Wolf-Rayet Stars
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Inversion of Intensity Profiles for Bubble Emissivity

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Under the assumption of spherical symmetry, the run of intensity with impact parameter for a spatially resolved and optically thin bubble can be inverted for an "effective emissivity" as a function of radius. The effective emissivity takes into account instrumental sensitivity and even interstellar absorption. This work was supported by a grant from NASA (G03-14008X).

1 The assumptions

Consider a spherically symmetric and optically thin bubble. For z the observer axis to the star at bubble center, and with p the cylindrical radius (or "impact parameter"), the observed intensity profile for a spatially resolved bubble is given by

$$I_\nu(p) = \int j_\nu(r)\, dz, \tag{1}$$

where j is the emissivity that is strictly a function of radius. This however is the emergent intensity. The measured intensity can be affected by interstellar extinction and instrumental response. So we define an "effective" emissivity $\tilde{j}$, and working with a passband intensity (to increase S/N of the data), this new parameter will be given by

$$\tilde{j}(r) = \frac{1}{A_0} \int\int \Lambda(E,T)\, \frac{dEM}{dT}\, e^{-\tau(E)}\, A(E)\, dE\, dT, \tag{2}$$

where for the case of X-ray emission: Λ is the temperature and energy dependent cooling function, T is temperature, EM is emission measure, dEM/dT is differential emission measure, τ is the energy-dependent interstellar absorption, $A(E)$ is the effective area as representing the X-ray telescope instrumental response, and A_0 is the total energy-integrated area. Our inversion recovers a passband-equivalent emissivity that takes these various factors into account.

2 The solution

Equation (1) is well known to yield an inversion solution in the form of Abel's equation (e.g., Craig & Brown 1986) The formal solution is analytic with

$$\tilde{j}(r) = \frac{-1}{\pi}\, \frac{d}{dr} \int_r^R r\, I(p)\, \frac{dp/p}{\sqrt{p^2 - r^2}}, \tag{3}$$

where R is the bubble radius, and the square root in the denominator acts like geometrical deprojection weighting factor. Note first that the integral for the inversion proceeds from the limb toward the center. Also, the solution ultimately derives from a derivative involving the data. It is well-known that this solution is ill-posed, and quite sensitive to the effects of noise. Applications will therefore generally require the use of regularization techniques.

In practice one would produce $I(p)$ by averaging the intensity in annuli. All things equal, this approach will increase the S/N of the averaged profile as $\sqrt{p}$, which means data toward the limb, where the inversion starts, will be of superior quality (unless $I(p)$ drops precipitously from center to limb), which should make the inversion more stable.

3 Some handy analytic examples

The advantage of inversion techniques is that the data guide the solution, as opposed to forward modeling and fits that ultimately involve preconceived notions regarding the source structure. The disadvantage is having to negotiate the sensitivity of solutions to the data noise.

Some useful insight can be derived from a consideration of simple profiles with analytic solutions. One might for example characterize intensity profiles as "flat" (i.e., uniform), "rounded", "triangular", "double-horned", and so on.

Take for example the case of a flat profile with $I(p) = I_0$, a constant. This implies an effective emissivity of the form

$$\tilde{j}(r) = (I_0/\pi)/\sqrt{R^2 - r^2}, \tag{4}$$

which increases steeply toward the limb. By contrast a parabolic profile with $I(p) = I_0\,(1 - p^2/R^2)$ would have a solution

$$\tilde{j}(r) = (2I_0/\pi R)\, \sqrt{R^2 - r^2}. \tag{5}$$

Interestingly, polynomial fits to $I(p)$ will generally yield these two types of square-root factors along with additional radial dependence and numerical weighting factors that derive from the best fit polynomial to the data.

References

Craig, I. J. D. & Brown, J. C. 1986, Inverse problems in astronomy: A guide to inversion strategies for remotely sensed data