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Optimal Spatial Patterns of Two, Three and Four Segregated Household Groups in a Monocentric City

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Abstract

Usually, in monocentric city models the spatial patterns of segregated household groups are assumed to be ring-shaped, while early in the 1930ies Hoyt showed that wedge-shaped areas empirically predominate. This contribution presents a monocentric city model with different household groups generating positive externalities within the groups. At first, border length is founded as a criterion of optimality. Secondly, it is shown that mixed patterns of concentric and wedge-shaped areas represent multiple equilibria if more than two groups of households are being considered. The welfare optimal segregated pattern depends on the relative purchasing power of different household groups.

Keywords: Monocentric city, segregation, spatial pattern, externalities
JEL Classification: D62, R14, R21, R31.

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1 Introduction

Empirical studies of current city structures show increasing segregation of ethnic or lifestyle groups (Sassen 1996, Harth/Herlyn/Scheller 1998, Schneider/Spellerberg 1999 or Wagner 2001). Ethnic or other non-economic segregation takes place where different household groups exist and if there are either negative externalities between households of different groups or positive externalities between households of the same group. An example of the former is racism while an example of the latter is the existence of social networks. Schelling (1978) shows that such externalities lead to a dynamic process of segregation because households choose their location so that either the number of households of the other group in the neighborhood is minimized or the number of households of the same group is maximized. This process is called *tipping-process.*

The analysis of urban segregation brought about two different spatial patterns of areas of different household groups. On the one hand, there is the well discussed ring-shaped pattern according to Alonso's (1960, 1964) description of households' location choice. On the other hand, in the 1930ies, Hoyt (1939) discovered empirically that the dominating spatial segregation pattern in American cities was more or less wedge-shaped. The basic difference lies in the direction of borderlines which can be either concentric, leading to a ring-shaped pattern, or radial, with wedge-shaped patterns.

In the discussion of segregation caused by ethnic or other non-economic characteristics, focusing on density and pricing structure in space, the spatial pattern is usually given. The arising density structure depends on different assumed causes for segregation. In *border models,* the border itself is such cause. Its influence on density and price structure at any given location decreases with distance. In *amenity models*, density and price structure are affected by the composition of the population in a certain neighborhood while the effects decrease by distance to the respective location.

Rose-Ackerman (1975) analyses the effects of racism within a border model. She assumes a ring-shape as the pattern with the shortest border length and thus the least connection between households of different groups. Yinger (1976) shows that, depending on population mix, a wedge-shaped segregation pattern may lead to a minimal border length as well as the lowest number of households on a border.

In this contribution a monocentric model is used to discuss the spatial segregation pattern of two, three and four household groups. There is a given city center which influences the location decision of households with regard to commuting between any location within the city and the city center. As in the model of Muth (1969, 37), the amount of commuting is a variable of the utility function, based on the idea that a local public good is available in the city center which can be consumed as often as a household commutes. Additionally, it is an element of the budget constraint due to transportation costs.

Furthermore, externalities between different types of households are assumed which affect the evaluation of a neighborhood by households according to their preferences. As a consequence the evaluation of a neighborhood varies with the household type. Later, positions of concentric and radial borders are examined. As in the amenity models, the externalities occur in the direct neighborhood of households while their effects disappear as soon as households are not located directly next to each other.

As a result of the model, the allocation efficiency and stability of different patterns are discussed. It is shown that urban space is divided into segregated areas of household groups according to the ratio of their purchasing power.

Section 2 presents a model of the housing market containing a local public good, a special production function and externalities between households of different types. Section 3 compares the efficiency of different spatial patterns of two, three and four household groups, and section 4 summarises.

2 The model

2.1 Assumptions

Assumption 1 *The population of the city is divided into different groups $j = 1, ..., i, i', ..., J$ of households H_j. The share of households of one group to the city population is b_j.*

These groups may be distinguished by family structure, race or lifestyle or other non-economic characteristics.

Assumption 2 *The city is open. The population may move without migration costs from outside into the city and vice versa.*

Assumption 3 *The households maximize their utility:*

$$U = z^{\alpha_z} S^{\alpha_s} x^{\alpha_x} \tag{1}$$

in which z represents a local public good, S the consumption of housing services and x the consumption of all other goods. The exponents $\alpha_z, \alpha_s, \alpha_x$ are exogenous and represent the preferences for the different goods.

The utility function is homogenous of the degree $\alpha_z + \alpha_s + \alpha_x$. It can be expected that $\alpha_z, \alpha_s, \alpha_x < 1$ and $\alpha_z + \alpha_s + \alpha_x \leq 1$. Therefore, there is a decreasing marginal utility for each good and for all goods, i.e. for income, according to the usual neoclassical framework.

Assumption 4 *The local public good can be obtained by commuting between the location of housing and the city center. The transportation cost for a unit of the local public good z is t per distance r inside the city and T outside.*

The public good z may either be the typical public service, like administration, infrastructure etc., or it may be interpreted as an immaterial good of the city itself, such as information, lifestyle etc. The main aspect is that it must be obtained via transport or commuting paid for by the households.

Assumption 5 *The housing service S is an aggregate of the lot size and the quality q. These characteristics are combined by a Leontief production function:*

$$S = Min(s, Q) \tag{2}$$

in which s represents land and Q characteristics of the lot's quality.

For simplicity housing service may be standardized to:

$$s = Min\,(1, q) \tag{3}$$

which is the "qualified land".

Assumption 6 *The quality includes neighborhood quality n which is affected by positive externalities among households of the same group and other producible characteristics a. They are perfect substitutes:*

$$q = a + n \tag{4}$$

Households obtain positive externalities which are generated by the share of the neighborhood occupied by households of their group.

Assumption 7 *The externalities vary with the household group. Three different cases shall be considered: The neighborhood with the radius g around a household is mixed* (n_{mix})*, the neighborhood is dominated by its group* (n_{dom})*, or the neighborhood is dominated by members of other groups* (n_0)*. It is* $n_0 < n_{mix} < n_{dom}$ *and* $n_0 = 0$*. The neighborhood quality at the outer border of the city is* n_{mix} *even if a group dominates a neighborhood because a part of that neighborhood does not generate externalities.*

Due to the form of externalities, neighborhood quality and the quality of a lot are not the same for households of different groups:

$$q_j = a + n_j. \tag{5}$$

Thus, externalities touch the neighborhood quality as well as the quality of the housing service and finally the utility of a household.

Assumption 8 *The budget* Y_j *can but doesn't have to vary among the different household types.*

Assumption 9 *The supply of space in relation to available land is inelastic.*

Assumption 10 *Housing service is produced by landlords who decide whether to supply qualified land and how many producible characteristics to add. They maximize profits.*

Assumption 11 *There is an alternative use of land which yields* p_b *per unit of land.*

The price for alternative land use may either be determined by rural land use or other alternative land uses. It is also the price for housing outside the city.

2.2 Demand for housing services in the city

Households maximize their utility subject to the budget constraint:

$$Y_i = trz + p_s s + p_x x \tag{6}$$

where p_x is the price for the consumption bundle x and p_s is the price for qualified land. The indirect utility function inside the city follows as:

$$U_j = \left(\frac{\alpha_z}{tr}\right)^{\alpha_z} \left(\frac{\alpha_s}{p_s}\right)^{\alpha_s} \left(\frac{\alpha_x}{p_x}\right)^{\alpha_x} \left(\frac{Y_j}{\alpha_z + \alpha_s + \alpha_x}\right)^{(\alpha_z+\alpha_s+\alpha_x)} \tag{7}$$

while outside the city the indirect utility is:

$$\overline{U}_j = \left(\frac{\alpha_z}{T}\right)^{\alpha_z} \left(\frac{\alpha_s}{p_b}\right)^{\alpha_s} \left(\frac{\alpha_x}{p_x}\right)^{\alpha_x} \left(\frac{Y_j}{(\alpha_s + \alpha_x + \alpha_z)}\right)^{(\alpha_s+\alpha_x+\alpha_z)}. \tag{8}$$

In the open city framework the households are willing to pay for housing as long as they obtain at least the amount of utility they can achieve outside the city. Thus, equalizing the utility and solving for p_s leads to the compensated price function $\psi_j(r)$, also known as bidprice function:

$$\Psi_i(r) = p_b \left(\frac{T}{tr}\right)^{\frac{\alpha_z}{\alpha_s}}. \tag{9}$$

This bidprice function shows a few interesting features. The shape bidprice function is similar for each group. Obviously there is no influence of income differences on the steepness of the bidprice function. The reason for this surprising result is that the amount of commuting is an element of the utility function, rather than a reduction of the income in the budget constraint as for example in Alonso (1964). The other characteristics of the bidprice function are as presented by Alonso (1964) and also by Wheaton (1974) who demonstrates the comparative statics.

2.3 Supply of housing services

With regard to the obtainable bidprice curves $\Psi_j(r)$ the landlords obtain a profit per unit of land or a rent ρ:

$$\begin{aligned}\rho_j(r) &= \Psi_j(r)s - p_a a \\ &= \Psi_j(r)Min(1, a + n_j) - p_a a \qquad (10)\end{aligned}$$

which is dependent on the bidding group and the neighborhood quality evaluated by this group. Thus, the profit maximizing level of produced characteristics of quality a follows as:

$$a_j = \left| \begin{array}{cc} 1 - n_j \text{ if } \psi_j(r) > p_a(1-n) \\ 0 \quad \text{otherwise} \end{array} \right., \tag{11}$$

and also varies with the household group. While for any unit of housing one unit of land is used, the amount of other quality characteristics a depends on the quality of the neighborhood n. In the bidprice function this leads to:

$$\rho_j(r) = \Psi_j(r) - p_a(1-n_j). \tag{12}$$

The profit, therfore, is a linear function of the neighborhood quality. The reason is that neighborhood quality perfectly substitutes other quality characteristics and reduces their costs. As a consequence the neighborhood quality is responsible for the profit.

2.4 Equilibria and welfare of the housing market

While the neighborhood's quality may be evaluated differently by households of different groups, the profit of the producer also varies with these households. Since the landlords have no influence on the bidprice curves $\Psi_j(r)$ they will choose households of a certain group to maximize their rent $\rho_j(r)$. They have strong incentives to select households in a way to minimize the costs for offering producible quality a. Those households will belong to the group which dominates the neighborhood. In their point of view, the neighborhood quality is n_{dom} which corresponds with a level of produced quality $a = 1 - n_{dom}$. If landlords did not discriminate in this way, they would have to add more produced quality characteristics a, namely $1 - n_0$, to the same amount of land without getting a higher price.

It is worth noting that this selection process could occur without open discrimination if landlords offered housing with a certain level of produced quality characteristics and only members of the dominating household group accepted this supply while members of other groups refused it because of neighborhood characteristics. By adjusting the amount of producible quality, landlords give the land to the households with the highest profit per land.

Under following conditions, this allocation leads to an equilibrium:

1. Locations are allocated to households of the group allowing the highest profit.

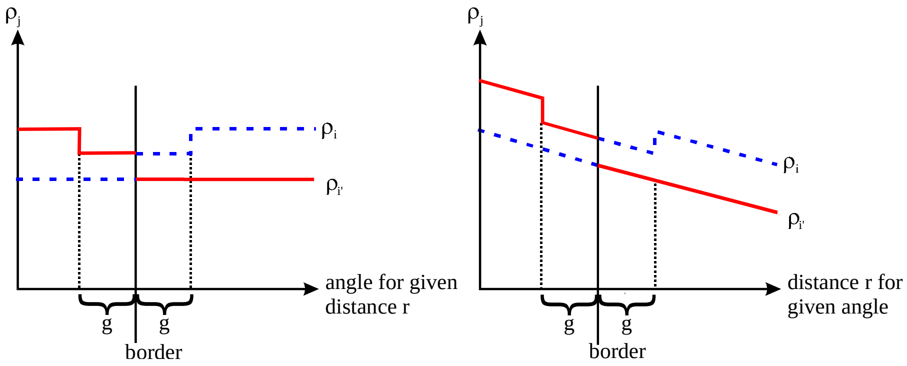

Figure 1: Bid price functions of two groups at a border

2. Within the city, households of a group get the same utility.
3. At borders of segregated areas, the profits allowed by different groups have to be equal.

This applies if landlords give their lots to households of the largest group or one of the largest groups of a neighborhood, enabling them to reduce the produced quality a to a minimum without reducing the bids and thus raising profits. At a border, for members of any two different groups (i and i') follows:

$$\begin{aligned} \rho_i(r) &= \rho_l(r) \\ \psi(r) - p_a n_i &= \psi(r) - p_a n_{i'}. \end{aligned} \tag{13}$$

Since with equation (9) the bid price functions are equal, it follows

$$n_i = n_{i'} = n_{mix} \tag{14}$$

at the border and profits are equal (see figure 1). If at the border (13) holds, then $\rho_i(r, n_{dom}) > \rho_{i'}(r, n_0)$ within an area of group i and $\rho_i(r, n_0) < \rho_{i'}(r, n_{dom})$ within an area of group i', which shows that pure segregation is stable. Consequently, in equilibrium there are only purely segregated areas and two different land rent functions $\psi(r) - p_a n_{dom}$ within purely segregated areas and $\psi(r) - p_a n_{mix}$ within a range g next to borders of segregated areas and the outer border of the city.

For the spatial pattern of the city there is not a unique solution. Any spatial pattern is stable when no landlord can increase his profit by choosing another household. This is the case when the borderlines are more or less straight. If a borderline was curved within a neighborhood with the radius g, one group would be larger and thus dominate. At the other side of the border landlords would give their locations to households of the larger group and thereby reshape the border to a straighter line. If a borderline becomes straight, the pattern is stable and an equilibrium is reached. Thus multiple equilibria are possible.

The welfare provided by the housing market in an open city is not dependent on households' utility because they get the same utility either in or outside the city. The only remaining aspect of welfare is the landlords' surplus. Equation (12) shows that it is dependent on the evaluation of the neighborhood which itself is dependent on the spatial structure. In pure segregated areas landlords get higher profits then in mixed neighborhoods and consequently the surplus is reduced in neighborhoods next to borders. As a result, the sum of border lengths in a city is a measure for the reduction of surplus.

As long as the population mix of the whole city is not explained, there is no reason for a mixed city. Without further assumptions the most efficient would be a city with only one household group and thus without any border, as in Miyao (1978) and Miyao, Shapiro and Knapp (1980). A reason for mixed population can be founded in the production of goods, assuming the units of labour provided by households to be imperfect substitutes or complements in production. Therefore, a groups' income can be positively related to the number of households of another group or the population mix is given by the production function.

In the next section, assuming the population mix being exogenous, the border length of stereotypically spatial patterns will be compared.

3 Comparison of spatial segregation patterns

The analysis is reduced to the comparison of stereotypical spatial patterns if the following assumption holds:

Assumption 12 *The directions of borderlines between segregated areas are either concentric or radial to a city center.*

With this assumption spatial patterns may be dereived. Thje spatial pattern is fully described when the outer border of the city and the locations of borders are known. As a first step the spatial distribution of households has to be derived. This, subsequently allows to locate the borderlines. The demand for qualified land follows as:

$$s_j = \frac{\alpha_s Y_j}{(\alpha_s + \alpha_x + \alpha_z)\, p_b} \left(\frac{tr}{T}\right)^{\frac{\alpha_z}{\alpha_s}}. \tag{15}$$

This leads to the density function of any household group j due to the relationship between demand and density $h_j = 1/s_j$:

$$h_j\,(r) = \frac{(\alpha_s + \alpha_x + \alpha_z)\, p_b}{\alpha_s Y_j} \left(\frac{T}{tr}\right)^{\frac{\alpha_z}{\alpha_s}}. \tag{16}$$

It is obvious that density decreases with distance r. Furthermore it is dependent on income. A higher income corresponds with a lower density due to a higher demand for qualified land. On the basis of these density functions the locations and thus the lengths of borders can be calculated if population mix b_j and income Y_j are known.

The outer border of the city is reached at the distance R to the city center where land rent $\rho_i\,(r)$ equals the alterative rent p_b. It follows:

$$R = \frac{p_b T}{(p_b + p_a n_{mix})\, t}. \tag{17}$$

Now the borderlines between segregated areas can be located. As a first pattern a radial borderline in an area will be derived. Such an area can either be a full circular city or a part f of it with radial and circular borders. In the latter case, the radii of an inner circular border r_{f-1} and an outer border r_f have to be taken into account which are set 0 and R if the full circular city is considered. This area, which is either the full ring around a city center or a wedge-shaped area of it, is divided into segregated parts by a radial borderline. It can be located by the angle of the border between the segregated parts and those of the considered area of the city. If the angle of the considered area is the radiant number γ, the angle of a border of a wedge-shaped segregated part of a group i can be represented by the share c_i of this angle and counts to $c_i\gamma$. The number of households belonging to group i with the share $\widetilde{b}_i$ can be written as:

$$\widetilde{H}_i = \int\limits_{r_{f-1}}^{r_f} c_i \gamma r h_i\,(r)\, dr \tag{18}$$

while the amount of all households of the whole area considered is:

$$\widetilde{H} = \sum_{j=1}^{J} \int_{r_f-1}^{r_f} c_j \gamma r h_i\left(r\right) dr \tag{19}$$

with $j = 1, .., i, .., J$ being all household groups living in the considered area. Considering equation (16) in

$$\widetilde{H}_i = \widetilde{b}_i \widetilde{H} \tag{20}$$

leads to:

$$\widetilde{b}_i = \frac{c_i Y_i^{-1}}{\sum\limits_{j=1}^{J} c_j Y_j^{-1}}. \tag{21}$$

Solved for c_i this yields:

$$c_i = \frac{Y_i \widetilde{b}_i}{\sum\limits_{j=1}^{J} Y_j \widetilde{b}_j}. \tag{22}$$

Therefore the share of the area of a household group within the whole city corresponds with its relative purchasing power.

The location of a circular border between segregated parts of an area can be derived as follows. The population of the considered area can be calculated as:

$$\widehat{H}_i = \int_{r_{i-1}}^{r_i} \gamma r h_j(r) dr. \tag{23}$$

Given radius r_i as the outer and r_{i-1} as the inner borderline of the segregated part of any group i, the number of households of this group follows as:

$$\widehat{H} = \sum_{j=1}^{J} \int_{r_{j-1}}^{r_j} \gamma r h_j(r) dr. \tag{24}$$

Its share $\widehat{b}_i$ of the considered area's population, containing $j = 1, .., i, .., J$ groups, can be written as:

$$\widehat{b}_i = \frac{\left(r_i^{\frac{2\alpha_s - \alpha_z}{\alpha_s}} - r_{i-1}^{\frac{2\alpha_s - \alpha_z}{\alpha_s}} \right) Y_i^{-1}}{\sum_{j=1}^{J} \left(r_j^{\frac{2\alpha_s - \alpha_z}{\alpha_s}} - r_{j-1}^{\frac{2\alpha_s - \alpha_z}{\alpha_s}} \right) Y_j^{-1}}. \tag{25}$$

The outer radius of any segregated part of group i in the considered area f follows as:

$$r_i = \left(r_{i-1}^{\frac{2\alpha_s - \alpha_z}{\alpha_s}} + \frac{\sum_{j=1}^{i} Y_i \widehat{b}_i}{\sum_{j=1}^{J} Y_j \widehat{b}_j} \left(r_{i+1}^{\frac{2\alpha_s - \alpha_z}{\alpha_s}} - r_{i-1}^{\frac{2\alpha_s - \alpha_z}{\alpha_s}} \right) \right)^{\frac{\alpha_s}{2\alpha_s - \alpha_z}} \tag{26}$$

with $i < J$. With i, here, groups are counted beginning at the inner border of the area r_{f-1}. For the segregated part closest to the city center $r_{i-1} = r_{f-1}$ which is 0 if the area is connected to the city center. For the segregated part farthest from the city center $r_{i+1} = r_f$ which is R, if the area reaches the city's border. Again, the angle γ of the area does not matter.

If the considered area is connected with the center and the border of the city, the radius of the outer border of any segregated part can be simplified as:

$$r_i = \left(\frac{\sum_{j=1}^{i} Y_j \widehat{b}_j}{\sum_{j=1}^{J} Y_j \widehat{b}_j} \right)^{\frac{\alpha_s}{2\alpha_s - \alpha_z}} R. \tag{27}$$

This shows that, as in wedge-shaped patterns, the shares of areas are based on the relative purchasing power of its inhabitants.

In addition to that, equation (27) reveals that the radius and thus the area increase with rising b_i if $\alpha_s / (2\alpha_s - \alpha_z) > 0$. This is the case for a limited value domain with $\alpha_z < 2\alpha_s$ only. Since $a_z > 0$, the expression $\alpha_s / (2\alpha_s - \alpha_z)$ is between 1/2 and infinity and the value domain for α_s is:

$$0 < \alpha_z < 2\alpha_s. \tag{28}$$

Ring and wedge-shaped segregation patterns are connected by the following relationship. The total population of a considered area is equal if the population mix is equal:

$$\widetilde{H} = \widehat{H} = H \tag{29}$$

if

$$\widetilde{b}_i = \widehat{b}_i = b_i. \tag{30}$$

To summarise, in the wedge-shaped as well as in the ring-shaped segregation patterns the constellation of purchasing power is responsible for the location of borderlines and vice versa.

3.1 Segregation patterns for two household groups

In this subsection the spatial pattern for two household groups will be examined, therefore set $J = 2$. The groups' shares of the population can be simplified to $b_1 = b$ and $b_2 = 1 - b$. For a ring-shaped segregation pattern border length G_R follows as:

$$G_R = 2\pi \left(\frac{Y_1 b}{Y_1 b + Y_2(1-b)} \right)^{\frac{\alpha_s}{2\alpha_s - \alpha_z}} R. \tag{31}$$

While the radius of the city border R is independent of total population, population mix etc., for a wedge-shaped segregation pattern the border length between two groups is constant:

$$G_S = 2R. \tag{32}$$

In order to obtain the most efficient segregation pattern, it is necessary to compare the border lengths of both cases. It follows:

$$G_R \gtreqless G_S \tag{33}$$

if

$$\frac{Y_1 b}{Y_1 b + Y_2 (1-b)} \gtreqless \frac{1}{\pi^{\frac{2\alpha_s - \alpha_z}{\alpha_s}}}. \tag{34}$$

The critical value for b follows as:

$$b^* = \frac{Y_2}{\left(\pi^{\frac{2\alpha_s - \alpha_z}{\alpha_s}} - 1 \right) Y_1 + Y_2}. \tag{35}$$

If the share of a group is larger than this critical value, the length of the border in a ring-shaped pattern is longer and the pattern less efficient than a wedge-shaped one and vice versa.

Note that the critical value of the share is a linear function of the ratio of income of the household groups $\theta = Y_2/Y_1$:

$$b^* = \frac{\theta}{\pi^{\frac{2\alpha_s - \alpha_z}{\alpha_s}} - 1 + \theta}. \tag{36}$$

Thus, with increasing differences in income distribution, the critical value of the share of the household group with the lower income rises. If the incomes are equal, however, the critical value is:

$$b^*_{\theta=1} = \frac{1}{\pi^{\frac{2\alpha_s - \alpha_z}{\alpha_s}}}, \tag{37}$$

which for $\alpha_s = 0.3$ and $\alpha_z = 0.1$ amounts to about 0.148. This finding is close to the results of Yinger (1976) who discussed this case with a different specification of the monocentric model.

In order to obtain the most efficient ring-shaped pattern, it is necessary that the group with the lower purchasing power lives closer to the city center than the other group. If α_z is restricted to values between 0 and $2\alpha_s$ following equation (28), the expression on the right hand side of equation (37) is of a value smaller than or equal to 0.361. Thus, depending on preferences for qualified land and the local public good, the ring-shaped pattern is more efficient only if there are strong differences in the purchasing power of the different household groups.

3.2 Spatial patterns for three and four household groups

Combining equations (27) and (22), any location and thus length of a radial or concentric borderline of a circular city with the range R around a city center can be located. With three different household groups ($J = 3$ in equation (22) and (26)), there are five possible spatial segregation patterns (figure 2) with 22 possible distributions of household groups. In the case of four household groups ($J = 4$) the number of possible patterns rises to 12 (figure 3). This, in turn, leads to many more possible different distributions of household groups to segregated areas.

In cases with more than two household groups critical values of population shares or ratios of production coefficients cannot be calculated as in the case

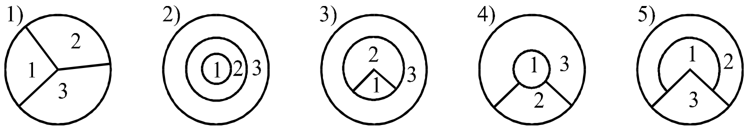

Figure 2: Possible patterns of segregation for three household groups

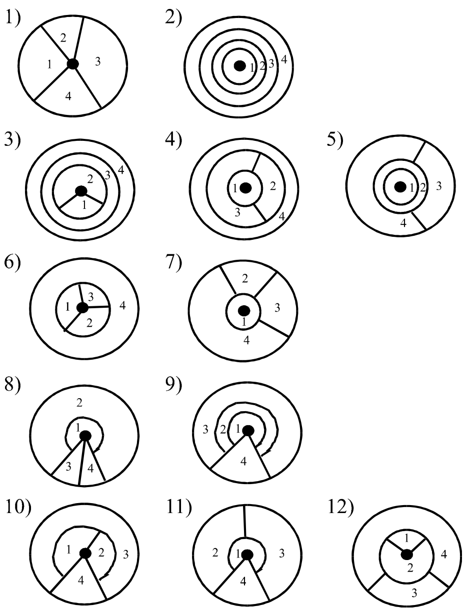

Figure 3: Possible segregation patterns for four household groups

of two household groups. Therefore it is appropriate to calculate numerical examples in order to examine the efficiency of different spatial patterns. The total border length of a certain pattern can be calculated by adding up all single border lines. The lengths are dependent on the ratios of purchasing powers.

The total length is dependent on the spatial order of groups. In any ring-shaped pattern, even if only in an area of the city, to obtain the minimum border length the respective groups tend to be located closer to the city center the lower their incomes are. For wedge-shaped parts of an area, the distribution of household groups is unimportant. Thus for every pattern one single obvious distribution of different household groups arises as the most efficient. In the fifth pattern the wedge-shaped area is occupied by the group with the highest purchasing power.

Case	$\alpha_{\mathbf{z}}$	Y_1b_1	Y_2b_2	Y_3b_3	$G_{3,1)}$	$G_{3,2)}$	$G_{3,3)}$	$G_{3,4)}$	$G_{3,5)}$
1	0.1	0.01	0.09	0.90	30.00	**19.75**	20.81	22.70	21.58
2		0.02	0.18	0.80	30.00	29.93	31.54	24.10	**23.16**
3		0.03	0.22	0.75	30.00	35.01	36.06	25.22	**24.40**
4		0.04	0.26	0.70	30.00	39.62	40.22	26.21	**25.63**
5		0.05	0.30	0.65	30.00	43.88	44.12	27.10	**26.84**
6		0.10	0.30	0.60	**30.00**	52.04	47.80	30.76	30.94
7		0.20	0.30	0.50	**30.00**	65.38	54.65	36.31	38.13
8		0.30	0.30	0.40	**30.00**	76.76	60.97	40.80	44.87
9	0.3	0.01	0.09	0.90	30.00	**6.91**	8.28	20.43	20.63
10		0.02	0.18	0.80	30.00	**13.82**	16.57	20.86	21.26
11		0.03	0.22	0.75	30.00	**17.59**	20.71	21.29	21.89
12		0.04	0.26	0.70	30.00	**21.36**	24.85	21.71	22.51
13		0.05	0.30	0.65	30.00	25.13	**28.99**	22.14	23.14
14		0.10	0.30	0.60	30.00	31.42	**33.13**	24.28	26.28
15		0.20	0.30	0.50	30.00	43.98	**41.42**	28.57	32.57
16		0.30	0.30	0.40	**30.00**	56.55	49.70	32.85	38.85

Table 1: Total border lengths of different patterns for three household groups

Table 1 presents the total border lengths $G_{J,k}$, k being the number of the pattern in figure (2) for different numeric situations with $J = 3$ household groups. While $R = 10$ and $a_s = 0.3$, different values for Y_j and a_z are examined. As result, the ranking of border lengths varies with the cases. The shortest border lengths are printed bold.

Therefore, the most efficient segregation patterns also vary from case to case. Especially in cases with strong differences in purchasing power, patterns with a mixture of ring and wedge-shaped areas are most efficient. For incomes converging to a common level, purely wedge-shaped patterns are efficient.

Tables 2a and 2b show the border lengths $G_{J,k}$ with k being the number of the pattern in figure (3) of numeric examples for four household groups. Different values for b_jY_j and a_z are examined for $R = 10$ and $a_s = 0.3$. Again the most efficient pattern depends on numerical values of the variable.

As with three household groups, for different values of production coefficients patterns with a mix of ring and wedge-shaped areas are more efficient whereas in cases with similar values the pure wedge-shaped pattern is the most efficient one.

Case	$\alpha_{\mathbf{z}}$	$Y_{\mathbf{1}}b_1$	$Y_{\mathbf{2}}b_2$	$Y_{\mathbf{3}}b_3$	$Y_{\mathbf{4}}b_4$	$G_{\mathbf{4,1)}}$	$G_{\mathbf{4,2)}}$	$G_{\mathbf{4,3)}}$	$G_{\mathbf{4,4)}}$
1	0.1	0.01	0.02	0.03	0.94	40.00	23,25	39.64	18.65
2		0.01	0.02	0.12	0.85	40.00	31,76	48.15	29.87
3		0.01	0.02	0.25	0.72	40.00	40,90	57.30	41.93
4		0.01	0.02	0.30	0.67	40.00	43,94	60.33	45.92
5		0.01	0.12	0.25	0.59	40.00	58,66	76.54	50.60
6		0.01	0.28	0.28	0.43	40.00	78,71	98.04	62.45
7		0.12	0.25	0.25	0.38	**40.00**	99,37	105.93	76.98
8		0.25	0.25	0.25	0.25	**40.00**	121,70	119.83	92.70
9	0.3	0.01	0.02	0.03	0.94	40.00	6.28	26.26	**5.50**
10		0.01	0.02	0.12	0.85	40.00	**11.94**	31.91	22.95
11		0.01	0.02	0.25	0.72	40.00	**20.11**	40.08	23.72
12		0.01	0.02	0.30	0.67	40.00	23.25	43.22	27.86
13		0.01	0.12	0.25	0.59	40.00	33.68	56.33	32.99
14		0.01	0.28	0.28	0.43	40.00	54.66	79.84	47.74
15		0.12	0.25	0.25	0.38	40.00	69.74	89.60	57.70
16		0.25	0.25	0.25	0.25	40.00	94.25	108.54	75.33

Table 2 a: Total border lengths of different patterns for four household groups (Part 1)

Case	$G_{\mathbf{4,5}})$	$G_{\mathbf{4,6}})$	$G_{\mathbf{4,7}})$	$G_{\mathbf{4,8}})$	$G_{\mathbf{4,9}})$	$G_{\mathbf{4,10}})$	$G_{\mathbf{4,11}})$	$G_{\mathbf{4,12}})$
1	30.37	**17.16**	32.07	30.98	23.77	29.09	31.19	27.66
2	30.37	29.74	32.07	30.98	**25.44**	27.40	31.76	27.66
3	30.37	43.25	32.07	30.98	**26.99**	27.22	32.28	27.66
4	30.37	47.73	32.07	30.98	27.46	**27.29**	32.44	27.66
5	41.57	52.91	32.11	**31.81**	35.71	38.19	32.67	38.81
6	52.60	66.26	32.07	**32.42**	47.04	50.54	33.07	49.90
7	66.61	69.68	39.20	41.83	63.12	55.92	43.34	54.60
8	80.10	78.12	44.29	50.73	81.32	64.79	51.88	61.45
9	22.31	5.57	30.33	30.63	22.51	26.89	30.53	21.89
10	22.31	13.93	30.33	30.63	22.51	23.89	30.53	21.89
11	22.31	25.99	30.33	30.63	22.51	22.96	30.53	21.89
12	22.31	30.64	30.33	30.63	22.51	22.79	30.53	**21.89**
13	28.86	36.37	30.34	30.65	29.07	31.84	30.55	**28.42**
14	38.65	52.91	30.33	**30.63**	38.85	43.31	30.53	38.22
15	48.39	57.56	33.94	**37.54**	50.79	49.22	36.34	43.25
16	62.12	69.62	38.21	**45.71**	67.12	58.08	43.21	51.42

Table 2 b: Total border lengths of different patterns for four household groups (Part 2)

4 Conclusion

In this contribution, within a monocentric model, spatial segregation caused by positive externalities between households of the same group, such as a social network, is discussed. When such externalities are introduced by a special production function for qualified land as housing service, segregation arises as a result of a selection by profit maximizing suppliers. In considering only the housing market, segregation leads to higher welfare than complete integration of different groups. The allocation process leads to equilibria if border lengths are more or less straight lines.

Since profits are the single criterion of an open city's surplus, efficiency depends on border lengths. Realized segregation patterns do not have to be efficient. However, since they are stable, they can only be changed by political regulation. The costs of such regulation have to be compared with the improvement of welfare.

Due to the assumption of radial or concentric borderlines, different possible spatial patterns of segregation emerge according to the number of household groups. The total border lengths of any pattern, besides households' preferences for housing services and for a local public good, depend on the relative purchasing power of different household groups. Examining the cases of three and four household groups, various possible spatial patterns containing a mix of concentric and radial borders between segregated areas emerge. The more similar households of different groups are to each other with regard to their relative purchasing power, the more likely it is that a pure wedge-shaped segregation pattern is the most efficient one. For less similar household groups mixed patterns with wedge and ring-shaped areas are efficient.

The results are highly limited by the partial and static framework. Dynamic implications as for example the development of groups and externalities between them are not considered here. While it is shown that pure segregation may lead to an improvement of negative externalities (Benabou 1993), it can be expected that in dynamic analysis negative implications of segregation will be observed. Thus a policy aimed at the integration of households belonging to different groups might be appropriate.

Nevertheless it can be shown that with only a few additional assumptions within the traditional monocentric city model the paradigm of ring-shaped patterns of residential land use can be transcended. This is the main goal of this approach. It is shown that there are arguments of efficiency for ring-shaped, wedge-shaped or mixed patterns. Since the discussion is a purely normative one in a context of stereotypical spatial patterns, it is unimportant that the assumption of pure radial and concentric borderlines does not meet reality. By introducing positive social externalities within household groups the gap between the ring-shaped patterns of monocentric models and the empirical results of Homer Hoyt (1939) can be closed.

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