devised what they called a "degradation function", used to correlate the ensemble of varying patterns in successive spectra in a statistical way. From this, they deduced that the (assumed constant) wind acceleration for any given line in any given WR star lies in the range 4-25 m/s^2, with the largest value for WR134, with its unusually broad CIRs. With these adopted acceleration values for each star, they then found from when the pattern correlation completely broke down, that the DWEE lifetimes are the same order as the propagation time through the LERs, i.e. typically 6-10 hours. Unfortunately, they could not constrain the wind accleration β parameter by itself; they could only constrain the product β R$_*$ to lie in the range 20-80 R$_\odot$. These values have been reduced by the patch-method simulations of Dessart & Owocki (2005) by a factor 2-3. Nevertheless, they still remain large enough to say that the β values where most WR lines are formed are >>1, i.e. much softer winds than in O stars. An exception to this is found in the Of-like WN3ha star WR3 (see Chené et al., these proceedings).

5 Not only classical WR-star winds are clumped!

Although the easiest, most obvious results have emerged for "classical" WR stars because of their intense, broad emission lines, it is now clear that virtually all hot-star winds likely show similar effects. One obvious general target to investigate was the WC-type central stars of planetary nebulae. Not many of these (they are faint!) have been observed sufficiently yet, but those that have, tend to show almost identical effects as do the clumps in population I WR stars, except that the timescales are much shorter in the CSPN [WC] stars, because their radii are smaller. The best current example is the [WC8] star NGC 40 (Grosdidier et al. 2001).

Among pop I O stars, only Of stars have strong enough winds to allow probing fine structures on their optical emission lines, where it is still easiest to obtain the highest S/N time-resolved spectra. This was demonstrated for the first time by Eversberg et al. (1998) for the bright (both apparently and intrinsically!) O4I(n)f star ζ Puppis. Although its optical emission lines (the strongest being at HeII 4686 and Hα) are an order of magnitude weaker than those of typical WR stars, with spectra of S/N > 1000 Eversberg et al. showed that its wind behaves almost identically with respect to LPV as any WR star. In fact, the relative stochastic LPV due to clumping is the same, i.e. several %, implying (for optically thin lines) that the overdensities in typical clumps in ζ Pup's wind are similar to those in the winds of WR stars. Another satisfying result of the Eversberg et al. study is that ζ Pup shows a normal wind-acceleration value ($\beta \sim$1) from the clump trajectories, as expected for OB stars.

In the meantime, Lépine & Moffat (in prep.) have observed ζ Pup again, along with another Of star (HD 93129A in the Tr14 cluster of the Carina Nebula), as well as the 2 WN6ha (Of-like) WR stars in the same Carina Nebula, WR24 and WR25. Again, the same general trend prevails as in the first monitoring of ζ Pup by Eversberg et al., suggesting that indeed clumping appears to be universal.

Other types of hot-star winds in which clumps have been seen recently include novae (Lépine et al. 1999) and supernovae (Matheson et al. 2000).

6 Importance of hot-star wind-clumping

The most important results relating to clumping (in the small-scale sense) in hot-star winds can probably be summarized as follows:

- The presence of clumping and its characteristics is universal in all hot-star winds.
- The root cause is likely connected to turbulence driven by radiative instabilities in the radial direction of the wind flow.
- Hot-star winds provide a unique astrophysical laboratory to study (anisotropic) supersonic compressible turbulence in action on tractable time scales.
- True mass-loss rates are a factor 2-5 (3 on average) lower than those based on Hα and thermal radio fluxes assuming smooth flows.
- Lower mass-loss rates have a significant impact on stellar evolution, e.g. decreasing the initial masses based on observed masses and increasing the importance of luminous blue variables (LBVs)to allow O stars to evolve into WR stars, since the winds of O stars can no longer be considered strong enough to do it by themselves.
- One should be able to empirically trace and tie down the wind expansion velocity-law using clumps in lines of various ionization/excitation potential.
- Clumps will soften collisions both between winds in hot-star binaries (where observed X-ray fluxes are low by orders of magnitude cf. those expected based on flow rates) and by interaction with the ISM.
- In binaries, the presence of clump overdensities will better allow one to understand how dust forms in the winds of some WC + O binaries, and possible even in some single WC stars; clumps also widen secondary eclipses and can cause X-ray flickering in MXRBs.
- Hot-star wind clumps could conceivably contribute to the clumpiness of the ISM.

7 Future needs

In order to advance our understanding of hot-star wind clumping, the most pressing needs in my view are:

- One needs ultimately to spatially resolve the clumps, starting where it may be easiest, i.e. for nearby objects in the radio, where large arrays are beginning to provide sufficient resolution combined with sensitivity.
- One also needs to carry out full (M?)H-D in 3D, given that turbulence is basically a 3D phenomenon.

References

Berghöfer, T.W. & Schmitt, J.H.M.M. 1995, AdSpR, 16, 163

Cherepashchuk, A.M., Khaliullin, K.F., Eaton, J.A. 1984, ApJ, 281, 774

Cranmer, S.R. & Owocki, S.P. 1996, ApJ, 462, 469

Drissen, L., et al. 1987, ApJ, 322, 888

Dessart, L. & Owocki, S.P. 2002, A&A, 393, 991

Dessart, L. & Owocki, S.P. 2005, A&A, 432, 281

Eversberg, T., Lépine, S. & Moffat, A.F.J. 1998, ApJ, 494, 799

Grosdidier, Y., et al. 1998, ApJ, 506, L127

Grosdidier, Y., Acker, A., Moffat, A.F.J. 2001, A&A, 370, 513

Haberl, F., White, N.E., Kallman, T.R. 1989, ApJ, 343, 409

Henriksen, R.N. 1991, ApJ, 377, 500

Henriksen, R.N. 1994, Ap&SS, 221, 137

Hillier, D.J. & Miller, D.L. 1999, ApJ, 519, 354

Lefèvre, L., et al. 2005, 634, L109

Lépine, S. 1998, PhD Thesis, Université de Montréal

Lépine, S. & Moffat, A.F.J. 1999, ApJ, 514, 909

Lépine, S., et al. 1999, ApJ, 522, L121

Lépine, S., et al. 2000, ApJ, 120, 3201

Matheson, T., et al. 2000, AJ, 120, 1499

McCandliss, S.R. 1988, PhD Thesis, Colorado Univ., Boulder

Moffat, A.F.J., et al. 1988, ApJ, 334, 1038

Moffat, A.F.J. & Robert, C. 1992, ASPC, 22, 203

Moffat, A.F.J. & Robert, C. 1994, ApJ, 421, 313

Moffat, A.F.J. 1994, Rev. Mod. Astronomy, 7, 51

Moffat, A.F.J., et al. 1994, ApSpSc, 216, 55

Owocki, S.P. 1994, ApSpSc, 221, 3

Owocki, S.P. & Cohen, D. 2006, ApJ, 648, 565

Robert, C. 1992, PhD Thesis, Université de Montréal

Runacres, M.C. & Owocki, S.P. 2002, A&A, 381, 1015

Schumann, J.D. & Seggewiss, W. 1975, IAUS, 67, 299

St-Louis, N., et al. 1987, ApJ, 322, 870

St-Louis, N., et al. 1988, ApJ, 330, 286

Stutzki, J. 1994, Rev. Mod. Astron., 6, 209

Taylor, M., et al. 1991, AJ, 102, 1197

Williams, P.M., et al. 1997, MNRAS, 289, 10

Feldmeier: How can you derive 10^4 clouds from only 100 spectral features seen? I guess you make the difference between model and observations ever smaller by increasing the cloud number. But what prevents you from going to infinity with the number of clouds in order to make this error go to zero?

Moffat: Actually one sees typically ~ 10 (not 100) spectral sub-peaks at any given time in WR spectral lines. Some $> 10^4$ DWEES are deduced in the context ot the simple Lépine et al. (1999) model to give the best match to what we actually see varying in the spectra. In reality, probably the number of large clumps, which dominate, will be considerably below 10^4.

Prinja: I think an important constraint will ultimately come from an understanding of how small-scale structure (clumping) coexists with large-scale spatial structure (such as diagnosed via DACs). Whilst the two forms of structure are known to coexist in OB stars, do you think the extensive clumping in WR stars may obliterate extended coherent structures in WR stars?

Moffat: No, the two are sometimes seen together, depending mainly on the quality of the data. In the case of WR stars, WR 134 in the Galaxy is a good example.

Cassinelli: You showed a slide in which there were large scatter on the polarization Q, U plane. This would mean there is a relatively small number of clumps and each is fairly massive. Have you tried to estimate clump masses from this?

Moffat: The time-sampling I showed ($\sim$ a day) is inadequate to say that one has a small number of clumps. In any case, spectra show typically only ~ 10 bumps, although one suspects $> 10^4$ substructures that overlap and combine to give a small number of apparent features.

Vink: Regarding Joe's comment: large polarization variations must be due to a few large clumps. But I will show tomorrow that there are two regimes, and recent polarization monitoring of LBVs favours the scenario of many small clumps.

Hillier: There does appear to be evidence that the volume filling factor you determine depends on v_∞. In particular, in LBVs (AG Car, P Cygni) the measured volume filling factors are clearly larger than the canonical value of 0.1 (e.g. 0.2 to 0.4) and thus different from many O and WR stars (which have higher terminal velocities). These values are determined from the strength of the electron-scattering wings, which are readily seen and measured. Such a scaling might be consistent with that expected from radiation driven instabilities.

Owocki: You emphasized a similarity between wind turbulence with that in the ISM. But I think that a key difference is that ISM turbulence is likely to be (statistically) isotropic, whereas your (and collaborators) observations show quite clearly that wind turbulence has a stronger velocity dispersion in the radial than lateral directions. The hydrodynamic simulations by Luc Dessart and myself are able to match the observed profile variations, including the variation of turbulent speed across the profile, without any fine tuning, except to choose a lateral scale size of about 3 degrees.